Sets

Quick Revision

A well-defined collection of objects, is called a **set**. Sets are denoted by capital letters A, B, C, X, Y, Z etc. and elements of a set are denoted by a, b, c, x, y, z etc.

If a is an element of set A, then we say that a belongs to A. The phrase 'belongs to' denoted by the Greek symbol $\in$ (epsilon). Thus, we write and written as $a \in A$ and b does not belongs to set A is written as $b \notin A$.

Representation of Sets

There are two ways of representing a set

(i) **Roster form or Tabular form or Listing method** In the roster form, we list all the elements of the set within curly braces {} and separate them by commas.

(ii) **Set-builder form or Rule method** In the set-builder form, we list the property or properties satisfied by all the elements of the sets.

Types of Sets

(i) **Empty set** A set which does not contain any element is called an empty set or the void set or the null set and it is denoted by {} or ϕ.

(ii) **Singleton set** A set consisting of a single element, is called a singleton set.

(iii) **Finite and infinite sets** A set which is empty or consists of a finite number of elements is called a finite set, otherwise, the set is called an infinite set.

(iv) **Equivalent sets** Two finite sets A and B are said to be equal, if they have equal number of elements, i.e. $n(A) = n(B)$.

(v) **Equal sets** Two sets A and B are said to be equal, if they have exactly the same elements and we write $A = B$. Otherwise, the sets are said to be unequal and we write $A \neq B$.

Subset

A set A is said to be a subset of a set B, if every element of A is also an element of B. In symbols, we can write

$$A \subset B \text{, if } x \in A \Rightarrow x \in B$$

Also, if $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

Note

(i) Every set is a subset of itself.

(ii) The empty set is a subset of every sets.

(iii) The total number of subsets of a finite set containing n elements is 2^n.

Subsets of the Set of Real Numbers

We know that, every real number is either a **rational** or an **irrational** number and the set of real numbers is denoted by R. There are many important subsets of set of real numbers which are given below

(i) **Natural numbers** The numbers being used in counting as 1, 2, 3, 4,..., called natural numbers.

The set of natural numbers is denoted by N.

Thus, $\quad N = \{1, 2, 3, 4, ...\}$

(ii) **Whole numbers** The natural numbers along with number 0 (zero) form the set of whole numbers i.e. 0, 1, 2, 3, ... , are whole numbers. The set of whole numbers is denoted by W.

Thus, $W = \{0, 1, 2, 3, ...\}$

(iii) **Integers** The natural numbers, their negatives and zero make the set of integers and it is denoted by Z.

$Z = \{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

(iv) **Rational numbers** A number of the form $\frac{p}{q}$, where p and q both are integers and $q \neq 0$ (division by 0 is not permissible), is called a rational number.

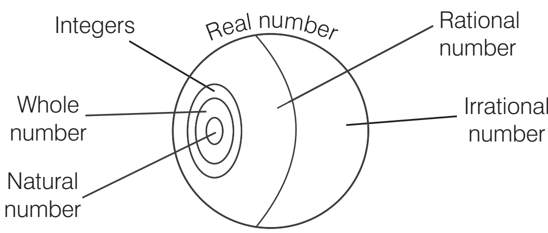
The set of rational numbers is generally denoted by Q.

Thus, $Q = \left\{\frac{p}{q} : p, q \in Z \text{ and } q \neq 0\right\}$

(v) **Irrational numbers** A number which cannot be written in the form p/q, where p and q both are integers and $q \neq 0$, is called an irrational number.

The set of irrational numbers is denoted by T. Thus, $T = \{x : x \in R \text{ and } x \notin Q\}$

Diagrammatical Representation All the subsets can be represented diagrammatically as given below



Intervals as Subsets of *R*

Let a and b be two given real numbers such that $a < b$, then

(i) the set of real numbers $\{x : a < x < b\}$ is called an **open interval** and is denoted by (a, b).

(ii) the set of real numbers $\{x : a \leq x \leq b\}$ is called a **closed interval** and is denoted by $[a, b]$.

(iii) intervals closed at one end and open at the other are known as **semi-open** or **semi-closed** intervals. $[a, b) = \{x : a \leq x < b\}$ is an open interval from a to b which includes a but excludes b. $(a, b] = \{x : a < x \leq b\}$ is an open interval from a to b which excludes a but includes b.

Universal Set

If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set and is denoted by U.

Power Set

The collection of all subsets of a set A is called the power set of A. It is denoted by $P(A)$. If the number of elements in A, i.e. $n(A) = m$, then the number of elements in $P(A)$ i.e. $n[P(A)] = 2^m$.

Properties of Power Sets

(i) If $A \subseteq B$, then $P(A) \subseteq P(B)$.

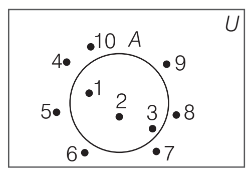
(ii) $P(A) \cap P(B) = P(A \cap B)$

(iii) $P(A \cup B) \neq P(A) \cup P(B)$

Venn Diagrams

Venn diagrams are the diagrams, which represent the relationship between sets. In Venn diagrams, the universal set is represented usually by a **rectangular region** and its subset are represented usually by **circle** or a **closed geometrical figure** inside the universal set. Also, an element of a set is represented by a **point** within the circle of set.

e.g. If $U = \{1, 2, 3, 4, \dots, 10\}$ and $A = \{1, 2, 3\}$, then its Venn diagram is as shown in the figure



Operations on Sets

(i) **Union of sets** The union of two sets A and B is the set of all those elements which belong to either in A or in B or in both A and B. It is denoted by $A \cup B$.
Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

(ii) **Intersection of sets** The intersection of two sets A and B is the set of all those elements, which are common to both A and B. It is denoted by $A \cap B$.

Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

(iii) **Disjoint sets** Two sets A and B are said to be disjoint sets, if they have no common elements i.e. if $A \cap B = \phi$.

