Chapter - Nuclei

Topic-1: Composition and Size of the Nuclei

1 MCQs with One Correct Answer

1. For uranium nucleus how does its mass vary with volume? **[2003S]**
   (a) $m \propto V$ (b) $m \propto 1/V$ (c) $m \propto \sqrt{V}$ (d) $m \propto V^2$

2. Order of magnitude of density of uranium nucleus is, $[m_p = 1.67 \times 10^{-27} \text{kg}]$ **[1999S - 2 Marks]**
   (a) 10^{20}kg/m^3 (b) 10^{17}kg/m^3
   (c) 10^{14}kg/m^3 (d) 10^{11}kg/m^3

5 True / False

3. The order of magnitude of the density of nuclear matter is 10^4 kg m^{-3}. **[1989 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

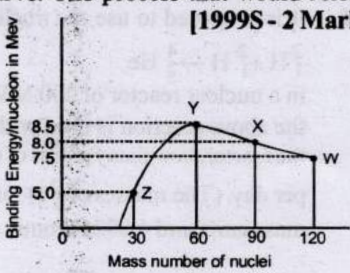
4. The mass number of a nucleus is **[1986 - 2 Marks]**
   (a) always less than its atomic number
   (b) always more than its atomic number
   (c) sometimes equal to its atomic number
   (d) sometimes more than and sometimes equal to its atomic number

Topic-2: Mass-Energy Equivalence and Nuclear Reactions

1 MCQs with One Correct Answer

1. If a star can convert all the He nuclei completely into oxygen nuclei, the energy released per oxygen nuclei is [Mass of He nucleus is 4.0026 amu and mass of Oxygen nucleus is 15.9994 amu] **[2005S]**
   (a) 7.6 MeV (b) 56.12 MeV (c) 10.24 MeV (d) 23.9 MeV

2. Binding energy per nucleon vs mass number curve for nuclei is shown in the Figure. *W, X, Y* and *Z* are four nuclei indicated on the curve. The process that would release energy is **[1999S - 2 Marks]**
   (a) $Y \to 2Z$
   (b) $W \to X + Z$
   (c) $W \to 2Y$
   (d) $X \to Y + Z$



3. Fast neutrons can easily be slowed down by **[1994 - 1 Mark]**
   (a) the use of lead shielding
   (b) passing them through water
   (c) elastic collisions with heavy nuclei
   (d) applying a strong electric field.

3 Numeric / New Stem Based Questions

4. In a radioactive decay chain reaction, $^{230}_{90}\text{Th}$ nucleus decays into $^{214}_{84}\text{Po}$ nucleus. The ratio of the number of α to number of β^- particles emitted in this process is ____. **[Adv. 2022]**

4 Fill in the Blanks

5. Consider the following reaction :
   $^2_1\text{H} + ^2_1\text{H} = ^4_2\text{He} + \text{Q}$
   Mass of the deuterium atom = 2.0141 *u*
   Mass of helium atom = 4.0024 *u*
   This is a nuclear .... reaction in which the energy *Q* released is .... MeV. **[1996 - 2 Marks]**

6. In the nuclear process, $_6\text{C}^{11} \longrightarrow {_5\text{B}^{11}} + \beta^+ + \text{X}$, X stands for .... **[1992 - 1 Mark]**

7. The binding energies per nucleon for deuteron ($_1\text{H}^2$) and helium ($_2\text{He}^4$) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus ($_2\text{He}^4$) is .... **[1988 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

8. The binding energy of nucleons in a nucleus can be affected by the pairwise Coulomb repulsion. Assume that all nucleons are uniformly distributed inside the nucleus. Let the binding energy of a proton be E_b^p and the binding energy of a neutron be E_b^n in the nucleus. **[Adv. 2022]**
Which of the following statement(s) is(are) correct?

(a) $E_b^p - E_b^n$ is proportional to $Z(Z-1)$ where Z is the atomic number of the nucleus.

(b) $E_b^p - E_b^n$ is proportional to $A^{-\frac{1}{3}}$ where A is the mass number of the nucleus.

(c) $E_b^p - E_b^n$ is positive.

(d) E_b^p increases if the nucleus undergoes a beta decay emitting a positron.

9. A heavy nucleus N, at rest, undergoes fission $N \to P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$ where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P and E_Q are the kinetic energies of P and Q, respectively. The speeds of P and Q are v_P and v_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct? **[Adv. 2021]**

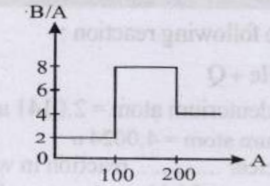
(a) $E_P + E_Q = c^2\delta$

(b) $E_P = \left(\dfrac{M_P}{M_P + M_Q}\right)c^2\delta$

(c) $\dfrac{v_P}{v_Q} = \dfrac{M_Q}{M_P}$

(d) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu = \dfrac{M_P M_Q}{(M_P + M_Q)}$

10. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below. **[2008]**



(a) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.

(b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy

(c) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments

(d) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments

11. Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}^{20}_{10}\text{Ne}$ nucleus and M_2 the mass of a ${}^{40}_{20}\text{Ca}$ nucleus. Then **[1998S - 2 Marks]**

(a) $M_2 = 2M_1$
(b) $M_2 > 2M_1$
(c) $M_2 < 2M_1$
(d) $M_1 < 10(m_n + m_p)$

12. Which of the following statement(s) is (are) correct? **[1994 - 2 Marks]**

(a) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons
(b) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons
(c) In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 amu)
(d) In nuclear fission, energy is released by fragmentation of a very heavy nucleus

13. During a nuclear fusion reaction **[1987 - 2 Marks]**

(a) a heavy nucleus breaks into two fragments by itself
(b) a light nucleus bombarded by thermal neutrons breaks up
(c) a heavy nucleus bombarded by thermal neutrons breaks up
(d) two light nuclei combine to give a heavier nucleus and possibly other products

14. From the following equations pick out the possible nuclear fusion reactions **[1984- 2 Marks]**

(a) ${}_6\text{C}^{13} + {}_1\text{H}^1 \to {}_6\text{C}^{14} + 4.3\text{MeV}$
(b) ${}_6\text{C}^{12} + {}_1\text{H}^1 \to {}_7\text{N}^{13} + 2\text{MeV}$
(c) ${}_7\text{N}^{14} + {}_1\text{H}^1 \to {}_8\text{O}^{15} + 7.3\text{MeV}$
(d) ${}_{92}\text{U}^{235} + {}_0n^1 \to {}_{54}\text{Xe}^{140} + {}_{38}\text{Sr}^{94} + {}_0n^1 + {}_0n^1 + \gamma + 200\text{MeV}$

10 Subjective Problems

15. In a nuclear reaction ${}^{235}\text{U}$ undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required. **[2001 - 5 Marks]**

16. It is proposed to use the nuclear fusion reaction

${}^2_1\text{H} + {}^2_1\text{H} \to {}^4_2\text{He}$ **[1990 - 8 Marks]**

in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day. (The masses of ${}^2_1\text{H}$ and ${}^4_2\text{He}$ are 2.0141 atomic mass units and 4.0026 atomic mass units respectively)

Topic-3: Radioactivity

1 MCQs with One Correct Answer

1. A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decays of Q in the first one hour is **[Adv. 2021]**

(a) 50 (b) 75 (c) 350 (d) 525

2. In a radioactive sample, $^{40}_{19}K$ nuclei either decay into stable $^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}Ca$ and $^{40}_{18}Ar$ nuclei are produced by the $^{40}_{19}K$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable $^{40}_{20}Ca$ and $^{40}_{20}Ca$ nuclei to the radioactive $^{40}_{19}K$ nuclei is 99, the value of t will be [Given: $ln\ 10 = 2.3$] **[Adv. 2019]**

(a) 9.2 (b) 4.6 (c) 1.15 (d) 2.3

3. A radioactive sample S_1 having an activity 5μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 can be **[2008]**

(a) 20 years and 5 years, respectively
(b) 20 years and 10 years, respectively
(c) 10 years each
(d) 5 years each

4. $^{221}_{87}$Ra is a radioactive substance having half life of 4 days. Find the probability that a nucleus undergoes decay after two half lives **[2006 - 3M, –1]**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

5. A 280 days old radioactive substance shows an activity of 6000 dps, 140 days later its activity becomes 3000 dps. What was its initial activity? **[2004S]**

(a) 20000 dps (b) 24000 dps
(d) 12000 dps (d) 6000 dps

6. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle **[2003S]**

(a) 4.4 MeV (b) 5.4 MeV (c) 5.6 MeV (d) 6.5 MeV

7. Which of the following processes represents a γ-decay? **[2002S]**

(a) $^{A}X_{z} + \gamma \longrightarrow {}^{A}X_{Z-1} + a + b$ dr

(b) $^{A}X_{z} + {}^{1}n_{o} \longrightarrow {}^{A-3}X_{Z-2} + c$

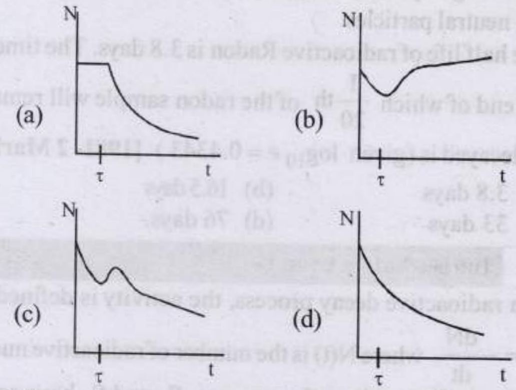
(c) $^{A}X_{z} \longrightarrow {}^{A}X_{Z} + f$

(d) $^{A}X_{z} + e_{-1} \longrightarrow {}^{A}X_{Z-1} + g$

8. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is **[2002S]**

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

9. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life time of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represent the form of this plot? **[2001S]**



10. The electron emitted in beta radiation originates from **[2001S]**

(a) inner orbits of atoms
(b) free electrons existing in nuclei
(c) decay of a neutron in a nucleus
(d) photon escaping from the nucleus

11. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time **[2000S]**

(a) $\frac{1}{10\lambda}$ (b) $\frac{1}{11\lambda}$
(c) $\frac{11}{10\lambda}$ (d) $\frac{1}{9\lambda}$

