Mensuration II: Volume and Surface Area

INTRODUCTION

Solids

A *solid* is a figure bounded by one or more surfaces. It has three dimensions namely length, breath or width, and thickness or height. The plane surfaces that bind it are called its *faces*.

The *volume* of any solid figure is the amount of space enclosed within its bounding faces. It is measured in cubic units, e.g., m^3, cm^3, etc.

The area of the plane surfaces that bind the solid is called its *surface area*.

For any regular solid,

Number of faces + Number of vertices

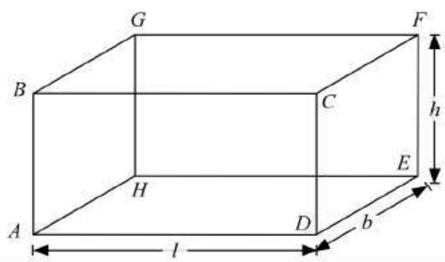
= Number of edges + 2.

We discuss below some important three dimensional figures and the formulae associated with them.

Some Basic Formulae

Cubic

It is a solid figure which has six rectangular faces. It is also called *rectangular parallelopiped*.



If l, b and h denote the length, breadth and height of the cuboid and d denotes the body diagonal (AF or BE or DG or CH), then

(i) Volume $= l \times b \times h = \sqrt{A_1 \times A_2 \times A_3}$,

where A_1 = area of base or top

A_2 = area of one side face

A_3 = area of other side face

(ii) Total surface area $= 2\,(lb + bh + lh)$
$= (l + b + h)^2 - d^2$

(iii) Diagonal of cuboid $= \sqrt{l^2 + b^2 + h^2}$

Note:

(i) For painting the surface area of a box or to know how much tin sheet is required for making a box, we use formula (ii).

(ii) To find how much a box contains or how much space a box shall occupy, we use formula (i). To find the length of the longest pole to be placed in a room, we use formula (iii).

(iii) The rise or fall of liquid level in a container

$$= \frac{\text{Total volume of objects submerged or taken out}}{\text{Cross-sectional area of container}}.$$

Illustration 1 Find the volume and the total surface area of a cuboid whose dimensions are 25 m, 10 m and 2 m.

Solution: Here $l = 25$ m, $b = 10$ m and $h = 2$ m.

Volume of the cuboid $= l \times b \times h$

$= 25 \times 10 \times 2$

$= 500 \text{ m}^3$

Total surface area of the cuboid

$= 2(lb + bh + lh)$

$= 2(25 \times 10 + 10 \times 2 + 25 \times 2)$

$= 2(250 + 20 + 50)$

$= 640 \text{ m}^2$

Illustration 2 Find the length of the longest bamboo that can be placed in a room 12 m long, 9 m broad and 8 m high

Solution: Length of the bamboo

= length of the diagonal of the room

$= \sqrt{12^2 + 9^2 + 8^2}$

$= \sqrt{289} = 17 \text{ m}$

Illustration 3 The area of a side of a box is 120 sq cm. The area of the other side of the box is 27 sq cm. If the area of the upper surface of the box is 60 sq cm, then find the volume of the box

Solution: Volume to the box

$= \sqrt{\text{area of base} \times \text{area of one face} \times \text{area of the other face}}$

$= \sqrt{60 \times 120 \times 72}$

$= \sqrt{518400} = 720 \text{ cm}^3$.

Illustration 4 The sum of length, breadth and height of a cuboid is 12 cm long. Find the total surface area of the cuboid

Solution: Total surface area

$= (\text{Sum of all three sides})^2 - (\text{Diagonal})^2$

$= 12^2 - 8^2 = 144 - 64 = 80$ sq cm.

Cube

It is a special type of cuboid in which each face is a square.

For a cube, length, breadth and height are equal and is called the edge of the cube.

If a be the edge of a cube, then

(i) Volume of the cube $= (\text{edge})^3 = a^3$

(ii) Total surface area of the cube $= 6(\text{edge})^2 = 6a^2$

(iii) Diagonal of the cube $= \sqrt{3}a$ (edge) $= \sqrt{3}a$

(iv) Volume of the cube $= \left(\frac{\text{diagonal}}{\sqrt{3}}\right)^3 = \left(\frac{d}{\sqrt{3}}\right)^2$

$= \left(\sqrt{\frac{\text{Surface area}}{6}}\right)^3$

(v) Total surface area of the cube

$= 2(\text{diagonal})^2 = 2d^2$

(vi) For two cubes

(*a*) Ratio of volumes = (ratio of sides)^3

(*b*) Ratio of surface areas = (Ratio of sides)^2

(*c*) (Ratio of surface areas)^3 = (Ratio of volumes)^2.

Illustration 5 Find the volume, surface area and the diagonal of a cube, each of whose sides measures 4 cm

Solution: Volume of the cube $= a^3 = (4)^3 = 64 \text{ cm}^3$.

Surface area of the cube $= 6a^2 = 6(4)^2 = 96 \text{ cm}^2$.

Diagonal of the cube $= \sqrt{3}a = 4\sqrt{3}$ cm

Illustration 6 The surface area of a cube is 216 sq cm. Find its volume.

Solution: Volume of the cube

$= \left(\sqrt{\frac{\text{Surface area}}{6}}\right)^3$

$= \left(\sqrt{\frac{216}{6}}\right)^3 = (6)^3 = 216 \text{ cm}^3$

Illustration 7 The diagonal of a cube is $8\sqrt{3}$ cm. Find its total surface area and volume.

Solution: We have,

Diagonal of cube $= \sqrt{3}$ (edge)

$\therefore$ Edge of cube $= \frac{\text{Diagonal of cube}}{\sqrt{3}}$

$= \frac{8\sqrt{3}}{\sqrt{3}} = 8$ cm.

Total surface area $= 6(\text{edge})^2 = 6(8)^2$

$= 384$ sq cm.

Volume of cube $= (\text{edge})^3$

$= (8)^3 = 512 \text{ cm}^3$

Illustration 8 If the volumes of two cubical blocks are in the ratio of 8:1, what will be the ratio of their edges?

Solution: We have,

Ratio of volumes = (Ratio of sides)^3

Since, ratio of volumes = 8:1, i.e., $2^3:1^3$

$\therefore$ ratio of sides = 2:1

Illustration 9 Volumes of the two cubes are in the ratio of 1:9. Find the ratio of their surface areas

Solution: (Ratio of the surface areas)^3

$$= (\text{Ratio of volumes})^2$$

$\therefore$ Ratio of surface areas = $\sqrt[3]{1:81}$ = 1:3 $(3)^{1/3}$

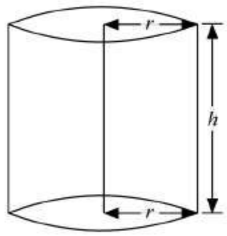
Illustration 10 Sides of two cubes are in the ratio of 2:3. Find the ratio of their surface areas

Solution: Ratio of surface areas

$$= (\text{Ratio of sides})^2$$
$$= (2:3)^2 = 4:9$$

Right Circular Cylinder

A *right circular cylinder* is a solid with circular ends of equal radius and the line joining their centres perpendicular to them. This is called axis of the cylinder. The length of the axis is called the height of the cylinder.



Note:

Take a rectangular sheet of paper and role it lengthwise or breadthwise in a round way, you will get a cylinder, i.e., a cylinder is generated by rotating a rectangle by fixing one of its sides.

If r is the radius of base and h is the height of the cylinder, then

(i) Volume of cylinder

= Area of the base × height

= $\pi r^2 \times h = \pi r^2 h$ cubic units

(ii) Area of the curved surface

= Circumference of the base × height

= $2\pi r \times h = 2\pi rh$ sq units

(iii) Area of the total surface

= Area of the curved surface

+ Area of the two circular ends

= $2\pi rh + 2\pi r^2$

= $2\pi r (h + r)$ sq units.

(iv) For two cylinders,

When radii are equal

(*a*) Ratio of volumes = Ratio of heights

(*b*) Ratio of volumes

= Ratio of curved surface areas

(*c*) Ratio of curved surface areas

= Ratio of heights

When heights are equal

(*a*) Ratio of volumes = (Ratio of radii)^2

(*b*) Ratio of volumes

= (Ratio of curved surface areas)^2

(*c*) Radii of curved surface areas

= Ratio of radii

When volumes are equal

(*a*) Ratio of radii = $\sqrt{\text{Inverse ratio of heights}}$

(*b*) Ratio of curved surface areas

= Inverse ratio of radii

(*c*) Ratio of curved surface areas

= $\sqrt{\text{Ratio of heights}}$

When curved surface areas are equal

(*a*) Ratio of radii = Inverse ratio of heights

(*b*) Ratio of volumes = Inverse ratio of heights

(*c*) Ratio of volumes = Ratio of radii

(v) For a cylinder

(*a*) Ratio of radii = (Ratio of curved surfaces)

× (Inverse ratio of heights)

(*b*) Ratio of heights = (Ratio of curved surfaces)

× (Inverse ratio of radii)

(*c*) Ratio of curved surfaces

= (Ratio of radii) × (Ratio of heights).

Illustration 11 The diameter of the base of a right circular cylinder is 28 cm and its height is 10 cm. Find the volume and area of the curved surface of the cylinder

Solution: Radius of the base = $\frac{28}{2}$ = 14 cm.

Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 10$$

$$= 6160 \text{ cm}^3$$

Area of the curved surface $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 10$$

$$= 880 \text{ sq cm.}$$

Illustration 12 A cylinder of height 21 cm has base of radius 4 cm. Find the total surface area of the cylinder

Solution: Total surface area $= 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 4 \times (21 + 4)$$

$$= \frac{4400}{7} = 628\frac{4}{7} \text{ sq cm}$$

Illustration 13 A rectangular piece of paper is 71 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its breadth. Find the volume of the cylinder $\left[\text{Take } \pi = \frac{355}{113}\right]$

Solution: Circumference of the paper = Breadth of the paper

$$\Rightarrow \quad 2\pi r = 10$$

$$\Rightarrow \quad r = \frac{10}{2\pi} = \frac{10 \times 113}{2 \times 355} = \frac{113}{71} \text{ cm.}$$

As the length of the paper becomes the height of the cylinder

∴ Volume of the cylinder

$$= \pi r^2 l$$

$$= \frac{355}{113} \times \frac{113}{71} \times \frac{113}{71} \times 71 = 565 \text{ cm}^3.$$

Illustration 14 Two circular cylinders of equal volume have their heights in the ratio of 9:16. Find the ratio of their radii

Solution: Ratio of radii $= \sqrt{\text{inverse ratio of heights}}$

$$= \sqrt{16:9} = 4:3$$

Illustration 15 Two circular cylinders of equal volume have their heights in the ratio of 16:25. Find the ratio of their curved surface areas.

Solution: Ratio of curved surface areas

$$= \sqrt{\text{Ratio of heights}} = \sqrt{16:25} = 4:5$$

Illustration 16 Two circular cylinders of equal volume have their radii in the ratio of 4:9. Find the ratio of their curved surface areas

Solution: Ratio of curved surface areas

= inverse ratio of radii = 9:4

Illustration 17 Two circular cylinders of equal heights have their radii in the ratio of 2:5. Find the ratio of their volumes

Solution: Ratio of volumes = (Ratio of radii)^2 = 4:25

Illustration 18 Two circular cylinders of equal heights have their curved surface areas in the ratio of 3:5. Find the ratio of their volumes

Solution: Ratio of volumes

= (Ratio of curved surface areas)^2

= 9:25

Illustration 19 Two circular cylinders of equal curved surface areas have their heights in the ratio of 4:7. Find the ratio of their volumes.

Solution: Ratio of volumes = Inverse ratio of heights

$$= \frac{1}{4}:\frac{1}{7} = 7:4$$

Illustration 20 Two circular cylinders of equal curved surface areas have their heights in the ratio of 4:5. Find the ratio of their volumes.

Solution: Ratio of volumes = Inverse ratio of heights

$$= \frac{1}{4}:\frac{1}{5} = 5:4$$

(vi) If the ratio of heights and the ratio of radii of two right circular cylinders are given, then

Ratio of curved surface areas = (ratio of radii) (ratio of heights).

Illustration 21 If the heights and the radii of two right circular cylinders are in the ratio 2:3 and 4:5, respectively. Find the ratio of their curved surface areas

Solution: Ratio of curved surface areas = (ratio of radii) (ratio of heights)

= (4:5) (2:3) = 8:15

(vii) If the ratio of heights and the ratio of curved surface areas of two right circular cylinders are given, then

Ratio of radii = (ratio of curves surface areas) (inverse ratio of heights).

Illustration 22 The heights and curved surface areas of two right circular cylinders are in the ratio 3:4 and 5:8, respectively. Find the ratio of their radii

Solution: Ratio of radii = (ratio of curved surface areas) (inverse ratio of heights)

$$= (5:8)\left(\frac{1}{3}:\frac{1}{4}\right) = (5:8)\ (4:3) = 5:6$$

(viii) If the ratio of radii and the ratio of curved surface areas of two right circular cylinders are given, then
Ratio of heights = (ratio of curved surface areas) (inverse ratio of radii)

Illustration 23 The radii of two right circular cylinders are in the ratio of 3:4 and their curved surface areas are in the ratio of 5:6. Find the ratio of their heights

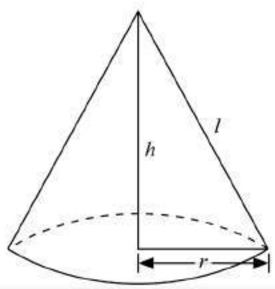
Solution: Ratio of heights = (ratio of curved surface areas) (inverse ratio of radii)

$$= (5:6)\left(\frac{1}{3}:\frac{1}{4}\right)$$

$$= (5:6)\ (4:3) = 10:9$$

Right Circular Cone

A *right circular cone* is a solid obtained by rotating a right-angled triangle around its height.



If r = radius of base; h = height,

l = slant height = $\sqrt{h^2 + r^2}$, then

(i) Volume of cone

$$= \frac{1}{3} \times \text{area of the base} \times \text{height}$$

$$= \frac{1}{3} \times \pi r^2 h \text{ cubic units}$$

(ii) Area of curved surface = πrl

$$= \pi r\sqrt{h^2 + r^2} \text{ sq. units}$$

(iii) Total surface area of cone

= Area of the base + area of the curved surface

$= \pi r^2 + \pi rl = \pi r(r + l)$ sq units.

(iv) For two cones

(*a*) When volumes are equal

Ratio of radii = $\sqrt{\text{inverse ratio of heights}}$

(*b*) When radii are equal

Ratio of volumes = Ratio of heights

(*c*) When heights are equal

Ratio of volumes = $(\text{ratio of radii})^2$

(*d*) When curved surface areas are equal

Ratio of radii = inverse ratio of slant heights.

Illustration 24 Find the slant heights of a cone whose volume is 1232 cm^3 and radius of the base is 7 cm

Solution: Volume of the cone = $\frac{1}{3}\pi r^2 h = 1232$

$$\Rightarrow \quad h = \frac{1232 \times 3}{\pi r^2} = \frac{1232 \times 3 \times 7}{22 \times 7 \times 7} = 24 \text{ cm}$$

Slant height l is given by the relation

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49}$$

$$= \sqrt{625} = 25 \text{ cm}$$

∴ Slant height of the cone is 25 cm.

Illustration 25 A tent is of diameter 12 m at the base and its height is 8 m.

(i) Find the slant height; and

(ii) The canvas required in sq metres

How many persons can the tent accommodate, at the most, if each person requires 18 m^3 of air?

Solution: Diameter of the base of a conical tent = 12 m

∴ Radius (r) = $\frac{12}{2}$ = 6 m and its height (h) = 8 m

(i) Slant height (l) $= \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2}$

$$= \sqrt{64 + 36} = \sqrt{100} = 10 \text{ m}$$

(ii) Area of canvas required

$$= \pi \times r \times l$$

$$= \frac{22}{7} \times 6 \times 10 = 188.57 \text{ m}$$

(iii) Volume of conical portion

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8 = 301.71 \text{ m}^3$$

Space required for each person = 18 m^3

$\therefore$ Number of persons that can be accommodated

$$= \frac{301.71}{18} = 16$$

Illustration 26 The height of a cone is 21 cm and radius of its base is 28 cm. Find its total surface area

Solution: We have, $r = 28$ cm and $h = 21$ cm.

$$\text{Slant height } (l) = \sqrt{r^2 + h^2} = \sqrt{(28)^2 + (21)^2}$$

$$= \sqrt{1225} = 35 \text{ cm}$$

$$\text{Total surface area} = \pi\, r\, (l + r)$$

$$= \frac{22}{7} \times 28 \times (35 + 28)$$

$$= \frac{22}{7} \times 28 \times 63$$

$$= 5544 \text{ sq cm}$$

Illustration 27 Two right circular cones of equal curved surface areas have their slant heights in the ratio of 3:5. Find the ratio of their radii

Solution: Ratio of radii = inverse ratio of slant heights

$$= \frac{1}{3} : \frac{1}{5} = 5:3$$

Illustration 28 Two right circular cones of equal volumes have their heights in the ratio of 4:9. Find the ratio of their radii

Solution: Ratio of radii = $\sqrt{\text{inverse ratio of heights}}$

$$= \sqrt{\frac{1}{4} : \frac{1}{9}} = \sqrt{9:4} = 3:2$$

Illustration 29 Two right circular cones of equal heights have their radii in the ratio of 1:3. Find the ratio of their volumes

Solution: Ratio of volumes = (Ratio of radii)^2

$$= (1:3)^2 = 1:9$$

(v) If the ratio of volumes and the ratio of heights of two right circular cones (or cylinders) are given, then

Ratio of radii

$$= \sqrt{(\text{ratio of volumes})\,(\text{inverse ratio of heights})}$$

$$= \sqrt{(3:2)\,(8:3)} : \sqrt{4:1} = 2:1$$

Illustration 30 The volumes of two cones are in the ratio 3:2 and their heights in the ratio 3:8. Find the ratio of their radii

Solution: Ratio of radii

$$= \sqrt{(\text{ratio of volumes})\,(\text{inverse ratio of heights})}$$

$$= \sqrt{(3:2)\,(8:3)} = \sqrt{4:1} = 2:1$$

(vi) If the ratio of heights and the ratio of diameters (or radii) of two right circular cones (or cylinders) are given, then

Ratio of volumes = (ratio of radii)^2 × (ratio of heights)

Illustration 31 The heights of two cones are in the ratio of 5:3 and their radii in the ratio 2:3. Find the ratio of their volumes

Solution: Ratio of volumes

$$= (\text{ratio of radii})^2 \times (\text{ratio of heights})$$

$$= (2:3)^2 \times (5:3)$$

$$= \frac{4}{9} \times \frac{5}{3} = 20:27$$

(vii) If the ratio of radii (or diameter) and the ratio of volumes of two right circular cones are given, then

Ratio of heights

= (inverse ratio of radii)^2 (ratio of volumes)

Illustration 32 The volumes of two cones are in the ratio of 1:4 and their diameters are in the ratio of 4:5. Find the ratio of their heights

Solution: Ratio of heights

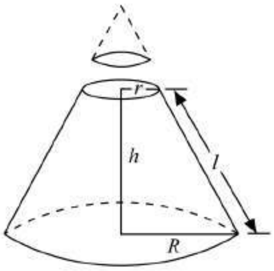
$$= (\text{inverse ratio of diameters})^2\,(\text{ratio of volumes})$$

$$= \left(\frac{1}{4} : \frac{1}{5}\right)^2 (1:4) = (5:4)^2\,(1:4)$$

$$= \frac{25}{16} \times \frac{1}{4} = 25:64$$

Frustum of a Right Circular Cone

A cone with some of its top portion cut off is called the *frustum* of the original cone.