Laws of Algebra of Sets

(i) **Idempotent laws** For any set A, we have

(a) $A \cup A = A$ (b) $A \cap A = A$

(ii) **Identity laws** For any set A, we have

(a) $A \cup \phi = A$ (b) $A \cap U = A$

(iii) **Commutative laws** For any two sets A and B, we have

(a) $A \cup B = B \cup A$ (b) $A \cap B = B \cap A$

(iv) **Associative laws** For any three sets A, B and C, we have

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

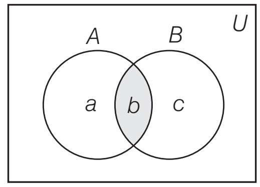
(v) **Distributive laws** If A, B and C are three sets, then

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

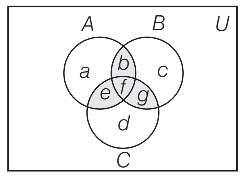
Results on Number of Elements in Sets

(i) When number of elements of two sets are given



Then, $n(A \cup B) = a + b + c$

(ii) When number of elements of three sets are given



Then,
$n(A \cup B \cup C) = a + b + c + d + e + f + g$

Objective Questions

Multiple Choice Questions

1. If $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then insert appropriate symbol $\in$ or $\notin$ in each of the following blank spaces.

(i) 4 ... A (ii) $-$ 4 ... A (iii) 12 ... A are

(a) $\in, \in, \in$ (b) $\in, \notin, \in$
(c) $\in, \notin, \notin$ (d) $\notin, \notin, \notin$

2. The following set in Roster form is $\{x : x$ is positive integer and a divisor of 9$\}$

(a) {1, 3, 9} (b) {1, 3, 8}
(c) {9, 8, 27} (d) None of these

3. The set of all natural numbers x such that $4x + 9 < 50$ in roster form is

(a) {1, 2, 4, 6, 8, 10}
(b) {1, 3, 5, 7, 9}
(c) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(d) None of the above

4. List of the elements of the following set $\{x : x = \dfrac{n}{1+n^2}$ and $1 \le n \le 3$, where $n \in N\}$, is

(a) $\left\{\frac{3}{10}, \frac{1}{5}, \frac{2}{3}\right\}$ (b) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{10}\right\}$
(c) $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{5}\right\}$ (d) $\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}\right\}$

5. The set $A = \{14, 21, 28, 35, 42, ..., 98\}$ in set-builder form is

(a) $A = \{x : x = 7n, n \in N$ and $1 \le n \le 15\}$
(b) $A = \{x : x = 7n, n \in N$ and $2 \le n \le 14\}$
(c) $A = \{x : x = 7n, n \in N$ and $3 \le n \le 13\}$
(d) $A = \{x : x = 7n, n \in N$ and $4 \le n \le 12\}$

6. If a set is denoted as $A = \phi$, then number of elements in A is

(a) 0 (b) 1
(c) 2 (d) 3

7. A set $B = \{5\}$ is called

(a) singleton set (b) empty set
(c) infinite set (d) None of these

8. The set of months of a year is …X… set. Here, X refers to

(a) empty (b) finite
(c) infinite (d) singleton

9. Let $A = \{x : x$ is a square of a natural number and x is less than $100\}$ and B is a set of even natural numbers. The cardinality of $A \cap B$ is

(a) 4 (b) 5
(c) 9 (d) None of these

10. The set $\{1, 2, 3, ...\}$ is …Y… set. Here, Y refers to

(a) null (b) finite
(c) infinite (d) singleton

11. From the following sets given below, pair the equivalent sets.
$A = \{1, 2, 3\}$, $B = \{t, p, q, r, s\}$, $C = \{\alpha, \beta, \gamma\}$ and $D = \{a, e, i, o, u\}$

(a) Sets A, C and A, D (b) Sets A, B and B, D
(c) Sets A, C and B, D (d) Sets A, C and B, C

12. If $A =$ the set of letters in 'ALLOY' and $B =$ the set of letters in 'LOYAL', then A and B are …X… . Here, X refers to

(a) equal (b) unequal
(c) disjoint (d) None of these

13. If $X = \{8^n - 7n - 1 \mid n \in N\}$ and $y = \{49n - 49 \mid n \in N\}$. Then, ***[NCERT Exemplar]***

(a) $X \subset Y$ (b) $Y \subset X$
(c) $X = Y$ (d) $X \cap Y = \phi$

14. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively ***[NCERT Exemplar]***

(a) 4, 7 (b) 7, 4 (c) 4, 4 (d) 7, 7

15. Write the $\{x : x \in R, -5 < x \leq 6\}$ as interval, then the length of the interval is

(a) 9 (b) 10 (c) 11 (d) 12

16. If $A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$ and $C = \{0, 7\}$, then the universal set will be

(a) $\{0, 7\}$
(b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{0, 1, 3, 5, 7\}$
(d) $\{0, 1, 2, 3, 4, 5, 6, 7\}$

17. Total number of elements in the power set of A containing 15 elements is

(a) 2^{15} (b) 15^2
(c) 2^{15-1} (d) $2^{15} - 1$

18. If $A = P(\{1, 2\})$, where P denotes the power set, then which one of the following is correct?

(a) $\{1, 2\} \subset A$ (b) $1 \in A$
(c) $\phi \notin A$ (d) $\{1, 2\} \in A$

19. If $A = \{1, 3, 5, 7\}$, then what is the cardinality of the power set $P(A)$?

(a) 8 (b) 15
(c) 16 (d) 17

20. If A and B are two sets, then $A \cap (A \cup B)$ equals to ***[NCERT Exemplar]***

(a) A (b) B
(c) ϕ (d) $A \cap B$

21. If $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$, then $A \cup B$ is

(a) $\{2, 4, 6, 8\}$ (b) $\{6, 8, 10, 12\}$
(c) $\{6, 8\}$ (d) $\{2, 4, 6, 8, 10, 12\}$