12. Which of the following is a correct statement? **[1999S - 2 Marks]**

(a) Beta rays are same as cathode rays
(b) Gamma rays are high energy neutrons
(c) Alpha particles are singly ionised helium atoms
(d) Protons and neutrons have exactly the same mass

13. A radioactive material decays by simultaneous emission of two particles with respective half-lives 1620 and 810 years. The time, in years, after which one-fourth of the material remains is **[1995S]**

(a) 1080 (b) 2430
(c) 3240 (d) 4860

14. Consider α particles, β particles and γ - rays, each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are: **[1994 - 1 Mark]**

(a) α, β, γ (b) α, γ, β
(c) β, γ, α (d) γ, β, α

15. The equation

$4\,{}^1_1H^+ \longrightarrow {}^4_2He^{2+} + 2e^- + 26\,\text{MeV}$ represents **[1983 - 1 Mark]**

(a) β-decay (b) γ-decay
(c) fusion (d) fission

16. Beta rays emitted by a radioactive material are **[1983 - 1 Mark]**

(a) electromagnetic radiations
(b) the electrons orbiting around the nucleus
(c) charged particles emitted by the nucleus
(d) neutral particles

17. The half life of radioactive Radon is 3.8 days. The time at the end of which $\frac{1}{20}$th of the radon sample will remain undecayed is (given $\log_{10} e = 0.4343$) **[1981- 2 Marks]**

(a) 3.8 days (b) 16.5 days
(c) 33 days (d) 76 days.

2 Integer Value Answer

18. In a radioactive decay process, the activity is defined as $A = -\frac{dN}{dt}$, where N(t) is the number of radioactive nuclei at time t. Two radioactive sources, S_1 and S_2 have same activity at time t = 0. At a later time, the activities of S_1 and S_2 are A_1 and A_2, respectively. When S_1 and S_2 have just completed their 3^{rd} and 7^{th} half-lives, respectively, the ratio A_1/A_2 is ________ **[Adv. 2023]**

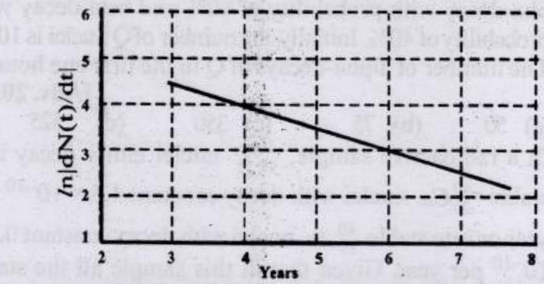
19. ^{131}I is an isotope of Iodine that B decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from person's body, and gives an activity of 115 Bq. The total volume of blood in the person¡¦s body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$). **[Adv. 2017]**

20. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is **[Adv.2015]**

21. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $ln2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is **[Adv. 2013-I]**

22. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is **[2011]**

23. To determine the half life of a radioactive element, a student plots a graph of $ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\left|\frac{dN(t)}{dt}\right|$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is **[2010]**



3 Numeric / New Stem Based Questions

24. The minimum kinetic energy needed by an alpha particle to cause the nuclear reaction ${}^{16}_7N + {}^4_2He \to {}^1_1H + {}^{19}_8O$ in a laboratory frame in n (in MeV). Assume that ${}^{16}_7N$ is at rest in the laboratory frame. The masses of ${}^{16}_7N$, ${}^4_2He$, ${}^1_1H$ and ${}^{19}_8O$ can be taken to be $16.006u$, $4.003u$, $1.008u$ and $19.003u$, respectively, where $1\ u = 930\ MeVc^{-2}$. The value of n is ______. **[Adv. 2022]**

4 Fill in the Blanks

25. When Boron nucleus $\left({}^{10}_5B\right)$ is bombarded by neutrons, α-particles are emitted. The resulting nucleus is of the element .... and has the mass number ...... **[1986 - 2 Marks]**

26. In the Uranium radioactive series the initial nucleus is ${}^{238}_{92}U$ and the final nucleus is ${}^{206}_{82}Pb$. When the Uranium nucleus decays to lead, the number of α-particles emitted is ......and the number of β-particles emitted is ..... **[1985 - 2 Marks]**

27. The radioactive decay rate of a radioactive element is found to be 10^3 disintegration/second at a certain time. If the half life of the element is one second, the decay rate after one second is ________ and after three seconds is ________. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

28. In a radioactive decay chain, ${}^{232}_{90}Th$ nucleus decays to ${}^{212}_{82}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true? **[Adv. 2018]**

(a) $N_\alpha = 5$ (b) $N_\alpha = 6$ (c) $N_\beta = 2$ (d) $N_\beta = 4$

29. The half-life period of a radioactive element X is same as the mean-life time of another radioactive element Y. Initially both of them have the same number of atoms. Then **[1999S - 3 Marks]**

(a) X and Y have the same decay rate initially
(b) X and Y decay at the same rate always
(c) Y will decay at a faster rate than X
(d) X will decay at a faster rate than Y

30. The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that **[1998S - 2 Marks]**

(a) no nucleus will decay before $t = 4$ days
(b) no nucleus will decay before $t = 8$ days
(c) all nuclei will decay before $t = 16$ days
(d) a given nucleus may decay at any time after $t = 0$

31. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are respectively given by **[1989 - 2 Marks]**

(a) $1/\lambda$ and $(\ln 2)/\lambda$ (b) $(\ln 2)/\lambda$ and $1/\lambda$
(c) $\lambda(\ln 2)$ and $1/\lambda$ (d) $\lambda/(\ln 2)$ and $1/\lambda$

32. A freshly prepared radioactive source of half life 2 hr emits radiation of intensity which is 64 times the permissibe safe level. The minimum time after which it would be possible to work safely with this source is **[1988 - 2 Marks]**

(a) 6 hr (b) 12 hr (c) 24 hr (d) 128 hr

33. During a negative beta decay **[1987 - 2 Marks]**

(a) an atomic electron is ejected
(b) an electron which is already present within the nucleus is ejected
(c) a neutron in the nucleus decays emitting an electron
(d) a part of the binding energy of the nucleus is converted into an electron

7 Match the Following

34. List–I shows different radioactive decay processes and List–II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option. **[Adv. 2023]**

List-I	List-II
(P) $^{238}_{92}U \to ^{234}_{91}Pa$	(1) one α particle and one β^+ particle
(Q) $^{214}_{82}Pb \to ^{210}_{82}Pb$	(2) three β^- particles and one α particle
(R) $^{210}_{81}Tl \to ^{206}_{82}Pb$	(3) two β^- particles and one α particle
(S) $^{228}_{91}Pa \to ^{224}_{88}Ra$	(4) one α particle and one β^- particle
	(5) one α particle and two β^+ particles

(a) $P \to 4, Q \to 3, R \to 2, S \to 1$
(b) $P \to 4, Q \to 1, R \to 2, S \to 5$
(c) $P \to 5, Q \to 3, R \to 1, S \to 4$
(d) $P \to 5, Q \to 1, R \to 3, S \to 2$

35. Match the nuclear processes given in column I with the appropriate option(s) in column II. **[Adv. 2015]**

Column I	Column II
(A) Nuclear fusion	(p) Absorption of thermal neutrons by $^{235}_{92}U$
(B) Fission in a nuclear reactor	(q) $^{60}_{27}Co$ nucleus
(C) β-decay	(r) Energy production in stars via hydrogen conversion to helium
(D) γ-ray emission	(s) Heavy water
	(t) Neutrino emission

36. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists: **[Adv. 2013-II]**

List I	List II
P. Alpha decay	1. $^{15}_{8}O \to ^{15}_{7}O + ...$
Q. β+ decay	2. $^{138}_{92}U \to ^{234}_{90}Th + ...$
R. Fission	3. $^{185}_{83}Bi \to ^{184}_{82}Pb + ...$
S. Proton emission	4. $^{239}_{94}Pu \to ^{140}_{57}La + ...$

Codes:

	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

37. Given below are certain matching type questions, where two columns (each having 4 items) are given. Immediately after the columns the matching grid is given, where each item of Column I has to be matched with the items of Column II, by encircling the correct match(es). Note that an item of column I can match with more than one item of column II. All the items of column II must be matched. Match the following : **[2006 - 6M]**

Column I	Column II
(A) Nuclear fusion	(p) Converts some matter into energy
(B) Nuclear fission	(q) Generally possible for nuclei with low atomic number
(C) β-decay	(r) Generally possible for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(s) Essentially proceeds by weak nuclear forces

8 Comprehension/Passage Based Questions

Passage-1

The mass of a nucleus ${}^{A}_{Z}X$ is less than the sum of the masses of (A-Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

${}^{1}_{1}H$	1.007825 u	${}^{2}_{1}H$	2.014102 u
${}^{3}_{1}H$	3.016050 u	${}^{4}_{2}He$	4.002603 u
${}^{6}_{3}Li$	6.015123 u	${}^{7}_{3}Li$	7.016004 u
${}^{70}_{30}Zn$	69.925325 u	${}^{82}_{34}Se$	81.916709 u
${}^{152}_{64}Gd$	151.919803 u	${}^{206}_{82}Pb$	205.974455 u
${}^{209}_{83}Bi$	208.980388 u	${}^{210}_{84}Po$	209.982876 u

($1u = 932$ MeV/c^2)

[Adv. 2013]

38. The kinetic energy (in keV) of the alpha particle, when the nucleus ${}^{210}_{84}Po$ at rest undergoes alpha decay, is

(a) 5319 (b) 5422 (c) 5707 (d) 5818

39. The correct statement is

(a) The nucleus ${}^{6}_{3}Li$ can emit an alpha particle

(b) The nucleus ${}^{210}_{84}Po$ can emit a proton

(c) Deuteron and alpha particle can undergo complete fusion

(d) The nuclei ${}^{70}_{30}Zn$ and ${}^{82}_{34}Se$ can undergo complete fusion

10 Subjective Problems

40. A radioactive sample of ^{238}U decays to Pb through a process for which the half-life is 4.5×10^9 years. Find the ratio of number of nuclei of Pb to ^{238}U after a time of 1.5×10^9 years. Given $(2)^{1/3} = 1.26$. **[2004 - 2 Marks]**

41. A radioactive sample emits n β-particles in 2 sec. In next 2 sec it emits $0.75\ n$ β-particle, what is the mean life of the sample? **[2003 - 2 Marks]**

42. A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_x = 0.1\ s^{-1}$. Y further decays to a stable nucleus Z with a decay constant $\lambda_Y = 1/30\ s^{-1}$. Initially, there are only X nuclei and their number is $N_o = 10^{20}$. Set up the rate equations for the populations of X, Y and Z. The population of Y nucleus as a function of time is given by $N_Y(t) = (N_o\lambda_X/(\lambda_X - \lambda_Y))\{\exp(-\lambda_Y t)-\exp(-\lambda_X t)\}$. Find the time at which N_Y is maximum and determine the populations X and Z at that instant. **[2001-5 Marks]**

43. Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element. **[1998 - 8 Marks]**

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of N as $t \to \infty$.

