Chapter 3

Stresses in Beams, Cylinders and Spheres

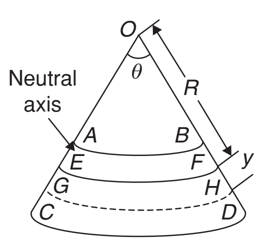
CHAPTER HIGHLIGHTS

- *Stresses in Beams and Bending Stresses*
- *Neutral Axis*
- *Moment of Inertia*
- *Centroid and Moment of Inertia of Some Plane Figures*
- *Centroids of Solid Figures*
- *Relationship Between Bending Moment and Radius of Curvature*
- *Moment of Resistance of a Section*
- *Shearing Stresses in Beams*
- *Shear Stress Distribution Across a Rectangular Section*
- *Shear Stress Distribution Across a Circular Section*
- *Shear Stress Distribution Across Triangular Cross Sections*
- *Composite Beams*
- *Strain Energy in Pure Bending*
- *Pressure Vessels (Cylinders and Spheres)*
- *Thin Walled Pressure Vessels*
- *Change in Dimensions of Cylinder*
- *Cylindrical Vessel with Hemispherical Ends*

Stresses in Beams

When beams are subjected to bending moment and shear forces, internal stresses are developed in them. Simple bending theory deals with finding stresses due to pure moment alone. When a beam is subjected bending moment it sags or hogs. When it sags fibres at the bottom are stretched and fibres at top are compressed. In another words tensile stresses are developed at the bottom and compressive stresses are developed at the top. When it hogs the reverse happens. We can identify a layer in between called neutral layer on which there shall be neither compression nor tension, and consequently stresses are zero.

Bending Stresses



After bending, cross sections AC and BD subtends an angle θ at O. Let R be the radius of the neutral plane represented by EF. Then $EF = R\theta$

Now, consider the layer GH at the bottom of neutral plane at a distance y from it.

$$GH = (R + y)\theta$$

Please note that before bending its length is equal to $EF = R\theta$.

Therefore,

$$\text{Strain in GH} = \frac{\text{Final length} - \text{original length}}{\text{original length}}$$

$$= \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

If f = the bending stress

$$\frac{\text{stress}}{\text{strain}} = E = \text{Young's modulus}$$

That is,

$$\frac{f}{\frac{y}{R}} = E$$

or

$$\frac{f}{E} = \frac{y}{R}$$

or

$$\boxed{f = \frac{E}{R}y}$$

Neutral Axis

After bending, there is tension in the convex surface and compression in the concave surface. Therefore between top and bottom, there is a layer at which there is no strain. This layer is known as neutral layer. The line of intersection of cross section with the neutral layer is called neutral axis.

Centroid or Centre of Area

Centroid or centre of area is the point where the whole area is assumed to be concentrated.

Moment of Inertia

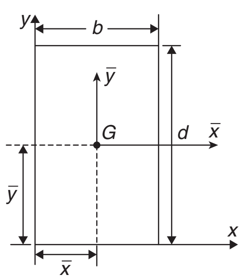
Area moment of inertia is the second moment of the area with respect to an axis. With respect to x-axis, it is

$I_{xx} = \int dAy^2$ and with respect to y-axis it is
$I_{yy} = \int dAx^2$

where x and y are the distance from the corresponding axes.
Mass moment of inertia
$I = mk^2$ where m = mass
and k = radius of gyration

Centroid and Moment of Inertia of Some Plane Figures

Rectangle



Position of centroid G

$$\bar{x} = \frac{b}{2}$$

$$\bar{y} = \frac{d}{2}$$

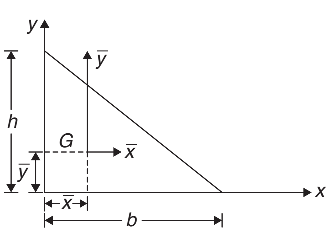
Moment of inertia about horizontal axis passing through G

$$I_{xx} = \frac{bd^3}{12}$$

Moment of inertia about vertical axis passing through G

$$I_{yy} = \frac{db^3}{12}$$

Triangle



$$\bar{x} = \frac{b}{3}$$

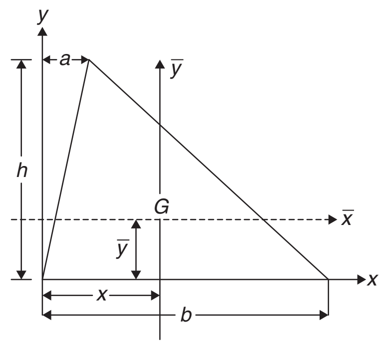
$$\bar{y} = \frac{h}{3}$$

Moment of inertia about axes through G

$$I_{\bar{x}} = \frac{bh^3}{36}$$

$$I_{\bar{y}} = \frac{hb^3}{36}$$

For any triangle as shown below,

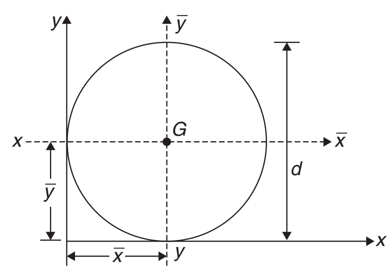


$$\bar{x} = \frac{(a+b)}{3}$$

$$\bar{y} = \frac{h}{3}$$

$$I_{\bar{x}} = \frac{bh^3}{36}$$

Circle



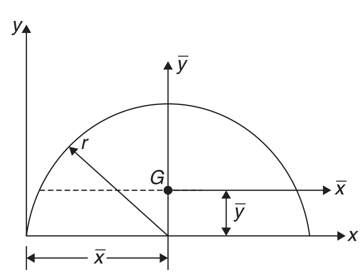
$$\bar{x} = \frac{d}{2},\ \bar{y} = \frac{d}{2}$$

$$I_{\bar{x}} = I_{\bar{y}} = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$$

where r = radius

$$= \frac{d}{2}$$

Semicircle



$$\bar{x} = r$$

$$\bar{y} = \frac{4r}{3\pi}$$

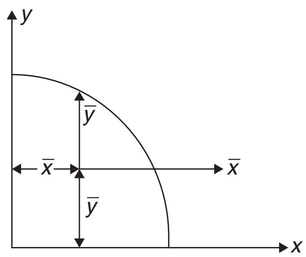
$$I_{\bar{y}} = \frac{\pi r^4}{8}$$

$$I_{\bar{x}} = (9\pi^2 - 64)\frac{r^4}{72\pi}$$

Moment of inertia about x-axis

$$I_x = \frac{\pi r^4}{8}$$

Quadrant



$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_{\bar{x}} = I_{\bar{y}} = \frac{(9\pi^2 - 64)r^4}{144\pi}$$

$$I_x = I_y = \frac{\pi r^4}{16}$$

Centroids of Solid Figures

Solid right circular cone of height h Height of centroid from base $= \frac{h}{4}$

Hollow right circular cone of height h Height of centroid from base $= \frac{h}{3}$

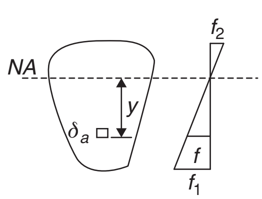
Solid hemisphere of radius R Height of centroid from base $= \frac{3R}{8}$

Hollow hemisphere of radius R Height of centroid from base $= \frac{R}{2}$

It is seen that stress is varying linearly with distance y from neutral axis. It can be shown that neutral axis coincides with the centroid of the cross section.

Position of Neutral Axis

Consider a beam with arbitrary cross section as shown in the figure.



Stress is varying linearly across the depth. Consider an element area δa at a distance y from the neutral axis. Let f be the stress on this area.