22. If $A = \{1, 3, 9\}$ and $B = \{2, 4, 5, 6\}$, then $A \cup B$ is

(a) $\{1, 3, 2, 4, 9\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3, 4, 5, 9\}$ (d) $\{1, 2, 3, 4, 5, 6, 9\}$

23. Let $A = \{a, e, i, o, u\}$ and $B = \{a, i, u\}$. Then, $A \cup B$ is …X… . Here, X refers to

(a) A (b) B
(c) A and B (d) None of these

24. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 = 144\}$, then $A \cap B$ contains

(a) one point (b) three points
(c) two points (d) four points

25. Let $X = \{\text{Ram, Geeta, Akbar}\}$ be the set of students of class XI, who are in school hockey team and $Y = \{\text{Geeta, David, Ashok}\}$ be the set of students from class XI, who are in the school football team. Then, $X \cap Y$ is

(a) {Ram, Geeta} (b) {Ram}
(c) {Geeta} (d) None of these

26. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{2, 3, 5, 7\}$. Then, which of following is true?

(a) $A \cap B = A$ (b) $A \cap B = B$
(c) $A \cap B \not\subset B$ (d) None of these

27. If $S = \{x \mid x$ is a positive multiple of 3 less than $100\}$ and $P = \{x \mid x$ is a prime number less than $20\}$. Then, $n(S) + n(P)$ is equal to ***[NCERT Exemplar]***

(a) 34 (b) 31
(c) 33 (d) 41

28. Are the given sets disjoint?

$A = \{x : x$ is the boys of your school$\}$

$B = \{x : x$ is the girls of your school$\}$

(a) Yes (b) No
(c) Can't say (d) Insufficient data

29. Let A and B be two sets such that $n(A) = 0.16, n(B) = 0.14$ and $n(A \cup B) = 0.25$. Then, $n(A \cap B)$ is equal to

(a) 0.3 (b) 0.5
(c) 0.05 (d) None of these

30. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, then the number of elements does Y have

(a) 10 (b) 20
(c) 30 (d) 40

31. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. The number of people who speak atleast one of these two languages, is

(a) 40 (b) 60
(c) 20 (d) 80

32. In a town with a population of 5000, 3200 people are egg-eaters, 2500 meat-eaters and 1500 eat both egg and meat. Then, the number of pure vegetarians is

(a) 800
(b) 1000
(c) 1600
(d) 2000

33. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. The number of students in the group is

(a) 50 (b) 125
(c) 75 (d) 175

34. In a school sports event, 50 students participate for football, 30 students participate for cricket and 15 students participate for both. Then, the number of students who participated for either football or for cricket, is

(a) 60 (b) 55
(c) 65 (d) 75

Assertion-Reasoning MCQs

Directions (Q. Nos. 35-49) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.
(b) A is true, R is true; R is not a correct explanation of A.
(c) A is true; R is false.
(d) A is false; R is true.

35. **Assertion** (A) 'The collection of all natural numbers less than 100' is a set.

Reason (R) A set is a well-defined collection of the distinct objects.

36. **Assertion (A)** The set $D = \{x : x$ is a prime number which is a divisor of 60$\}$ in roster form is $\{1, 2, 3, 4, 5\}$.

Reason (R) The set $E =$ the set of all letters in the word 'TRIGONOMETRY', in the roster form is {T, R, I, G, O, N, M, E, Y}.

37. **Assertion (A)** The set $\{1, 4, 9, \ldots 100\}$ in the set-builder form is $\{x : x = n^2$, where $n \in N$ and $1 \le n \le 10\}$.

Reason (R) In roster form, the order in which the elements are listed is, immaterial.

38. **Assertion (A)** The set $\{x : x$ is a month of a year not having 31 days$\}$ in roster form is {February, April, June, September, November}.

Reason (R) The set $F = \{x : x$ is a consonant in the English alphabet which precedes $k\}$ in roster form is $F = \{b, c, d, f, g, h, j\}$.

39. **Assertion (A)** The set $A = \{x : x$ is an even prime number greater than 2$\}$ is the empty set.

Reason (R) The set $B = \{x : x^2 = 4, x$ is odd$\}$ is not an empty set.

40. **Assertion (A)** The set $A = \{a, b, c, d, e, g\}$ is finite set.

Reason (R) The set $B = \{$ men living presently in different parts of the world$\}$ is finite set.

41. **Assertion (A)** The set of positive integers greater than 100 is infinite.

Reason (R) The set of prime numbers less than 99 is finite.

42. **Assertion (A)** If $A =$ set of prime numbers less than 6 and $B =$ set of prime factors of 30, then $A = B$.

Reason (R) If $P = \{1, 2, 3\}$ and $Q = \{2, 2, 1, 3, 3\}$, then P and Q are not equal.

43. **Assertion (A)** Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Then, $A \subset B$.

Reason (R) If every element of X is also an element of Y, then X is a subset of Y.

44. **Assertion (A)** The interval $\{x : x \in R, -4 < x \le 6\}$ is represented by $(-4, 6]$.

Reason (R) The interval $\{x : x \in R, -12 < x < -10\}$ is represented by $[-12, -10]$.

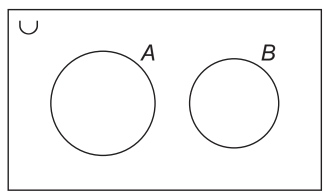
45. **Assertion (A)** Set of English alphabets is the universal set for the set of vowels in English alphabets.

Reason (R) The set of vowels is the subset of set of consonants in the English alphabets.

46. **Assertion (A)** The power set of the set $\{1, 2\}$ is the set $\{\phi, \{1\}, \{2\}, \{1,2\}\}$.

Reason (R) The power set is set of all subsets of the set.

47. **Assertion (A)** If $A \subset B$ for any two sets A and B.



Then, above Venn diagram represents correct relationship between A and B.