44. At a given instant there are 25% undecayed radio-active nuclei in a sample. After 10 seconds the number of undecayed nuclei reduces to 12.5%. Calculate (i) mean-life of the nuclei, and (ii) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number. **[1996 - 3 Marks]**

45. A small quantity of solution containing Na^{24} radio nuclide (half life = 15 hour) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hour show an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that radioactive solution mixes uniformly in the blood of the person. (1 curie = 3.7×10^{10} disintegrations per second) **[1994 - 6 Marks]**

46. A nucleus X, initially at rest, undergoes alpha decay according to the equation, ${}^{A}_{92}X \to {}^{228}_{Z}Y + \alpha$ **[1991 - 4 + 4 Marks]**

(a) Find the values of A and Z in the above process.

(b) The alpha particle produced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 Tesla. Find the energy (In MeV) released during the process and the binding energy of the parent nucleus X.

Given that : $m(Y) = 228.03\ u$; $m\left({}^{1}_{0}n\right) = 1.009\ u$.

$m\left({}^{4}_{2}He\right) = 4.003\ u$; $m\left({}^{1}_{1}H\right) = 1.008\ u$

Topic-4: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? **[Adv. 2016]**

(a) 64 (b) 90 (c) 108 (d) 120

2. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron ${}^{1}_{1}H$, ${}^{15}_{7}N$ and ${}^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_{7}N$ and ${}^{15}_{8}O$

nuclei are same, 1 u = 931.5 Me V/c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0)$ = 1.44 MeV fm. Assuming that the difference between the binding energies of $^{15}_{7}N$ and $^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = 10^{-15} m) **[Adv. 2016]**

(a) 2.85 fm (b) 3.03 fm (c) 3.42 fm (d) 3.80 fm

3. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is **[1999S - 2 Marks]**

(a) nitrogen (b) carbon (c) boron (d) oxygen

4. The isotope $^{12}_{5}B$ having a mass 12.014 u undergoes β–decay to $^{12}_{6}C$. $^{12}_{6}C$ has an excited state of the nucleus $\left(^{12}_{6}C^*\right)$ at 4.041 MeV above its ground state. If $^{12}_{5}E$ decays to $^{12}_{6}C^*$, the maximum kinetic energy of the β–particle in units of MeV is (1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum). **[Adv. 2016]**

5. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is **[Adv. 2015]**

4 Fill in the Blanks

6. Atoms having the same .... but different .... are called isotopes. **[1986 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

7. A fission reaction is given by $^{236}_{92}U \to {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering $^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2 MeV) and K_y(2 MeV), respectively. Let the binding energies per nucleon of $^{236}_{92}U$, $^{140}_{54}Xe$ and $^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is(are) **[Adv. 2015]**

(a) $x = n, y = n, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(b) $x = p, y = e^-, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(c) $x = p, y = n, K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV
(d) $x = n, y = n, K_{Sr} = 86$ MeV, $K_{Xe} = 129$ MeV

8. A star initially has 10^{40} deuterons. It produces energy via the processes $_1H^2 + {}_1H^2 \to {}_1H^3 + p$, and $_1H^2 + {}_1H^3 \to {}_2He^4 + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of **[1993-2 Marks]**

(a) $10^6 s$. (b) $10^8 s$. (c) $10^{12} s$. (d) $10^{16} s$.

The masses of the nuclei are as follows :

$M(H^2) = 2.014$ amu;

$M(p) = 1.007$ amu; $M(n) = 1.008$ amu; $M(He^4) = 4.001$ amu.

7 Match the Following

9. Four physical quantities are listed in Column I. Their values are listed in Column II in a random order: **[1987 - 2 Marks]**

Column I	Column II
(a) Thermal energy of air molecules at room temp	(e) 0.02 eV
(b) Binding energy of heavy nuclei per nucleon	(f) 2 eV
(c) X-ray photon energy	(g) 1 keV
(d) Photon energy of visible light	(h) 7 MeV

The correct matching of Columns I and II is given by

(a) $a-e, b-h, c-g, d-f$ (b) $a-e, b-g, c-f, d-h$
(c) $a-f, b-e, c-g, d-h$ (d) $a-f, b-h, c-e, d-g$.

10. **Column-II** gives certain systems undergoing a process. **Column-I** suggests changes in some of the parameters related to the system. Match the statements in **Column-I** to the approapriate process(es) from **Column-II**. **[2009]**

Column-I	Column-II
(A) The energy of the system is increased	(p) *System :* A capacitor, initially uncharged *Process :* It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q) *System :* A gas in an adiabatic container fitted with an adiabatic piston *Process:* The gas is compressed by pushing the piston
(C) Internal energy of the system is converted into its mechanical energy	(r) *System:* A gas in a rigid container *Process:* The gas gets cooled due to colder atmosphere surrounding it
(D) Mass of the system is decreased	(s) *System:* A heavy nucleus, initially at rest *Process:* The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t) *System:* A resistive wire loop *Process:* The loop is placed in a time varying magnetic field perpendicular to its plane

11. In the following, column I lists some physical quantities and the column II gives approximate energy values associated with some of them. Choose the appropriate value of energy from column II for each of the physical quantities in column I and write the corresponding letter p, q, r, etc.

against the number (A), (B), (C), (D) etc. of the physical quantity in the answer book. In your answer, the sequence of column I should be maintained. **[1997 - 4 Marks]**

Column I	Column II
(A) Energy of thermal neutrons	(p) 0.025 eV
(B) Energy of X-rays	(q) 0.5 eV
(C) Binding energy per nucleon	(r) 3 eV
(D) Photoelectric threshold of a metal	(s) 20 eV
	(t) 10 k eV
	(u) 8 M eV

8 Comprehension/Passage Based Questions

Passage-1

The β -decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \to p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

12. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron? **[2012]**

(a) $0 \le K \le 0.8 \times 10^6 eV$

(b) $3.0\ eV \le K \le 0.8 \times 10^6 eV$

(c) $3.0\ eV \le K < 0.8 \times 10^6 eV$

(d) $0 \le K < 0.8 \times 10^6 eV$

13. What is the maximum energy of the anti-neutrino? **[2012]**

(a) Zero

(b) Much less than 0.8×10^6 eV.

(c) Nearly 0.8×10^6 eV

(d) Much larger than 0.8×10^6 eV

Passage-2

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_1^2$H, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is ${}_1^2\text{H} + {}_1^2\text{H} \to {}_2^3\text{He} + n +$ energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_1^2H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away form the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3. It may be helpful to use the following: Boltzmann constant

$k = 8.6 \times 10^{-5}$ eV/K; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm **[2009]**

14. In the core of nuclear fusion reactor, the gas becomes plasma because of

(a) strong nuclear force acting between the deuterons

(b) coulomb force acting between the deuterons

(c) coulomb force acting between deuteron-electron pairs

(d) the high temperature maintained inside the reactor core

15. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(a) $1.0 \times 10^9\,K < T < 2.0 \times 10^9\,K$

(b) $2.0 \times 10^9\,K < T < 3.0 \times 10^9\,K$

(c) $3.0 \times 10^9\,K < T < 4.0 \times 10^9\,K$

(d) $4.0 \times 10^9\,K < T < 5.0 \times 10^9\,K$

16. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(a) deuteron density = 2.0×10^{12} cm^{-3}, confinement time = 5.0×10^{-3} s

(b) deuteron density = 8.0×10^{14} cm^{-3}, confinement time = 9.0×10^{-1} s

(c) deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11} s

(d) deuteron density = 1.0×10^{24} cm^{-3}, confinement time = 4.0×10^{-12} s

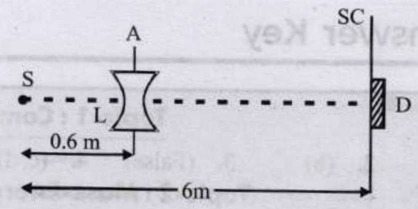
10 Subjective Problems

17. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. Find (a) atomic number of the nucleus. (b) the frequency of K_α line of the X-ray produced. ($R = 1.1 \times 10^7$ m^{-1} and $c = 3 \times 10^8$ m/s) **[2005 - 4 Marks]**

18. A nucleus at rest undergoes a decay emitting an α particle of de-Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α particles is 4.002 amu, determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in amu. (1 amu = 931.470 MeV/c^2) **[2001-5 Marks]**

19. The element Curium $^{248}_{96}$Cm has a mean life of 10^{13} seconds. Its pirmary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in α-decay are as follows: $^{248}_{96}$Cm = 248.072220*u*, $^{244}_{94}$Pu = 244.064100*u* and $^{4}_{2}$He=4.002603 *u*. Calculate the power output from a sample of 10^{20} Cm atoms. (1 u = 931 MeV/c^2.) **[1997 - 5 Marks]**

20. A monochromatic point source radiating wavelength 6000 Å, with power 2 watt, an aperture *A* of diameter 0.1 m and a large screen *SC* are placed as shown in fig. A photoemissive detector *D* of surface area 0.5 cm^2 is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9. **[1991 - 2 + 4 + 2 Marks]**



(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.
(b) If the concave lens *L* of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.
(c) If the work function of the photoemissive surface is 1eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