Force on the elemental area $= f\delta a$

Total force on the cross section $= \Sigma f\delta a$

But $f = \frac{E}{R}y$

$$\therefore \quad \text{Total force} = \Sigma\frac{E}{R}y\delta a$$

$$= \frac{E}{R}\Sigma y\delta a$$

Since there is no axial forces acting on the beam, for equilibrium

$$\frac{E}{R}\sum y\delta a = 0$$

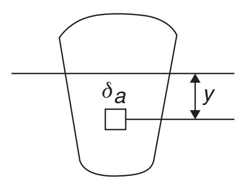
or $\quad \Sigma y\,\delta a = 0$

If A is the total area,

$$\frac{\Sigma y\delta a}{A} = 0$$

But $\frac{\Sigma y\delta a}{A}$ is the distance of centroid from neutral axis. Therefore neutral axis coincides with centroid of the cross section.

Relationship Between Bending Moment and Radius of Curvature



Considering an element of area δa at a distance y from the neutral axis,

Stress on the element $f = \frac{E}{R}y$

Force on the element $= f\delta a = \frac{E}{R}y\delta a$

Moment of the force about neutral axis $= \frac{E}{R}y^2\delta a$

Total moment of resistance $= M' = \Sigma\frac{E}{R}y^2\delta a$

$$= \frac{E}{R}\Sigma y^2\delta a$$

But $\Sigma y^2\delta a = I$

= Moment of inertia or second moment of area about centroid

$$\therefore \quad M' = \frac{E}{R}\cdot I$$

But $M' = M$, the applied moment. So,

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{f}{y}}$$

Moment of Resistance of a Section

The stress is maximum on the extreme end or cross-section where y is maximum.

Let f_p be the maximum permissible stress of the material. Then the maximum stress should not exceed the maximum permissible stress

or $$f_{max} \le f_p$$

or $$\frac{M}{I} y_{max} \le f_p$$

or $$M = \frac{I}{y_{max}} f_p$$

where M is the moment carrying capacity of the section.

$\frac{I}{y_{max}} = z$ is the section modulus of the cross-section

$\therefore$ $$\boxed{M = f_p \cdot z}$$

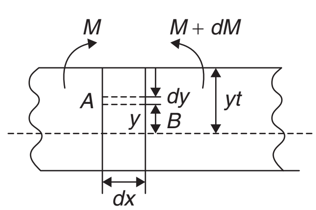
Section modulus of different section

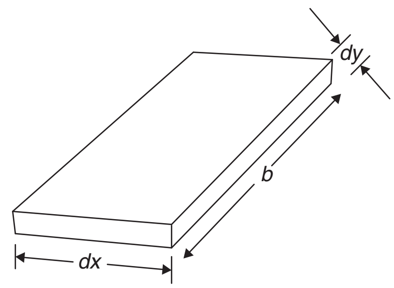
1. Rectangular section: $\frac{bd^2}{6}$
2. Hollow section: $\frac{1}{6}\frac{(BD^3 - bd^3)}{D}$
3. Circular section: $\frac{\pi d^3}{32}$
4. Triangular section: $\frac{bh^2}{24}$

Application of Bending Equation

The equation $\frac{M}{I} = \frac{E}{R} = \frac{f}{y}$ is on the assumption that bending moment M is constant in a section through out and no shear force is acting on the sections. But this is not the actual situation. There is shear force and bending moment varies. So application of bending equation has some limitations. As shear force is zero when bending moment is maximum, the equation can be applied at this situation.

Shearing stresses in beams





In a beam, consider an elemental length of length dx. Moments acting at the two sides of this element = M and $M + dM$

Bending stress at left side of the element $= \frac{My}{I}$

Corresponding force on the element $= \frac{M}{I} ybdy$

where b = breadth of the beam

Force on the right side of the element due to the moment

$$M + dM = \frac{M + dM}{I} ybdy$$

Unbalanced force towards the right

$$= \frac{M + dM}{I} ybdy - \frac{M}{I} ybdy = \frac{dM}{I} ybdy$$

Total unbalanced forces acting above the section

$$AB = \int_y^{yt} \frac{dM}{I} ybdy$$

This force is resisted by shearing stresses in the plane at y i.e., at AB. If the intensity of shearing stress is q,

$$qb\ dx = \int_y^{yt} \frac{dM}{I} ybdy$$

$$q = \frac{dM}{dx} \frac{1}{bI} \int_y^{yt} da$$

where $da = bdy$ = area of the element

But $\int_y^{yt} da = a\bar{y}$

= Moment of area above the section

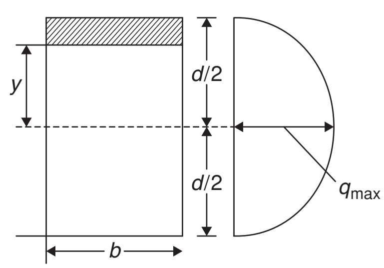
about the neutral axis and $\frac{dM}{dx} = F$

$\therefore$ $$q = \frac{F}{bI} a\bar{y}$$

Shear stress distribution across a rectangular section We know that

$$q = \frac{F}{bI} a\bar{y}$$

Consider a rectangular section as shown in the figure.



In the above expression, q, the shear stress is at a distance y from the neutral axis $a\bar{y}$ is the moment of area above this section.

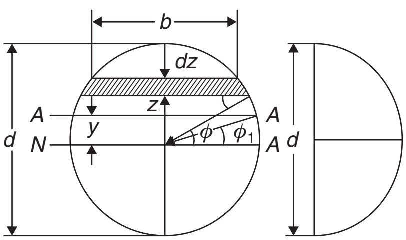
i.e.,
$$a = \left(\frac{d}{2} - y\right)b$$
$$\bar{y} = y + \left(\frac{d}{2} - y\right)\frac{1}{2} = \frac{1}{2}\left(\frac{d}{2} + y\right)$$
$$I = \frac{1}{12}bd^3$$

Substituting we get $q = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right)$

Variation of shear stress is parabolic as shown in figure.

When $y \pm \frac{d}{2}$, $q = 0$ also when $y = 0$, q is maximum
$$\int q_{\max} = \frac{6F}{bd^3}\frac{d^2}{4}$$
$$= 1.5\frac{F}{bd} = 1.5q_{av}$$
$$\left(\frac{F}{bd} = \frac{\text{shearing force}}{\text{area}} = \text{average shearing stress}\right)$$

Shear stress distribution across a circular section



In the given circular section of diameter d, shear stress is to be determined at a section AA at a distance y from the neutral axis NA.

For finding the moment of area above AA an element of thickness dz at a vertical distance z from neutral axis is considered.