Reason (R) If $A \subset B$, then all elements of A is also in B.

48. **Assertion (A)** Let $A = \{a, b\}$ and $B = \{a, b, c\}$. Then, $A \not\subset B$.

Reason (R) If $A \subset B$, then $A \cup B = B$.

49. **Assertion (A)** If $n(A) = 3$, $n(B) = 6$ and $A \subset B$, then the number of elements in $A \cup B$ is 9.

Reason (R) If A and B are disjoint, then $n(A \cup B)$ is $n(A) + n(B)$.

Case Based MCQs

50. A class teacher Mamta Sharma of class XI write three sets A, B and C are such that $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$ and $C = \{2, 3, 5, 7, 11\}$.

Answer the following questions which are based on above sets.

(i) Find $A \cap B$.

(a) $\{3, 5, 7\}$ (b) ϕ
(c) $\{1, 5, 7\}$ (d) $\{2, 5, 7\}$

(ii) Find $A \cap C$.

(a) $\{3, 5, 7\}$ (b) ϕ
(c) $\{1, 5, 7\}$ (d) $\{3, 4, 7\}$

(iii) Which of the following is correct for two sets A and B to be disjoint?

(a) $A \cap B = \phi$ (b) $A \cap B \neq \phi$
(c) $A \cup B = \phi$ (d) $A \cup B \neq \phi$

(iv) Which of the following is correct for two sets A and C to be intersecting?

(a) $A \cap C = \phi$ (b) $A \cap C \neq \phi$
(c) $A \cup C = \phi$ (d) $A \cup C \neq \phi$

(v) Write the $n[P(B)]$.

(a) 8 (b) 4
(c) 16 (d) 12

51. The school organised a farewell party for 100 students and school management decided three types of drinks will be distributed in farewell party i.e. Milk (M), Coffee (C) and Tea (T).

Organiser reported that 10 students had all the three drinks M,C,T. 20 students had M and C; 30 students had C and T; 25 students had M and T. 12 students had M only; 5 students had C only; 8 students had T only.

Based on the above information, answer the following questions.

(i) The number of students who did not take any drink, is

(a) 20 (b) 30
(c) 10 (d) 25

(ii) The number of students who prefer Milk is

(a) 47 (b) 45
(c) 53 (d) 50

(iii) The number of students who prefer Coffee is

(a) 47 (b) 53
(c) 45 (d) 50

(iv) The number of students who prefer Tea is

(a) 51 (b) 53
(c) 50 (d) 47

(v) The number of students who prefer Milk and Coffee but not tea is

(a) 12 (b) 10 (c) 15 (d) 20

52. In a library, 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.

Based on the above information, answer the following questions.

(i) The number of students who reading only chemistry is

(a) 5 (b) 4
(c) 2 (d) 1

(ii) The number of students who reading only mathematics is

(a) 4 (b) 3
(c) 5 (d) 11

(iii) The number of students who reading only one of the subjects is

(a) 5 (b) 8
(c) 11 (d) 6

(iv) The number of students who reading atleast one of the subject is

(a) 20 (b) 22
(c) 23 (d) 21

(v) The number of students who reading none of the subject is

(a) 2 (b) 4
(c) 3 (d) 5

53. In an University, out of 100 students 15 students offered Mathematics only, 12 students offered Statistics only, 8 students offered only Physics, 40 students offered Physics and Mathematics, 20 students offered Physics and Statistics, 10 students offered Mathematics and Statistics, 65 students offered Physics.

Based on the above information answer the following questions

(i) The number of students who offered all the three subjects is

(a) 4 (b) 3
(c) 2 (d) 5

(ii) The number of students who offered Mathematics is

(a) 62 (b) 65
(c) 55 (d) 60

(iii) The number of students who offered statistics is

(a) 31 (b) 35
(c) 39 (d) 34

(iv) The number of students who offered mathematics and statistics but not physics is

(a) 7 (b) 6
(c) 5 (d) 4

(v) The number of students who did not offer any of the above three subjects is

(a) 4 (b) 1
(c) 5 (d) 3

54. The school organised a cultural event for 100 students. In the event, 15 students participated in dance, drama and singing. 25 students participated in dance and drama; 20 students participated in drama and singing; 30 students participated in dance and singing. 8 students participated in dance only; 5 students in drama only and 12 students in singing only.

Based on the above information, answer the following questions.

(i) The number of students who participated in dance, is

(a) 18 (b) 30
(c) 40 (d) 48

(ii) The number of students who participated in drama, is

(a) 35 (b) 30 (c) 25 (d) 20

(iii) The number of students who participated in singing, is

(a) 42 (b) 45
(c) 47 (d) 37

(iv) The number of students who participated in dance and drama but not in singing, is

(a) 20 (b) 5
(c) 10 (d) 15

(v) The number of students who did not participate in any of the events, is

(a) 20 (b) 30
(c) 25 (d) 35

55. In a company, 100 employees offered to do a work. In out of them, 10 employees offered ground floor only, 15 employees offered first floor only, 10 employees offered second floor only, 30 employees offered second floor and ground floor to work, 25 employees offered first and second floor, 15 employees offered ground and first floor, 60 employees offered second floor.

Based on the above information answer the following questions

(i) The number of employees who offered all three floors.

(a) 5 (b) 3 (c) 4 (d) 6

(ii) The number of employees who offered ground floor.

(a) 50 (b) 60 (c) 65 (d) 70

(iii) The number of employees who offered first floor.

(a) 40 (b) 45 (c) 50 (d) 55

(iv) The number of employees who offered ground and first floor but not second floor.

(a) 10 (b) 15 (c) 20 (d) 25

(v) The number of employees who did not offer any of the above three floors.