If R = Radius of the base of frustum
r = Radius of the top of the frustum
h = Height of the frustum
l = Slant height of the frustum, then

(*a*) Slant height = $\sqrt{h^2 + (R - r)^2}$ units

(*b*) Area of the curved surface = $\pi (R + r) l$ sq units

(*c*) Total surface area of the frustum
$= \pi [(R^2 + r^2) + l(R + r)]$ sq units

(*d*) Volume of the frustum = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$ cu units.

Illustration 33 A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across the bottom. It is 6 m deep. Find the area of its curved surface, total surface area and also its volume

Solution: Here $R = 4$, $r = 2$ and $h = 6$.

$\therefore$ Slant height $(l) = \sqrt{h^2 + (R - r)^2}$

$$= \sqrt{(6)^2 + (4-2)^2} = \sqrt{40}$$

$\therefore$ Area of the curved surface

$$= \pi (R + r) l$$

$$= \frac{22}{7}(4 + 2)\sqrt{40}$$

$$= 18.8 \times 6.3 = 118.4 \text{ m}$$

Total surface area $= \pi [(R^2 + r^2) + l(R + r)]$

$$= \frac{22}{7}[(4^2 + 2^2) + \sqrt{40}(4 + 2)]$$

$$= \frac{22}{7}(20 + 6\sqrt{40}) = 181.6 \text{ sq m}$$

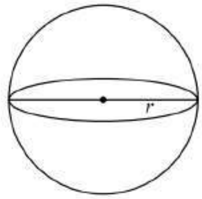
Volume of the frustum $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$

$$= \frac{22}{7} \times \frac{6}{3}(4^2 + 2^2 + 4 \times 2)$$

$$= \frac{44}{7}(20 + 4 + 8) = 176 \text{ m}^3$$

Sphere

A *sphere* is the solid figure formed by revolving a semi-circle on its diameter.

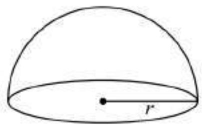


The mid-point of the diameter is called centre of the sphere and the radius of the semi-circle is called the radius of the sphere.

If r = radius of the spheres, then

(i) Volume of sphere = $\frac{4}{3}\pi r^3$ cubic units

(ii) Surface area = $4\pi r^2$ sq units.



(iii) Volume of hemisphere = $\frac{2}{3}\pi r^2$ cubic units

(iv) Area of curved surface = $2\pi r^2$ sq units of hemisphere

(v) Total surface area of hemisphere = $3\pi r^2$ sq units.

Illustration 34 Diameter of a sphere is 28 cm. Find its surface area and volume

Solution: Radius of the sphere $(r) = \frac{28}{2} = 14$ cm

Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times 14 \times 14$

$$= 2464 \text{ sq cm}$$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

$$= 11498.6 \text{ cm}^3$$

Illustration 35 Find the volume, curved surface area and total surface area of a hemisphere of radius 21 cm

Solution: Volume of the hemisphere

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times 21\times 21\times 21$$

$$= 19494 \text{ cm}^3$$

$$\text{Curved surface area} = 2\pi r^2$$

$$= 2\times\frac{22}{7}\times 21\times 21$$

$$= 2772 \text{ sq cm}$$

$$\text{Total surface area} = 3\pi r^2$$

$$= 3\times\frac{22}{7}\times 21\times 21$$

$$= 4158 \text{ sq cm}$$

(vi) For two spheres

(*a*) $(\text{Ratio of radii})^2$ = Ratio of surface areas

(*b*) $(\text{Ratio of radii})^3$ = Ratio of volumes

(*c*) $(\text{Ratio of surface areas})^3 = (\text{Ratio of volumes})^2$.

Illustration 36 The radii of two spheres are in the ratio of 2:3. What is the ratio of their surface areas?

Solution: Ratio of surface areas = $(\text{ratio of radii})^2$

$$= (2:3)^2 = 4:9$$

Illustration 37 The surface areas of two spheres are in the ratio 1:2. Find the ratio of their volumes

Solution: We have

$$(\text{Ratio of surface areas})^3 = (\text{Ratio of volumes})^2$$

$$\Rightarrow \quad (1:2)^3 = (\text{Ratio of volumes})^2$$

$$\therefore \quad \text{Ratio of volumes} = \sqrt{1:8} = 1:2\sqrt{2}$$

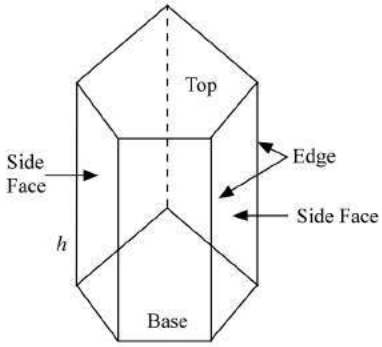
Illustration 38 The radii of two spheres are in the ratio of 2:45. Find the ratio of their volumes

Solution: Ratio of volumes = $(\text{Ratio of radii})^3$

$$= (2:5)^3 = 8:125$$

Prism

A solid having top and bottom faces identical and side faces rectangular is a prism.



In a prism with a base of n sides,

Number of vertices = $2n$

and Number of faces = $n + 2$.

Volume of the prism = area of base × height

Lateral surface area = perimeter of base × height

Total surface area = 2 × Base area + Lateral surface area.

Illustration 39 Find the volume and the total surface area of a triangular prism whose height is 30 m and the sides of whose base are 21 m, 20 m and 13 m, respectively

Solution: Perimeter of base = 21 + 20 + 13 = 54 m, height = 30 m

$$\text{Area of base} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27\,(27-21)\,(27-20)\,(27-13)}$$

$$= \sqrt{27\times 6\times 7\times 14}$$

$$= 126 \text{ sq m}$$

∴ Volume of the prism

= area of base × height

= 126 × 54

= 6804 m^3.

Also, surface area of the prism

= 2 × Base area + lateral surface area

= 2 Base area + perimeter of base × height

= 2 × 126 + 54 × 30

= 1872 sq m

SOLIDS INSCRIBED/CIRCUMSCRIBING OTHER SOLIDS

1. If a largest possible sphere is circumscribed by a cube of edge 'a' cm, then the radius of the sphere $= \frac{a}{2}$.

Illustration 40 Find the volume of largest possible sphere circumscribed by a cube of edge 8 cm

Solution: Radius of the sphere $= \frac{a}{2} = \frac{8}{2} = 4$ cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 4 \times 4 \times 4$

$= 26.81 \text{ cm}^3$

2. If a largest possible cube is inscribed in a sphere of radius 'a' cm, then the edge of the cube $= \frac{2a}{\sqrt{3}}$.

Illustration 41 Find the surface area of largest possible cube inscribed in a sphere of radius 4 cm

Solution: Edge of the cube $\frac{2a}{\sqrt{3}} = \frac{2 \times 4}{\sqrt{3}} = \frac{8}{\sqrt{3}}$

$\therefore$ Surface area of the cube = 6 (edge)^2

$= 6 \times \frac{64}{3}$

$= 128$ sq cm

3. If a largest possible sphere is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then

radius of the sphere $= \begin{cases} a \text{ for } h > a \\ \frac{h}{2} \text{ for } a > h \end{cases}$

Illustration 42 Find the surface area of largest possible sphere inscribed in a cylinder of radius 14 cm and height 17 cm

Solution: Radius of the sphere = 14 cm $(\because h > a)$

$\therefore$ Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 14 \times 14$

$= 2464$ sq cm

4. If a largest possible sphere is inscribed in a cone of radius 'a' cm and slant height equal to the diameter of the base, then radius of the sphere $= \frac{a}{\sqrt{3}}$

Illustration 43 Find the surface area of largest possible sphere inscribed in a cone of radius 21 cm and slant height equal to the diameter of the base

Solution: Radius of the sphere $= \frac{a}{\sqrt{3}} = \frac{21}{\sqrt{3}}$ cm

$\therefore$ Surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times \frac{21}{\sqrt{3}} \times \frac{21}{\sqrt{3}}$

$= 1848$ sq cm

5. If a largest possible cone is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then radius of the cone = a and height = h.

Illustration 44 Find the volume of largest possible cone inscribed in a cylinder of radius 6 cm and height 14 cm

Solution: Radius of the cone (r) = 6 cm

and, height of the cone (h) = 14 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$

$= 528$ sq cm.

6. If a largest possible cube is inscribed in a hemisphere of radius 'a' cm, then the edge of the cube $= a\sqrt{\frac{2}{3}}$.

Illustration 45 Find the length of the diagonal of largest possible cube inscribed in a hemisphere of radius cm

Solution: Edge of the cube $= a\sqrt{\frac{2}{3}} = 4\sqrt{2} \times \sqrt{\frac{2}{3}}$

$= \frac{8}{\sqrt{3}}$ cm

$\therefore$ Diagonal of the cube $= \sqrt{3}$ (edge)

$= \sqrt{3} \times \frac{8}{\sqrt{3}} = 8$ cm

SOME USEFUL SHORT-CUT METHODS

1. If all three measuring dimensions of a sphere, cuboid, cube, cylinder or cone are increased or decreased by $x\%$, $y\%$ and $z\%$, respectively, then the volume of the figure will increase or decrease by

$$\left(x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{100^2}\right)\%$$

For cuboid, the three measuring dimensions are length, breadth and height.

For cube, all three measuring dimensions are equal, i.e., $x = y = z$.

For sphere also (or diameter) all three measuring dimensions are equal and is given by radius, i.e., $x = y = z = r$.

For cylinder or a cone two measuring dimensions are equal to radius and third measuring dimension is height

i.e., $x = y = r$ and $z = h$.

Illustration 46 The length, breadth and height of a cuboid are increased by 5%, 10% and 20%, respectively. Find the percentage increase in its volume

Solution: Here $x = 5$, $y = 10$ and $z = 20$

∴ Percentage increase in volume

$$= \left[x + y + z + \frac{xy + xz + yz}{100} + \frac{xyz}{(100)^2}\right]\%$$

$$= \left[5 + 10 + 20 + \frac{(5\times10) + (5\times20) + (10\times20)}{100} + \frac{5\times10\times20}{(100)^2}\right]\%$$

$$= \left(35 + \frac{350}{100} + \frac{1000}{(100)^2}\right)$$

$$= (35 + 3.5 + 0.1)\%$$

$$= 38.6\%$$

Illustration 47 The sides of a cube are decreased by 10% each. Find the percentage change in its volume.

Solution: Here $x = y = z$

∴ Percentage change in volume

$$= \left[3x + \frac{3x^2}{100} + \frac{x^3}{(100)^2}\right]\%$$

$$= \left[3(-10) + \frac{3(-10)^2}{100} + \frac{(-10)^3}{(100)^2}\right]\%$$

$$= (-30 + 3 - 0.1)\% = -27.1\%$$

–ve sign indicates decrease in volume, that is, there is a decrease in volume by 27.1%

Illustration 48 The diameter of a sphere is increased by 20%. What is the percentage increase in its volume?

Solution: Percentage increase in volume

$$= \left[3x + \frac{3x^2}{100} + \frac{x^3}{(100)^2}\right]\% \quad [\text{Here } x = y = z]$$

$$= \left[3\times20 + \frac{3(20)^2}{100} + \frac{(20)^3}{(100)^2}\right]\%$$

$$= (60 + 12 + 0.8)\% = 72.8\%.$$

Illustration 49 The radius of a right circular cylinder is decreased by 5% but its height is increased by 10%. What is the percentage change in its volume?

Solution: Here $x = y = -15$ and $z = 10$

∴ Percentage change in volume

$$= \left[-5 - 5 + 10 + \frac{(-5)(-5) + (-5)(10) + (-5)(10)}{100} + \frac{(-5)(-5)(10)}{(100)^2}\right]\%$$

$$= (0 - 0.75 + 0.025)\% = -0.725\%$$

Therefore, volume decrease by 0.725%

Illustration 50 Each of the radius and the height of a cone is increased by 25%. Find the percentage increase in volume.

Solution: Here $x = y = 25$ and $z = 25$

∴ Percentage increase in volume

$$= \left[25 + 25 + 25 + \frac{25\times25 + 25\times25\times25\times25}{100} + \frac{25\times25\times25}{(100)^2}\right]\%$$

$$= (75 + 18.75 + 1.56)\% = 95.3\%$$

2. If the two measuring dimensions which are included in the surface area of a sphere, cuboid, cube, cylinder or cone are increased or decreased by $x\%$ and $y\%$, then the surface area of the figure will increase or decrease by $\left(x + y + \frac{xy}{100}\right)\%$

> Note that in case of percentage increase, values of x, y and z are positive and in case of percentage decrease, values of x, y and z are negative.

Illustration 51 Each edge of a cube is increased by 20%. What is the percentage increase in its surface area?

Solution: Here $x = y = 20$

$\therefore$ Percentage increase in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(20 + 20 + \frac{20 \times 20}{100}\right)\%$$

$$= (40 + 4)\%$$

$$= 44\%$$

Illustration 52 The radius of a hemisphere is decreased by 10%. Find the percentage change in its surface area

Solution: Here $x = y = -10$

$\therefore$ Percentage change in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-10 - 10 + \frac{(-10)(-10)}{100}\right)\%$$

$$= (-20 + 1)\%$$

$$= -19\%$$

Therefore, surface area of hemisphere decreases by 19%.

Illustration 53 The radius of a right circular cone is increased by 25% and slant height is decreased by 30%. Find the percentage change in curved surface area of the cone

Solution: Here $x = 25$ and $y = -30$

$\therefore$ Percentage change in curved surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left[25 - 30 + \frac{(25)(-30)}{100}\right]\%$$

$$= (-5 - 7.5)\%$$

$$= -12.5\%$$

Therefore, curved surface area decreases by 12.5%

Illustration 54 The radius and height of a cylinder are increased by 10% and 20%, respectively. Find the percentage increase in its surface area

Solution: Here $x = 10$ and $y = 20$

$\therefore$ Percentage increase in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(10 + 20 + \frac{10 \times 20}{100}\right)\% = 32\%$$

> **3.** If a sphere of radius R is melted to form smaller spheres each of radius r, then
>
> The number of smaller spheres
>
> $$= \frac{\text{Volume of the bigger sphere}}{\text{Volume of the smaller sphere}}$$
>
> $$= \left(\frac{R}{r}\right)^3.$$

Illustration 55 Find the number of lead balls of radius 1 cm each that can be made from a sphere of radius 4 cm

Solution: Number of lead balls $= \left(\frac{R}{r}\right)^3 = \left(\frac{4}{1}\right)^3 = 64$

> **4.** If by melting n spheres, each of radius r, a big sphere is made, then
>
> Radius of the big sphere $= r.\sqrt[3]{n}$.

Illustration 56 If by melting 8 spheres, each of radius 5 cm, a big sphere is made, what will be the radius of the big sphere?

Solution: Radius of the big sphere $= r.\sqrt[3]{n}$

$$= 5.\sqrt[3]{8} = 5.2 = 19 \text{ cm}$$

> **5.** If a cylinder is melted to form smaller spheres each of radius r, then
>
> The number of small spheres $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$.

Illustration 57 How many bullets can be made out of a loaded cylinder 24 cm high and 5 cm diameter, each bullet being 2 cm in diameter?

Solution: Number of bullets $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$

$$= \frac{\pi \times \frac{5}{2} \times \frac{5}{2} \times 24}{\frac{4}{3} \times \pi \times 1 \times 1 \times 1} = 450$$

6. If a sphere of radius r is melted and a cone of height h is made, then

Radius of the cone $= 2 \times \sqrt{\frac{r^3}{h}}$

or,

If a cone of height h is melted and a sphere of radius r is made, then

Radius of the cone $= 2 \times \sqrt{\frac{r^3}{h}}$

Illustration 58 A solid cone of copper of height 3 cm is melted and a solid sphere of radius 3 cm is made. What is the diameter of the base of the cone?