Width of the element
$$b = d\cos\phi,\ z = \frac{d}{2}\sin\phi,\ dz = \frac{d}{2}\cos\phi\, d\phi$$

Area of element $= bdz = d\cos\phi \times \frac{d}{2}\cos\phi\, d\phi$
$$= \frac{d^2}{2}\cos^2\phi\, d\phi$$

Moment of area of the element = (area) . z
$$= \frac{d^2}{2}\cos^2\phi\, d\phi \times \frac{d}{2}\sin\phi$$

Moment of area above section AA about neutral axis
$$a\bar{y} = \int_{\phi_1}^{\frac{\pi}{2}} \frac{d^3}{4}\cos^2\phi \sin\phi\, d\phi$$
$$= \frac{d^3}{4}\left[\frac{-\cos^3\phi}{3}\right]_{\phi_1}^{\frac{\pi}{2}}$$
$$= \frac{d^3}{12}\left[-\cos^3\frac{\pi}{2} + \cos^3\phi_1\right]$$
$$= \frac{d^3}{12}\cos^3\phi_1$$

Shearing stress of on AA is $q = \frac{F}{bI}a\bar{y}$
$$= \frac{16}{3} \times \frac{F}{\pi d^2}\cos^2\phi_1$$
$$= \frac{16}{3} \times \frac{F}{\pi d^2}(1 - \sin^2\phi_1)$$
$$= \frac{16}{3} \times \frac{F}{\pi d^2}\left[1 - \left(\frac{y}{\frac{d}{2}}\right)^2\right]$$
$$= \frac{16}{3} \times \frac{F}{\pi d^2}\left(1 - \frac{4y^2}{d^2}\right)$$

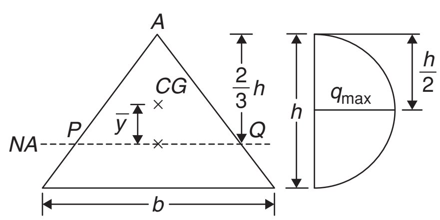
That is, shear stress varies parabolically. Its value is maximum when $y = 0$, i.e., at neutral axis and is given by
$$q_{\max} = \frac{16}{3}\frac{F}{\pi d^2} = \frac{4}{3}\frac{F}{\frac{\pi}{4}d^2}$$
$$= \frac{4}{3}\frac{F}{A} = \frac{4}{3}q_{av}$$

where A = area of cross section

Q_{av} = average shear stress

Shear stress distribution across triangular cross sections Let width at neutral axis PQ be b'.

Then, $q_{NA} = \frac{E}{b'I}a\bar{y}$



Area above neutral axis (NA)
$$A = \frac{1}{2}PQ \times \frac{2}{3}h$$
$$= \frac{1}{2} \times \frac{2}{3}b \times \frac{2}{3}h$$
$$= \frac{2}{9}bh$$

$$\bar{y} = \frac{2}{3}h \times \frac{1}{3} = \frac{2}{9}h$$

$$b' = \frac{2}{3}b$$

$$I = \frac{bh^3}{36}$$

$$\therefore \quad q_{NA} = \frac{F \times \frac{2}{9}bh \times \frac{2}{9}h}{\frac{2}{3}b \times \frac{bh^3}{36}}$$

$$= \frac{4}{3} \times \frac{F}{\left(\frac{1}{2}bh\right)}$$

$$= \frac{4}{3}q_{av}$$

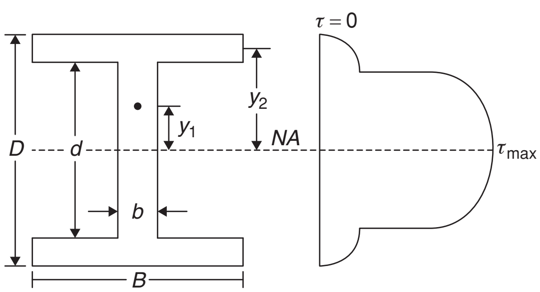
It can be shown that the shear stress distribution is

$$q = \frac{12F}{bh^3}y(h-y)$$

where y is the distance from the top fibre.

q_{max} occurs at $y = \frac{h}{2}$ and $q_{max} = 1.5q_{av}$

Shear stress distribution across symmetric I-section



The **I**-section is symmetric about neutral axis and shear stress q is maximum at neutral axis (NA).

In the formula for shear stress,

$$q = \frac{F}{b'I}a\bar{y}$$

b' is the width where the shear stress is to be calculated. For calculation of shear stress at neutral axis, $b' = b$.

Moment of inertia for the section is

$$I = \frac{BD^2}{12} - \frac{(B-b)d^2}{12}$$

$$a\bar{y} = a_1y_1 + a_2y_2$$

where $a_1 = \dfrac{bd}{2}$

$$y_1 = \frac{d}{4}$$

$$a_2 = B\frac{(D-d)}{2}$$

$$y_2 = \frac{d}{2} + \frac{(D-d)}{4}$$

$$= \frac{D+d}{4}$$

$$\therefore \quad a\bar{y} = \frac{bd}{2}\cdot\frac{d}{4} + \frac{B}{2}(D-d)\frac{(D+d)}{4}$$

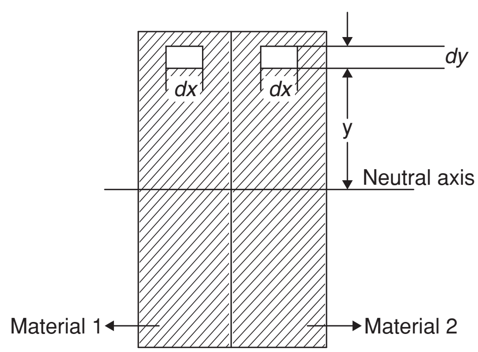
$$= \frac{1}{8}[bd^2 + B(D^2 - d^2)]$$

$$q_{max} = q_{NA} = \frac{F[B(D^2-d^2)+bd^2]}{8bI}$$

At $y = \frac{d}{2}$, there are two widths B and b and correspondingly there are two values for the shear stress. The shear stress distribution for the sections will be as shown in the figure.

Composite Beams

Composite beams are beams of more than one material rigidly connected so that there is no slip at the common faces. So when subjected to stresses, the strain in each part will be same. They are also called flitched beams.



Since the strains are same, $\dfrac{f_1}{E_1} = \dfrac{f_2}{E_2}$ or

$$f_1 = f_2 \times \frac{E_1}{E_2} = f_2 \times m$$

where $m = \dfrac{E_1}{E_2}$ the modular ratio. Comparing the moments of resistances of the elemental identical areas

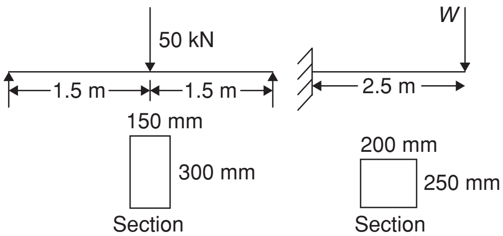
$\delta M_1 = (f_1dx\ dy)y = (f_2mdx\ dy)y = m\ (f_2dx\ dy)y$

but, $\delta M_2 = (f_2dx\ dy)y$

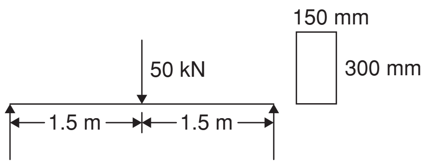
we see that δM_1 is m times of δM_2.

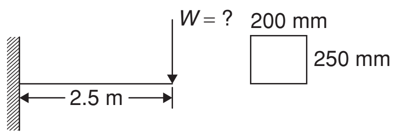
Solved Examples

Example 1: A beam of cross section 150 × 300 mm can support a maximum load of 50 kN at its centre when it is used as a simply supported beam of span 3 m. If the same material is used for a cantilever of length 2.5 m and cross-section 200 × 250 mm as shown in the figure, what is the maximum load it can support at its free end?



