11. TURBULENT FLOW

SHEAR STRESS IN TURBULENT FLOW

$$\tau = \mu \frac{d\bar{v}}{dy} + \eta \frac{d\bar{v}}{dy}$$

where, μ = dynamic coefficient of viscosity (*fluid* characteristic)

η = eddy viscosity coefficient (*flow* characteristic)

- Eddy viscosity comes in picture due to the turbulence effect.

HYDRO DYNAMICALLY SMOOTH AND ROUGH PIPES

- If the ***average height of irregularities*** (k) is much less the thickness of ***laminar*** sublayer (δ'), then the boundary is called hydrodynamically smooth.
- If the average height of irregularities (k) is much greater than the thickness of laminar sublayer (δ'), then the boundary is called hydrodynamically rough.
- On the basis of NIKURADSE'S EXPERIMENT the boundary is classified as:

Hydrodynamically smooth: $\frac{k}{\delta'} < 0.25$

Boundary in transition: $0.25 < \frac{k}{\delta'} < 6.0$

Hydrodynamically Rough: $\frac{k}{\delta'} > 6.0$

- $\frac{R}{k}$ is known as specific roughness. where 'k' is average height of roughness and 'R' is radius of the pipe.

VELOCITY DISTRIBUTION FOR TURBULENT FLOW IN PIPES

- **Prandtl's universal velocity distribution equation**

$$v = v_{max} + 2.5V^* \log_e\left(\frac{y}{R}\right)$$

where $V^* = \sqrt{\frac{\tau_0}{\rho}}$ = shear or friction velocity.

y = distance from pipe wall

ρ = Density of fluid

- The above equation is valid for both smooth and rough pipe boundaries.
- Shear velocity is still an ambigous quantity.

- **Karman - Prandtl Velocity distribution equation**

(i) Hydro Dynamically Smooth pipe

$$\frac{v}{V^*} = 5.75 \log_{10}\left(\frac{V^* y}{\nu}\right) + 5.5$$

(ii) Hydro Dynamically Rough pipe

$$\frac{v}{V^*} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5$$

where

V^* = shear velocity

y = distance from pipe wall

k = average height of roughness

ν = kinematic viscosity.

v = Average velocity

- **Velocity distribution in terms of mean velocity**

$$\frac{v - V^*}{V^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

The above equation is for ***both rough and smooth*** pipes.

FRICTION FACTOR

- **Friction factor 'f' for laminar flow**

$$f = \frac{64}{Re} \text{ where Re = Reynolds number}$$

- **Friction factor (f) for turbulent flow in smooth pipes**

$$f = \frac{0.316}{(Re)^{1/4}} \quad \left(4 \times 10^3 < Re < 10^5\right)$$

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}\left(Re\sqrt{f}\right) - 0.8 \quad \left(5 \times 10^4 < Re < 4 \times 10^7\right)$$

- **Friction factor (f) for turbulent flow in Rough pipes**

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}\left(\frac{R}{k}\right) + 1.74$$

This eq. shows that for rough pipes friction factor depends only on $\frac{R}{K}$ (Relative smoothness) and not on Reynolds number (Re)

- **Friction factor for commercial pipes**

$$\frac{1}{\sqrt{f}} - 2.0 \log_{10}\left(\frac{R}{k}\right) = 1.74 - 2.0 \log_{10}\left(1 + 18.7\right)\frac{R/k}{Re\sqrt{f}}$$

In this equation, K = equivalent sand grain roughness.

There is no specific relationship between f and R_e for transition flow in pipes.

■■■