Solution: Radius of the base of the cone

$$= 2 \times \sqrt{\frac{r^3}{h}} = 2 \times \sqrt{\frac{3^3}{3}} = 6.$$

$\therefore$ Diameter of the base of the cone

$$= 2 \times 6 = 12 \text{ cm}$$

Illustration 59 If a solid cone of copper of height 2 cm is melted and a solid sphere of radius 2 cm is made, what is the diameter of the base of the cone?

Solution: Radius of the base of the cone

$$= 2 \times \sqrt{\frac{r^3}{h}} = 2 \times \sqrt{\frac{(2)^3}{2}}$$

$$= 2 \times 2 = 4 \text{ cm}$$

$\therefore$ Diameter of the base of the cone $= 2 \times 4 = 8$ cm

Practice Exercises

Difficulty Level-1
(Based on Memory)

1. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cm side. What is the ratio of the total surface areas of the smaller cubes and the large cube?

(*a*) 2:1 (*b*) 3:2

(*c*) 25:18 (*d*) 27:20

[Based on MAT, 2004]

2. A cylinder 6 cm in diameter is partially filled with water. A sphere 3 cm in diameter is gently dropped into the cylinder. To what further height will the water in the cylinder rise?

(*a*) 6 cm (*b*) 2 cm

(*c*) 1/2 cm (*d*) None of these

[Based on MAT, 2004]

3. A copper sphere is drawn into a cylindrical wire of 4 m length. If the diameter of the sphere is ten times the diameter of the wire, then what is the radius of the sphere?

(*a*) 3 cm (*b*) 3 mm

(*c*) 6 cm (*d*) 6π mm

[Based on MAT, 2004]

4. How many bricks, each measuring 24 cm × 11.5 cm × 8 cm, will be needed to construct a wall 8 m long, 6 m high and 23 cm wide, while 5% of the total wall contains mortar?

(*a*) 5000 (*b*) 5250

(*c*) 4750 (*d*) 4250

[Based on MAT, 2003]

5. The dimensions of an open box are 52 cm × 40 cm × 29 cm. Its thickness is 2 cm. If 1 cu cm of metal used in the box weighs 0.5 g, then the weight of the box is:

(*a*) 6.832 kg (*b*) 7.576 kg

(*c*) 7.76 kg (*d*) 8.56 kg

[Based on MAT, 2003]

6. How many metres of cloth 2.5 m wide will be needed to make a conical tent with base radius 7 m and height 24 m?

(*a*) 120 m (*b*) 180 m

(*c*) 220 m (*d*) 550 m

[Based on MAT, 2003]

7. Sixteen cylindrical cans, each with a radius of 1 unit, are placed inside a cardboard box four in a row. If the cans touch the adjacent cans and or the walls of the box, then which of the following could be the interior area of the bottom of the box in square units?

(*a*) 16 (*b*) 32

(*c*) 64 (*d*) 128

[Based on MAT, 2003]

8. In a right circular cone of vertical angle of 60° and height of 6 cm, a sphere of maximum volume is inserted. If the radius of this sphere is 2.33 cm, find the volume of the cone.

(*a*) 18 π (*b*) 15 π

(*c*) 24 π (*d*) Cannot be determined

9. A cylindrical bucket of height 36 cm and radius 21 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed. The height of the conical heap is 12 cm. The radius of the heap at the base is:

(*a*) 63 cm (*b*) 53 cm

(*c*) 56 cm (*d*) 66 cm

[Based on MAT, 2003]

10. A hemispherical bowl is made of steel 0.5 cm thick. The inside radius of the bowl is 4 cm. The volume of the steel used in making the bowl is:

(*a*) 55.83 cm^2 (*b*) 56.83 cm^2

(*c*) 57.83 cm^3 (*d*) 58.83 cm^3

[Based on MAT, 2003]

11. A metallic sheet is of rectangular shape with dimensions 48 m × 36 m. From each of its corners, a square is cut off so as to make an open box. The volume of the box is $X\,m^3$, when the length of the square is 8 m, the value of X is:

(*a*) 5120 (*b*) 8960

(*c*) 4830 (*d*) 5120

[Based on MAT, 2003]

12. The sum of length, breadth and height of a room is 19m. The length of the diagonal is 11 m. The cost of painting the total surface area of the room at the rate of ₹10 per m^2 is:

(*a*) ₹240 (*b*) ₹2400

(*c*) ₹420 (*d*) ₹4200

[Based on MAT, 2003]

13. The cost of painting the walls of a room at the rate of ₹1.35 per square metre is ₹340.20 and the cost of matting the floor at the rate of Re 0.85 per square metre is ₹91.80. If the length of the room is 12 m, then the height of the room is:

(*a*) 6 m (*b*) 12 m

(*c*) 1.2 m (*d*) 12.6 m

[Based on MAT, 2003]

14. A solid sphere of radius 7 cm is melted to form a number of small cones and cylinders. The requirement is such that the number of cones should be twice the number of cylinders. Also the radius of the cone must be equal to its height, which should be equal to the radius of the cylinder and also be half the height of the cylinder. If the height of one such cylinder is 4 cm, find the maximum number of cones which can be made out of the sphere.

(*a*) 18 (*b*) 21

(*c*) 36 (*d*) 42

15. A cone, a hemisphere and a cylinder have equal bases and same heights. Their volumes will be in the ratio:

(*a*) 1:2:3 (*b*) 3:4:1

(*c*) 3:2:1 (*d*) None of these

16. Find the number of coins, 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

(*a*) 430 (*b*) 440

(*c*) 450 (*d*) 460

17. The volume of a cylindrical tank is 12320 litres. Its radius and height are in the ratio of 7:10, respectively. What is the height of the tank?

(*a*) 1.4 m (*b*) 2.8 m

(*c*) 2 m (*d*) None of these

[Based on IRMA, 2002]

18. A garden is 24 m and 14 m wide. There is a path 1 m wide outside the garden along its sides. If the 20 cm × 20 cm, the number of tiles required to cover then path is:

(*a*) 1800 (*b*) 200

(*c*) 2000 (*d*) 2150

[Based on MAT, 2008]

19. If 600 men dig a 5.5 m wide, 4 m deep and 405 m long canal in half an hour, then how long a canal will 2500 men, working for 6 h, dig if it is 10 m wide and 8 m deep?

(*a*) 6,452 m (*b*) $5{,}568\frac{3}{4}$ m

(*c*) $2{,}694\frac{1}{3}$ m (*d*) 4,082 m

20. A conical vessel of base radius 2 cm and height 3 cm is filled with kerosene. This liquid leaks through a hole in the bottom and collects in a cylindrical jar of radius 2 cm. The kerosene level in the jar is:

(*a*) π cm (*b*) 1.5 cm

(*c*) 1 cm (*d*) 3 cm

[Based on MAT, 2008]

21. The volume of a cuboid is 1120 cm^3 and its height is 5 cm while the length and the breadth of the cuboid are in the ratio 8:7. The length of this cylinder exceeds the breadth by:

(*a*) 2 cm (*b*) 4 cm

(*c*) 7 cm (*d*) 5 cm

[Based on MAT (Feb), 2011]

22. 1 m^3 piece of copper is melted and recast into a square cross section bar 36 m long. An exact cube is cut off from this bar. If 1 m^3 of copper cost ₹108, then the cost of the cube is:

(*a*) 50 paise (*b*) 25 paise

(*c*) 75 paise (*d*) ₹1

[Based on MAT (Feb), 2011]

23. The volume of a rectangular block of stone is 10368 dm^2, its dimensions are in the ratio of 3:2:1. If its entire surface is polished at 2 paise per dm^2, then what is the total cost?

(*a*) ₹31.68 (*b*) ₹31.50

(*c*) ₹63 (*d*) ₹63.36

[Based on MAT (Feb), 2011]

24. A school room is to be built to accommodate 70 children, so as to allow 2.2 m^2 of floor and 11 m^3 of space for each child. If the room be 14 m long, what must be its breadth and height?

(*a*) 12 and 5.5 m (*b*) 13 and 6 m

(*c*) 11 and 5 m (*d*) 11 and 4 m

[Based on MAT (Dec), 2010]

25. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across the bottom. It is 6 m deep. Find the area of its curved surface.

(*a*) 118.4 m^2 (*b*) 162.3 m^2
(*c*) 452 m^2 (*d*) 181.6 m^2

[Based on MAT (Dec), 2010]

26. Two cans have the same height equal to 21 cm. One can is cylindrical, the diameter of whose base is 10 cm. The other can has square base of side 10 cm. What is the difference in their capacities?

(*a*) 350 cm^3 (*b*) 250 cm^3
(*c*) 450 cm^3 (*d*) 300 cm^3

[Based on MAT (Dec), 2010]

27. What is the total surface area of a triangular prism whose height is 30 m and the sides of whose base are 21 m, 20 m and 13 m, respectively?

(*a*) 1872 sq m (*b*) 1725 sq m
(*c*) 1652 sq m (*d*) 1542 sq m

[Based on MAT (Sept), 2010]

28. It is required to design a circular pipe such that water flowing through it at a speed of 7 m/min fills a tank of capacity 440 cu m in 10 minutes. The inner radius of the pipe should be:

(*a*) 2 m (*b*) $\sqrt{2}$ m
(*c*) 1/2 m (*d*) $1/\sqrt{2}$ m

[Based on MAT (Sept), 2010]

29. A cube of white chalk is painted red, and then cut parallel to the sides to form two rectangular solids of equal volume. What per cent of the surface area of each of the new solids is not painted red?

(*a*) 20% (*b*) $16\frac{2}{3}\%$
(*c*) 15% (*d*) 25%

30. A toy is in the shape of a hemisphere surmounted by a cone. If radius of base of the cone is 3 cm and its height is 4 cm, the total surface area of the toy is:

(*a*) 33 π cm^2 (*b*) 42 π cm^2
(*c*) 66 π cm^2 (*d*) 56 π cm^2

[Based on MAT (Feb), 2010]

31. A cylindrical container of height 14 m and base diameter 12 m contains oil. This oil is to be transferred to one cylindrical can, one conical can and a spherical can. The base radius of all the containers is same. The height of the conical can is 6 m. While pouring some oil is dropped and hence only three-fourths of cylindrical can could be filled. How much oil is dropped?

(*a*) 54 π m^3 (*b*) 36 π m^3
(*c*) 46 π m^3 (*d*) 50 π m^3

[Based on MAT (Feb), 2010]

32. The ratio of the volume of a cube to that of the sphere which can fit inside the cube is:

(*a*) 3:4 (*b*) 21:11
(*c*) 11:22 (*d*) 4:3

[Based on MAT (Feb), 2010]

33. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface area of the remainder is eight-ninths of the curved surface of the whole cone, the ratio of the line segments into which the cone's altitude is divided by the plane is given by:

(*a*) 2:3 (*b*) 1:3
(*c*) 1:2 (*d*) 1:4

[Based on MAT (Feb), 2010]

34. A cylindrical tub of radius 12 cm contains water up to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. The radius of the ball is:

(*a*) 7.25 cm (*b*) 6 cm
(*c*) 4.5 cm (*d*) 9 cm

[Based on MAT (May), 2009]

35. A spherical ball of lead, 3 cm in diameter is melted and recast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm, respectively. The diameter of the third ball is:

(*a*) 3 cm (*b*) 2.66 cm
(*c*) 2.5 cm (*d*) 3.5 cm

[Based on MAT (May), 2009]

36. A hemispherical bowl is 176 cm round the brim. Supposing it to be half full, how many persons may served from it in hemispherical glasses 4 cm in diameter at the top?

(*a*) 1372 (*b*) 1272
(*c*) 1172 (*d*) 1472

[Based on MAT (May), 2009]

37. A 4 cm cube is cut into 1 cm cubes. Find the percentage increase in surface area.

(*a*) 200% (*b*) 100%
(*c*) 400% (*d*) 300%

[Based on MAT (Feb), 2009]

38. A well has to be dug out that is to be 22.5 m deep and of diameter 7 m. Find the cost of plastering the inner curved surface at ₹3 per square metre.

(*a*) ₹1,465 (*b*) ₹1,485
(*c*) ₹1,475 (*d*) ₹1,495

39. A wooden box of dimensions 8 m × 7 m × 6 m is to carry rectangular boxes of dimensions 8 cm × 7 cm × 6 cm. The maximum number of boxes that can be carried in the wooden box is:

(*a*) 9800000 (*b*) 1000000
(*c*) 7500000 (*d*) 1200000

[Based on MAT (Dec), 2008]

40. A cylinder is filled to four-fifths of volume. It is, then tilted so that the level of water coincides with one edge of its bottom and top edge of the opposite side. In the process, 30 cc of the water is spilled. What is the volume of the cylinder?

(*a*) 75 cc (*b*) 96 cc
(*c*) Data insufficient (*d*) 100 cc

[Based on MAT (Sept), 2008]

41. The number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the mortar occupies 5% of the volume of the wall, is:

(*a*) 5740 (*b*) 6080
(*c*) 3040 (*d*) 8120

[Based on MAT (May), 2008]

42. 1496 cm^3 of metal is used to cast a pipe of length 28 cm. If the internal radius of the pipe is 8 cm, the outer radius of the pipe is:

(*a*) 7 cm (*b*) 10 cm
(*c*) 9 cm (*d*) 12 cm

[Based on MAT (Dec), 2007]

43. A monument has 50 cylindrical pillars each of diameter 50 cm and height 4 m. What will be the labour charges for getting these pillars cleared at the rate of 50 paise per sq m? (Use $\pi = 3.14$):

(*a*) ₹237 (*b*) ₹157
(*c*) ₹257 (*d*) ₹353

[Based on MAT (Dec), 2007]

44. A solid cube with an edge of 10 cm is melted to form two equal cubes. The ratio of the edge of the smaller cube to the bigger cube is:

(*a*) $\left(\frac{1}{3}\right)^{1/3}$ (*b*) $\frac{1}{2}$

(*c*) $\left(\frac{1}{2}\right)^{1/3}$ (*d*) $\left(\frac{1}{4}\right)^{1/3}$

[Based on MAT (Sept), 2007]

45. How many small cubes, each of 96 cm^2 surface area, can be formed from the material obtained by melting a larger cube if 384 cm^2 surface area?

(*a*) 8 (*b*) 5
(*c*) 800 (*d*) 8000

[Based on MAT (Sept), 2007]

46. Consider the volumes of the following:

A. A parallelepiped of length 5 cm, breadth 3 cm and height 4 cm.
B. A cube having each side 4 cm.
C. A cylinder of radius 3 cm and length 3 cm.
D. A sphere of radius 3 cm.

The volumes of these in the decreasing order is

(*a*) A, B, C and D (*b*) A, C, B and D
(*c*) D, B, C and A (*d*) D, C, B and A

[Based on MAT (Dec), 2006]

47. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of the cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. The area of the canvas required for the tent is:

(*a*) 1300 m^2 (*b*) 1320 m^2
(*c*) 1310 m^2 (*d*) 1330 m^2

[Based on MAT (May), 2006]

48. An iron pipe 20 cm long has exterior diameter equal to 25 cm. If the thickness of the pipe is 1 cm, the whole surface of the pipe is:

(*a*) 3068 cm^2 (*b*) 3268 cm^2
(*c*) 3168 cm^2 (*d*) 3368 cm^2

[Based on MAT (Feb), 2006]

49. A tank 30 m long, 20 m wide and 12 m deep is dug in a field 500 m long and 30 m wide. By how much will the level of the field rise if the earth dug out of the tank is evenly spread over the field?

(*a*) 0.33 m (*b*) 0.5 m
(*c*) 0.25 m (*d*) 0.4 m

[Based on MAT, 1997]

50. The length of an edge of a hollow cube open at one face is $\sqrt{3}$ m. What is the length of the largest pole that it can accommodate?

(*a*) $\sqrt{3}$ m (*b*) $3\sqrt{3}$ m
(*c*) 3 m (*d*) $3/\sqrt{3}$ m

[Based on MAT, 1997]

51. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end, and the other end as its base. The volume of the cylinder, hemisphere and the cone are respectively in the ratio:

(*a*) 2:3:2 (*b*) 3:2:1
(*c*) 3:1:2 (*d*) 1:2:3

[Based on MAT, 1998]

52. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and resolidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to:

(*a*) 3 (*b*) 3.5
(*c*) 4 (*d*) 4.5

53. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by:

(*a*) $\frac{9}{2}$ cm (*b*) $\frac{9}{4}$ cm

(*c*) $\frac{4}{9}$ cm (*d*) $\frac{2}{9}$ cm

[Based on MAT, 1998]

54. A conical cavity is drilled in a circular cylinder of height 15 cm and base radius 8 cm. The height and the base radius of the cone are also same. Then the whole surface of the remaining solid is:

(*a*) 440 π sq cm (*b*) 240 π sq cm

(*c*) 640 π sq cm (*d*) 960 π sq cm

[Based on MAT, 1998]

55. The height of a room is 40% of its semi-perimeter. It costs ₹260 to paper the walls of the room with paper 50 cm wide @ ₹2 per metre allowing an area of 15 sq m for doors and windows. The height of the room is:

(*a*) 2.6 m (*b*) 3.9 m

(*c*) 4.0 m (*d*) 4.2 m

[Based on MAT, 1998]

56. A conical flask has base radius '*a*' cm and height '*h*' cm. It is completely filled with milk. The milk is poured into a cylindrical thermos flask whose base radius is '*p*' cm. What will be the height of the solution level in the flask?