Solution:





Maximum moment in simply supported beam $= \dfrac{WL}{4}$

$$= \frac{50 \times 3}{4} = 37.5 \text{ kNm} = 37.5 \times 10^6 \text{ Nmm}$$

If f is the stress at failure, $M = fz$

That is, $37.5 \times 10^6 = f \times \dfrac{1}{6} bd^2$

$$= f \times \frac{150 \times 300^2}{6}$$

$$\therefore \quad f = \frac{37.5 \times 10^6 \times 6}{150 \times 300^2}$$

$$= \frac{375 \times 6}{15 \times 9} = 16.67 \text{ N/mm}^2$$

Maximum load the cantilever can take is calculated using the above stress

Max moment $= WL$

$= W \times 2.5 \times 10^6$ Nmm (W in kN)

$= fz$

$$z = \frac{1}{6} \times 200 \times 250^2$$

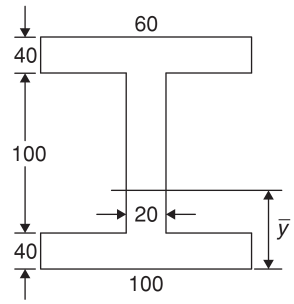
$$\therefore \quad W_x 2.5 \times 10^6 = 16.67 \times \frac{1}{6} \times 200 \times 250^2$$

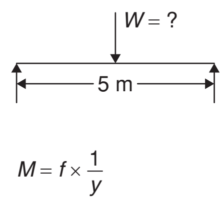
$$W \times 250 = \frac{16.67}{6} \times 2 \times 25^2$$

$$W = 13.89 \text{ kN}$$

Example 2: A cast iron beam of cross-section as shown with length 5 m, is simply supported at the ends. Find out the maximum concentrated central load it can take if permissible stresses are 28 N/mm^2 (tensile) and 80 N/mm^2 (compressive).

Solution:





Moment carrying capacity of the beam is to be found out. For this distance y from centroid is to be found out for both tension and compression. So position of centroid is to be located first.

$$\bar{y} = \frac{\Sigma ay}{A}$$

$$= \frac{(60 \times 40 \times 160) + (100 \times 20 \times 90) + (100 \times 40 \times 20)}{60 \times 40 + 100 \times 20 + 40 \times 100}$$

$$= 76.67 \text{mm}$$

$$I = \frac{1}{12} \times 60 \times 40^3 + (160 - 76.67)^2 \times 60 \times 40$$

$$+ \frac{1}{12} \times 20 \times 100^3 + (90 - 76.67)^2 \times 100 \times 20$$

$$+ \frac{1}{12} \times 100 \times 40^3 + (20 - 76.67)^2 \times 100 \times 40$$

$= 16{,}985{,}333.36 + 2{,}022{,}044.47 + 13{,}379{,}288.93$

$= 32{,}386{,}666.76$

$$M = f \times \frac{I}{y}$$

For tensile stress,

$$M = \frac{28 \times 32{,}386{,}667}{76.67} = 11827660$$

For compressive stress,

$$M = \frac{80 \times 32{,}386{,}667}{(180 - 76.67)} = 25{,}074{,}358$$

Choosing the smaller,

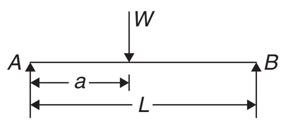
$M = 11{,}827{,}660$ Nmm

$= \dfrac{WL}{4}$, where W = load and L = span = 5 m

$$\therefore \quad W = \frac{11{,}827{,}660 \times 4}{10^3 \times 5} \text{ N} = 9462.13 \text{ N.}$$

Example 3: A wooden beam 200 mm × 250 mm is simply supported over a span of 6 m. When a concentrated load of W is placed at a distance 'a' from the support, the maximum bending stress is 12 N/mm^2 and maximum shear stress is 0.8 N/mm^2. Determine values of W and 'a'.

Solution:



$$R_A = \frac{W(L-a)}{L}$$

$$R_B = \frac{Wa}{L}$$

R_A will be greater than R_B if $a < L - a$

Maximum shear $F = \dfrac{W(L-a)}{L}$

Maximum bending moment, $M = \dfrac{w(L-a)}{L}a$

Maximum shear stress = 1.5 × average shear stress

$$= 1.5 \times \frac{F}{\text{Area}}$$

$$= 1.5 \times \frac{F}{200 \times 250}$$

$$= 0.8 \text{ N/mm}^2 \text{ (given)}$$

$$\therefore \quad F = \frac{0.8 \times 200 \times 250}{1.5}$$

That is, $\dfrac{W(L-a)}{L} = \dfrac{0.8 \times 200 \times 250}{1.5}$

$$= 26{,}667 \qquad (1)$$

Maximum moment $M = fz$

where, f = maximum bending stress $= 12 \text{ N/mm}^2$ (given)

$$Z = \text{section modulus } = \frac{bd^2}{6}$$

That is, $\dfrac{W(L-a)a}{L} = \dfrac{12 \times 200 \times (250)^2}{6}$

$$= 25 \times 10^6 \qquad (2)$$

Dividing (2) by (1)

$$a = \frac{25 \times 10^6}{26{,}667} = 937.49 \text{ mm}$$

Substituting in (1)

$$W\frac{(6000 - 937.49)}{6000} = 26{,}667$$

or $W = 31{,}605$ N

Strain Energy in Pure Bending

We have seen that normal stress in the cross-section of a beam varies linearly from the neutral axis.

Considering an infinitesimal beam element area da and length dx, the strain energy is given by

$$U = \int_v \frac{f^2}{2E} dv$$

$$= \int_v \frac{1}{2E}\left(\frac{My}{I}\right)^2 dx \cdot da$$

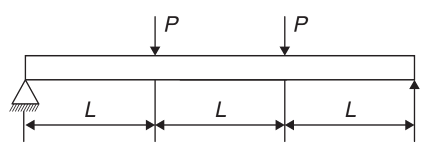
M at a section is constant, therefore, on rearranging,

$$U = \int_{\text{Length}} \frac{M^2}{2EI^2} \cdot dx \cdot \int_{\text{area}} y^2 da$$

$$= \int_0^L \frac{M^2}{2EI^2} \cdot dx \cdot I$$

$$\boxed{U = \int_0^L \frac{M^2}{2EI} dx}$$

Example 4: A beam with flexural rigidity EI is loaded as shown in the figure.

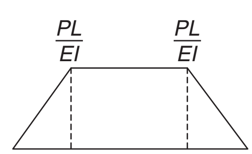


Strain energy stored in thm e beam is

(A) $\dfrac{2P^2L^3}{3EI}$ (B) $\dfrac{5P^2L^3}{6EI}$

(C) $\dfrac{4P^2L^3}{3EI}$ (D) $\dfrac{7P^2L^3}{6EI}$

Solution:



Strain energy stored $= \int \frac{M^2}{2EI} dx$

$$= 2\int_O^L \frac{(Px)^2\,dx}{2EI} + \int_O^L \frac{(PL)^2\,dx}{2EI}$$

$$= \frac{P^2}{2EI}\left[\frac{2x^3}{3} + L^2 \cdot x\right]_0^L$$

$$= \frac{P^2}{2EI}\left[\frac{2}{3}L^3 + L^3\right]$$

$$= \frac{P^2}{2EI} \times \frac{5}{3}L^3$$

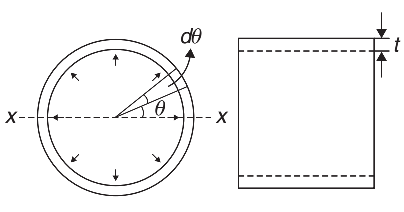
$$= \frac{5P^2L^3}{6EI}$$

Pressure Vessels (Cylinders and Spheres)

Cylinders and spheres are pressure vessels mostly used in chemical plants for various purposes. They are subjected to internal or external pressures. When wall thickness of cylinder is less than $\frac{1}{10}$ of its radius it is called a thin cylinder. Radial stress in thin cylinder is negligible. Hence, for stress analysis radial stress is neglected in thin cylinders.