(a) 15 (b) 10 (c) 5 (d) 0

ANSWERS

Multiple Choice Questions

1. (c) 2. (a) 3. (c) 4. (d) 5. (b) 6. (a) 7. (a) 8. (b) 9. (a) 10. (c)
11. (c) 12. (a) 13. (a) 14. (b) 15. (c) 16. (d) 17. (a) 18. (d) 19. (c) 20. (a)
21. (d) 22. (d) 23. (a) 24. (d) 25. (c) 26. (b) 27. (d) 28. (a) 29. (c) 30. (c)
31. (b) 32. (a) 33. (b) 34. (c)

Assertion-Reasoning MCQs

35. (a) 36. (d) 37. (b) 38. (b) 39. (c) 40. (b) 41. (b) 42. (c) 43. (a) 44. (c)
45. (c) 46. (a) 47. (d) 48. (d) 49. (d)

Case Based MCQs

50. (i) - (b); (ii) - (a); (iii) - (a); (iv) - (b); (v) - (c)
51. (i) - (a); (ii) - (a); (iii) - (c); (iv) - (b); (v) - (b)
52. (i) - (a); (ii) - (a); (iii) - (c); (iv) - (c); (v) - (a)
53. (i) - (b); (ii) - (a); (iii) - (c); (iv) - (a); (v) - (b)
54. (i) - (d); (ii) - (a); (iii) - (c); (iv) - (c); (v) - (b)
55. (i) - (a); (ii) - (a); (iii) - (c); (iv) - (a); (v) - (c)

SOLUTIONS

1. Given, $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(i) Since, 4 is an element of A, therefore $4 \in A$.

(ii) Since, -4 is not an element of A, therefore $-4 \notin A$.

(iii) Since, 12 is not an element of A, therefore $12 \notin A$.

2. Here, x is a positive integer and a divisor of 9. So, x can take values 1, 3, 9.

$\therefore$ $\{x : x$ is a positive integer and a divisor of $9\}$
$= \{1, 3, 9\}$

3. We have, $4x + 9 < 50$

$\Rightarrow \quad 4x + 9 - 9 < 50 - 9$ [subtracting 9 from both sides]

$\Rightarrow \quad 4x < 41 \Rightarrow x < \dfrac{41}{4}$

$\therefore \quad x < 10.25$

Since, x is a natural number, so x can take values 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

$\therefore$ Required set $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

4. Given, $\{x : x = \dfrac{n}{1+n^2}$ and $1 \le n \le 3$, where $n \in N\}$

Here, $x = \dfrac{n}{1+n^2}, 1 \le n \le 3, n \in N$

$\Rightarrow \quad x = \dfrac{n}{1+n^2}, n = 1, 2, 3$

$\Rightarrow \quad x = \dfrac{1}{1+1^2}, \dfrac{2}{1+2^2}, \dfrac{3}{1+3^2} = \dfrac{1}{2}, \dfrac{2}{5}, \dfrac{3}{10}$

$\therefore$ Required set is $\left\{\dfrac{1}{2}, \dfrac{2}{5}, \dfrac{3}{10}\right\}$.

5. Let x represents the elements of given set.

Given numbers are natural numbers greater than 13, less than 99 and multiples of 7.

Thus, $A = \{x : x$ is a natural number greater than 13, less than 99 and a multiple of $7\}$, which is the required set-builder form of given set.

This can also be written as
$A = \{x : x$ is a natural number, a multiple of 7 and $13 < x < 99\}$
or $A = \{x : x = 7n, n \in N$ and $2 \le n \le 14\}$

6. The empty set contains no element.

Hence, the number of elements in A will be 0.

7. If a set A has only one element, we call it a singleton set. Thus, $\{a\}$ is a singleton set.

8. A year contains 12 months. So, the set of months of a year is finite.

9. Given, $A = \{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

and $\quad B = \{2, 4, 6, \dots\}$

Now, $A \cap B = \{4, 16, 36, 64\}$

$\therefore$ Cardinality of $(A \cap B)$
$=$ Number of elements in $(A \cap B) = 4$

10. The set $\{1, 2, 3, \dots\}$ contains infinite number of elements. So, it is infinite.

11. Given, $A = \{1, 2, 3\} \Rightarrow n(A) = 3$

$B = \{t, p, q, r, s\} \Rightarrow n(B) = 5$

$C = \{\alpha, \beta, \gamma\} \Rightarrow n(C) = 3$

$D = \{a, e, i, o, u\} \Rightarrow n(D) = 5$

Here, $n(A) = n(C) = 3$ and $n(B) = n(D) = 5$

$\therefore$ The sets A, C and B, D are equivalent sets.

12. $A = \{\text{A, L, O, Y}\}$, $B = \{\text{L, O, Y, A}\}$

Thus, A and B are equal.

13. $X = \{8^n - 7n - 1 \mid n \in N\} = \{0, 49, 490, \dots\}$

$Y = \{49n - 49 \mid n \in N\} = \{0, 49, 98, 147, \dots\}$

Clearly, every elements of X is in Y but every element of Y is not in X.

$\therefore \quad X \subset Y$

14. Since, number of subsets of a set containing m elements is 112 more than the subsets of the set containing n elements.

$\because \quad 2^m - 2^n = 112$

$\Rightarrow \quad 2^n \cdot (2^{m-n} - 1) = 2^4 \cdot 7$

$\Rightarrow \quad 2^n = 2^4$ and $2^{m-n} - 1 = 7$

$\Rightarrow \quad n = 4$ and $2^{m-n} = 8$

$\Rightarrow \quad 2^{m-n} = 2^3$

$\Rightarrow \quad m - n = 3$

$\Rightarrow \quad m - 4 = 3 \Rightarrow m = 4 + 3$

$\therefore \quad m = 7$

15. $\{x : x \in R, -5 < x \le 6\}$ is the set that does not contain -5 but contains 6. So, it can be written as a semi-closed interval whose first end point is open and last end point is closed i.e. $(-5, 6]$.

Length of the interval is $6-(-5)=11$.