21. How many electron, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24 ? **[1982 - 2 Marks]**

(i) number of electrons =
(ii) number of protons =
(iii) number of neutrons =

Answer Key

Topic-1 : Composition and Size of the Nuclei

1. (a) **2.** (b) **3.** (False) **4.** (c, d)

Topic-2 : Mass-Energy Equivalence and Nuclear Reactions

1. (c) **2.** (c) **3.** (b) **4.** (2) **8.** (a, b, d) **9.** (a, c, d)
10. (b, d) **11.** (c, d) **12.** (a, d) **13.** (d) **14.** (b, c)

Topic-3 : Radioactivity

1. (d) **2.** (a) **3.** (a) **4.** (c) **5.** (b) **6.** (b) **7.** (c) **8.** (a) **9.** (d) **10.** (c)
11. (d) **12.** (a) **13.** (a) **14.** (a) **15.** (c) **16.** (c) **17.** (b) **18.** (16) **19.** (5) **20.** (2)
21. (4) **22.** (1) **23.** (8) **24.** (2.33) **28.** (a, c) **29.** (c) **30.** (d) **31.** (b) **32.** (b) **33.** (c)
34. (a) **35.** A→r, t; B→p, s; C→p, q, r, t; D→p, q, r, t **36.** (c)
37. A→p, q; B→p, r; C→p, s; D→p, q, r **38.** (a) **39.** (c)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (c) **2.** (c) **3.** (b) **4.** (9) **5.** (3) **7.** (a) **8.** (c) **9.** (a)
10. A→p, q, t; B→q; C→s; D→s **11.** A→p; B→t; C→u; D→r **12.** (d) **13.** (c) **14.** (d) **15.** (a)
16. (b)

Hints & Solutions

Topic-1: Composition and Size of the Nuclei

1. **(a)** We know that radius of the nucleus $R = R_0 A^{1/3}$, where A is the mass number.

$\therefore R^3 = R_0^3 A$

Volume, $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \quad \therefore$ Mass $\propto$ volume

2. **(b)** Nuclear density of an atom

$$d = \frac{\text{mass}}{\text{volume}} = \frac{A(1.67\times10^{-27})}{\frac{4}{3}\pi[1.25\times10^{-15}A^{1/3}]^3} \; (A = \text{mass number})$$

$$\left[\because V = \frac{4}{3}\pi R^3, R = R_0 A^{1/3}, R_0 = 1.25\times10^{-15}\right]$$

$\therefore \; d = 2\times10^{17}\text{ kg/m}^3$.

3. **False;** The order of nuclear density is 10^{17} kg/m^3. Density

$$= \frac{m}{V} = \frac{A\times1.67\times10^{-27}}{\frac{4}{3}\pi\left[R_0 A^{1/3}\right]^3} = \frac{1.67\times10^{-27}}{1.33\times3.14\times(1.1\times10^{-15})}$$

$= 3\times10^{17}\text{ kg/m}^3$

4. **(c, d)** In the case of hydrogen, atomic number = mass number (= 1) In the other atoms, atomic number (z) < mass number (A = Z + n).

Topic-2: Mass-Energy Equivalence and Nuclear Reactions

1. **(c)** $4\,{}_2^4\text{He} \longrightarrow {}_8^{16}\text{O}$

B.E. $= \Delta m \times 931.5$ MeV

$= (4\times4.0026 - 15.9994)\times931.5 = 10.24$ MeV

2. **(c)** Energy is released when stability increases. This will happen when binding energy per nucleon increases i.e.,

$$\left(\frac{B.E.}{A}\right)_{\text{Product}} > \left(\frac{B.E.}{A}\right)_{\text{Reactant}}.$$

$\because \quad |M_1V_1| = |M_2V_2|$

Reactant	Product
Reaction (a) $60\times8.5\text{MeV} = 510\text{MeV}$	$2\times30\times5 = 300$ MeV
Reaction (b) $120\times7.5 = 900$ MeV	$(90\times8 + 30\times5) = 870$ MeV
Reaction (c) $120\times7.5 = 900$ MeV	$2\times60\times8.5 = 1020$ MeV
Reaction (d) $90\times8 = 720$ MeV	$(60\times8.5 + 30\times5) = 600$ MeV

3. **(b)** Fast neutrons can be easily slowed down by passing them through water. In nuclear reactors heavy water in used as moderator.

4. **(2)** ${}_{90}^{230}\text{Th} \longrightarrow {}_{84}^{214}\text{Po} + n\,{}_2^4\text{He} + m\,{}_{-1}^{0}e$

$\Rightarrow \; 230 = 214 + 4n$, and $90 = 84 + 2n - m$

$\Rightarrow \; 16 = 4n$ and $6 = 2n - m$

$\Rightarrow \; n = 4 \qquad \Rightarrow 6 = 8 - m\,[\because n = 4]$

$\Rightarrow m = 2$

So, $\frac{n}{m} = \frac{4}{2} = 2$

5. This is a nuclear fusion reaction. In a nuclear fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus.

Energy released $Q = (\Delta m)\,[931.5\; MeV/u]$

$= [2\times2.0141 - 4.0024]\times931.5\; MeV \approx 24\; MeV$

6. ${}_6^{11}C \to {}_5^{11}B + \beta^+ + X \Rightarrow {}_6^{11}C \to {}_5^{11}B + {}_{+1}^{0}e + \nu$ (neutrino)

When balancing atomic number and mass number both sides of arrow then, X stands for neutrino.

7. ${}_1^2\text{H} + {}_1^2\text{H} \longrightarrow {}_2^4\text{He}$

Binding energy of two deuterons

$= 2\,[1.1\times2] = 4.4$ MeV

Binding energy of helium nucleus $= 4\times7.0 = 28\; MeV$

$\therefore$ Energy released $= 28 - 4.4 = 23.6\; MeV$

8. **(a,b,d)** Binding energy of proton and neutron due to nuclear force is same. So difference is due to electrostatic potential repulsion energy and it is +ve. So $E_b^p - E_b^n$ = electrostatic potential energy.

Number of proton pair $= {}^{Z}C_2 = \frac{Z(Z-1)}{2}$

So, repulsion energy $\propto \frac{Z(Z-1)}{2}\times\frac{1}{4\pi\epsilon_0}\frac{e^2}{R}$,

R = radius of nuclei

$E_b^p - E_b^n \propto Z(Z-1) \Rightarrow$ (a) is correct.

As $R = R_0 A^{1/3}$

So, $E_b^p - E_b^n \propto A^{-1/3} \Rightarrow$ (b) is correct.

Because of repulsion

$E_b^p < E_b^n \Rightarrow$ (c) is incorrect.

Since in β^+ decay, number of proton decrease $\Rightarrow$ repulsions decreases $\Rightarrow E_b^p$ increases (d) is correct.

9. **(a, c, d)** For nuclear fission reaction,

$N \rightarrow P + Q$

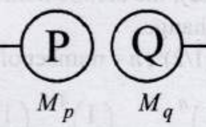
Energy released $= \Delta Mc^2 = (M_N - M_P - M_Q)\, c^2 = \delta c^2$

This will be distributed as kinetic energy of *P* and *Q*

$\therefore \quad Ep + Eq = \delta c^2 \qquad \ldots \text{(i)}$

By conservation of momentum

$V_P M_P = V_Q M_Q$

$V_P = \dfrac{P}{M_P}$  $\dfrac{P}{M_q} = V_q$

$\therefore \quad \dfrac{V_P}{V_q} = \dfrac{M_q}{M_p} \qquad \ldots \text{(ii)}$

Kinetic energy be written as $KE = \dfrac{P^2}{2m}$

Hence divided in inverse ratio of masses

$E_P = \dfrac{M_q}{M_P + M_q} c^2 \delta \qquad \ldots \text{(iii)}$

By equation (i)

$\Rightarrow \quad \dfrac{P^2}{2M_P} + \dfrac{P^2}{2M_q} = \delta c^2$

$\Rightarrow \quad \dfrac{P^2}{2\mu} = \delta c^2 \Rightarrow P = c\sqrt{2\mu\delta} \quad \left[\because \mu = \dfrac{M_P M_Q}{M_P + M_Q}\right]$

10. **(b, d)** In fusion two or more lighter nuclei combine to form a comparatively heavier nucleus. When binding energy per nucleon increases for a nuclear process, energy is released. In fission, a heavy nucleus breaks into two or more lighter nuclei.

(a) For $1 < A < 50$, on fusion mass number of the resulting nucleus will be less than 100.

(b) For $51 < A < 100$, on fusion mass number the resulting nucleus is between 100 and 200. B/A increases, energy will be released.

(c) On fission for $100 < A < 200$, the mass number for fission nuclei will be between 50 to 100. B/A decreases, no energy will be released.

(d) On fission for $200 < A < 260$, the mass number for fission nuclei will be between 100 to 130, B/A will increase, energy will be released.

11. **(c, d)** Due to mass defect which is finally responsible for the binding energy of the nucleus, mass of a nucleus is always less than the sum of masses of its constituent particles. i.e., protons and neutrons.

${}^{20}_{10}\text{Ne}$ is made up of 10 protons and 10 neutrons.

$\therefore$ Mass of ${}^{20}_{10}\text{Ne}$ nucleus

$M_1 < 10\,(m_p + m_n)$

Heavier the nucleus, more is the mass defect

$20\,(m_n + m_p) - M_2 > 10\,(m_p + m_n) - M_1$

or, $10\,(m_n + m_p) > M_2 - M_1$

or, $M_2 < M_1 + 10\,(m_p + m_n)$

Now, since $M_1 < 10\,(m_p + m_n)$

$\therefore \quad M_2 < 2M_1$

12. **(a, d)** In nuclear fission, a heavy nucleus breaks into two nuclei and more products and released energy.

13. **(d)** In nuclear fusion reaction two light nuclei combine to give a heavier nucleus and possibly other products and huge amount of energy.

14. **(b, c)** Nuclear fusion occurs when two or more lighter nuclei combine to form a comparatively heavier nucleus with release of a huge amount of energy.