Thin Walled Pressure Vessels

Stresses in Thin Cylinders Subjected to Internal Pressure



Due to the internal fluid pressure on the wall of the cylinder the following types of stresses are developed:

1. Circumferential stress (hoop stress)
2. Longitudinal stress
3. Radial stress (neglected)

Let the internal pressure be P, thickness be t and r, the radius. Consider an elemental length subtending an angle $d\theta$.

$\therefore$ Normal pressure on the element $= P \cdot rd\theta \cdot L$

Bursting force on the element normal to section XX $= prL \cos\theta\, d\theta$

Total bursting force

$$= 2\int_0^{\frac{\pi}{2}} prl\cos\theta d\theta = 2prl[\sin\theta]_0^{\frac{\pi}{2}}$$

$$= p\,2rL$$

$$= p\,dL$$

$$= p \times \text{the projected area}$$

where d = diameter

Resisting force on the walls $= f_c \times t \times L \times 2$

$$= 2f_c\,tL$$

where f_c = circumferential stress

$$\therefore \quad p\,dL = 2f_c\,tL$$

That is, $\boxed{f_c = \frac{pd}{2t}}$

Let f_ℓ be the longitudinal stress on the walls resisting the force in the longitudinal direction.

Force in the longitudinal direction $= p \times$ the projected area

$$= p \times \frac{\pi d^2}{4}$$

Resisting force $= \pi dt \times f_\ell$

$$\therefore \quad \pi dt f_\ell = p \times \frac{\pi d^2}{4}$$

That is, $\boxed{f_\ell = \frac{pd}{4t}}$

Thus it is found that longitudinal stress is half of the hoop stress.

Increase in volume due to the circumferential and longitudinal stresses Net circumferential strain e_1 = direct strain–lateral strain

$$= \frac{f_c}{E} - \frac{f_\ell}{mE}$$

$$= \frac{f_c}{E} - \frac{f_c}{2mE} \quad as\ f_\ell = \frac{f_c}{2}$$

$$= \frac{f_c}{E}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right)$$

Net longitudinal strain $e_2 = \frac{f_\ell}{E} - \frac{f_c}{mE}$

$$= \frac{f_\ell}{E} - \frac{2f_\ell}{mE}$$

$$= \frac{f_\ell}{E}\left(1-\frac{2}{m}\right)$$

$$= \frac{pd}{4tE}\left(1-\frac{2}{m}\right)$$

Volumetric strain = net longitudinal strain + 2 × Net circumferential strain

$$= \frac{pd}{4tE}\left(1-\frac{2}{m}\right)+2\times\frac{pd}{2tE}\left(1-\frac{1}{2m}\right)$$

$$= \frac{pd}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)+\frac{pd}{2tE}\left(2-\frac{1}{m}\right)$$

$$= \frac{pd}{4tE}(5-4\mu),\ \text{where } \mu = \frac{1}{m}$$

Percentage increase in volume $= \frac{\delta_v}{V}\times 100$

where $\frac{\delta_v}{v}$ = Volumetrics strain.

Change in Dimensions of Cylinder

We have seen that the circumferential strain

$$e_1 = \frac{pd}{2tE}\left(1-\frac{1}{2m}\right)$$

$$= \frac{pd}{4Et}\left(2-\frac{1}{m}\right) \text{ and longitudinal strain}$$

$$e_2 = \frac{pd}{4tE}\left(1-\frac{2}{m}\right)$$

Now, $e_1 = \frac{\delta d}{d}$

∴ Change in diameter

$$= \delta d$$

$$= d \times e_1$$

$$= \frac{pd^2}{2tE}\left(1-\frac{1}{2m}\right)$$

$$e_2 = \frac{\delta L}{L}$$

∴ Change in length

$$= \delta L$$

$$= L \times e_2$$

$$= \frac{pdL}{4tE}\left(1-\frac{2}{m}\right)$$

Thin Spherical Vessels

Stress produced at any diametrical section of a thin spherical vessel is $\frac{pd}{4t}$.

Therefore principal stresses are

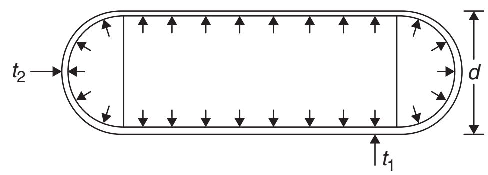
$$f_1 = f_2 = \frac{pd}{4t}$$

Hoop or circumferential strain

$$e = \frac{f_1}{E} - \frac{\mu f_2}{E}$$

$$= \frac{pd}{4tE}(1-\mu)$$

Cylindrical Vessel with Hemispherical Ends



Consider a cylindrical pressure vessel with hemispherical ends of internal diameter d subjected to an internal pressure p. Let t_1 be the thickness of cylindrical portions and t_2 be the thickness of the hemispherical portion.

Maximum stress is the hoop stress.

$$\sigma_{max} = \frac{pd}{2t_1} \text{ for cylindrical portion}$$

$$= \frac{pd}{4t_2} \text{ for hemispherical ends}$$

$$\therefore\ \frac{t_1}{t_2} = 2 \text{ when maximum stress is equal.}$$

Hoop strain or circumferential strain for the cylindrical portions,

$$e_1 = \frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)$$

where μ = Poisson's ratio $= \frac{1}{m}$

$$= \frac{pd}{4tE}(2-\mu)$$

Circumferential strain in hemispherical portion

$$e_2 = \frac{pd}{4tE}(1-\mu)$$

To avoid distortion at the junctions of cylindrical and hemispherical portions circumferential strain in both should be same.

Therefore,

$$\frac{pd}{4t_1E}(2-\mu) = \frac{pd}{4t_2E}(1-\mu)$$

$$\Rightarrow\ \frac{t_1}{t_2} = \frac{2-\mu}{1-\mu}$$

This means that $t_1 > t_2$.

Thick Cylinders

In thick cylinders, it cannot be assumed that hoop stress is uniform across the cross section and radial stress is negligible.

Stresses in thick cylinders can be analyzed using Lames' equations. It is assumed that the longitudinal strain is uniform across the cross-section of the cylinder. Lames' equations are:

$$p_x = \frac{b}{x^2} - a$$

$$f_1 = \frac{b}{x^2} + a$$

where
p_x = radial pressure
f_1 = hoop stress
x = radial distance from centre 'a'
b = arbitrary constants.

Example 5: A thin cylindrical shell of length 2 m has 200 mm diameter and thickness 10 mm. It is completely filled with a liquid at atmospheric pressure. If an additional quantity of 30,000 mm^3 liquid is pumped in, determine the internal pressure developed. (Take $E = 2\times 10^5\ \text{N/mm}^2 \mu = 0.3)$.

Solution: Let p be the pressure developed. Then circumferential stress

$$f_1 = \frac{pd}{2t} = \frac{p\times 200}{2\times 10} = 10p$$

Longitudinal stress

$$f_2 = \frac{pd}{4t} = \frac{f_1}{2} = 5p$$

Circumferential strain = diametral strain

$$= e_1 = \frac{\delta d}{d}$$

$$= \frac{f_1}{E} - \mu\frac{f_2}{E}$$

$$= \frac{1}{E}[10p - 0.3\times 5p]$$

$$= \frac{8.5p}{E}$$

Longitudinal strain $e_2 = \frac{1}{E}(f_2 - \mu f_1)$

$$= \frac{1}{E}[5p - 0.3\times 10p] = \frac{2p}{E}$$

Volumetric strain $= \frac{\delta v}{v} = 2e_1 + e_2$

$$= \frac{2\times 8.5p}{E} + \frac{2p}{E}$$

$$= \frac{19p}{E}$$

$$\frac{30{,}000}{\frac{\pi}{4}\times 200^2\times 2000} = \frac{19p}{2\times 10^5}$$

$$\therefore \quad p = 5.03\ \text{N/mm}^2.$$

Example 6: A spherical shell of diameter 500 mm and thickness 8 mm is completely filled with a liquid at atmospheric pressure. Taking efficiency of joint as 75% and permissible stress as 80 N/mm^2; determine the maximum pressure that can be permitted.