16. If there are some sets under consideration, then a set can be chosen arbitrarily which is a superset of each one of the given sets. Such a set is known as the universal set and it is denoted by U.

$\because\ A=\{2, 4, 6\}, B=\{1, 3, 5\}$ and $C=\{0, 7\}$

$\therefore\ U=\{0, 1, 2, 3, 4, 5, 6, 7\}$

17. If a set A has n elements, then its power set will contain 2^n elements.

$\therefore$ Total number of elements in power set of $A=2^{15}$.

18. Let $B=\{1, 2\}$

Then, $A=P(B)=\{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Clearly, $\{1, 2\} \in A$

19. Given that, $A=\{1, 3, 5, 7\}$

Here, $n(A)=4$

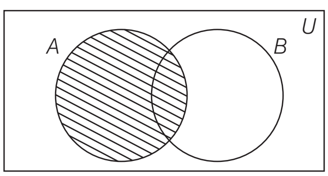
$\therefore$ Number of elements on power set of A

$$=2^{n(A)}=2^4=16$$

$\therefore$ Cardinality of the power set

$$P(A)=16$$

20. $\because \qquad A \cap (A \cup B)=A$



$A \cap (A \cup B)$

21. $A=\{2, 4, 6, 8\}$ and $B=\{6, 8, 10, 12\}$

The common elements of A and B are 6 and 8 which have been taken only once.

$\therefore \qquad A \cup B=\{2, 4, 6, 8, 10, 12\}$

22. $A=\{1, 3, 9\}, B=\{2, 4, 5, 6\}$,

then $A \cup B=\{1, 2, 3, 4, 5, 6, 9\}$

23. $A=\{a, e, i, o, u\}$

$B=\{a, i, u\}$

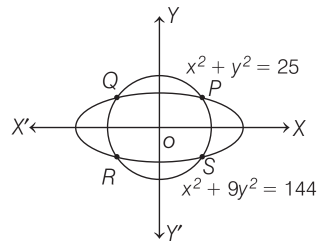
$A \cup B=\{a, e, i, o, u\}=A$

We observe that $B \subset A$.

Hence, if $B \subset A$, then $A \cup B=A$.

24. Clearly, A is the set of all points on the circle $x^2+y^2=25$ and B is the set of all points on the ellipse $x^2+9y^2=144$.

These two intersect at four points P, Q, R and S.



Hence, $A \cap B$ contains four points.

25. Here, $X=\{\text{Ram, Geeta, Akbar}\}$

and $\quad Y=\{\text{Geeta, David, Ashok}\}$

Then, $X \cap Y=\{\text{Geeta}\}$

26. $A=\{1, 2, 3, 4, \dots, 10\}$

and $B=\{2, 3, 5, 7\}$

$\therefore\ A \cap B=\{2, 3, 5, 7\}=B$

27. $\because\ S=\{x \mid x$ is a positive multiple of 3 less than $100\}$

$\therefore\ n(S)=33$

and $\ P=\{x \mid x$ is a prime number less than $20\}$

$\therefore \qquad n(P)=8$

$n(S)+n(P)=33+8=41$

28. Here, $A=\{b_1, b_2, \dots b_n\}$ and $B=\{g_1, g_2, \dots g_m\}$

where, $b_1, b_2, \dots, b_n$ are the boys and $g_1, g_2, \dots, g_m$ are the girls of school.

Clearly, $\qquad A \cap B=\phi$

Hence, this pair of set is disjoint set.

29. Given, $n(A)=0.16$, $n(B)=0.14$

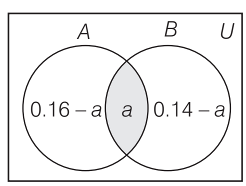
and $n(A \cup B)=0.25$

Let $n(A \cap B)=a$

$\therefore\ \ n(A \cup B)=0.16-a+a+0.14-a$

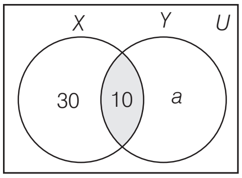
$\Rightarrow \qquad 0.25=0.30-a$

$\Rightarrow \qquad a=0.5$



30. Given, $n(X) = 40$, $n(X \cup Y) = 60$

and $\quad n(X \cap Y) = 10$



Clearly, $n(X \cup Y) = 30 + 10 + a$

$\Rightarrow \quad 60 = 30 + 10 + a$

$\Rightarrow \quad 60 = 40 + a$

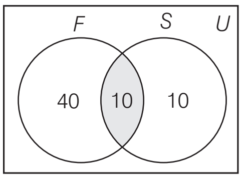
$\Rightarrow \quad a = 60 - 40$

$\therefore \quad a = 20$

Hence, Y have $10 + a = 10 + 20 = 30$ elements.

31. Given, $n(F) = 50$, $n(S) = 20$

and $n(F \cap S) = 10$



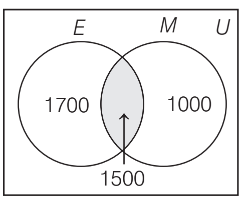
$\therefore n(F \cup S) = 40 + 10 + 10 = 60$

Hence, there are 60 people who speak atleast one of these two languages.

32. Let E be the set of people who are egg-eaters M be set of people who are meat-eaters and U be the set of people in the town.

$\therefore n(E) = 3200$, $n(M) = 2500$

and $n(E \cap M) = 1500$



$\therefore n(E \cup M) = 1700 + 1500 + 1000 = 4200$

$\because \quad n(U) = 5000$

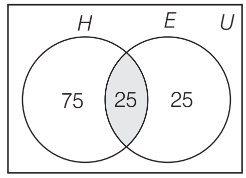
$\therefore$ Number of pure vegetarians $= 5000 - 4200$

$= 800$

33. Let H be the set of those students who know Hindi and E be the set of those students who know English.

Then, $n(H) = 100$, $n(E) = 50$

and $n(H \cap E) = 25$



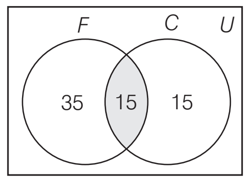
$\therefore \quad n(H \cup E) = 75 + 25 + 25$

$= 125$

Hence, the number of students in the group is 125.