(*a*) $\frac{a^2h}{3p^2}$ cm (*b*) $\frac{3hp^2}{a^2}$ cm

(*c*) $\frac{p^2}{3h^2}$ cm (*d*) $\frac{3a^2}{hp^2}$ cm

[Based on MAT, 1998]

57. A swimming pool 9 m wide and 12 m long is 1 m deep on the shallow side and 4 m deep on the deeper side. Its volume is:

(*a*) 408 m^3 (*b*) 360 m^3

(*c*) 270 m^3 (*d*) 208 m^3

[Based on MAT, 1998]

58. Three cubes of a metal are of edges 3 cm, 4 cm and 5 cm. These are melted together and from the melted material another cube is formed. The edge of this cube is:

(*a*) 8 cm (*b*) 10 cm

(*c*) 9 cm (*d*) 6 cm

[Based on MAT, 1998]

59. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

(*a*) 6 cm (*b*) 9 cm

(*c*) 8 cm (*d*) None of these

[Based on MAT, 1999]

60. A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm. Find the total surface area (use $\pi = 22/7$).

(*a*) 137.5 cm^2 (*b*) 214.5 cm^2

(*c*) 154 cm^2 (*d*) 291.5 cm^2

[Based on MAT, 1999]

61. A right circular cone of height *h* is cut by a plane parallel to the base at a distance $\frac{h}{3}$ from the base, then the volumes of the resulting cone and the frustum are in the ratio:

(*a*) 1:3 (*b*) 8:19

(*c*) 1:4 (*d*) 1:7

62. The length, breadth and height of a cuboid are in the ratio 1:2:3. The length, breadth and height of the cuboid are increased by 100%, 200% and 200%, respectively. Then the increase in the volume of the cuboid is:

(*a*) 5 times (*b*) 6 times

(*c*) 12 times (*d*) 17 times

[Based on MAT, 1999]

63. The volume of a cube is numerically equal to the sum of its edges. What is its total surface area in square units?

(*a*) 66 (*b*) 183

(*c*) 36 (*d*) 72

[Based on MAT, 1999]

64. A solid cylinder and a solid cone have equal base and equal height. If the radius and the height be in the ratio of 4:3, the ratio of the total surface a area of the cylinder to that of the cone is in the ratio of:

(*a*) 10:9 (*b*) 11:9

(*c*) 12:9 (*d*) 14:9

[Based on MAT, 1999]

65. A sphere of radius 3 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 6 cm. If the sphere is submerged completely, then the surface of the water is raised by:

(*a*) 1/4 cm (*b*) 1/2 cm

(*c*) 1 cm (*d*) 2 cm

[Based on MAT, 1999]

66. A colourless cube is painted blue and then cut parallel to sides to form two rectangles solids of equal volume.

What percentage of surface area of each of new solids is not painted blue?

(*a*) 25 (*b*) 16

(*c*) 20 (*d*) 18

[Based on SNAP, 2008]

67. A cube of side 6 cm is cut to a number of cubes each of side 2 cm. The number of cubes will be:

(*a*) 6 (*b*) 9

(*c*) 12 (*d*) 27

[Based on FMS, 2006]

68. The sum of the length, breadth and depth of a cuboid is 19 cm and its diagonal is $5(\sqrt{3})$ cm. Its surface area is:

(*a*) 361 cm^2 (*b*) 125 cm^2

(*c*) 236 cm^2 (*d*) 486 cm^2

[Based on GBO Delhi University, 2011]

69. The trunk of a tree is a right cylinder 1.5 m in radius and 10 m high. What is the volume of the timber which remains when the trunk is trimmed just enough to reduce it to a rectangular parallelepiped in a square base?

(*a*) 25 m^3 (*b*) 12 m^3
(*c*) 45 m^3 (*d*) 14 m^3

[Based on MAT, 2013]

70. A wire of length 22 cm and 0.2 cm in diameter is melted and recast into small balls of diameter 0.1 cm. The number of balls made is:

(*a*) 1225 (*b*) 1350
(*c*) 1320 (*d*) 1280

[Based on MAT, 2011]

71. A hemispherical bowl of thickness 1cm and external diameter 10 cm is to be painted all over. What is the cost of painting at the rate of ₹0.70 per cm^2?

(*a*) ₹200 (*b*) ₹400
(*c*) ₹800 (*d*) ₹100

[Based on MAT, 2011]

72. A cylindrical container is filled with ice cream. Its diameter is 12cm and height is 15cm. The whole ice cream is distributed among 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of its base, the diameter of the ice cream cone is:

(*a*) 8 cm (*b*) 5 cm
(*c*) 7 cm (*d*) 6 cm

[Based on MAT, 2011]

73. A circus tent is cylindrical to a height of 3 m and conical above it. If the diameter of the base is 140 m and the slant height of the conical portion is 80 m, the length of canvas 2 m wide required to make the tent is:

(*a*) 8960 m (*b*) 9660 m
(*c*) 9460 m (*d*) 9860 m

[Based on MAT, 2012]

74. The ratio between the length and breadh of a rectangular park is 3:2. If a man cycling along the boundary of the park at the speed of 12 Km/h completes one round in 8 minutes, then the area of the park in sq m is:

(*a*) 15360 (*b*) 153600
(*c*) 30720 (*d*) 307200

[Based on MAT, 2012]

75. If the numbers representing volume and surface area of a cube are equal , then the length of the edge of the cube in terms of the unit of measurement will be:

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

[Based on MAT, 2012]

76. A metal sheet 27 cm long, 8 cm broad and 1 cm thick is meited into a cube. The difference between the surface area of the two solids, is:

(*a*) 284 cm^2 (*b*) 296 cm^2
(*c*) 286 cm^2 (*d*) 300 cm^2

[Based on MAT, 2012]

77. A shuttlecock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, the height of the entire shuttlecock is 7 cm. The external surface area of the shuttlecock is:

(*a*) 67.98 cm^2 (*b*) 74.26 cm^2
(*c*) 70 cm^2 (*d*) 72 cm^2

[Based on MAT (feb), 2012]

78. A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. The fraction of water that overflows is:

(*a*) 1:4 (*b*) 1:2
(*c*) 3:8 (*d*) 5:8

[Based on MAT (feb), 2012]

79. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. The surface area of the capsule is:

(*a*) 420 mm^2 (*b*) 222 mm^2
(*c*) 220 mm^2 (*d*) 440 mm^2

[Based on MAT (feb), 2012]

80. Two cubes have their faces painted either red or blue. The first cube has 5 red face and 1 blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. How many red faces are there on the second cube?

(*a*) 3 (*b*) 2
(*c*) 4 (*d*) 5

[Based on MAT, 2013]

81. The dimensions of a room are 10 m × 7 m × 5 m. There are 2 doors and 3 windows in the room. The dimensions of the doors are 1 m × 3 m. One window is of size 2 m × 1.5 m and the other two windows are of size 1 m × 1.5 m. The cost of painting the walls at ₹3 per m^2 is:

(*a*) ₹578.50 (*b*) ₹474
(*c*) ₹684 (*d*) ₹894

[Based on MAT, 2013]

82. A cylindrical tub of radius 12 cm contains water up to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. The radius of the ball is:

(*a*) 6 cm (*b*) 4.5cm
(*c*) 7.25 cm (*d*) 9 cm

[Based on MAT, 2013]

83. The length, breadth and height of a room are, in the ratio of 3:2:1. If its volume be 1296 m^3, find its breadth.

(*a*) 24 m (*b*) 15 m
(*c*) 16 m (*d*) 12 m

[Based on MAT, 2014]

84. Seven equal cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

(*a*) 750 cm^3 (*b*) 1500 cm^3

(*c*) 2250 cm^3 (*d*) 700 cm^3

[Based on MAT, 2014]

85. If the curved surface area of a cone is thrice that of another cone and slant height of the second cone is thrice that of the first, find the ratio of the area of their base.

(*a*) 9:1 (*b*) 81:1

(*c*) 3:1 (*d*) 27:1

[Based on MAT, 2014]

Difficulty Level-2 (Based on Memory)

1. Let *A* and *B* be two solid spheres such that the surface area of *B* is 300% higher than the surface area of *A*. The volume of *A* is found to be *K*% lower than the volume of *B*. The value of *K* must be:

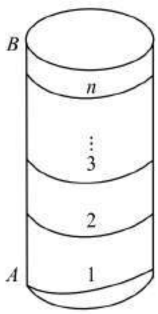
(*a*) 85.5% (*b*) 92.5%

(*c*) 90.5% (*d*) 87.5%

[Based on CAT, 2003]

Directions (Q. 2 to 4): *Answer the questions on the basis of the information given below. Consider a cylinder of height h cm and radius* $r = \frac{2}{\pi}$ *cm as shown in the figure (not drawn to scale). A string of certain length, when wound on its cylindrical surface, starting at a point A and ending at point B, gives a maximum of n turns (in other words, the string length is the minimum length of wind n turns).*

2. What is the vertical spacing in cm between consecutive turns?



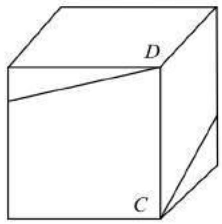
(*a*) $\frac{h}{n}$ (*b*) $\frac{h}{\sqrt{n}}$

(*c*) $\frac{h}{n^2}$

(*d*) Cannot be determined with the given information

[Based on CAT, 2004]

3. The same string, when wound on the exterior four walls of a cube of side *n* cm, starting at point *C* and ending at point *D*, can give exactly one turn (see figure, not drawn to scale). The length of the string, in cm, is:



(*a*) $\sqrt{2}n$ (*b*) $\sqrt{17}n$

(*c*) n (*d*) $\sqrt{13}n$

[Based on CAT, 2004]

4. In the setup of the previous two questions, how is *h* related to *n*?

(*a*) $h = \sqrt{2}n$ (*b*) $h = \sqrt{17}n$

(*c*) $h = n$ (*d*) $h = \sqrt{13}n$

[Based on CAT, 2004]

5. A square tin sheet of side 12 inches is converted into a box with open top in the following steps: The sheet is placed horizontally. Then, equal-sized squares, each of side *x* inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If *x* is an integer, then what value of *x* maximizes the volume of the box?

(*a*) 3 (*b*) 4

(*c*) 1 (*d*) 2

[Based on CAT, 2004]

6. The number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the mortar occupies 5% of the volume of the wall, is:

(*a*) 6080 (*b*) 5740

(*c*) 3040 (*d*) 8120

[Based on FMS (Delhi), 2003]

7. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume is 1/27 of the given cone, then the height of the smaller cone is:

(*a*) 13.5 cm (*b*) 11 cm

(*c*) 10 cm (*d*) 12 cm

[Based on IIFT, 2003]

8. A milkman has 3 jars containing 57 litres, 129 litres and 177 litres of pure milk respectively. A measuring can, after a different number of exact measurements of milk in each jar, leaves the same amount of milk unmeasured in each jar. What is the volume of the largest such can ?

(*a*) 12 litres (*b*) 16 litres
(*c*) 24 litres (*d*) None of these

9. The formula E = *sqrt* of (A/7) describes the relationship between the length of the edge E of a pyramid and the surface area A of the pyramid. How much longer is the edge of the pyramid with a surface area 3,087 than the edge of one with a surface area 2,023?

(*a*) 1,064 (*b*) 152
(*c*) 250 (*d*) 17

10. An ice-cream company makes a popular brand of ice-cream in rectangular shaped bar 6 cm long, 5 cm wide and 2 cm thick. To cut costs, the company had decided to reduce the volume of the bar by 20%, the thickness will remain the same, but the length and wide will be decreased by the same percentage amount. The new length l will satisfy:

(*a*) $5.5 < l < 6$ (*b*) $5 < l < 5.5$
(*c*) $4.5 < l < 5$ (*d*) $4 < l < 4.5$

11. A water tank in the form of a cuboid has its base 20 m long, 7 m wide and 10 m deep. Initially, the tank is full but later when water is taken out of it, the level of water in the tank reduces by 2 m. The volume of the water left in the tank is:

(*a*) 1,120 m^3 (*b*) 400 m^3
(*c*) 280 m^3 (*d*) 140 m^3

12. The length of a room is double the breadth. The cost of colouring the selling at ₹25 per square metre is ₹5,000 and the cost of painting the four walls at ₹240 per square metre is ₹64,800. Find the height of the room.

(*a*) 4.5 m (*b*) 4 m
(*c*) 3.5 m (*d*) 5 m

13. A conical cavity is drilled in a circular cylinder of 15 cm height and 16 cm base diameter. The height and the base diameter of the cone are same as those of the cylinder. Determine the total surface area of the remaining solid.

(*a*) 440 π cm^2 (*b*) 215 π cm^2
(*c*) 542 π cm^2 (*d*) 376 π cm^2

14. It is required to fix a pipe such that water flowing through it at a speed of 7 m per minute fills a tank of capacity 440 cubic metres in 10 minutes. The inner radius of the pipe should be:

(*a*) $\sqrt{2}$ m (*b*) 2 m
(*c*) $\frac{1}{2}$ m (*d*) $\frac{1}{\sqrt{2}}$ m

15. For a sphere of radius 10 cm, what per cent of the numerical value of its volume would be the numerical value of the surface area?

(*a*) 26.5% (*b*) 24%
(*c*) 30% (*d*) 45%

[Based on FMS, 2005]

16. Two rectangular sheets of paper, each 30 cm *x* 18 cm are made into two right circular cylinders, one by rolling the paper along its length and the other along the breadth. The ratio of the volumes of the two cylinders, thus formed, is:

(*a*) 2:1 (*b*) 3:2
(*c*) 4:3 (*d*) 5:3

[Based on FMS, 2006]

17. A rectangular water tank is open at the top. Its capacity is 24 m^3. Its length and breadth are 4 m and 3 m respectively. Ignoring the thickness of the material used for building the tank, the total cost of painting the inner and outer surfaces of the tank at the rate of ₹10 per m^2 is:

(*a*) ₹400 (*b*) ₹500
(*c*) ₹600 (*d*) ₹800

[Based on FMS, 2006]

18. Suresh, who runs a bakery, uses a conical shaped equipment to write decorative labels (e.g., Happy Birthday etc.) using cream. The height of this equipment is 7 cm and the diameter of the base is 5 mm. A full charge of the equipment will write 330 words on an average. How many words can be written using three-fifths of a litre of cream?

(*a*) 45090 (*b*) 45100
(*c*) 46000 (*d*) None of the above

[Based on XAT, 2012]

19. A right circular hollow cylinder, kept vertically on its circular base has a height of 20 cm and radius of 10 cm. A sugar grain is kept inside this cylinder on its circular base at the periphery. If an ant is at the top rim of the same cylinder and diagonally opposite the sugar grain, the minimum distance the ant should travel to reach the sugar grain is approximately:

(*a*) 82.86 cm (*b*) 51.43 cm
(*c*) 37.25 cm (*d*) 65.96 cm

[Based on JMET, 2009]

20. Madan is going from Mumbai to Delhi in order to join a new job there. He has a glass memento of right circular conic shape under his possession and he does not want it to break during transportation. So, he purchases a cubic metal box from the market spending ₹500. The cone is exactly fitted in the metal cube in such a way that while the edges of the base of the cone are touching the edges of all the sides of the cube, the vertex of it touches the opposite face of the cube. After inserting the memento in the box, he packed the metal box from outside with wallpaper costing ₹1.5 per sq cm. Given that the volume of the glass memento is $718^{2/3}$cc, mark all the correct statements.

(*a*) Madan had incurred total expenditure of ₹2264 on the metal box.

(*b*) Madan had incurred an expenditure of ₹1754 on packing the metal box.

(*c*) The area of any one side of the metal box is 196 sq cm.

(*d*) The volume of the metal box is 2644 cc.

[Based on IIFT, 2006]

21. A cylinder, a hemisphere and a cone stand on the same base and have the same heights. The ratio of the areas of their curved surface is:

(*a*) 2:2:1 (*b*) 2: $\sqrt{2}$:1

(*c*) $\sqrt{2}$:3:1 (*d*) None of these

[Based on IIFT, 2008]

22. A right circular cone is enveloping a right circular cylinder such that the base of the cylinder rests on the base of the cone. If the radius and the height of the cone is 4 cm and 10 cm respectively, then the largest possible curved surface area of the cylinder of radius *r* is:

(*a*) $20\pi r^2$ (*b*) $5\pi r(4 - r)$

(*c*) $5\pi r(r - 4)$ (*d*) $5\pi r(2 - r)$

[Based on IIFT, 2009]

23. A regular pyramid has a square base with side 10 cm and a vertical height of 20 cm. If the height increases by 10% of its original value and the volume is constant, the percentage change in the side of the square base with respect to its original value is approximately:

(*a*) +5% (*b*) +10%

(*c*) –5% (*d*) –10%

[Based on JMET, 2009]

24. In a rocket shape firecracker, explosive powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with the base of radius 8 cm. The ratio of heigth of cylinder and cone is 5:3. A cylindrical hole is drilled through the metal solid with height one-third the height of metal solid. What should be the radius of the hole, so that volume of the hole (in which gun powder is to be filled up) is half of the volume of metal solid after drilling?

(*a*) $4\sqrt{3}$ cm (*b*) 4 cm

(*c*) 3 cm (*d*) None of these

[Based on IIFT, 2010]

25. A tank internally measuring 150 cm × 120 cm × 100 cm has 1281600 cm^3 water in it. Porous bricks are placed in the water until the tank is full up to its brim. Each brick absorbs one tenth of its volume of water. How many bricks of 20 cm × 6 cm × 4 cm, can be put in the tank without spilling over the water?

(*a*) 1100 (*b*) 1200

(*c*) 1150 (*d*) 1250

[Based on XAT, 2010]

26. A child consumed an ice cream of inverted right-circular conical shape from the top and left only 12.5% of the cone for her mother. If the height of the ice cream-cone was 8 cm, what was the height of the remaining ice-cream cone?

(*a*) 2.5 cm (*b*) 3.0 cm

(*c*) 3.5 cm (*d*) 4.0 cm

[Based on JMET, 2009]

27. A cube of white chalk is painted red, and then cut parallel to the sides to form two rectangular solids of equal volume. What per cent of the surface area of each of the new solids is not painted red?

(*a*) 20% (*b*) $16\frac{2}{3}$%

(*c*) 15% (*d*) 25%

[Based on NMAT, 2005]

28. The dimensions of a room are 12.5 m by 9 m by 7 m. There are 2 doors and 4 windows in the room; each door measure 2.5 m by 1.2 m and each window 1.5 m by 1 m. Find the cost of paining the walls at ₹3.50 per square meter.