Solution:

$$f = \frac{pd}{4t\eta}$$

$$80 = \frac{p\times 500}{4\times 8\times 0.75}$$

$$p = 3.84\ \text{N/mm}^2.$$

Example 7: A pressure vessel is to be fabricated using 12 mm thick plate with permissible tensile stress of 12 kN/cm^2. Determine the maximum permissible diameter of the vessel for an internal pressure of 180 N/cm^2. if the longitudinal and circumferential joints have efficiencies 70% and 30%, respectively.

Solution: Since the efficiency of longitudinal joint is more and have considerable difference with efficiency of circumferential joint, both stresses are to be analyzed.

Circumferential stress as limiting value

$$f_c = \frac{pd}{2t\eta_L}$$

$$d = \frac{f_c\times 2t\times \eta_L}{p}$$

$$= \frac{12{,}000\times 2\times 1.2\times 0.7}{180} = 112\ \text{cm}$$

Longitudinal stress as limiting value

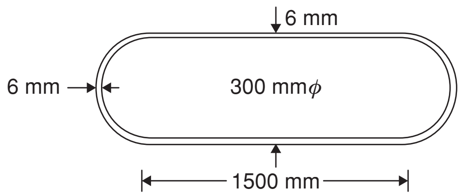
$$f_1 = \frac{pd}{4t\eta_L}$$

$$d = \frac{f_L\times 4t\times 0.3}{p}$$

$$= \frac{12{,}000\times 4\times 1.2\times 0.3}{180} = 96\ \text{cm}$$

Therefore, 96 cm is the maximum permissible diameter of the vessel.

Example 8: A cylindrical pressure vessel has hemispherical ends and dimensions are as shown in the figure. Determine the change in volume if the internal pressure is increased to 2 N/mm^2. Take Young's modulus = 2×10^5 N/mm^2 and Poisson's ratio = 0.25.



Solution: Circumferential strain in hemispherical ends

$$e_s = \frac{pd}{4tE}(1-\mu)$$

Volumetric strain

$$e_{vs} = \frac{\delta V_s}{V_s} = 3e = \boxed{\frac{3pd}{4tE}(1-\mu)}$$

$$= \frac{3\times2\times300}{4\times6\times2\times10^5}(1-0.25)$$

$$= 28.125 \times 10^{-5}$$

Change in volume

$$\delta V_s = e_{vs} \times V_s$$

$$= 28.125\times10^{-5}\times\frac{\pi}{6}\times300^3$$

$$= 397.6 \text{ mm}^3$$

For cylindrical portion

Diametral strain $e_1 = \dfrac{\delta d}{d} = \dfrac{pd}{4tE}(2-\mu)$

Longitudinal strain $e_2 = \dfrac{\delta \ell}{\ell}$

$$= \frac{pd}{4tE}(1-2\mu)$$

Volumetric strain $\dfrac{\delta V_c}{V_c} = 2e_1 + e_2$

$$= \frac{2pd}{4tE}(2-\mu)+\frac{pd}{4tE}(1-2\mu)$$

$$= \frac{pd}{4tE}[2(2-\mu)+(1-2\mu)]$$

$$= \frac{pd}{4tE}(5-4\mu)$$

$$= \frac{2\times300}{4\times6\times2\times10^5}[5-4\times0.25]$$

$$= 50 \times 10^{-5}$$

$$\delta V_c = 50\times10^{-5}\times\frac{\pi}{4}\times300^2\times1500 = 53014 \text{ mm}^3$$

Total change in volume = $\delta v_s + \delta v_c$

= 53412 mm^3.

Exercises

Practice Problems I

Direction for questions 1 to 15: Select the correct alternative from the given choices.

Direction for questions 1 to 3: A composite beam consisting of rectangular timber section 150 mm × 200 mm and, 150 mm wide and 10 mm deep steel plates attached on top and bottom of the timber section. Maximum stress in timber is 7 N/mm^2.

$$E_{\text{steel}} = 2\times10^5 \text{ N/mm}^2 \text{ and } E_{\text{Timber}} = 1\times10^4 \text{ N/mm}^2.$$

1. The maximum stress in steel plate is
(A) 150 N/mm^2 (B) 154 N/mm^2
(C) 145 N/mm^2 (D) 160 N/mm^2

2. If the timber section is transformed to an equivalent steel section so as to form a symmetrical steel I-section, the moment of inertia of the transformed beam about the neutral axis is
(A) 47.2×10^6 mm^4 (B) 45.87×10^6 mm^4
(C) 38.1×10^6 mm^4 (D) 33.2×10^6 mm^4

3. The moment of resistance of the composite section is
(A) 53.34×10^6 Nmm (B) 45.07 Nmm
(C) 38.79×10^6 Nmm (D) 35.82 Nmm

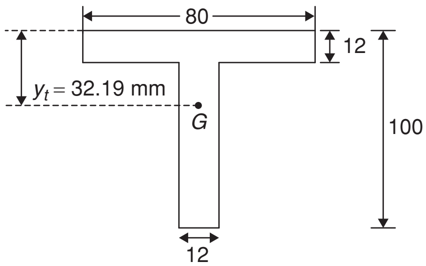
Direction for questions 4 and 5: A vessel 80 cm in diameter and 1.8 m in length having thickness of 12 mm is subjected to an internal pressure of 1.5 N/mm^2. Take $E = 2\times10^5$ N/mm^2 and $\mu = 0.25$.

4. The changes in the length and diameter of the vessel are
(A) 0.218 mm and 0.2238 mm
(B) 0.1125 mm and 0.175 mm
(C) 0.1987 mm and 0.2018 mm
(D) 0.1012 mm and 0.0983 mm

5. The change in the volume of the vessel is
(A) 388.5 cm^3
(B) 172.8 cm^3
(C) 452.4 cm^3
(D) 289.3 cm^3

6. A simply supported beam with rectangular cross section 100 mm × 200 mm has a span of 5 m. The permissible bending and shearing stress are 12 N/mm^2 and 0.8 N/mm^2, respectively. The maximum uniformly distributed load it can carry is
(A) 3.15 kN/m (B) 2.56 kN/m
(C) 2.82 kN/m (D) 5.33 kN/m

7. The cross-section of a beam which is subjected to a shear force of 20 kN is shown in the following figure:



The shear stress at the bottom of the flange is

(A) 4.265 N/mm^2 (B) 3.682 N/mm^2
(C) 3.895 N/mm^2 (D) 3.223 N/mm^2

Direction for questions 8 to 10: A cylindrical vessel 100 mm long and 250 mm diameter has a thickness of 10 mm. The vessel is filled with a fluid at atmospheric pressure. An additional quantity of 40×10^3 mm^3 of fluid is pumped into the cylinder. Take E = 2×10^{-5} N/mm^2 and $\mu = 0.3$.