34. Given, $n(F) = 50$, $n(C) = 30$ and $n(F \cap C) = 15$

$\therefore$ The number of students who participated for either football or for cricket is 65.



35. **Assertion** 'The collection of all natural numbers less than 100', is a well-defined collection. So, it is a set.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

36. **Assertion** We can write $60 = 2 \times 2 \times 3 \times 5$

$\therefore$ Prime factors of 60 are 2, 3 and 5.

Hence, the set D in roster form is {2, 3, 5}.

Reason There are 12 letters in the word 'TRIGONOMETRY' out of which three letters T, R and O are repeated.

Hence, set E in the roster form is {T, R, I, G, O, N, M, E, Y}.

Hence, Assertion is false and Reason is true.

37. **Assertion** We see that each member in the given set is the square of a natural number. Hence, the given set in set-builder form is $\{x : x = n^2$, where $n \in N$ and $1 \le n \le 10\}$.

Therefore, Assertion is true.

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

38. **Assertion** The months not containing 31 days are February, April, June, September, November. So, the given set in roster form is {February, April, June, September, November}.

Reason The given set can be represented in roster form as $F = \{b, c, d, f, g, h, j\}$.

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

39. **Assertion** 2 is the only even prime number. So, set A is the empty set.

Reason The equation $x^2 = 4$ is not satisfied by any odd value of x. So, set B is the empty set.

Hence, Assertion is true and Reason is false.

40. **Assertion** We know that, a set which is empty or consists of a definite number of elements, is called finite, otherwise the set is called infinite. Since, set A contains finite number of elements. So, it is a finite set.

Reason We do not know the number of elements in B, but it is some natural number. So, B is also finite.

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

41. **Assertion** There are infinite positive integer greater than 100. So, the set of positive integers greater than 100 is infinite.

Reason There are 25 prime numbers less than 99. So, the set of prime numbers less than 99 is finite.

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

42. **Assertion** $A = \{2, 3, 5\}$

Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

So, $\quad B = \{2, 3, 5\}$

$\therefore \quad A = B$

Reason We know that, a set does not change, if one or more elements of the set are repeated.

Hence, P and Q are equal.

Hence, Assertion is true and Reason is false.

43. **Assertion** Since, every element of A is in B, so $A \subset B$.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

44. **Assertion** The interval $\{x : x \in R, -4 < x \le 6\}$ is represented by $(-4, 6]$.

Reason The interval $\{x : x \in R, -12 < x < -10\}$ is represented by $(-12, -10)$.

Hence, Assertion is true and Reason is false.

45. **Assertion** Since, the set of vowels is the subset of the set of English alphabets.

So, the set of English alphabets is the universal set for set of vowels in English alphabets.

Reason We know set of vowels is not the subset of the set of consonants.

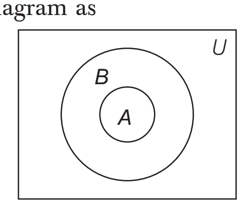
Hence, Assertion is true and Reason is false.

46. **Assertion** The subsets of the sets {1, 2} are ϕ, {1}, {2} and {1, 2}.

So, $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

47. **Assertion** If $A \subset B$, then every element of A is in B. Hence, this can be represented by Venn diagram as



Hence, Assertion is false and Reason is true.

48. **Assertion** $A = \{a, b\}, B = \{a, b, c\}$

Since, all the elements of A are in B. So,

$$A \subset B$$

Reason $\because \quad A \subset B$

$\Rightarrow \quad A \cup B = B$

Hence, Assertion is false and Reason is true.

49. **Assertion** $A \subset B$

$\Rightarrow \quad n(A \cup B) = n(B) = 6$

Reason If A and B are disjoint, then

$$n(A \cup B) = n(A) + n(B)$$

Hence, Assertion is false and Reason is true.

50. We have, $A = \{1, 3, 5, 7, 9\}$,

$B = \{2, 4, 6, 8\}$

and $C = \{2, 3, 5, 7, 11\}$

(i) $A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\}$

$= \phi$

(ii) $A \cap C = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7, 11\}$

$= \{3, 5, 7\}$

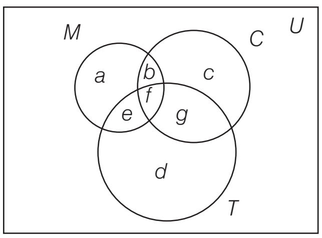
(iii) Here, $A \cap B = \phi$

(iv) The correct option for intersecting of two sets A and C is

$$A \cap C \neq \phi$$

(v) The number of elements in set B are 4. Therefore, the number of elements in $n[P(B)]$ are 2^4 i.e. 16.

51. Consider the following Venn diagram



where,

$a =$ Number of students who had M only

$b =$ Number of students who had M and C only

$c =$ Number of students who had C only

$d =$ Number of students who had T only

$e =$ Number of students who had M and T only

$f =$ Number of students who had three drinks M, C, T

and $g =$ Number of students who had C and T only

Then, we have

$a = 12$, $b + f = 20$, $c = 5$, $d = 8$, $e + f = 25$

$f = 10$ and $g + f = 30$

$\Rightarrow$ $a = 12$, $b = 10$, $c = 5$, $d = 8$, $e = 15$, $f = 10$ and $g = 20$

(i) Number of students who did not take any drink

$= 100 - (a + b + c + d + e + f + g)$

$= 100 - (12 + 10 + 5 + 8 + 15 + 10 + 20)$

$= 100 - 80 = 20$

(ii) Number of students who prefer Milk

$= a + b + f + e = 12 + 10 + 10 + 15 = 47$

(iii) Number of students who prefer Coffee

$= b + c + f + g = 10 + 5 + 10 + 20 = 45$

(iv) Number of students who prefer Tea

$= d + e + f + g = 8 + 15 + 10 + 20 = 53$

(v) Number of students who prefer Milk and Coffee but not Tea is b, i.e. 10.