(*a*) ₹1101.50 (*b*) ₹1050.20

(*c*) ₹1011.50 (*d*) Cannot be determined

[Based on NMAT, 2005]

29. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 Km/h?

(*a*) 200 (*b*) 300

(*c*) 250 (*d*) 350

[Based on NMAT, 2005]

30. A swimming bath is 24 m long and 15 m broad. When a number of men dive into the bath, the height of the water rises by 1 cm. If the average amount of water displaced by one of the men be 0.1 cu. m, how many men are there in the bath?

(*a*) 42 (*b*) 46

(*c*) 32 (*d*) 36

[Based on NMAT, 2005]

31. A tank is 7 m wide and 4 m of length water run through a pipe 5 cm broad and 4 cm deep so that in 5 hrs and 18 minutes water level in the tank rise by 4.5. The flow rate of water is:

(*a*) 12 Km/h (*b*) 10 Km/h

(*c*) 14 Km/h (*d*) None of these

[Based on NMAT, 2006]

32. It took 15 hrs and 40 minutes for Rakesh to paint four walls and the ceiling of a room of size 900 cu ft. The ceiling height of the room is 10 ft. If Rakesh painted at a constant rate of 0.5 sq ft/min, how long will it take for him to paint the walls?

(*a*) 12 hrs 40 minutes (*b*) 11 hrs

(*c*) 13 hrs (*d*) 11 hrs 10 minutes

[Based on JMET, 2009]

33. The carpeting of a room twice as long as it is broad at the rate of 50 p per square meter cost ₹12.25 and cost of painting its walls at the rate of 9 p per square metre is ₹6.30. Find the height of the room.

(*a*) 7 m (*b*) $3\frac{1}{8}$ m

(*c*) $4\frac{1}{3}$ m (*d*) None of these

[Based on NMAT, 2006]

34. The circumference of a cylinder is 3 ft and its height is 16 ft. An insect climbs the pole such that its motion is spiral, and one complete spiral helps it to cover 4 ft in height. Thus, when the insect reaches the top, what is the total distance covered by it?

(*a*) 16 ft (*b*) 18 ft

(*c*) 20 ft (*d*) 25 ft

[Based on CAT, 2009]

35. A solid sphere of radius 12 inches is melted and cast into a right circular cone whose base diameter is $\sqrt{2}$ times its slant height. If the radius of the sphere and the cone are the same, how many such cones can be made and how much material is left out?

(*a*) 4 and 1 cubic inch (*b*) 3 and 12 cubic inches

(*c*) 4 and 0 cubic inch (*d*) 3 and 6 cubic inches

[Based on CAT, 2012]

36. Diameter of the base of a water-filled inverted right circular cone is 26 cm. A cylindrical pipe, 5 mm in radius, is attached to the surface of the cone at a point. The perpendicular distance between the point and the base (the top) is 15 cm. The distance from the edge of the base to the point is 17 cm. along the surface. If water flows at the rate of 10 meters per minute through the pipe, how much time would elapse before water stops coming out of the pipe?

(a) <4.5 minutes

(*b*) ≥4.5 minutes but <4.8 minutes

(*c*) ≥4.8 minutes but <5 minutes

(*d*) ≥5 minutes but <5.2 minutes

[Based on XAT, 2014]

37. A rectangular swimming pool is 48 m long and 20 m wide. The shallow edge of the pool is 1 m deep. For every 2.6 m that one walks up the inclined base of the swimming pool, one gains an elevation of 1 m. What is the volume of water (in cubic meters), in the swimming pool? Assume that the pool is filled up to the brim.

(*a*) 528 (*b*) 960

(*c*) 6790 (*d*) 10560

[Based on XAT, 2014]

38. The radius of the base of a conical tent is 5 cm. If the tent is 12 m high then the area of the canvas required in making the tent is:

(*a*) 300π m^3 (*b*) 60π m^2

(*c*) 90π m^2 (*d*) None of these

[Based on MAT, 2013]

Answer Keys

Difficulty Level-1

1. (*c*)	**2.** (*c*)	**3.** (*b*)	**4.** (*c*)	**5.** (*a*)	**6.** (*d*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*b*)	**11.** (*a*)	**12.** (*b*)	**13.** (*a*)
14. (*d*)	**15.** (*a*)	**16.** (*c*)	**17.** (*d*)	**18.** (*c*)	**19.** (*b*)	**20.** (*c*)	**21.** (*a*)	**22.** (*a*)	**23.** (*d*)	**24.** (*c*)	**25.** (*a*)	**26.** (*c*)
27. (*a*)	**28.** (*b*)	**29.** (*d*)	**30.** (*a*)	**31.** (*b*)	**32.** (*b*)	**33.** (*c*)	**34.** (*d*)	**35.** (*c*)	**36.** (*a*)	**37.** (*d*)	**38.** (*b*)	**39.** (*b*)
40. (*d*)	**41.** (*b*)	**42.** (*c*)	**43.** (*b*)	**44.** (*c*)	**45.** (*a*)	**46.** (*d*)	**47.** (*b*)	**48.** (*a*)	**49.** (*b*)	**50.** (*c*)	**51.** (*b*)	**52.** (*b*)
53. (*b*)	**54.** (*a*)	**55.** (*c*)	**56.** (*a*)	**57.** (*c*)	**58.** (*d*)	**59.** (*d*)	**60.** (*b*)	**61.** (*b*)	**62.** (*d*)	**63.** (*d*)	**64.** (*d*)	**65.** (*c*)
66. (*a*)	**67.** (*d*)	**68.** (*c*)	**69.** (*c*)	**70.** (*c*)	**71.** (*a*)	**72.** (*d*)	**73.** (*c*)	**74.** (*b*)	**75.** (*d*)	**76.** (*c*)	**77.** (*b*)	**78.** (*c*)
79. (*c*)	**80.** (*a*)	**81.** (*b*)	**82.** (*d*)	**83.** (*d*)	**84.** (*a*)	**85.** (*b*)						

Difficulty Level-2

1. (*d*)	**2.** (*a*)	**3.** (*b*)	**4.** (*c*)	**5.** (*d*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*b*)	**11.** (*a*)	**12.** (*a*)	**13.** (*a*)
14. (*a*)	**15.** (*c*)	**16.** (*d*)	**17.** (*d*)	**18.** (*d*)	**19.** (*c*)	**20.** (a, *c*)	**21.** (*d*)	**22.** (*b*)	**23.** (*c*)	**24.** (*a*)	**25.** (*b*)	**26.** (*d*)
27. (*d*)	**28.** (*c*)	**29.** (*c*)	**30.** (*d*)	**31.** (*c*)	**32.** (*a*)	**33.** (*d*)	**34.** (*c*)	**35.** (*c*)	**36.** (*d*)	**37.** (*d*)	**38.** (*d*)	

Explanatory Answers

Difficulty Level-1

1. (*c*) Let r be the side of the larger cube.

$\therefore$ Volume of the larger cube

= Sum of the volumes of the smaller cubes

i.e., $r^3 = 3^3 + 4^3 + 5^3$

$= 27 + 64 + 125 = 216$

$\Rightarrow r = 6$

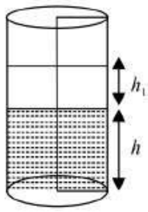
$\therefore$ Total surface area of the larger cube = 216 sq.cm

Total surface area of the three smaller cubes

$= 6\,[3^2 + 4^2 + 5^2] = 300$ sq.cm.

$\therefore$ Required ratio = 300:216 = 25:18.

2. (*c*) Let h_1 be the height to which the water in the cylinder will rise after gently dropping the sphere of radius $\frac{3}{2}$ cm in the cylinder of radius 3cm, which is partially filled with water. Let h be the height to which the water was filled in the cylinder initially.



$$\therefore \pi \times 3^2 \times h_1 = \frac{4}{3} \times \pi \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$$

$$\Rightarrow h_1 = \frac{1}{2}.$$

3. (*b*) Let the radius of the cylindrical wire be r m.

$\therefore$ Radius of the sphere = $10r$ m

$$\therefore \frac{4}{3}\pi \times (10r)^3 = \pi \times r2 \times 4$$

$$\Rightarrow \frac{4}{3} \times 1000r^3 = 4r^2$$

$$\Rightarrow r = \frac{3}{1000} \text{ m} = 3 \text{ mm}.$$

4. (*c*) No. of bricks

$$= \frac{800 \times 600 \times 23 - 5\% \text{ of } (800 \times 600.23)}{24 \times 11.5 \times 8}$$

$$= \frac{11040000 - 552000}{2208} = 4750.$$

5. (*a*) Volume of the metal used in the box

$= 52 \times 40 \times 29 - 48 \times 36 \times 27$

$= 60320 - 46656$

$= 13664$ cu. cm.

$\therefore$ Weight of the box $= 13664 \times 0.5$

$= 6832$ gm $= 6.832$ kg.

6. (*d*) Area of the cloth required to make a conical tent

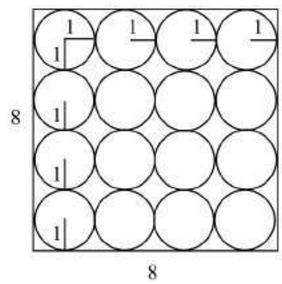
= Surface area of the cone with height 24 m and base radius 7 m

$= \pi r\sqrt{r^2 + h^2}$ $[r = 7, h = 24]$

$= \frac{22}{7} \times 7 \times \sqrt{49 + 576} = 22 \times 25 = 550$ sq. m.

7. (*c*) Let r be the radius of the base of each can

$\Rightarrow r = 1$



$\therefore$ Length of the box = 8 units

Width of the box = 8 units

$\therefore$ Interior area of the bottom of the box

= 64 sq. units.

8. (*c*) Since the vertical angle and height are given,

Radius $= 6 \times (\tan 30^\circ) = 2\sqrt{3}$

Volume $= \left(\frac{1}{3}\right)\pi(2\sqrt{3})^2 6 = 24\pi$

All the data regarding the sphere is redundant.

9. (*a*) Volume of the land $= \pi \times (21)^2 \times 36$

Let r be the radius of the conical heap.

$\therefore\ \frac{1}{3}\pi r^2 \times 12 = \pi \times (21)2 \times 36$

$\Rightarrow\ r^2 = (21)^2 \times 9$

$\Rightarrow\ r = 63.$

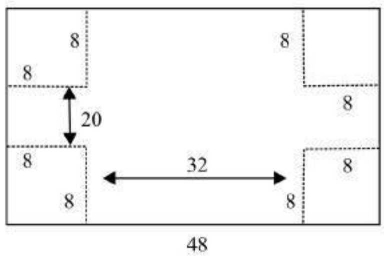
10. (*b*) Volume of the steel $= \frac{2}{3}\pi(4.5)^3 - \frac{2}{3}\pi(4)^3$

$= \frac{2}{3}\pi[(4.5)^3 - 4^3]$

$= \frac{2}{3} \times \frac{22}{7} \times (27.125)$

$= 56.83\ \text{cm}^2.$

11. (*a*)



$\therefore$ Dimensions of the box are 32, 20 and 8.

$\therefore$ Volume of the box $= 32 \times 20 \times 8 = 5120\ \text{m}^3$.

12. (*b*) $L + B + H = 19$

Let, $L = x, B = y$ and $H = z$

$\therefore\ x^2 + y^2 + z^2 = 11^2 = 121$

Also, $x + y + z = 19$ (Given)

$\therefore\ (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow 2(xy + yz + zx) = (19)^2 - 121 = 240$

Surface are of the room

$= 2(xy + yz + zx) = 240\ \text{m}^2$

$\therefore$ Cost of painting the total surface area of the room @ ₹10/m^2 = ₹2400.

13. (*a*) Let the height of the room be h metres.

Area of four walls of the room

$= 2(L + B) \times h = \frac{340.20}{1.35} = 252\ \text{m}^2$

$\Rightarrow\ (L + B)h = 125$

Also, $L \times B = \frac{91.80}{0.85} = 108$

Since $L = 12$, therefore $B = 9$

Hence, $h = \frac{126}{L + B}$

$= \frac{126}{21} = 6.$

14. (*d*) Volume of sphere $= \frac{4}{3} \times \pi \times 7^3$

Let the number of cylinders be k. Then, $2k$ is the number of cones.

Also, $R_{\text{cylinder}} = R_{\text{cone}} = H_{\text{cone}} = \frac{1}{2}\ H_{\text{cylinder}}$

and, $H_{\text{cylinder}} = 4$

$\Rightarrow\ \frac{4}{3}\pi \times 7^3 = k(\pi \times 22 \times 4) + 2k\left(\frac{1}{3}\pi \times 2^2 \times 2\right)$

$\Rightarrow\ k = \frac{7^3}{16} = 21.4375 = 21$

So, number of cones = 42

15. (*a*) Let r be the radius of the base and h be the height in each case. [$r = h$ = height of the hemisphere]

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{\pi r^2 h}{3} = \frac{\pi r^3}{3}$

Volume of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$

Volume of the cylinder $= \pi r^2 h = \pi r^3 = .\frac{1}{3} \times 3\pi r^3$

$\therefore$ Required ratio = 1 : 2 : 3.

16. (*c*) Volume of cylinder $= \pi r^2 h$

$= 2.25 \times 2.25 \times 10\pi$

$= 50.625\pi$

Volume of coin $= \pi r^2 h$

$= 0.75 \times 0.75 \times 0.2\pi = 0.1125\pi$

$\therefore$ Number of coins $= \frac{50.625\pi}{0.1125\pi} = 450.$

17. (*d*) Suppose Radius = $7K$, Height = $10K$

$\therefore$ Volume of the cylinderical tank

$= \pi \times (7K)^2 \times 10K = 12320$

$\Rightarrow\ 490K^3 = \frac{12320 \times 7}{22}$

$\Rightarrow\ K^3 = \frac{86240}{10780} = 8 \Rightarrow K = 2.$

$\therefore$ Height = 20 m.

18. (*c*) Area of path $= 26 \times 10 - 24 \times 14$

$416 - 336 = 80\ \text{m}^2$

$\therefore$ required number of tiles

$= \frac{80\ \text{m}^2}{(20 \times 20)}\text{cm}^2$

$= \frac{80 \times 100 \times 100}{20 \times 20} = 2000.$

19. (*b*) 600 men dig $5.5 \times 4 \times 405 = 8910\ \text{m}^3$ in half an hour.

600 men dig 106920 m^3 in six hours.

2500 men dig 445500 m^3 in six hours.

Also 2500 men dig $80x$ m^3 in six hours,

where x = length of the canal dug.

$\therefore \quad 80x = 445500$

$\Rightarrow \quad x = 5,568.75$ m.

20. (*c*) Volume of conical vessel = Volume of cylindrical jar

$$\frac{1}{3}\pi \times 2^2 \times 3 = \pi \times 2^2 \times h$$

$$h = \frac{1 \times \pi \times 2^2 \times 3^3}{3 \times 2^2 \times \pi} = 1 \text{ cm.}$$

21. (*a*) Let the length and breadth of a cuboid be $8x$ and $7x$. Volume of cuboid = $l \times b \times h$

$\Rightarrow \quad 1120 = 8x \times 7x \times 5$

$\Rightarrow \quad x^2 = \frac{1120}{8 \times 7 \times 5} = 4$

$\Rightarrow \quad x = 2.$

$\therefore$ Length and breadth of a cuboid are 16 cm and 14 cm.

Hence, length is exceed by breadth is 2 cm.

22. (*a*) Let length of square section of bar be l.

$\therefore \quad 1 = l \times l = 36$

$\Rightarrow \quad l = \frac{1}{6}$ m

$\therefore$ Volume of exact cube

$$= \left(\frac{1}{6}\right)^3 = \frac{1}{216} \text{ m}^2$$

Since, cost of 1 m^3 = 108

$\therefore$ Cost of $\frac{1}{216}$ m^3 = $\frac{108}{216}$ = ₹0.50 = 50 paise

23. (*d*) Let dimensions of a stone be $3x$, $2x$, x.

$\therefore$ Volume of stone = 10368

$\Rightarrow \quad 3x \times 2x \times x = 10368$

$\Rightarrow \quad 6x^3 = 10368$

$\Rightarrow \quad x^3 = 1728$

$\Rightarrow \quad x = 12$

$\therefore$ Dimensions are 36 dm, 24 dm, 12 dm.

$\therefore$ Entire surface area of a stone

$= 2(lb + bh + hl)$

$= 2(36 \times 24 + 24 \times 12 + 12 \times 36)$

$= 2(864 + 288 + 432)$

$= 3168$ dm^2

$\therefore$ Total polished cost

$= 3168 \times 0.02$

$= 63.36.$

24. (*c*) Let the breadth and height be b and h, respectively. Then,

$70 \times 2.2 = 14 \times b$

$b = 11$ m

Also, $14 \times b \times h = 70 \times 11$

$\therefore \quad h = \frac{70 \times 11}{14 \times 11} = 5$ m.

25. (*a*) Curved surface area of a frustum = $\pi(r + R)L$

Here, $\quad R = \frac{8}{2} = 4$ m, $r = \frac{4}{2} = 2$ m

and, $\quad l = \sqrt{h^2 + (R - r)^2}$

$l = \sqrt{6^2 + 2^2} = \sqrt{40}$

$\therefore$ Curved surface area

$= \frac{22}{7} \times (2 + 4) \times \sqrt{40}$

$= 119.26 \approx 118.4$ m^2

26. (*c*) Required difference = $l \times b \times h - \pi r^2 h$

$= 10 \times 10 \times 21 - \frac{22}{7} \times \left(\frac{10}{2}\right)^2 \times 21$

$= 2100 - 1650 = 450$ cm^3

27. (*a*) Total surface area of prism

= lateral surface area + 2 × (area of base)

Here, $\quad s = \frac{a + b + c}{2}$

$= \frac{21 + 20 + 13}{2} = 27$

$\therefore$ Required area = $(21 + 20 + 13) \times 30 + 2 \times \sqrt{27(27 - 21)(27 - 20)(27 - 13)}$

$= 54 \times 30 + 2\sqrt{27 \times 6 \times 7 \times 14}$

$= 1620 + 2 \times 126 = 1872$ sq m

28. (*b*) A tank of 440 cu m is filled in 10 minutes.