8. The changes in the length and diameter of the vessel are

(A) 0.0125p and 0.0133p
(B) 0.0225p and 0.0213p
(C) 0.0315p and 0.0295p
(D) 0.2965p and 0.2813p

9. The pressure in the cylinder is

(A) 7.52 N/mm^2 (B) 5.84 N/mm^2
(C) 6.86 N/mm^2 (D) 8.12 N/mm^2

10. The hoop stress induced is

(A) 76.86 N/mm^2 (B) 85.75 N/mm^2
(C) 92.13 N/mm^2 (D) 79.05 N/mm^2

11. A steel tube of 6 mm bore and 0.6 mm wall thickness is 1 m long and is full of mercury. It is placed horizontally, supported at its ends. Steel and mercury weigh 7.7×10^{-5} N/mm^3 and 1.36×10^{-4} N/mm^3, respectively. The maximum stress in the tube is

(A) 28.92 N/mm^2 (B) 31.65 N/mm^2
(C) 32.74 N/mm^2 (D) 29.25 N/mm^2

Direction for questions 12 and 13: A simply supported beam of 4 m span carries concentrated loads 2 kN at distances 1 m, 2 m and 3 m from the support.

The beam has a cross-section of 100 mm width and 240 mm depth. At a section 0.75 m from the support,

12. Bending stress at a point 60 mm above neutral axis will be

(A) 1.172 N/mm^2 (B) 0.141 N/mm^2
(C) 0.176 N/mm^2 (D) 1.192 N/mm^2

13. Shearing stress at a point 60 mm above neutral axis will be

(A) 1.172 N/mm^2 (B) 0.1875 N/mm^2
(C) 0.141 N/mm^2 (D) 1.134 N/mm^2

14. A riveted boiler of 1.5 m diameter has to withstand a pressure of 2 N/mm^2. If the efficiency of longitudinal joints is 85% and that of circumferential joints is 40% and the permissible stress is 150 N/mm^2, the plate thickness of the boiler is

(A) 10.4 mm (B) 12.5 mm
(C) 12.8 mm (D) 11.6 mm

15. A spherical shell with internal diameter 320 mm and external diameter 640 mm is subjected to an internal fluid pressure of 75 N/mm^2. The hoop stress developed at the outer surface will be

(A) 15.132 N/mm^2 (B) 16.071 N/mm^2
(C) 14.067 N/mm^2 (D) 17.173 N/mm^2

Practice Problems 2

Direction for questions 1 to 10: Select the correct alternative from the given choices.

1. Two beams, one having a square cross-section and another a circular cross-section are subjected to the same amount of bending moment. If the cross-sectional area as well as the material of both the beams are the same, then

(A) maximum bending stress developed in both the beams are same.
(B) circular beam has more bending stress induced.
(C) square beam has more bending stress induced.
(D) bending stress does not depend on the cross-section.

2. A cylindrical pipe of diameter 400 mm and thickness 100 mm is subjected to an internal pressure of 80 N/mm^2. Maximum and minimum hoop stresses developed in N/mm^2 are

(A) 208 and 128 (B) 227 and 135
(C) 192 and 112 (D) 222 and 131

Direction for questions 3 and 4: A rectangular section of 250 mm × 500 mm is used as a cantilever with a span of 4 m. The beam carries a load of 15 kN/m and a point load of 30 kN at the free end.

3. The maximum moment and moment of inertia about the neutral axis is

(A) 201 kNm and 2.01×10^9 mm^4
(B) 210 kNm and 1.98×10^9 mm^4
(C) 240 kNm and 2.6×10^9 mm^4
(D) 235 kNm and 2.4×10^9 mm^4

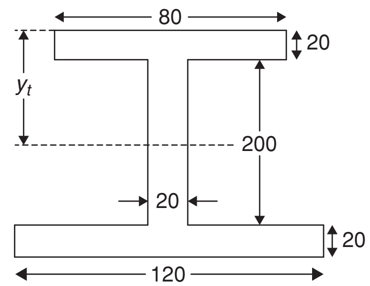
4. The maximum bending stress is

(A) 23.04 N/mm^2
(B) 19.45 N/mm^2
(C) 21.13 N/mm^2
(D) 22.34 N/mm^2

5. The air pressure in a cylindrical vessel of internal diameter 1.2 m is 15 N/mm^2. If the maximum permissible stress induced is 150 N/mm^2, then the thickness of the cylinder is
   (A) 40 mm (B) 50 mm
   (C) 60 mm (D) 45 mm

Direction for questions 6 and 7: A cylindrical vessel is to be designed for an internal pressure of 10 N/mm^2. The internal diameter of the vessel is 160 mm. The maximum hoop stress is not to exceed 40 N/mm^2.

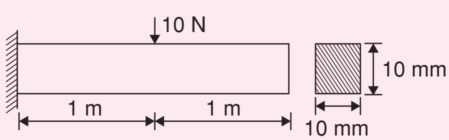
6. The constants for Lami's equation are
   (A) 25 and 175000
   (B) 15 and 160000
   (C) 20 and 158000
   (D) 17 and 165000
7. The thickness of the metal required is
   (A) 36.17 mm (B) 20.35 mm
   (C) 23.38 mm (D) 18.98 mm
8. A cantilever beam of span 1 m has a circular cross-section of diameter 200 mm. If a concentrated load of 50 kN is applied at the free end of the beam, maximum shear stress developed is
   (A) 2.68 N/mm^2 (B) 2.12 N/mm^2
   (C) 1.84 N/mm^2 (D) 2.92 N/mm^2
9. The I-section (dimensions in mm) shown in the figure is subjected to a shear force of 40 kN.
   (Take $I = 65 \times 10^6$ mm^4)

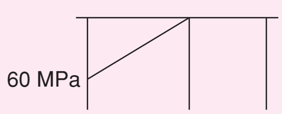


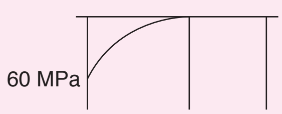
   The shear stress at the bottom of the top flange will be
   (A) 1.489 N/mm^2 (B) 1.378 N/mm^2
   (C) 1.402 N/mm^2 (D) 1.517 N/mm^2
10. A steel spherical vessel of radius 1000 mm and wall thickness 10 mm is subjected to an internal pressure of 0.8 MPa. The change in diameter of the spherical vessel is
    (Take E = 200 GPa and Poisson's ratio = 0.25)
    (A) 3 mm (B) 0.5 mm
    (C) 0.3 mm (D) 0.2 mm

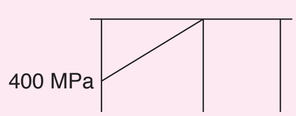
Previous Years' Questions

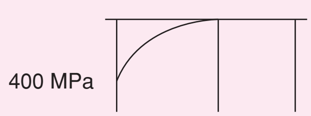
1. A cantilever beam has the square cross-section of 10 mm × 10 mm. It carries a transverse load of 10 N. Considering only the bottom fibres of the beam, the correct representation of the longitudinal variation of the bending stress is **[2005]**



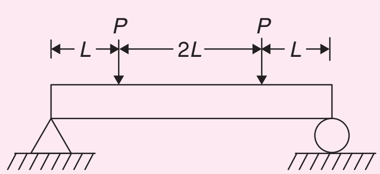
(A)


(B)


(C)


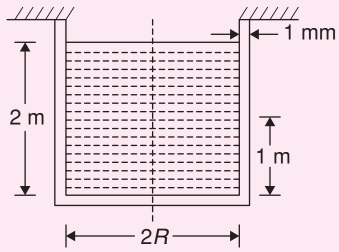
(D)


2. A simply supported beam of span length 6 m and 75 mm diameter carries a uniformly distributed load of 1.5 kN/m. What is the maximum value of bending stress? **[2006]**
   (A) 162.98 MPa (B) 325.95 MPa
   (C) 625.95 MPa (D) 651.90 MPa
3. The strain energy stored in the beam with flexural rigidity EI and loaded as shown in the figure is **[2008]**



   (A) $\dfrac{P^2L^3}{3EI}$ (B) $\dfrac{2P^2L^3}{3EI}$
   (C) $\dfrac{4P^2L^3}{3EI}$ (D) $\dfrac{8P^2L^3}{3EI}$

Direction for questions 4 and 5: A cylindrical container of radius R = 1 m, wall thickness 1 mm is filled with water up to a depth of 2 m and suspended along its upper rim. The density of water is 1000 kg/m^3 and acceleration due to gravity is 10 m/s^2. The self-weight of the cylinder is negligible. The formula for hoop stress in a thin-walled cylinder can be used at all points along the height of the cylindrical container.