15. Efficiencys $\eta = \dfrac{P_{\text{out}}}{P_{\text{in}}}$

$\therefore \quad P_{\text{in}} = \dfrac{P_{\text{out}}}{\eta} = \dfrac{1000 \times 10^6}{0.1} = 10^{10}\,\text{W}$

Energy $E \qquad = P \times t$

$= 10^{10} \times 86{,}400 \times 365 \times 10$

$= 3.1536 \times 10^{18}\,\text{J}$

$200 \times 1.6 \times 10^{-13}$ J of energy is released by 1 fission

$\therefore 3.1536 \times 10^{18}$ J of energy is released by

$\dfrac{3.1536 \times 10^{18}}{200 \times 1.6 \times 10^{-13}}$ fission $= 0.9855 \times 10^{29}$ fission

$= 0.985 \times 10^{29}$ of U^{235} atoms.

6.023×10^{23} atoms of Uranium has mass 235g

$\therefore 0.9855 \times 10^{29}$ atoms of Uranium has mass

$\dfrac{235 \times 0.9855 \times 10^{29}}{6.023 \times 10^{23}}\,\text{g} = 38451\,\text{kg}$

16. Energy required per day to run the reactor

$E = P \times t = 200 \times 10^6 \times 24 \times 60 \times 60$

$= 1.728 \times 10^{13}\,\text{J}$

Energy released in the given nuclear fusion reaction
= (mass defect) × (931.3) mev
= [2 (2.0141) − 4.0026] × 931.5 MeV
= 23.85 MeV = 23.85 × 106 × 1.6 × 10^{-19} = 38.15 × 10^{-13}J
∴ No. of fusion reactions required

$$= \frac{1.728 \times 10^{13}}{38.15 \times 10^{-13}} = 0.045 \times 10^{26}$$

∴ No. of deuterium required = 2 × 0.045 × 10^{26} = 0.09 × 10^{26}

$$\text{Number of moles of deuterium} = \frac{0.09 \times 10^{26}}{6.02 \times 10^{23}} = 14.95$$

∴ Mass in gram of deuterium = 14.95 × 2 = 29.9 g
Since the energy from the reaction is used with a 25% efficiency in the reactor
= (4 × 29.9)g = 119.6 g

Topic-3: Radioactivity

1. (d) Out of 1000 nuclei of Q, 60% may go α-decay
i.e, 600 nuclei may have α-decay
Decay constant
t = 1 hour = 60 minutes

From $N = N_0 e^{-\lambda t} = 600 \times e^{\frac{\ln 2}{20} \times 60}$
∴ $N = 75$
i.e., 75 Nuclei are left afdter one hour.
So, number of nuclei decayed = 600 − 75 = 525

2. (a) Here $-\frac{dN}{dt} = \lambda_1 N + \lambda_2 N$
In tegrating on both sides

$$t = \frac{2.303}{\lambda_1 + \lambda_2} \log_{10} \frac{N_0}{N} \Rightarrow t = \frac{2.303}{5 \times 10^{-10}} \log_{10} \frac{100}{1}$$

$$\left[\because \lambda_1 + \lambda_2 = 4.510^{-10} + 0.5 \times 10^{-10} \text{ and } N_0 / N = 100\right]$$

∴ $t = 9.2 \times 10^9$ year

3. (a) Let λ_1 and λ_2 be the decay constants and N_1 and N_2 be the number of active nuclei present for the two samples S_1 and S_2 respectively.
i.e., $\lambda_1 N_1 = 5\mu Ci$ and $\lambda_2 N_2 = 10\mu Ci \Rightarrow \lambda_2 N_2 = 2\lambda_1 N_1$
Also, $N_1 = 2N_2$

$\therefore \lambda_2 N_2 = 2\lambda_1 (2N_2)$ or $\lambda_2 = 4\lambda_1 \Rightarrow \frac{\lambda_2}{\lambda_1} = 4$ Also $T_{1/2} = 0.693$

$\therefore (T_{1/2})_1 = 4(T_{1/2})_2$.

4. (c) For a nucleus to disintegrate in two half life, $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ i.e., The probability is $\frac{3}{4}$ as 75% of the nuclei will disintegrate in this time.

5. (b) In two half lives, the activity will remain $\frac{1}{4}$ of its initial activity. ∴ Initial activity = 4 × 6000 = 24000 dps.

6. (b) By conservation of momentum, $p_1 = p_2$

$$\sqrt{2K_1 m_1} = \sqrt{2K_2 m_2} \qquad \left[\because P = \sqrt{2\,km}\right]$$

$$\Rightarrow \sqrt{2K_1(216)} = \sqrt{2K_2(4)}$$

∴ $K_2 = 54K_1$ ...(i)
And given $K_1 + K_2 = 5.5$ MeV ...(ii)
Solving equation (i) and (ii) we get $K_\alpha = K_2 = 5.4$ MeV

7. (c) In γ-decay, the atomic number (Z) and mass number (A) do not change.

8. (a) $A = A_0(1/2)^n$; n = number of half lives.

$$\frac{A_0}{16} = A_0\left(\frac{1}{2}\right)^n \quad \therefore \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n \quad \therefore n = 4$$

Therefore time taken to decay $\frac{1}{16^{th}}$ of its initial value.

$$\therefore t = n \times t_{1/2} = (4 \times 100)\mu s = 400\ \mu s$$

9. (d) $N_1 = N_0 e^{-\lambda_1 t} = N_0 e^{-\frac{t}{\tau}}$ ....(i)

Mean life time $\tau = \frac{1}{\lambda_1}$

Similarly, $N_2 = N_0 e^{-\lambda_2 t} = N_0 e^{-\frac{t}{5\tau}}$ ....(ii) as $5\tau = \frac{1}{\lambda_2}$

Adding eq. (i) and (ii) we get

$$N = N_1 + N_2 = N_0(e^{-t/\tau} + e^{-t/5\tau})$$

The total number of radioactive nuclei (N) as a function of time only decreases exponentially hence graph (d) correctly depicts.

10. (c) In a nucleus a neutron converts into a proton as follows $n \rightarrow p^+ + e^{-1}$
Therefore, decay of neutron is responsible for β-radiation origination

11. (d) Number of nuclei of X_1, $N_1 = N_0 e^{-10\lambda t}$ and number of nuclei of X_2 $N_2 = N_0 e^{-\lambda t}$

$$\therefore \frac{N_1}{N_2} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = \frac{1}{e^{9\lambda t}}$$

Given $\frac{N_1}{N_2}=\frac{1}{e}$; $\therefore \frac{1}{e^{9\lambda t}}=\frac{1}{e}\left(\because \frac{N_1}{N_2}=\frac{1}{e}\text{given}\right)$

$\Rightarrow 9\lambda t=1 \quad \therefore t=\left(\frac{1}{9\lambda}\right)$

i.e., After time $t=\frac{1}{9\lambda}$ the ratio of the number of nuclei of X_1 to that of X_2 will be $\frac{1}{e}$.

12. **(a)** Beta rays are same as cathode rays as both are stream of electrons.

13. **(a)** Let after 't' years $\frac{1}{4}th$ of the material remains.

$$\frac{-dN}{dt}=\lambda_1 N+\lambda_2 N \quad \Rightarrow \log_e \frac{N}{N_o}=-(\lambda_1+\lambda_2)t$$

when N_o is initial number of atoms

Decay constant $\lambda=\frac{0.693}{t_{1/2}}$

$\therefore \lambda_1=\frac{0.693}{1620}$ and $\lambda_2=\frac{0.693}{810}$;

$$\frac{N}{N_o}=\frac{1}{4}\Rightarrow \log_e\frac{1}{4}=-\left(\frac{0.693}{1620}+\frac{0.693}{810}\right)t$$

$\Rightarrow t=1080$ years

14. **(a)** The penetrating power is dependent on velocity. For a given energy, the velocity of γ radiation is highest and α-particle is least.

15. **(c)** $4\,{}_1^1H^+ \rightarrow {}_2^4He^{2+}+2e^-+26\,MeV$

represent a fusion reaction.

In a nuclear fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus and releases energy.

16. **(c)** β-particles $({}_{-1}B^0)$ are charged particles emitted by the nucleus.

17. **(b)** Half life, $t_{1/2}=3.8$ day

$\therefore$ Decay constant, $\lambda=\frac{0.693}{t_{1/2}}=\frac{0.693}{3.8}=0.182$

If initial number of atom $N=A_0$ then after time t the number of atoms is $N/20=A$.

$$\therefore t=\frac{2.303}{\lambda}\log\frac{A_0}{A}=\frac{2.303}{0.182}\log\frac{N}{N/20}\simeq 16.5\text{ days}$$

18. **(16)** When radioactive sources just completed their 3rd and 7th half – lives then the ratio

$$\frac{A_1}{A_2}=\frac{A_0 e-3\ell n2}{A_0 e-7\ell n2}=\frac{2^{-3}}{2^{-7}}=\frac{1}{2^{(-4)}}=2^4=16$$

19. **(5)** According to question,

$$I^{131}\xrightarrow{T_{1/2}=8\text{ days}}Xe^{131}+\beta$$

$A_0=2.4\times 10^5\,Bq=\lambda N_0$

Let the volume is V

Given: At $t=0$, $A_0=\lambda N_0=2.4\times 10^5\,Bq$

$t=11.5$ hrs, $A=\lambda N$

After $t=11.5\,h$, 2.5 ml of blood is drawn from person's body and gives an activity of 115 Bq.

$$\therefore 115=\lambda\left(\frac{N}{V}\times 2.5\right);115=\frac{\lambda}{V}\times 2.5\times N_0e^{\lambda t}$$

$$\Rightarrow 115=\frac{\lambda N_0}{V}\times 2.5\times e^{-\frac{\ln 2}{8\text{ days}}(11.5\text{ hrs})}$$

$$\Rightarrow V=\frac{2.4\times 10^5}{115}\times 2.5\left[1-\frac{1}{24}\right]$$

[from approximation $e^x\approx 1+x$]

$$\Rightarrow V=\frac{2.4\times 10^5}{115}\times 2.5\times\frac{23}{24}=5\times 10^3\,\text{ml}=5\text{ litres.}$$

20. **(2)** Activity,

$$R=-\frac{dA}{dt}=-\frac{d}{dt}\left[-\frac{dN}{dt}\right]=\frac{d^2N}{dt^2}=\frac{d^2\left(N_oe^{-\lambda t}\right)}{dt^2}$$