$\therefore$ Flow of water = 44 cu m/min

$\therefore \quad \pi r^2 h = 44$

$\Rightarrow \frac{22}{7} \times r^2 \times 7 = 44$

$\Rightarrow \quad r^2 = \frac{44}{22} \Rightarrow r = \sqrt{2}$ m.

29. (*d*) When the cube is cut into two rectangular solids of equal volumes, sides of rectangular solids are

$$L = \frac{a}{2}, B = a, H = a$$

S.A. not painted $= B \times H = a \times a\ a^2$

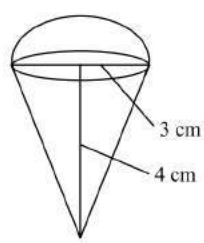
T.S.A. of each solid $= 2\left[\frac{a}{2} \times a + a \times a + a \times \frac{a}{2}\right]$

$= 4a^2$

% are not painted $= \frac{a^2}{4a^2} \times 100 = 25.$

30. (*a*) Surface area of hemisphere $= 2\pi r^2$

$= 2\pi(3)^2 = 18\pi$ cm^2



Now $\quad l = \sqrt{4^2 + 3^2} = 5.$

Surface area of cone $= \pi r l$

$= \pi \times 3 \times 5 = 15\pi$ cm^2

Total surface area $= 18\pi + 15\pi = 33\pi$ cm^2.

31. (*b*) Volume of oil $= \pi \times (6)^2 \times 14 = 504\pi$ m^3

Volume of conical can

$= \frac{1}{3} \times \pi \times (6)^2 \times 6 = 72\pi$ m^3

Volume of spherical can

$= \frac{4}{3} \times \pi (6)^3 = 288\pi$ m^3

$\therefore$ Remaining oil $= 504\pi - (288\pi + 72\pi)$

$= 144\pi$ m^3

Volume of cylindrical can

$= \pi \times (6)^2 \times h$

$144\pi = \pi \times 36 \times h$

$\Rightarrow \quad h = 4$ m

Now, $\frac{3}{4}$ th of cylindrical can is filled.

$\therefore$ Oil dropped $= \frac{1}{4} \times \pi \times (6)^2 \times 4 = 36\pi$ m^3.

32. (*b*) Let the side of cube $= x$

Volume of cube $= x^3$

Let the radius of sphere $= r$

Now, sphere can fit inside the cube, so

$$r = \frac{x}{2}$$

$$= \frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{x^3}{\frac{4}{3} \times \frac{22}{7} \times \frac{x^3}{8}}$$

$$= \frac{3 \times 7 \times 8}{4 \times 22} = \frac{21}{11}.$$

33. (*c*) Curved surface area of cone $= \pi R(l_1 + l_2)$

Curved surface area of frustum $= \pi(R + r)l_2$

$\therefore \frac{8}{9} \times \pi R(l_1 + l_2) = \pi(R + r)l_2$

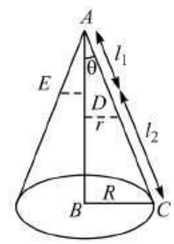
$\Rightarrow 8 \times Rl_1 + 8Rl_2 = 9Rl_2 + 9rl_2$

$\Rightarrow \quad 8Rl_1 = Rl_2 + 9rl_2$

$\Rightarrow \quad 8Rl_1 = l_2(R + 9r)$

$\Rightarrow \quad \frac{l_1}{l_2} = \frac{R + 9r}{8R} = \left(\frac{1}{8} + \frac{9r}{8R}\right) \quad (1)$

Now, in ΔABC and ΔADE,



According to sin rule,

$$\frac{R}{l_1 + l_2} = \frac{r}{l_1}$$

$$\frac{r}{R} = \frac{l_1}{l_1 + l_2}$$

From Eq. (1),

$$\frac{l_1}{l_2} = \left(\frac{1}{8}+\frac{9}{8}\times\frac{l_1}{l_1+l_2}\right)$$

$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{1}{8}\left(1+\frac{9l_1}{l_1+l_2}\right)$$

$$\Rightarrow \quad \frac{8l_1}{l_2}-\frac{9l_1}{l_1+l_2} = 1$$

$$\Rightarrow \quad 8l_1^2+8l_1l_2-9l_1l_2 = l_1l_2+l_2^2$$

$$\Rightarrow \quad 8l_1^2-2l_1l_2-l_2^2 = 0$$

$$\Rightarrow \quad 8l_1^2-4l_1l_2+2l_1l_2-l_2^2 = 0$$

$$\Rightarrow \quad 4l_1[2l_1-l_2]+l_2[2l_1-l_2] = 0$$

$$\Rightarrow \quad (2l_1-l_2)(4l_1+l_2) = 0$$

$$\Rightarrow \quad 2l_1-l_2 = 0$$

$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{1}{2}.$$

34. (*d*) Volume of the ball = Volume of raised water

$$= \pi\times(12)^2\times 6.75$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi\times 144\times 6.75$$

$$\Rightarrow \quad r^3 = 729$$

$$\Rightarrow \quad r = 9 \text{ cm.}$$

35. (*c*) Let the diameter of third ball be $2r$.

$$\frac{4}{3}\pi\times\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\times\left(\frac{1.5}{2}\right)^3 + \frac{4}{3}\pi\times\left(\frac{2}{2}\right)^3+\frac{4}{3}\pi\left(\frac{2r}{2}\right)^3$$

$$\Rightarrow \quad \frac{27}{8} = \frac{27}{64}+1+r^3$$

$$\Rightarrow \quad r^3 = \frac{27}{8}-\frac{27}{64}-1$$

$$\Rightarrow \quad r^3 = \frac{216-27-64}{65} = \frac{125}{64}$$

$$\Rightarrow \quad r = \frac{5}{4}\text{ cm}$$

$$\therefore \quad 2r = \frac{10}{4} = 2.5 \text{ cm.}$$

36. (*a*) $\quad 2\pi r = 176$

$$\therefore \quad r = \frac{176\times 7}{2\times 22} = 28 \text{ cm}$$

Let there are n number of persons.

$$\therefore \quad n\times\frac{2}{3}\times\pi\times(2)^3 = \frac{1}{2}\times\frac{2}{3}\times\pi\times(28)^3$$

$$\Rightarrow \quad n = \frac{(28)^3}{(2)^3\times 2} = 1372.$$

37. (*d*) Total number of smaller cubes $= \dfrac{4\times4\times4}{1\times1\times1} = 64$

Increase in surface area

$$= 64\times 6\times(1)^2-6\times(4)^2$$

$$= 384-96 = 288 \text{ cm}^2$$

$\Rightarrow$ Required percentage

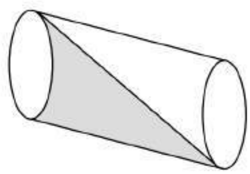
$$= \frac{288}{96}\times 100 = 300\%$$

38. (*b*) Total inner cost $= 2\times\dfrac{22}{7}\times\dfrac{7}{2}\times 22.5\times 3$ = ₹1,485.

39. (*b*) Required number of boxes

$$= \frac{8\times7\times6\times100\times100\times100}{8\times7\times6} = 1000000.$$

40. (*d*) Volume of cylinder $= xr^2h$



Given,

$$\frac{4}{5}\pi r^2h-30 = \frac{1}{2}\pi r^2h$$

$$\Rightarrow \quad \left(\frac{4}{5}-\frac{1}{2}\right)\pi r^2h = 30$$

$$\Rightarrow \quad \frac{3}{10}\pi r^2h = 30$$

$$\Rightarrow \quad \pi r^2h = 100 \text{ cc.}$$

41. (*b*) Total number of bricks

$$= \frac{6\times5\times0.5}{25\times12.5\times7.5}\times 10^6\times\frac{95}{100}$$

$$= 6080$$

42. (*c*) Let the outer radius of pipe be r.

Then,

$$\pi\times 28\times(r^2-8^2) = 1496$$

$\Rightarrow \quad r^2 - 64 = \frac{1496}{28 \times 22} \times 7$

$\Rightarrow \quad r^2 = 17 + 64 = 81$

$\Rightarrow \quad r = 9$ cm.

43. (*b*) Curved surface area of 50 pillars

$= 50 \times 2 \times 3.14 \times \frac{50}{2} \times 400$

$= 314 \times 10^4$ sq cm = 314 sq m

$\therefore$ Labour charges = 314×0.50 = ₹157

44. (*c*) Volume of bigger cube

$= 10 \times 10 \times 10$

$= 1000 \text{ cm}^3$

Volume of smaller cube $= \frac{1000}{2} \text{ cm}^3$

$\therefore$ Edge of smaller cube $= \frac{10}{(2)^{1/3}}$ cm

and, edge of smaller cube = 10 cm

$\therefore$ Required ratio $= \left(\frac{1}{2}\right)^{1/3}$.

45. (*a*) Let edge of bigger cube be x and smaller cube be y.

Then, $6x^2 = 384 \Rightarrow x^2 = 64$

$\therefore \quad x = 8$ cm

Volume of bigger cube

$= 8 \times 8 \times 8 = 512 \text{ cm}^3$

$6y^2 = 96$

$\Rightarrow \quad y^2 = 16$

$\therefore \quad y = 4$ cm

Volume of smaller cube

$= 4 \times 4 \times 4 = 64 \text{ cm}^3$

$\therefore$ Total number of smaller cubes

$= \frac{512}{64} = 8.$

46. (*d*) Volume of the parallelepiped $= 5 \times 4 \times 3$

$= 60 \text{ cm}^3$

Volume of the cube $= 4 \times 4 \times 4 = 64 \text{ cm}^3$

Volume of the cylinder $= \pi \times 3 \times 3 \times 3$

$= 27\pi$

$= 84.8 \text{ cm}^3$

Volume of the sphere $= \frac{4}{3}\pi (3)^3$

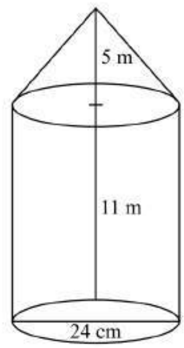
$= 36\pi = 113.04 \text{ cm}^3$

The required decreasing order is D, C, B and A.

47. (*b*) Curved surface area of the figure

= curved surface area of cone

+ curved surface area of cylinder



$= \pi r l \times 2\pi r h$

$= \pi \times 12\sqrt{12^2 + 5^2} + 2\pi \times 12 \times 11$

$= \pi \times 12 \times 13 + \pi \times 24 \times 11$

$= 156\pi + 264\pi = 420\pi$

$= 420 \times \frac{22}{7} = 1320$ sq m.

48. (*a*) Whole surface of pipe

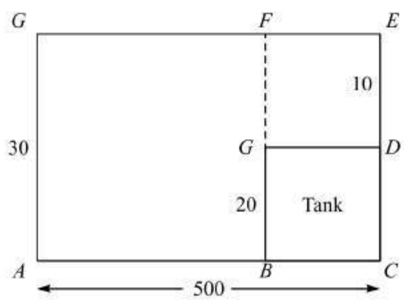
$= 2\pi h\,(r_1 + r_2) + 2\pi(r_1^2 - r_2^2)$

$= 2 \times 3.14 \times 20\left(\frac{25}{2} + \frac{23}{2}\right) + \left[\left(\frac{25}{2}\right)^2 - \left(\frac{23}{2}\right)^2\right]$

$= 3014.4 + 6.28(24)$

$= 3165.12 \approx 3068 \text{ cm}^2$.

49. (*b*)



Volume of the earth dug

$= 30 \times 20 \times 12$

$= 7200$ cu. m.

Area of $ABGDEFGA$

$=$ Area $ABFG$ + Area $GDEF$

$= 470 \times 30 + 30 \times 10$

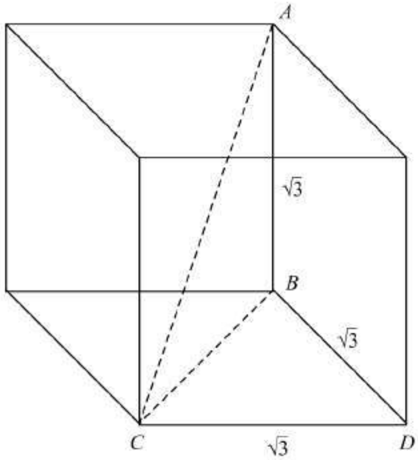
$= 14100 + 300 = 14400$

$\therefore \quad 14400\, h = 7200$

where h is the level to which the field is raised by spreading the earth dug out of the tank, evenly over the remaining field.

$\therefore \quad h = 0.5$ m.

50. (c)



$BC^2 = BD^2 + CD^2 = 3 + 3 = 6$

$AC^2 = AB^2 + BC^2 = 3 + 6 = 9$

$\therefore \quad AC = 3$

(AC is the largest pole that can be accommodated by the cube).

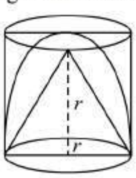
51. (b) Let r = radius of the base of the cylinder

= radius of the hemisphere

= radius of the base of the cone

= height of the cylinder

= height of the cone



Required ratio $= \pi r^3 : \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^3$

$= 1 : \frac{2}{3} : \frac{1}{3} = 3:2:1$.

52. (b) Volume of the given ice cuboid $= 8 \times 11 \times 2 = 176$.

Let the length of the required rod be l

$\therefore \quad \pi l \frac{8^2}{4} = 176$

$\Rightarrow \quad l = 3.5$ inches.

53. (b) Radius of the cylinderical vessel = 4 cm

$\therefore$ Volume of water in the vessel $= \pi \times (4)^2 \times h$

$= 16\,\pi\, h$ where h is the height of water in it.

When a sphere is lowered in water, suppose that the water level rises by k.

$\therefore$ Volume of water and the sphere in the vessel

$= 16\,\pi\,(h + k)$

$16\,\pi\,(h + k) - 16\,\pi\, h = \frac{4}{3}\pi(3)^3$

$16\,\pi\, k = 36\,\pi$

$\Rightarrow \quad k = \frac{9}{4}$.

54. (a) Surface area of the circular cylinder

$= 2\pi rh + 2\pi r^2$,

where r is the radius of the base and h is the height

$= 2\pi \times 8 \times 15 + 2 \times \pi \times (8)^2 - \pi \times (8)^2$

$= 240\pi + 128\pi - 64\,\pi = 304\pi$

(Because when conical cavity is drilled in the circular cylinder, the surface area of one circular base is deleted). Now surface area of the conical cavity = $\pi r\sqrt{r^2 + h^2}$ (Because radius of the base and height are same for circular cylinder and conical cavity.

$\sqrt{r^2 + h^2}$ = Slant height of the conical cavity.

$= \pi \times 8 \times \sqrt{64 + 225}$

$= \pi \times 8 \times 17 = 136\pi$

$\therefore$ Surface area of the remaining solid

$= 304\pi + 136\pi = 440\pi$ sq cm.

55. (c) Suppose length and breadth of the room be x and y metres respectively.

$\therefore$ Height = 40% of $(x + y) = \frac{2}{5}(x + y)$

$\therefore$ Area of the four walls of the room

$$= 2 \times \frac{2}{5}(x + y) \times x + 2 \times \frac{2}{5}(x + y) \times y$$

$$= \frac{4}{5}(x + y)^2 \text{ sq m}$$

$\therefore$ Area of the paper required

$$= \left[\frac{4}{5}(x + y) - 15\right] \text{sq m}$$

$\Rightarrow$ Length of the paper required

$$= \frac{\text{Area}}{\text{Width}} \quad \frac{-(x + y) \ -15}{1/2} \text{metres}$$

$$= \left[\frac{8}{5}(x + y)^2 - 30\right] \text{metres}$$

$\therefore$ Cost of the paper @ ₹2 per metre

$$= ₹\, 2\left[\frac{8}{5}(x + y)^2 - 30\right]$$

Hence,

$$2 \times \left[\frac{8}{5}(x + y)^2 - 30\right] = 260$$

$$\Rightarrow \qquad x + y = 10$$

$$\therefore \qquad \text{Height} = \frac{2}{5}(x + y)$$

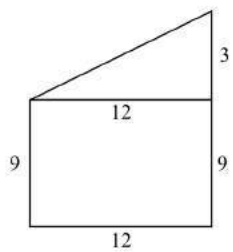
$$= 4 \text{ metres.}$$

56. (*a*) Let the height level in the flask be k cm.

$$\therefore \qquad \pi p^2 k = \frac{1}{3}\pi a^2 h$$

$$\Rightarrow \qquad k = \frac{1}{3}\frac{a^2 h}{p^2} = \frac{ha^2}{3p^2}$$

57. (*c*)



Required volume

= Volume of the rectangular portion

+ Volume of the triangular portion.

$$= 12 \times 9 + \left(\frac{1}{2} \times 12 \times 3\right) \times 9$$

= 108 + 162 = 270 cu m.