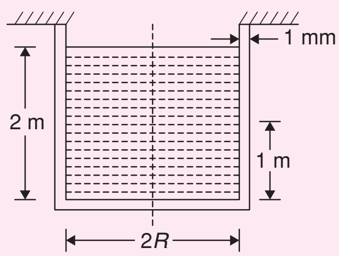
4. The axial and circumferential stress (σ_a, σ_c) experienced by the cylinder wall at mid-depth (1 m as shown) are **[2008]**

(A) (10, 10) MPa (B) (5, 10) MPa
(C) (10, 5) MPa (D) (5, 5) MPa

5. If the Young's modulus and Poisson's ratio of the container material are 100 GPa and 0.3, respectively, the axial strain in the cylinder wall at mid-depth is **[2008]**

(A) 2×10^{-5} (B) 6×10^{-5}
(C) 7×10^{-5} (D) 1.2×10^{-4}

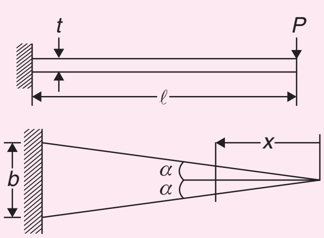
6. A cylindrical container of radius $R = 1$ m, wall thickness 1 mm is filled with water up to a depth of 2 m and suspended along its upper rim. The density of water is 1000 kg/m^3 and acceleration due to gravity is 10 m/s^2. The self-weight of the cylinder is negligible. The formula for hoop stress in a thin-walled cylinder can be used at all points along the height of the cylindrical container.



The maximum magnitude of bending stress (in MPa) is given by **[2010]**

(A) 60.0 (B) 67.5
(C) 200.0 (D) 225.0

7. A triangular-shaped cantilever beam of uniform thickness is shown in the figure. The Young's modulus of the material of the beam is E. A concentrated load P is applied at the free end of the beam.



The maximum deflection of the beam is

(A) $\dfrac{24P\ell^3}{Ebt^3}$ (B) $\dfrac{12P\ell^3}{Ebt^3}$

(C) $\dfrac{8P\ell^3}{Ebt^3}$ (D) $\dfrac{6P\ell^3}{Ebt^3}$

8. A thin-walled spherical shell is subjected to an internal pressure. If the radius of the shell is increased by 1% and the thickness is reduced by 1%, with the internal pressure remaining the same, the percentage change in the circumferential (hoop) stress is **[2012]**

(A) 0 (B) 1
(C) 1.08 (D) 2.02

9. A long thin-walled cylindrical shell, closed at both the ends, is subjected to an internal pressure. The ratio of the hoop stress (circumferential stress) to longitudinal stress developed in the shell is **[2013]**

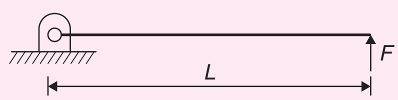
(A) 0.5 (B) 1.0
(C) 2.0 (D) 4.0

10. A simply supported beam of length L is subjected to a varying distributed load $\sin(3\pi x/L)$ Nm^{-1}, where the distance x is measured from the left support. The magnitude of the vertical reaction force in N at the left support is **[2013]**

(A) zero (B) $\dfrac{L}{3\pi}$

(C) $\dfrac{L}{\pi}$ (D) $\dfrac{2L}{\pi}$

11. A pin jointed uniform rigid rod of weight W and length L is supported horizontally by an external force F as shown in the figure below. The force F is suddenly removed. At the instant of force removal, the magnitude of vertical reaction developed at the support is **[2013]**



(A) zero (B) $\dfrac{W}{4}$

(C) $\dfrac{W}{2}$ (D) W

12. Consider a simply supported beam of length, $50h$, with a rectangular cross-section of depth, h and width, $2h$. The beam carries a vertical point load, P at its mid-point. The ratio of the maximum shear stress to the maximum bending stress in the beam is **[2014]**

(A) 0.02 (B) 0.10
(C) 0.05 (D) 0.01

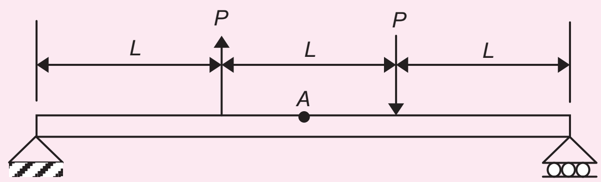
13. A thin gas cylinder with an internal radius of 100 mm is subject to an internal pressure of 10 MPa. The maximum permissible working stress is restricted to 100 MPa. The minimum cylinder wall thickness (in mm) for safe design must be ______. **[2014]**

14. A gas is stored in a cylindrical tank of inner radius 7 m and wall thickness 50 mm. The gage pressure of the gas is 2 MPa. The maximum shear stress (in MPa) in the wall is: **[2015]**

(A) 35 (B) 70
(C) 140 (D) 280

15. A cylindrical tank with closed ends is filled with compressed 2 m, and it has wall thickness of 10 mm. The magnitude of maximum inplane shear stress (in MPa) is _____. **[2015]**

16. A simply-supported beam of length $3L$ is subjected to the loading shown in the figure.



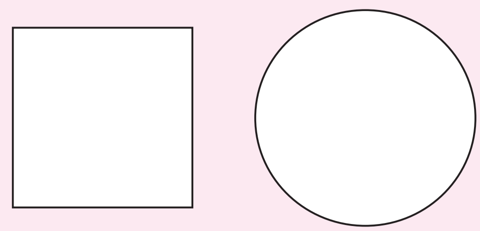
It is given that $P = 1$ N, $L = 1$ m and Young's modulus $E = 200$ GPa. The cross-section is a square with dimension 10 mm × 10 mm. The bending stress (in Pa) at the point A located at the top surface of the beam at a distance of 1.5 L from the left end is ________. **[2016]**

17. A thin cylindrical pressure vessel with closed-ends is subjected to internal pressure. The ratio of circumferential (hoop) stress to the longitudinal stress is: **[2016]**

(A) 0.25 (B) 0.50
(C) 1.0 (D) 2.0

18. The cross-sections of two solid bars made of the same material are shown in the figure as given on next page. The square cross-section has flexural (bending) rigidity I_1, while the circular cross-section has flexural rigidity I_2. Both sections have the same cross-sectional area. The ratio I_1/I_2 is **[2016]**

(A) $1/\pi$ (B) $2/\pi$
(C) $\pi/3$ (D) $\pi/6$



ANSWER KEYS

EXERCISES

Practice Problems 1

1. B **2.** C **3.** A **4.** B **5.** C **6.** B **7.** D **8.** A **9.** C **10.** B
11. B **12.** A **13.** C **14.** B **15.** B

Practice Problems 2

1. B **2.** A **3.** C **4.** A **5.** C **6.** B **7.** C **8.** B **9.** A **10.** C

Previous Years' Questions

1. A **2.** A **3.** C **4.** A **5.** C **6.** B **7.** B **8.** D **9.** C **10.** B
11. B **12.** D **13.** 9.8 to 10.6 **14.** C **15.** 25 **16.** 0 **17.** D **18.** C