$\therefore \quad R=N_o\lambda^2 e^{-\lambda t}=(N_o\lambda)\lambda e^{-\lambda t}=A_o\lambda e^{-\lambda t} \quad [\because A_o=N_o\lambda]$

$$\therefore \quad \frac{R_P}{R_Q}=\frac{\lambda_P e^{-\lambda_P t}}{\lambda_Q e^{-\lambda_Q t}}=\frac{\lambda P}{\lambda Q}\times\frac{e^{\lambda_Q t}}{e^{\lambda_P t}}=\frac{2\tau}{\tau}\frac{e^{\frac{2\tau}{2\tau}}}{e^{\frac{2\tau}{\tau}}}=\frac{2}{e}=\frac{n}{e}$$

$\therefore \quad n=2$

21. **(4)** For a radioactive decay

$N=N_0(1-e^{-\lambda t})$

$\therefore \frac{N}{N_0}=e^{-\lambda t} \qquad \therefore 1-\frac{N}{N_0}=1-e^{-\lambda t}$

$$\therefore \frac{N_0-N}{N}=1-e^{-\frac{0.693}{t_{1/2}}\times t}=1-e^{-0.04}=1-(1-0.04)$$

$[\because e^{-x}=1-x \; x<<1]$

% decayed $\approx 0.04\times 100=4\%$

22. **(1)** We know that, $\left|\frac{dN}{dt}\right| = \lambda N = \frac{1}{T_{mean}} N \left[\because \lambda = \frac{1}{T_{mean}}\right]$

$\therefore \ 10^{10} = \frac{1}{10^9} \times N \ \therefore \ N = 10^{19}$

i.e. 10^{19} radioactive atoms are present in the freshly prepared sample.

$\therefore$ Mass of the sample = N × mass of me atom

$= 10^{19} \times 10^{-25}$ kg $= 10^{-6}$ kg = 1 mg

23. **(8)** We know that $N = N_0 e^{-\lambda t}$

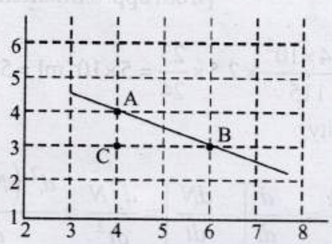
$\therefore \frac{dN}{dt} = N_0 e^{-\lambda t}(-\lambda) = -N_0 \lambda e^{-\lambda t}$

Taking log on both sides

$\log_e \frac{dN}{dt} = \log_e(\lambda N_0) - \lambda t$

Comparing it with the graph line,

Decay constant, $\lambda = \frac{1}{2}\text{yr}^{-1}$ $\left[\frac{AC}{BC} = \frac{1}{2}\right]$



$\therefore T_{1/2} = \frac{0.693}{\lambda} = 0.693 \times 2 = 1.386$ years

$n\,(t_{1/2}) = 4.16 \ \therefore n = \frac{4.16}{1.386} \approx 3$

$\therefore \ N = N_0 \left(\frac{1}{2}\right)^3 \ \therefore \ \frac{1}{P} = \frac{1}{8} \quad [\therefore \ P = 8]$

24. **(2.33)** We have

$Q = \Delta mc^2$

$= (m_N + m_{He} - m_H - m_O) \times 930$ MeV

$= (16.006 + 4.003 - 1.008 - 19.003) \times 930$ MeV

$= -1.86$ MeV = 1.86 MeV energy absorbed

This Q is equal to maximum loss in kinetic energy of α-particle.

Considering collision as inelastic, we get maximum loss in

$\text{K.E.} = \frac{1}{2} \times \left(\frac{4m \times 16m}{4m + 16m}\right) \times v^2$

$\Rightarrow Q = \left(\frac{1}{2} \times 4m \times v^2\right) \times \frac{16m}{20m} \Rightarrow Q = (\text{K.E.})_{min} \times \frac{4}{5}$

$\Rightarrow (\text{K.E.})_{min} = \frac{5}{4}Q = \left(\frac{5}{4} \times 1.86\right)\text{MeV} = 2.325 \text{ MeV}$

$\therefore \quad n = 2.33$

25. When boron nucleus ($^{10}_{5}B$) is bombarded by nentron $\left(^{1}_{0}n\right)$

$^{10}_{5}B + ^{1}_{0}n \longrightarrow ^{4}_{2}He + ^{7}_{3}Li$

the resulting nucleus is of element lithium and mass number A = 7.

26. $^{238}_{92}U \rightarrow ^{206}_{82}Pb + x\ ^{4}_{2}He + y\ ^{0}_{-1}e$

No. of α – particles emitted = 8 and no. of β-particles emitted = 6, α – particle is $_2He^4$ so by emission of 1 α – particle mass number (A) decreases by

4 units and a tomic number (Z) decreases by 2 units.

β - particle is electron –1C° there is no change in mass number (A) but atomic number increases by 1 unit by emission of 1 beta particle.

27. Using $A = A_0\left(\frac{1}{2}\right)^n$ where A_0 = initial activity = 1000 dps (given)

A = Activity after n number of half lives

At $t = 1, n = 1 \therefore A = 1000\left(\frac{1}{2}\right)^1 = 500$ dps

At $t = 3, n = 3 \ \therefore A = 1000\left(\frac{1}{2}\right)^3 = 125$ dps

28. **(a, c)** No. of α-particles emitted, $N\alpha = \frac{232 - 212}{4} = \frac{20}{4} = 5$

$\therefore \ ^{232}_{90}Th \longrightarrow ^{212}_{82}Pb + 5\ ^{4}_{2}He + 2\ ^{0}_{-1}\beta$

Considering the laws of conservation of mass number (*A*) and atomic number (*Z*) number of β–particles emitted $N_\beta = 2$

29. **(c)** According to question, $(t_{1/2})_x = (t_{mean})_Y$

$\Rightarrow \frac{0.693}{\lambda_x} = \frac{1}{\lambda_Y} \quad \therefore \lambda_x = 0.693\,\lambda_Y$ i.e., $\lambda_x < \lambda_Y$

Now, rate of decay $\left(\frac{-dN}{dt}\right) = \lambda N$

Initially, number of atoms (*N*) of both *x* and *y* are equal but since $\lambda_Y < \lambda_x$, therefore element *Y* will decay at a faster rate than element *x*.

30. **(d)** The result follows from the formula based on laws of radioactive decay $N = N_0 e^{-\lambda t}$ And rate of decay $\left(\frac{-dN}{dt}\right) = \lambda N$

The nucleus start decaying after time $t = 0$

31. **(b)** Half-life $T_{1/2} = \frac{\ell n\, 2}{\lambda}$ and Mean life, $\tau = \frac{1}{\lambda}$

32. **(b)** The intensity of radiation emitted is proportional to the rate of decay of radioactive meterial.

$$t = \frac{2.303}{\lambda} \log_{10} \frac{N_0}{N} \quad \text{And } \lambda = \frac{0.693}{t_{1/2}}$$

$$\therefore t = \frac{2.303}{0.693/2} \log_{10} \frac{N_0}{N_0/64} \quad \text{or, } t = 12 \text{ hours.}$$

33. **(c)** For negative beta decay

$$_0n^1 \to {}_1H^1 + \left({}_{-1}B^0\right)$$
(Neutron) (*Electron*)

34. **(a)** In α–decaymass number (A) decreases by 4 unit and atomic number (Z) decreases by 2 unit.
In β^- decay A does not change but Z increases by 1 unit.
In β^+ decay A does not change but Z decreases by 1 unit.

(P) $_{92}U^{238} \to _{91}Pa^{234}$

$$N_1 = \frac{238 - 234}{4} = 1 \to 1\alpha$$

$$N_2 - N_3 = (92 - 91) - \left(\frac{4}{2}\right) = -1 \to 1\beta^-$$

$\therefore$ 1α and $1\ \beta^-$ emission.

(Q) $_{82}Pb^{214} \to _{82}Pb^{210}$

$$N_1 = \frac{214 - 210}{4} = 1 \to 1\alpha$$

$$N_2 - N_3 = (82 - 82) - \left(\frac{4}{2}\right) = -2 \to 2\beta^-$$

$\therefore$ 1α and $2\ \beta^-$ emission.

(R) $_{81}T\ell^{210} \to _{82}Pb^{206}$

$$N_1 = \frac{210 - 206}{4} = 1 \to 1\alpha$$

$$N_2 - N_3 = (81 - 83) - \frac{4}{2} = -3 \to 3\beta^-$$

$\therefore$ 1α and $3\ \beta^-$ emission.

(S) $_{91}Pa^{228} \to _{88}Ra^{224}$

$$N_1 = \frac{228 - 224}{4} = 1\alpha$$

$$N_2 - N_3 = (91 - 88) - \frac{4}{2} = 1\beta^+$$

$\therefore$ 1α and $1\ \beta^+$ emission.

35. For A $\to$ r, t; B $\to$ p, s; C $\to$ p, q, r, t; D $\to$ p, q, r, t

36. **(c)** In B^+-decay mass number (*Z*) decreases by 1 and mass number (*A*) remains unchanged.

$$^{15}_{8}O \longrightarrow {}^{15}_{7}N + \underset{\beta^+ \text{ particle}}{{}^{0}_{1}\beta}$$

In α-decay mass number (*A*) decreases by 4 unit and atomic number (*Z*) by 2 unit.

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + \underset{\alpha-\text{particle}}{{}^{4}_{2}He}$$

In proton $\left({}^1_1H\right)$ emission both (*A*) and (*Z*) decreases by 1.

$$^{185}_{83}Bi \longrightarrow {}^{184}_{82}Pb + \underset{\text{proton}}{{}^{1}_{1}H}$$

In fission process heavier nucleus breaks into two fragments.

$$^{239}_{94}Pu \xrightarrow{(i)} {}^{140}_{57}La + {}^{99}_{37}X$$

37. **A $\to$ p, q; B $\to$ p, r; C $\to$ p, s; D $\to$ p, q, r**
In a nuclear fusion reaction two or more lighter nuclei combine to give a comparatively heavier nucleus and some matter is converted into energy.
In a nuclear fission reaction a heavy nucleus breaks into two or more lighter nuclei and some matter is converted into energy.
β-decay essentially proceeds by weak nuclear forces and converts some matter into energy.
Exothermic nuclear reaction possible for both nuclei with low and high atomic number and releases energy.