58. (*d*) Total volume of the three cubes

$$= (3)^3 + (4)^3 + (5)^3$$

$$= 27 + 64 + 125$$

$$= 216 \text{ cu cm.}$$

$\therefore$ Volume of the bigger cube = 216 cu cm

$\therefore$ Edge of the bigger cube formed

$$= \sqrt[3]{216} = 6 \text{ cm}$$

59. (*d*) Volume of the water in the cylinder before dropping the ball = $\pi \times (6)^2 \times 20$

$$= 720\,\pi \text{ cu cm}$$

Volume of water after dropping the ball

$$= \pi \times (6)^2 \times 26.75$$

$$= 963\,\pi \text{ cu cm}$$

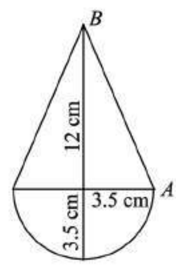
$\therefore$ Volume of the spherical ball

$$= \frac{4}{3}\pi r^3 = 963\pi - 720\pi$$

$$= 243\,\pi \text{ cu cm}$$

$$\therefore \qquad r^3 = \frac{243\pi \times 3}{4\pi} = \frac{729}{4}.$$

60. (*b*)



Surface area of the given figure

$$= \pi r^2 \sqrt{r^2 + h^2} + 2\pi r^2$$

Here $r = 3.5$ cm and $h = 12$ cm

$$= \frac{22}{7} \times 3.5 \times \sqrt{12.25 + 144} + 2 \times \frac{22}{7} \times 12.25$$

$$= \frac{22}{7} \times 3.5 \times 12.5 + 2 \times \frac{22}{7} \times 12.50$$

$$= \frac{22}{7} \times 43.75 + \frac{22}{7} \times 24.50$$

$$= \frac{22}{7} \times 68.25 = 214.5 \text{ cm}^2.$$

61. (*b*) The volume of the original cone is $V = \pi R^2 h/3$

The height and the radius of the smaller cone are $2h/3$ and $2R/3$, respectively.

$$\frac{1}{3}\pi\left(\frac{2R}{3}\right)^2 \times \frac{2h}{3} = \frac{8V}{27}$$

$\therefore$ Volume of the frustum $= \left(V - \frac{8V}{27}\right)$

$$= \frac{19V}{27}$$

$\therefore$ Required ratio is 8:19.

62. (*d*) Let Length, Breadth and Height of a cuboid be L, B and H, respectively.

$\therefore$ $L = K, B = 2K, H = 3K$

Volume $= LBH = 6K^3$

If, $L = 2K, B = 6K, H = 9K$,

then volume $= 108K^3$

Increase in volume $= 102\ K^3$

= 17 times the original volume

63. (*d*) $a^3 \approx 12a \Rightarrow a^2 \approx 12$

where a is an edge of the cube

$\therefore$ Total surface area $= 6a^2 = 72$

64. (*d*) Suppose radius $= r = 4k$ and height $= h = 3k$

$$\therefore \text{ Required ratio} = \frac{2\pi rh + 2\pi r^2}{\pi r\sqrt{r^2 + h^2} + \pi r^2}$$

$$= \frac{14k}{9k} = \frac{14}{9}$$

65. (*c*) Let h be the height upto the level of water in the cylinder.

$\therefore$ Volume of the water in the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 36 \times h$$

Let h_1 be the height to which the water level is raised.

$$\therefore \frac{22}{7} \times 36 \times (h_1 - h) = \frac{4}{3} \times \frac{22}{7} \times 27$$

$$\therefore h_1 - h = 1,$$

i.e., water level is raised by 1 cm.

66. (*a*) Let the $2x$ side of cube be 100 units,

when cut, then surface area of one cube

$= 2[100 \times 50 + 50 \times 100 + 100 \times 100]$

$= 40000$ unit.

Total surface area of two blocks = 40000 units

Area of unpainted region = 10000

$\therefore$ Required percentage $= \frac{10000}{40000} \times 100 = 25\%$

67. (*d*) Number of cubes $= \frac{6 \times 6 \times 6}{2 \times 2 \times 2} = 27.$

68. (*c*) Let the length, breadth and height be a, b and c, respectively.

$a + b + c = 19$

$\sqrt{a^2 + b^2 + c^2} = 5\sqrt{5}$

or, $a^2 + b^2 + c^2 = 125$

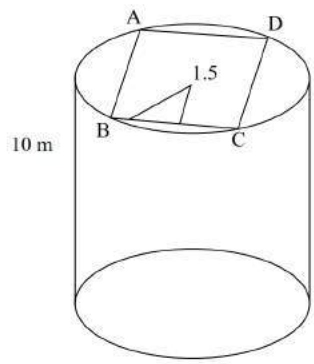
Surface area $= 2\ (ab + bc + ca)$

Now $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow$ $19^2 = 125 + 2\ (ab + bc + ca)$

or, $2\ (ab + bc + ca) = 361 - 125 = 236$ cm^2.

69. (*c*)



From the figure, $BD^2 = BC^2 + DC^2$

$\Rightarrow (3)^2 = BC^2 + BC^2$ ($\because$ it has square base, $BC = DC$)

$\Rightarrow 9 = 2BC^2$

$\Rightarrow BC = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$ m

$\therefore$ Volume of parallelopiped

$= \frac{3}{\sqrt{2}} \times \frac{3}{\sqrt{2}} \times 10$

$= \frac{9 \times 10}{2} = 45$ m^3.

70. (*c*) If the number of balls are n, then, the volume of wire $= n \times$ volume of ball

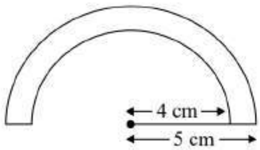
$$\Rightarrow \pi \times \frac{0.2}{2} \times \frac{0.2}{2} \times 22$$

$$= n \times \frac{4}{3} \times \pi \times \frac{0.1}{2} \times \frac{0.1}{2} \times \frac{0.1}{2}$$

$$\Rightarrow n = \frac{22 \times 3 \times 2 \times 2 \times 2 \times 10}{4}$$

$$\Rightarrow n = 1320.$$

71. (*a*) We have $r_1 = \frac{10}{2} = 5$ cm and $r_2 = \frac{8}{2} = 4$ cm



$\therefore$ Area to be painted

$$= 2\pi r_1^2 + 2\pi r_2^2 + (\pi r_1^2 - \pi r_2^2)$$

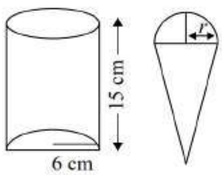
$$= 3\pi_1^2 + \pi r_2^2$$

$$= \pi(3r_1^2 + r_2^2)$$

$$= \frac{22}{7}[3 \times 25 + 16] = \frac{22}{7} \times 91\text{cm}^2$$

$$\therefore \text{cost of painting} = ₹\frac{22}{7} \times 91 \times 0.70 = ₹200.2 = ₹200$$

72. (*d*)



Let radius of cone $= r$

Volume of cylinder = 10 [Volume of cone + Volume of hemisphere]

$$\Rightarrow \pi \times 6 \times 6 \times 15 = 10\left[\frac{1}{3}\pi \times r^2 \times 4r + \frac{2}{3}\pi r^3\right]$$

[$\because$ height of cone $= 4 \times$ radius]

$$\Rightarrow 36 \times 15\pi = 10 \times \pi \times 2 \times r^3$$

$$\Rightarrow r^3 = 27$$

$$\Rightarrow r = 3$$

Hence, diameter of icecream cone = 2 × 3 = 6 cm.

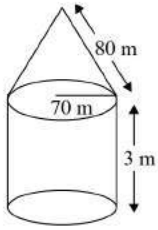
73. (*c*) Surface area of the tent

$= 2\pi rh + \pi rl$

$= \pi r\,(2h + l)$

$$= \frac{22}{7} \times 70(6 + 80)$$

$$= 220 \times 86 = 18920$$



Width of canvas = 2 m

$\therefore$ length of canvas

$$= \frac{18920}{2} = 9460 \text{ m}$$

74. (*b*) Let length and breadth of the rectangular park be 3x and 2x, respectively.

Then, using the formula, distance = speed × time, we get

$$10x = 12 \times \frac{5}{18} \times 8 \times 60$$

$$\Rightarrow x = 160$$

$\therefore$ Area of rectangular park is $3x \times 2x = 6x^2$

$= 6 \times (160)^2$

$= 6 \times 25600$

$= 153600$ sq m.

75. (*d*) Let the length of the edge of the cube be x. Then, we are given,

$a^3 = 6a^2 \Rightarrow a = 6$ unit.

76. (*c*) Surface area of metal sheet

$= 2\,(27 \times 8 + 8 \times 1 + 1 \times 127)$

$= 2\,(216 + 8 + 27)$

$= 2 \times 251 = 502\ \text{cm}^2$

Let edge of cube which we form be x.Then, we are given,

$x^3 = 27 \times 8 \times 1$

$\Rightarrow x = 3 \times 2 = 6$ cm

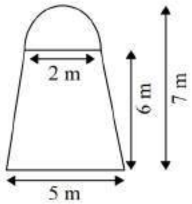
$\therefore$ surface area of the cube $= 6x^2$

$= 6 \times 6^2$

$= 216\ \text{cm}^2$

$\therefore$ Required difference $= 502 - 216 = 286\ \text{cm}^2$.

77. (*b*) External surface area of the shuttlecock = External surface area of frustrum + External surface area of hemisphere



$= \pi(R+r)l = 2\pi r^2$

$= \pi(R+r)(\sqrt{h^2+(R-r)^2} + 2\pi r^2$

Where $R = 2.5$ cm, $r = 1$ cm, $h = 6$ cm

$\therefore$ Required surface area

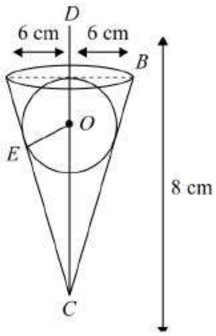
$= \frac{22}{7} \times 3.5\sqrt{36+1.5^2+2\times\frac{22}{7}\times 1}$

$= 22 \times 0.5\sqrt{38.25} + 6.28$

$= 11 \times 6.18 + 6.28$

$= 74.26 \text{ cm}^2.$

78. (*c*) Here, $BD = 6$ cm



$DC = 8$ cm

$\therefore \quad BC = \sqrt{(6)^2+(8)^2} = \sqrt{36+64} = 10$ cm

In ΔACD and ΔEOC

$\angle ADC = \angle OEC = 90°$

$\angle ACD = \angle OCE$ (common angle)

$\angle CAD = \angle EOC$ (remaining angle)

$\therefore \quad \Delta ACD \sim \Delta EOC$

Also, $AD = AE = 6$ cm

($\because$ the length of two tangents drawn from an external point to circle are equal)

$\therefore \quad EC = AC - AE = 10 - 6 = 4$ cm

In similar ΔACD and ΔEOC

$\frac{DC}{AD} = \frac{EC}{OE}$

$\Rightarrow \quad OE = \frac{AD \times EC}{DC} = \frac{6\times 4}{8} = 3$ cm

$\therefore$ Required of sphere = 3 cm.

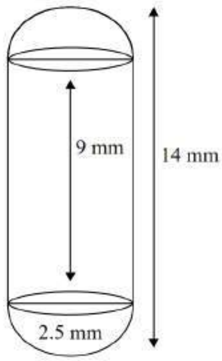
Now, volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\pi\times 36\times 8$

And volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times 27$

$\therefore$ Required fraction of water

$= \frac{\frac{4}{3}\pi\times 27}{\frac{1}{3}\pi\times 36\times 8} = \frac{4\times 27}{36\times 8} = \frac{3}{8} = 3:8.$

79. (*c*) Required surface area



$= 2\times 2\pi r^2 + 2\pi rh$

$= 4\times\frac{22}{7}\times 2.5\times 2.5 + 2\times\frac{22}{7}\times 2.5\times 9$

$= 2\times\frac{22}{7}\times 2.5(5+9)$

$= 220 \text{ mm}^2.$

80. (*a*) For first cube, probability of getting red face $= \frac{5}{6}$

and probability of getting blue face $= \frac{1}{6}$

Now, let the number of faces which are red in colour $= x$

$\therefore$ Number of blue faces $= 6 - x$

Similarly, for second cube,

Probability of getting red face $= \frac{x}{6}$

and probability of getting blue face $= \frac{6-x}{6}$

Then, we are given ,

$\frac{5}{6}\times\frac{x}{6}+\frac{1}{6}\times\frac{6-x}{6} = \frac{1}{2}$

$\frac{5x}{36}+\frac{6-x}{36} = \frac{1}{2}$

$\frac{5x+6-x}{36} = \frac{1}{2}; 4x+6 = 18$

$4x = 18-6 = 12; x = \frac{12}{4} = 3$

$\therefore$ Number of red faces on second cube = 3.

81. (*b*) Dimensions of room = 10 m × 7 m × 5 m

$\therefore$ Area of walls $= 2\times h(l+b) = 2\times 5(10+7) = 170 \text{ m}^2$

Area covered by doors and windows

$= 2(1 \times 3) + (2 \times 1.5) + 2(1 \times 1.5)$

$= 6 + 3 + 3 = 12 \text{ m}^2$

$\therefore$ Area to be painted $= (170 - 12) = 158 \text{ m}^2$

The Cost of painting = ₹3 per m^3

$\therefore$ Total cost of painting the wall $= 3 \times 158 =$ ₹474.

82. (*d*) We have, radius of cylindrical tub = 12 cm

Depth of cylindrical tub = 20 cm

Let the radius of spherical iron ball be R cm.

According to the question,

$$\pi \times (12)^2 \times 20 + \frac{4}{3}\pi(R)^3 = \pi(12)^2 \times (26.75)$$

$$\Rightarrow \frac{3}{4}(R)^3 = (12)^2 \times (26.75) - (12)^2 \times 20$$

$$\Rightarrow \frac{4}{3}R^2 = 144 \times 6.75$$

$$\Rightarrow R^3 = \frac{144 \times 6.75 \times 3}{4}$$

$$\Rightarrow R^3 = 729$$

$$\therefore R = 9 \text{ cm.}$$

83. (*d*) Let length, breadth and height of a room are $3x$, $2x$ and x, respectively.

Then, volume of room = 1296

$$\Rightarrow 3x \times 2x \times x = 1296$$

$$\Rightarrow 6x^3 = 1296$$

$$\Rightarrow x^3 = 216$$

$$\therefore x = 6$$

$\therefore$ Breadth of room $= 2x = 2 \times 6 = 12$ m.

84. (*a*) Since, seven equal cubes are joined and made a cuboid.

So, length of cuboid $= 7 \times 5 = 35$ cm

breadth of cuboid = 5 cm

and height of cuboid = 5 cm

Now, surface area of resulting cuboid $= 2(lb + bh + hl)$

$= 2(35 \times 5 + 5 \times 5 + 5 \times 35)$

$= 2(175 + 25 + 175)$

$= 2 \times 375$

$= 750 \text{ cm}^3$.

85. (*b*) Let curved surface area of first cone is CS_1, and curved surface area of second cone is CS_2. Then, we are given

$$CS_1 = 3CS_2$$

$$\Rightarrow \pi r_1 l_1 = 3(\pi r_2 l_2)$$

$$\Rightarrow \pi r_1 l_1 = 3\pi r_2 l_2$$

$$\Rightarrow r_1 l_1 = 3r_2 l_2$$

$$\Rightarrow r_1 l_1 = 3r_2 \times 3l_1$$

$$\Rightarrow r_1 l_1 = 9r_2 l_1$$

$$\therefore r_1 = 9r_2$$

Therefore, the required ratio of base area $= \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$

$$= \left(\frac{9r_2}{r_2}\right)^2 = \left(\frac{9}{1}\right)^2$$

$$= \frac{81}{1} = 81:1.$$

Difficulty Level-2

1. (*d*) Let r be the radius of sphere A and R be the radius of sphere B.

Surface area of $A = S_A = 4\pi r^2$

Surface area of $B = S_B = 4\pi R^2$

According to the question,

$$4\pi R^2 = 4\pi r^2\left(1 + \frac{300}{100}\right)$$

$$\Rightarrow 4\pi R^2 = 16\pi r^2$$

$$\Rightarrow R = 2r$$

Therefore,

Volume of $A = V_A = \frac{4}{3}\pi r^3$ and

Volume of $B = V_B = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$

$$K = \frac{\frac{32}{3}\pi r^3 - \frac{4}{3}\pi r^3}{\frac{32}{3}\pi r^3} \times 100 = 87.5\%$$

Quicker Method:

	A	:	B	
Area	100	:	400	
Side	$\sqrt{100}$	:	$\sqrt{400}$	= 1:2
Volume	$(1)^3$	:	$(2)^3$	= 1:8

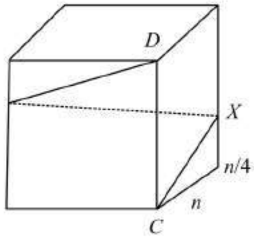
Value of $K = \frac{8-1}{8} \times 100 = 87.5\%$

2. (*a*) A string of a certain length, when wound on a cylindrical surface, gives a maximum of n turns. It means that every turn is equidistant, starting from 1 to n.

$\therefore$ vertical distance between two consecutive turns.

$$= \frac{\text{height of cylinder}}{\text{number of turns}} = \frac{h}{n}.$$

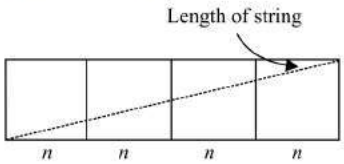
3. (*b*) The string of minimum length, if starting from C, touches next corner at height $n/4$ on the completion of one turn, starting from height $n/4$ touches next corner at height $n/2$ in the second turn, and so on.



$$CX = \sqrt{n^2 + \left(\frac{n}{4}\right)^2} = \frac{\sqrt{17} \times n}{4}$$

$\therefore$ Length of string $= 4 \times \frac{\sqrt{17} \times n}{4} = \sqrt{17}n$.

Aliter (Quicker Method): Opening up the four vertical sides of the cube of side n,



Length of string $= \sqrt{(4n)^2 + (n)^2} = \sqrt{17} \times n$.

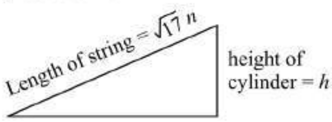
4. (*c*) With the help of the string, we wound on the curved surface of the cylinder n times, thus reaching height h of the cylinder. Opening up the cylinder, we get that a length of $\sqrt{17}n$ of string reaches height h after wounding the cylinder's circumference n times.