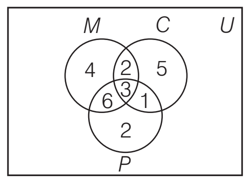
52. Let M denotes set of student who reading mathematics books, P denotes who reading Physics books and C denotes who reading chemistry books.

We have,

$n(U) = 25$, $n(M) = 15$, $n(P) = 12$, $n(C) = 11$

$n(M \cap C) = 5$, $n(M \cap P) = 9$, $n(P \cap C) = 4$,

$n(M \cap P \cap C) = 3$



(i) The number of students who reading only Chemistry is 5.

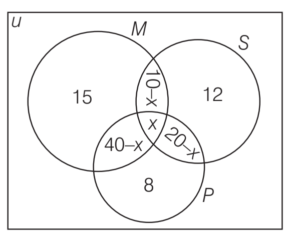
(ii) The number of students who reading only Mathematics is 4.

(iii) The number of students who reading only one of the subject is $4 + 5 + 2$ i.e. 11.

(iv) The number of students who reading atleast one of the subject is $4 + 6 + 3 + 2 + 5 + 1 + 2$ i.e. 23.

(v) The number of students who reading none of the subject is $25 - 23$ i.e. 2.

53. Let M, S and P be the sets of students wo offered Mathematics, Statistics and Physics respectively. Let x be the number of students who offered all the three subjects, then the number of members in different regions are shown in the following diagram.



From the Venn diagram, we get, the number of students who offered Physics.

$= (40 - x) + x + (20 - x) + 8 = 65$ [given]

$\Rightarrow \quad 68 - x = 65$

$\Rightarrow \quad x = 3$

(i) The number of students who offered all the three subjects are 3.

(ii) The number of students who offered Mathematics

$= 15 + (10 - x) + x + (40 - x)$

$= 65 - x$

$= 65 - 3 = 62$ [$\because x = 3$]

(iii) The number of students who offered Statistics

$= 12 + (10 - x) + x + (20 - x)$

$= 42 - x$

$= 42 - 3 = 39$ [$\because x = 3$]

(iv) $10 - x = 10 - 3 = 7$

(v) The number of students who offered anyone of the three subjects

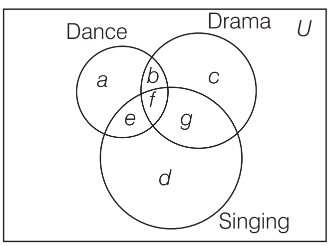
$= 15 + 12 + 8 + (10 - x) + (40 - x) + (20 - x) + x$

$= 105 - 2x$

$= 105 - 2 \times 3 = 99$ [$\because x = 3$]

$\therefore$ The number of students who did not offer anyone of the three subjects $= 100 - 99 = 1$

54. Consider the following Venn diagram



Where,

a = Number of students who participated in dance only

b = Number of students who participated in dance and drama only

c = Number of students who participated in drama only

d = Number of students who participated in singing only

e = Number of students who participated in dance and singing only

f = Number of students who participated in all three events dance, drama and singing

and g = Number of students who participated in drama and singing only

Then, we have

$a = 8, b + f = 25, c = 5, d = 12, e + f = 30, f = 15$ and $g + f = 20$

$\Rightarrow \quad a = 8, b = 10, c = 5, d = 12, e = 15, f = 15$ and $g = 5$

(i) The number of students who participated in dance $= a + b + e + f$

$= 8 + 10 + 15 + 15$

$= 48$

(ii) The number of students who participated in drama $= b + c + f + g$

$= 10 + 5 + 15 + 5$

$= 35$

(iii) The number of students who participated in singing $= d + e + f + g$

$= 12 + 15 + 15 + 5$

$= 47$

(iv) The number of students who participated in dance and drama but not in singing $= b = 10$

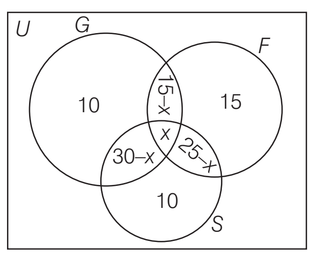
(v) The number of students who did not participats in any of the events

$= 100 - (a + b + c + d + e + f + g)$

$= 100 - (8 + 10 + 5 + 12 + 15 + 15 + 5)$

$= 100 - (70) = 30$

55. Let G, F and S be the sets of employees who offered ground floor, first floor and second floor respectively. Let x be the number of employees who offered all three floors, then the number of members in different regions are shown in the following diagram.



(i) From the Venn diagram, we get the number of employees who offered second floor

$$= (30 - x) + x + (25 - x) + 10 = 60 \quad \text{[given]}$$

$$\Rightarrow \quad 65 - x = 60$$

$$\Rightarrow \quad x = 5$$

(ii) The number of employees who offered ground floor

$$= 10 + (15 - x) + x + (30 - x)$$
$$= 55 - x$$
$$= 55 - 5$$
$$= 50$$

(iii) The number of employees who offered first floor

$$= 15 + (15 - x) + x + (25 - x)$$
$$= 55 - x$$
$$= 55 - 5 = 50$$

(iv) The number of employees who offered ground and first floor but not second floor

$$= 15 - x$$
$$= 15 - 5$$
$$= 10$$

(v) The number of employees who offer anyone of the three floors

$$= 10 + 15 + 10 + (15 - x) + (25 - x) + (30 - x) + x$$
$$= 105 - 2x$$
$$= 105 - 2 \times 5$$
$$= 95$$

$\therefore$ The number of employees who did not offer any of the three floors

$$= 100 - 95$$
$$= 5$$