38. **(a)** $^{210}_{84}Po \longrightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$
Mass defect
$\Delta m = [209.982876 - (205.974455 + 4.002603)] = 0.005818\mu$
$= 5.422$ MeV $= 5422$ keV
$\therefore Q = (\Delta m) \times 932 = (0.005818) \times 932$

$$K_\alpha = \frac{(A-4)}{A} Q = \left(\frac{210-4}{210}\right) \times 5422$$

$\therefore K_\alpha = 5319$ keV

39. **(c)** In case of (*c*) only, mass defect (Δm) is positive in all other cases (*a*) (*b*) and (*d*) Δm is negative.
Hence Deuteron and α-particle can undergo complete fusion.

40. Let initial Uranium atom = a

$\therefore$ atoms left

$$(a-x)=a\left(\frac{1}{2}\right)^n \text{ and } n=\frac{t}{t_{1/2}}=\frac{1.5\times10^9}{4.5\times10^9}=\frac{1}{3}$$

$$\therefore a-x=a(\frac{1}{2})^{1/3}$$

$$\Rightarrow \frac{a}{a-x}=\frac{1}{(1/2)^{1/3}}=\frac{2^{1/3}}{1}=1.26$$

$$\Rightarrow \frac{x}{a-x}=1.259-1=0.259 \Rightarrow \frac{a-x}{x}=\frac{1}{0.259}=3.86$$

41. Using, $\lambda=\dfrac{\log_e \dfrac{A_0}{A}}{t}=\dfrac{1}{2}\log_e \dfrac{n}{0.75n}$

Mean Life $=\dfrac{1}{\lambda}=\dfrac{2}{\log_e 4/3}=6.94$ s

42. $X \xrightarrow[\lambda_x=0.1\text{s}^{-1}]{T_{1/2}=10\text{sec}} Y \xrightarrow[\lambda_y=\frac{1}{30}\text{s}^{-1}]{T_{1/2}=30\text{sec}} Z$

Here $\dfrac{dN_x}{dt}=-\lambda_x N_x$ ...(i)

$\dfrac{dN_y}{dt}=-\lambda_y N_y+\lambda_x N_x$ ...(ii)

$\dfrac{dN_z}{dt}=-\lambda_y N_y$ ...(iii)

Integrating, we get

$N_x=N_0 e^{-\lambda_x t}$ ...(iv)

Given $N_y=\dfrac{\lambda_x N_0}{\lambda_x-\lambda_Y}\left[e^{-\lambda_y t}-e^{\lambda_x t}\right]$

To determine the maximum N_Y, $\dfrac{dN_Y}{dt}=0$

From eq. (ii)

$-\lambda_y N_y+\lambda_x N_x=0$

$\Rightarrow \lambda_x N_x=\lambda_y N_y$ ....(v)

$$\Rightarrow \lambda_x(N_0 e^{-\lambda_x t})=\lambda_y\left[\frac{\lambda_x N_0}{\lambda_x-\lambda_y}\left(e^{-\lambda_y t}-e^{\lambda_x t}\right)\right]$$

$$\Rightarrow \frac{\lambda_x-\lambda_y}{\lambda_y}=\frac{e^{-\lambda_y t}-e^{-\lambda_x t}}{e^{-\lambda_x t}} \Rightarrow \frac{\lambda x}{\lambda_y}=e^{(\lambda_x-\lambda_y)t}$$

$$\Rightarrow \log_e\frac{\lambda_x}{\lambda_y}=\left(\lambda_x-\lambda_y\right)t$$

$$\Rightarrow t=\frac{\log_e(\lambda_x/\lambda\gamma)}{\lambda_x-\lambda_\gamma}=\frac{\log_e\left[0.1/\left(\frac{1}{30}\right)\right]}{0.1-\frac{1}{30}}=15\log_e 3$$

$$\therefore N_x=N_0 e^{-0.1(15\log_e 3)}=N_0 e^{\log_e(3^{-1.5})}$$

$$\Rightarrow N_x=N_0 3^{-1.5}=\frac{10^{20}}{3\sqrt{3}}=1.9\times10^{19}$$

Since, $\dfrac{dN_y}{dt}=0$ at $t=15\log_e 3$, $\therefore N_y=\dfrac{\lambda_x N_x}{\lambda_y}=\dfrac{10^{20}}{\sqrt{3}}$

$=5.77\times10^{19}$

and $N_z=N_0-N_x-N_y$

$$=10^{20}-\left(\frac{10^{20}}{3\sqrt{3}}\right)-\frac{10^{20}}{\sqrt{3}}=10^{20}\left(\frac{3\sqrt{3}-4}{3\sqrt{3}}\right)=2.32\times10^{19}$$

43. (a) Let at time 't' number of radioactive nuclei = N.

Net rate of formation of nuclei of A.

$$\frac{dN}{dt}=\alpha-\lambda N \text{ or } \frac{dN}{\alpha-\lambda N}=dt \quad [\propto=\text{rate of production}]$$

$$\text{or } \int_{N_0}^{N}\frac{dN}{\alpha-\lambda N}=\int_0^t dt$$

Solving, we get

$N=\dfrac{1}{\lambda}\left[\alpha-(\alpha-\lambda N_0)e^{-\lambda t}\right]$ ...(i)

(b) Substituting $\alpha=2\lambda N_0$ and $t=t_{1/2}=\dfrac{\ln(2)}{\lambda}$ in (i),

$N=\dfrac{3}{2}N_0$

(ii) Substituting $\alpha=2\lambda N_0$ and $t\to\infty$ in equation (i), we get

$N=\dfrac{\alpha}{\lambda}=\dfrac{2\lambda N_0}{\lambda}=2N_0$.

44. (i) From the given information, number of nuclei reduced to half 25% to 12.50% m 10s $\therefore$ half life $T_{1/2}=10$s

Mean life $\tau=\dfrac{1}{\lambda}=\dfrac{1}{0.693/t_{1/2}}=\dfrac{t_{1/2}}{0.693}=\dfrac{10}{0.693}=14.43$ sec.

(ii) $N=N_0 e^{-\lambda t}\Rightarrow \dfrac{N}{N_0}=\dfrac{6.25}{100}$

$$\frac{6.25}{100} = e^{-0.0693t} \Rightarrow e^{+0.0693t} = \frac{100}{6.25} = 16$$

$$(\because \lambda = 0.0693 s^{-1})$$

$$0.0693t = \ln 16 = 2.773 \text{ or } \quad t = \frac{2.733}{0.0693} = 40 \text{ sec.}$$

45. Given: Half-life, $t_{1/2} = 15$ hours
Activity initially $A_0 = 10^{-6}$ Curie $= 3.7 \times 10^4$ dps
After 5 hours, $A = 296$ dpm $= 296/60$ dps
The initial activity can be found by the formula

$$t = \frac{2.303}{\lambda}\log_{10}\frac{A_0}{A} \Rightarrow 5 = \frac{2.303}{0.693/15} \times \log_{10}\frac{A_0}{296}$$

$$\Rightarrow \log_{10}\frac{A_0}{296} = \frac{5 \times 0.693}{2.303 \times 15} = \frac{0.3010}{3} = 0.10033$$

$$\Rightarrow \frac{A_0}{296} = 1.26 \Rightarrow A_0 = 373 \text{ dpm} = \frac{373}{60} \text{ dps}$$

This is the activity level in blood volume 1 cm^3.
$\therefore$ Activity 3.7×10^4 dps in blood volume

$$V = \frac{3.7 \times 10^4}{373/60} = 5951.7 \text{ cm}^3 = 5.95 \text{l litre}$$

46. (a) ${}^{A}_{92}X \to {}^{228}_{X}Y + {}^{4}_{2}He \quad (\because \alpha = {}^{4}_{2}He)$

$\therefore A = 228 + 4 = 232$ and $92 = Z + 2 \therefore Z = 90$

(b) Let v be the velocity with which α - particle is emitted.

The magnetic force qvB provides centripetal force $\frac{mv^2}{r}$ to α-particle for its circular motion.

$$\therefore \frac{mv^2}{r} = qvB \Rightarrow v = \frac{qrB}{m} = \frac{2 \times 1.6 \times 10^{-19} \times 0.11 \times 3}{4.003 \times 10^{-27}}$$

$\therefore v = 1.59 \times 10^7 ms^{-1}$.

Applying law of conservation of linear momentum during α-decay

$m_Y v_Y = m_\alpha v_\alpha$ ...(i)

Total kinetic energy,

$$E = K.E._\alpha + K.E._Y = \frac{1}{2}m_\alpha {v_\alpha}^2 + \frac{1}{2}m_Y {v_Y}^2$$

$$= \frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_Y\left[\frac{m_\alpha v_\alpha}{m_Y}\right]^2 = \frac{1}{2}m_\alpha {v_\alpha}^2 + m_\alpha v_\alpha^2 + \frac{m_\alpha^2 v_\alpha^2}{2m_Y}$$

$$= \frac{1}{2}m_\alpha v_\alpha^2\left[1 + \frac{m_\alpha}{m_Y}\right]$$

$$= \frac{1}{2} \times 4.033 \times 1.6 \times 10^{-27} \times (1.59 \times 10^7)^2\left[1 + \frac{4.003}{228.03}\right] J$$

$$= 8.55 \times 10^{-13} J = \frac{8.55 \times 10^{-13}}{1.6 \times 10^{-19}} = 5.34 \text{ MeV}$$

$$\therefore \text{Mass equivalent of this energy} = \frac{5.34}{931.5} = 0.0051 \text{ a.m.u.}$$

Also, $m_x = m_Y + m_\alpha$ + mass equivalent
$= 228.03 + 4.003 + 0.0057 = 232.0387$ u.
Number of nucleons = 92 protons + 140 neutrons.
$\therefore$ Binding energy of nucleus X
$= [92 \times 1.008 + 140 \times 1.009] - 232.0387] \times 931.5$ Mev
$= 1.9571 \times 931.5 = 1823$ MeV.