Cylindrical circumference

$= n$ times the circumference of cross-sectional area

$= n \times 2 \times \pi \times \frac{2}{\pi} = 4n$

For the sake of simplicity, we can represent the relationship as below:



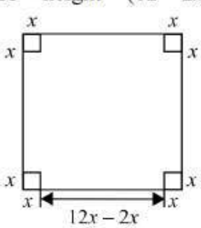
Cylindrical circumference = $4n$

From the above figure,

$(4n)^2 = (h)^2 = (\sqrt{17}n)^2 \Rightarrow h = n.$

5. (*d*) Volume of the cuboid formed

= Area of base × height $= (12 - 2x)^2 \times x$

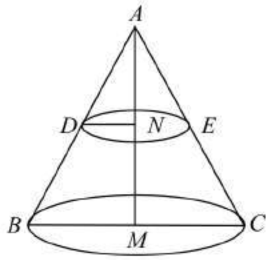


Now, substituting the value of x from the options, we get the maximum value of cuboid when $x = 2$.

6. (*a*) $\frac{6 \times 5 \times 0.5 - 5\% \text{ of } (6 \times 5 \times 0.5)}{0.25 \times 0.125 \times 0.075}$

$$= \frac{15 - 0.05 \times 15}{0.25 \times 0.125 \times 0.075} = 6080.$$

7. (*c*) $AM = 30$ cm (Given)



Let the cone ADE is cut off from the cone ABC.

Let DN = radius of the base of the cone $ADE = r_1$cm.

Let MC = Radius of the base of cone $ABC = r$

$$\therefore \quad \frac{h}{30} = \frac{r_1}{r} \Rightarrow r_1 = \frac{hr}{30} \quad (1)$$

Also volume of the cone ADE

$$= \frac{1}{27} \times \text{Volume of the cone } ABC$$

$$\Rightarrow \frac{1}{3}\pi r_1^2 h = \frac{1}{27} \times \frac{1}{3}\pi r^2 \times 30$$

$$\Rightarrow r_1{}^2 h = \frac{1}{27} \times 30r^2$$

$$\Rightarrow \frac{h^2 r^2}{(30)^2} \times h = \frac{30}{27} r2$$

$$\Rightarrow h^3 = \frac{(30)^3}{(3)^3}$$

$$\Rightarrow h = \frac{30}{3} = 10.$$

8. (*c*)

9. (*d*) $E = \sqrt{\frac{A}{7}}$

If $A = 3087$

$$E = \sqrt{\frac{3087}{7}} = \sqrt{441} = 21$$

If $A = 2023,$

$$E = \sqrt{\frac{2023}{7}} = \sqrt{289} = 17.$$

10. (*b*) $l \times b \times 2 = 48$

$\Rightarrow \quad l \times b = 24$

Now, $6 - 6 \times 10\% = 5.4$

$5 - 5 \times 10\% = 4.5$

$5.4 \times 4.5 = 24.3$

Clearly, $5 < l < 5.5.$

11. (*a*) Volume of the tank $= l \times b \times h$

$$= 20 \times 7 \times (10 - 2)$$
$$= 1{,}120 \text{ m}^3.$$

12. (*a*) Let length $= l$, so, breadth $= l/2$

Now, $l \times \frac{l}{2} = \frac{5000}{25}$

$\Rightarrow$ $l^2 = 400$ or $l = 20$

and, $2lh + 2 \times \frac{1}{2} \times h = \frac{64800}{240}$

$\Rightarrow$ $3lh = 270$

$\Rightarrow$ $h = \frac{270}{3 \times 20} = 4.5$ m.

13. (*a*) Total surface area of the remaining solid = curved surface area of the cylinder + area of base + curved surface area of cone

$$= 2\pi rh + \pi r^2 + \pi rl$$
$$= 2\pi \times 8 \times 15 + \pi \times (8)^2 + \pi \times 8 \times 17$$
$$= 240\pi + 64\pi + 136\pi = 440\,\pi \text{ cm}^2$$

14. (*a*) Let the inner radius of pipe be r m.

Then, $440 = \frac{22}{7} \times r^2 \times 7 \times 10$

$\Rightarrow$ $r^2 = \frac{440}{22 \times 10} = 2$

$\therefore$ $r = \sqrt{2}$ m.

15. (*c*) $\frac{\text{Surface area}}{\text{Volume}} \times 100\% = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} \times 100\%$

$$= \frac{3}{r} \times 100\%$$
$$= \frac{3}{10} \times 100\% = 30\%$$

16. (*d*) Radius of first cylinder $= \frac{15}{\pi}$

Height $= 18$

Radius of second cylinder $= \frac{9}{\pi}$

Height $= 30$

$\therefore$ Ratio $= \frac{\pi\left(\frac{15}{\pi}\right)^2 \times 18}{\pi\left(\frac{9}{\pi}\right)^2 \times 30} = 5{:}3.$

17. (*d*) Surface area of the tank

= area of 4 walls + area of base

$= 3 \times 2 \times 2 + 4 \times 2 \times 2 + 4 \times 3 = 40 \text{ m}^2$

Total surface area $= 40 + 40 = 80 \text{ m}^2$

$\therefore$ Cost $= 80 \times 10 =$ ₹800.

18. (*d*) Volume of the conical shaped equipment used by

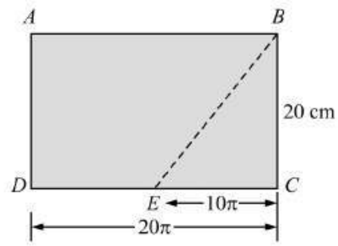
Suresh $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 7$$
$$= \frac{11}{24} \text{ c.c.}$$

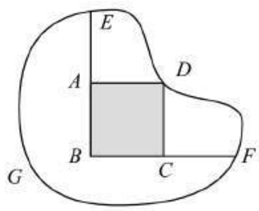
Number of words that can be written using $\frac{3}{5}$ litres of cream

$$= 330 \times \frac{24}{11} \times \frac{3}{5} \times 1000$$
$$= 432000.$$

19. (*c*) On opening the cylinder into a rectangular sheet the sugar grain is at E and ant is at B and BE the shortest route



$\therefore$ $BE = \sqrt{20^2 + (10\pi)^2} \approx 37.25$ cm



20. (*a, c*) Memento is a right circular cone. The circumference of the cone's base touches each side of the base of the box. The vertex of the cone touches the opposite face of the box.

Height of the cone = diameter of the cone = side of the cube.

The box is a cube of side 14 cm.

The total surface area of the box

$$= 6 \times (14)^2 = 1176 \text{ cm}^2$$

Expenditure incurred in packing the box

$= 1176 \times 1.5 = ₹1764$

Option (*a*) Madan's total expenditure on the box

$= 500 + 1764 = ₹2264$

Hence, option (*a*) is correct.

Option (*b*) is wrong as expenditure was ₹1764

Option (*c*) is correct as the area $= 14^2 = 196 \text{ cm}^2$

Option (*d*) volume of the box $= 14^3 = 2744 \text{ cm}^3$

Hence, option (*d*) is wrong.

21. (*d*) As all of them have same base so they have same radius.

Suppose their radius is r and height is h.

$\therefore$ curved surface area of cylinder $= 2\pi rh$

$= 2\pi r^2$

curved surface are of hemisphere $= 2\pi r^2$

and curved surface area of cone

$$= \pi r\sqrt{r^2 + h^2}$$

$$= \pi r\sqrt{r^2 + r^2}$$

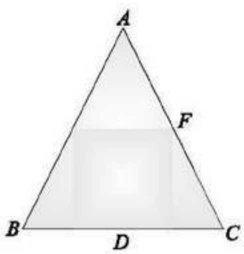
$$= \pi r^2\sqrt{2}$$

$\therefore$ Ratio between that

$$= 2\pi r^2 ; 2\pi r^2 : \sqrt{2}\,\pi r^2$$

$$= \sqrt{2} : \sqrt{2} : 1$$

22. (*b*) Let in this 2-dimensional figure $AE = h$, $EF = r$



$\therefore \quad ED = (10 - h) \quad [\because AD - AE]$

$\because$ ΔAEF and DAC are similar

$$\frac{AF}{EF} = \frac{AD}{CD}$$

$$\Rightarrow \quad \frac{h}{r} = \frac{10}{4}$$

$\Rightarrow h = \frac{5r}{2}$; height of cylinder $(H) = \left(10 - \frac{5r}{2}\right)$

$\therefore$ Curved surface area of cylinder

$$= 2\pi r\left(10 - \frac{5r}{2}\right) = 5\pi r(4 - r).$$

23. (*c*) Volume of pyramid $= \frac{1}{3} \times 100 \times 20 \text{ cm}^3$

New height of pyramid = 22 cm

Let the new side of square be x cm

$\therefore$ new volume $= \frac{1}{3} \times x^2 \times 22 \text{ cm}^3$

$$\therefore \frac{1}{3} \times x^2 \times 22 = \frac{1}{3} \times 100 \times 20$$

$$x^2 = \frac{100 \times 20}{22} = 90.90$$

$$x \approx = 9.5$$

$\therefore$ Percentage change in the side of square base

$$= \frac{10 - 9.5}{10} \times 100 = 5\% \text{ less.}$$

24. (*a*) As the ratio of height of the cylindrical base to that of the conical top is 5:3, let their hugs be $5K$ and $3K$, respectively. Let R be the radius of the cylinder (as well as the cone) and let r be the radius of the cylindrical hole.

Given that the height of cylindrical hole $= \frac{8K}{3}$

$$= \frac{x}{3}(R^2)3K + x(R^2)5K$$

$$= 6xKR^2$$

Given that the volume of the solid left after the hole is made = 2 (volume of the cylindrical hole)

$$= \frac{6xKR^2}{3} = 2xKR^2$$

$$= x(r^2)\frac{8}{3}K$$

$$r^2 = \frac{3}{4}R^2 \Rightarrow r = \frac{\sqrt{3}}{2}R$$

But it is given that $R = 8$

$$r = \frac{\sqrt{3}}{2} \times 8 = 4\sqrt{3}.$$

25. (*b*) Let x bricks are placed into water.

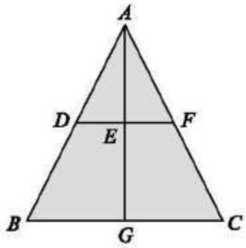
$\therefore$ x bricks absorb $48x$ cm^3 water.

$\therefore \quad 1281600 - 48x + x(480) = 150 \times 120 \times 100$

$\Rightarrow \quad x = 1200.$

26. (*d*) It is the two-dimensional figure of the cone *ADF* is the part which is left

$AG = 8$ cm



Let $GC = r$ cm, $EF = r_1$ cm, $AE = h$ cm.

$$\frac{AE}{EF} = \frac{AG}{GC}$$

[$\because$ ΔAEF and ΔAGC are similar]

$$\therefore \quad \frac{h}{r_1} = \frac{8}{r}$$

$$\Rightarrow \quad r_1 = \frac{hr}{8}$$

Volume of the smaller cone

$$= \frac{1}{3}\pi(r_1)^2 h$$

$$= \frac{1}{3}\pi\left(\frac{hr}{8}\right)^2 h = \frac{1}{3}\frac{\pi h^3 r^2}{64}$$

Volume of bigger cone $= \frac{1}{3}\pi r^2 \cdot 8$

$\because$ smaller cone is 12.5% of the bigger cone

$$\therefore \quad \frac{1}{3}\pi r^2 \times 8 \times \frac{12\cdot5}{100} = \frac{1}{3} \times \frac{\pi r^2 h^3}{64}$$

$$\Rightarrow \quad h = 4$$

27. (*d*) Let each side of the given cube = 2 units

It is cut from the middle therefore sides of cuboid will be 2, 2, 1 unit and total surface area

$= 2(2 \times 2 + 2 \times 1 + 2 \times 1)$

$= 16$ unit^2

Area not painted $= 2 \times 2$ unit^2 $= 4$ unit^2

$\therefore$ Percentage of area not painted red

$$= \frac{4}{16} \times 100 = 25\%$$

28. (*c*) Area of 4 walls $= 2(l + b) \times h$

$= 2(12.5 + 9)7 = 301$ m^2

Area of the doors $= 2 \times 2.5 \times 1.2 = 6$ m^2

Area of the window $= 4 \times 1.5 \times 1 = 6$ m^2

$\therefore$ Cost of painting $= (301 - 6 - 6) \times 3.5 =$ ₹1011.5

29. (*c*) Distance in 60 minutes = 66000 m

Distance in 1 minutes $= \frac{66000}{60} = 1100$ m

Hence, number of revolution per minute

$$= \frac{1100}{2 \times \frac{22}{7} \times 0.70} = 250.$$

30. (*d*) Let number of men $= x$

$\therefore$ Increase in volume = Total displacement

$\Rightarrow \quad 24 \times 15 \times 0.01 = 0.1 \times x$

$\Rightarrow \quad x = 36.$

31. (*c*) Length = 4 m, breadth = 7 m height 4.5 m

$$\therefore \ 7 \times 4 \times 4.5 = \left(\frac{5}{100} \times \frac{4}{100} \times x\right) \times \frac{53}{10}$$

(where x is the rate).

On solving, we get $x = 14$ Km/h

32. (*a*) $\because$ Rakesh paints 0.5 sq ft in 1 minutes

$\therefore$ Rakesh paints 470 sq ft in 15 hrs and 40 minutes

Thus, the dimensions of the room is $(10 \times 10 \times 9)$ Cu ft

$\therefore$ Area of 4 walls = 380 sq ft.

$\therefore$ Time taken to paint it = 12 hrs and 40 minutes.

33. (*d*) Let x be the breadth, then $2x \times x \times 0.50 = 12.25$

$\therefore \quad x = 3.5$ thus, length 7

$\therefore \quad$ Area of 4 walls $= 2(l + b)h$

$\Rightarrow \quad 2(3.5 + 7)h \times 0.09 = 6.30$

$$\therefore \quad h = \frac{30}{9}.$$

34. (*c*)

ΔPQR is right angled at R.

Therefore, for 1 spiral, it moves 5 ft.

From above, we can say that 4 ft height is equivalent to 5 ft.

So, 16 ft height $\rightarrow \frac{16 \times 5}{4} = 20$ ft

35. (*c*) Volume of the sphere $= \frac{4}{3}\pi r^3$ (1)

Volume of the right circular cone $= \frac{1}{3}\pi r^2 H$

Diameter of the base of the cone

$$= \sqrt{2}\,(\text{slant height})$$

$$= \sqrt{2}l \text{ (say)}$$

$$\Rightarrow 4R^2 = 2l^2 = 2\left(H^2 + R^2\right)$$

$$\Rightarrow 2R^2 = 2H^2 \Rightarrow R = H$$

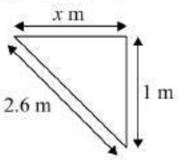
Volume of the cone $= \frac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi r^2 \cdot r$$

$$= \frac{1}{3}\pi r^3 \qquad (2)$$

From (i) and (ii), the material creates exactly 4 cones of the specified dimensions.

36. (*d*)



In right $\Delta CBD, BC = \sqrt{(17)^2 - (15)^2} = 8\text{cm}$

$\because \Delta OED \sim \Delta OAC$

$$\therefore \frac{ED}{AC} = \frac{OE}{OA}$$

$$\Rightarrow \frac{5}{13} = \frac{h}{h+15}$$

$$\Rightarrow \quad h = 9.375\text{cm}$$

Volume of water that overflows

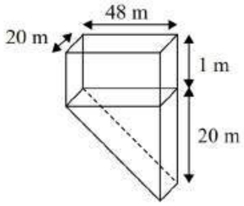
$$= \frac{1}{3}\pi\left[\left(13^2 \times (15+9.375)\right) - \left(5^2 \times 9.375\right)\right]$$

$$= \frac{1}{3}\pi\left[\left(13^2 \times 24.375)\right) - \left(5^2 \times 9.375\right)\right]$$

$$= \frac{1}{3}\pi[4119.375 - 234.375]$$

$$= 1295\pi \text{ cm}^3$$

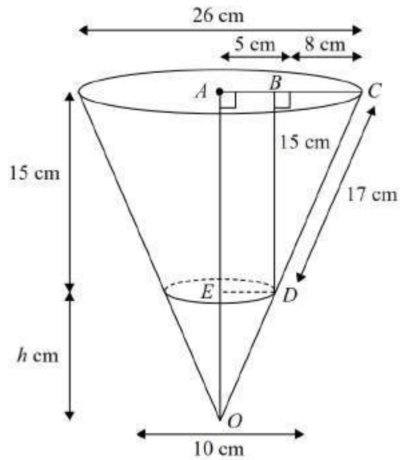
Radius of the hole = 5 mm = 0.5 cm



The volume of water flowing in 1 minute $= (0.5^2 \times 1000)\pi \text{ cm}^3$

Hence, the required time $= \frac{1295\pi}{0.5^2 \times 1000\pi} = 5.18$ minutes

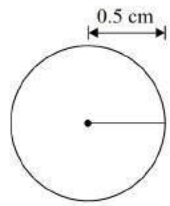
37. (*d*) Elevation increases by 1m for every 2.6 m,



$$x = \sqrt{2.6^2 - 1} = \sqrt{6.76 - 1} = \sqrt{5.76} = 2.4m$$

Now, $\frac{48}{2.4} = 20$

The height of the deeper end of the pool is 20 m. This can be represented as follows:



The total volume of water in the pool

$$= \left(\frac{1}{2} \times 48 \times 20\right) \times 20 + (48 \times 20 \times 1) = 10560 \text{ m}^3.$$

38. (*d*) We have, radius, $r = 5$ m and height $h = 12$ m

$\therefore$ slant height, $l = \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2} = 13$ m

$\therefore$ Area of canvas required $= \pi rl$

$$= \pi \times 5 \times 13$$

$$= 65\pi \text{ m}^2.$$