Topic-4: Miscellaneous (Mixed Concepts) Problems

1. **(c)** Using, $\frac{A}{A_0} = \frac{1}{2^n}$

n = number of half lives

$$\therefore 2^n = \frac{A_0}{A} = \frac{64}{1} = 2^6 \Rightarrow n = 6$$

$\therefore$ Time $= n(t_{1/2}) = 6 \times t_{1/2} = 6 \times 18 = 108$ days

Hence after 108 days the laboratory can be considered safe for use.

2. **(c)** Binding energy of nitrogen atom
$[8M_n + 7M_p - M_N] \times 931$
$= [8 \times 1.008665 + 7 \times 1.007825 - 15.000109] \times 931$
Binding energy of oxygen atom
$[8M_n + 8M_p - M_o] \times 931$
$= [7 \times 1.008665 + 8 \times 1.007825 - 15.003065] \times 931$
$\therefore$ Difference $= 0.0037960 \times 931$ MeV ...(i)

$$\text{Also } E_O = \frac{3}{5} \times \frac{8 \times 7}{R} \times \frac{e^2}{4\pi \epsilon_0} = \frac{3}{5} \times \frac{56}{R} \times 1.44\text{MeV}$$

$$E_N = \frac{3}{5} \times \frac{7 \times 6}{R} \times \frac{e^2}{4\pi \epsilon_0} = \frac{3}{5} \times \frac{42}{R} \times 1.44\text{MeV}$$

$$\therefore E_O - E_N = \frac{3}{5} \times \frac{14}{R} \times 1.44\text{MeV} \quad ...(ii)$$

From eq. (i) & (ii)

$$\frac{3}{5} \times \frac{14}{R} \times 1.44 = 0.0037960 \times 931 \therefore R = 3.42 \text{ fm}$$

3. (b) $^{22}_{10}\text{Ne} \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} + {}^{14}_{6}\text{X}$

Atomic number of neon Ne is 10 and α-particle is helium $^{4}_{2}\text{He}$.

The new element X has atomic number 6. Therefore, it is carbon atom A = 14 and C = 6.

4. (9) Here, $^{12}_{5}\beta \longrightarrow {}^{13}_{6}C + {}^{0}_{-1}e + \bar{\nu}$

Maximum kinetic energy of β-particle

$= [\text{mass of } ^{12}_{5}\beta - \text{mass of } ^{12}_{6}C\,] \times 931.5 - 4.041$

$= [12.014 - 12] \times 931.5 - 4.041] = 9\text{MeV}$

5. (3) $N_o \xrightarrow{T} \frac{N_o}{2} \xrightarrow{T} \frac{N_o}{4} \xrightarrow{T} \frac{N_o}{8}$

100% 50% 25% 12.5%

So, three half lives are required.

$\therefore t = n(T) = 3T \therefore n = 3$

6. Atomic number, mass number

7. (a) For the given fission reaction,

$^{236}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y$

From conservation laws of mss number (A) and atomic number (Z).

$x = n, y = n$ i.e., $x = {}^{1}_{0}n$ and $y = {}^{1}_{0}n$

From conservation of momentum, $|P_{xe}| = |P_{st}|$

From $K = \frac{P^2}{2m} \Rightarrow K \propto \frac{1}{M} \therefore \frac{K^{st}}{K_{xe}} = \frac{m^{xe}}{m_{st}}$

$\therefore K_{sr} = 129$ MeV and $K_{xe} = 86$ MeV

8. (c) ${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_1\text{H}^3 + p$

${}_1\text{H}^2 + {}_1\text{H}^3 \rightarrow {}_2\text{He}^4 + n$

By adding given two equation $3\,{}_1\text{H}^2 \rightarrow {}_2\text{He}^4 + p + n$

$\Delta m = 3\,(2.014) - [4.001 + 1.007 + 1.008] = 0.026$

3 deuterons release 3.87×10^{-12} J

$\therefore 10^{40}$ deuterons release $= \frac{3.87 \times 10^{-12} \times 10^{40}}{3}$

$= 1.29 \times 10^{28}$ J

Power, $P = \frac{E}{t} \Rightarrow t = \frac{E}{P} = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12}$

9. (a) At room temperature, thermal energy of air molecule = 0.02 eV

photon energy of visible light (λ = 4000Å to 700Å) = $2eV$.

10. **A → p, q, t; B → q; C → s; D → s**

(p) When an uncharged capacitor is connected to a battery, it becomes charged and energy stored $E = \frac{1}{2}QV$ in the capacitor.

(q) When a gas in an adiabatic container fitted with an adiabatic piston is compressed by pushing the piston

(i) the internal energy of the system increases

$\Delta U = Q - W = 0 - (-PdV) = +PdV$

(ii) Mechanical energy is proceeded to the piston which is converted into kinetic energy of the gas molecules.

(r) When the gas in a rigid container gets cooled, the internal energy of the system will decrease. Due to its conversion into mechanical energy.

(s) When a heavy nucleus initially at rest splits into two nuclei of nearly equal masses and some neutrons are emitted as in case of nuclear fission internal energy of the system is converted into mechanical energy and converts some matter into energy.

(t) When a resistive wire loops is placed in a time varying magnetic field perpendicular to its palne then energy of system is increased due to induced current.

11. **A → p; B → t; C → u; D → r**

(A) Energy of thermal neutrons	(p) 0.025 eV
(B) Energy of X-rays	(t) 10 keV
(C) Binding energy per nucleon	(u) 8 MeV
(D) Photoelectric threshold of a metal	(r) 3 eV

12. (d) Kinetic energy (K) of electron will be minimum or zero when total energy is shared by proton and anti-neutrino

$\therefore 0 \le K < 0.8 \times 10^6$ eV

13. (c) $\because K_p^- + K_e^- + K_\nu^- = 0.8 \times 10^6$ eV

When $K_e^- = 0$ then $K_p^- + K_\nu^- = 0.8 \times 10^6$ eV

Mass of $\bar{\nu} << P \therefore$ maximum energy of anti-neutrino is nearly 0.8×10^6 eV.

14. (d) In the core of nuclear fusion reactor the gas becomes plasma a collection of $^{2}_{1}\text{H}$ nuclei and electron which is formed due to high temperature main tained inside for the reactor core. High temperature is required for nuclear fusion.

15. (a) From conservation of mechanical energy

Loss of kinetic energy of two deuteron nuclei = gain in their potential energy.

$$2 \times 1.5kT = \frac{1}{4\pi\varepsilon_0}\frac{e \times e}{r} = \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{r}$$

$$\Rightarrow 2 \times 1.5 \times \left(8.6 \times 10^{-5}\frac{eV}{k}\right) \times T = \frac{(1.44 \times 10^{-9}\, eVm)}{4 \times 10^{-15} m}$$

$$\therefore \quad T = \frac{1.44 \times 10^{-9}}{2 \times 1.5 \times 8.6 \times 10^{-5} \times 4 \times 10^{-15}} = 1.4 \times 10^9 \text{K}$$

16. (b) As given in the passage, the product of the deutron density (n) and confinement time (t_0) $nt_o > 5 \times 10^{14}$ which is the Lawson criterion for a reactor to work successfully.

$\therefore$ Here $n = 8.0 \times 10^{14}$ cm^{-3} and $t_0 = 9.0 \times 10^{-1}$s

17. Radius of nucleus $r = r_0 A^{1/3}$
where r_0 = constt, and A = mass number.
Here $r_1 = r_0 4^{1/3}$
and unknown nucleus $r_2 = r_0 (A)^{1/3}$

$$\therefore \frac{r_2}{r_1} = \left(\frac{A}{4}\right)^{1/3}, (14)^{1/3} = \left(\frac{A}{4}\right)^{1/3} \Rightarrow A = 56$$

$\therefore$ No of proton = mass number, A – no. of neutrons = $56 - 30 = 26$

$\therefore$ Atomic number, $Z = 26$

(b) Using $\nu = Rc(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$R = 1.1 \times 10^7, c = 3 \times 10^8, Z = 26$
$b = 1$ (for K_α), $n_1 = 1, n_2 = 2$

$$\therefore \nu = 1.1 \times 10^7 \times 3 \times 10^8 [26-1]^2 \left[\frac{1}{1} - \frac{1}{4}\right]$$

$$= 3.3 \times 10^{15} \times 25 \times 25 \times \frac{3}{4} = 1.546 \times 10^{18} \text{ Hz}$$

18. Here reaction ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$
$m_y = 223.61$ amu and $m_\alpha = 4.002$ amu

Momentum, $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.76 \times 10^{-15}} = 1.15 \times 10^{-19}$ kg m/s

This will be the momentum of α-particle as well as y (law of conservation of linear momentum

$$\therefore \quad \text{K.E.} = \frac{p^2}{2m_y} + \frac{p^2}{2m_\alpha} \Rightarrow \text{K.E.} = \frac{p^2}{2}\left[\frac{1}{m_y} + \frac{1}{m_\alpha}\right]$$

$$\therefore \quad \text{K.E.} = \frac{(1.15 \times 10^{-19})^2}{2 \times 1.66 \times 10^{-27}}\left[\frac{1}{223.61} + \frac{1}{4.002}\right] = 10^{-12} \text{ J}$$

From $E = \Delta mc^2$

$$\therefore \Delta m = \frac{E}{c^2} = \frac{10^{-12}}{(3 \times 10^8)^2} \text{kg} = \frac{10^{-28}}{3^2} \times \frac{1}{1.67 \times 10^{-27}} \text{amu}$$

$= 0.00665\, u$
Mass of the parent nucleus X
$m_x = m_y + m_\alpha + \Delta m$
$= 223.61 + 4.002 + 0.00665 = 227.62$ amu

19. In α-decay ${}^{248}_{96}Cm \rightarrow {}^{244}_{94}Pu + {}^{4}_{2}He$
Mass defect

Δm = Mass of ${}^{248}_{96}Cm$ – Mass of ${}^{244}_{94}Pu$ – Mass of ${}^{4}_{2}He$
$= (248.072220 - 244.064100 - 4.002603)\,u = 0.005517u$
$\therefore$ Energy released in α-decay

$E_\alpha = (0.005517 \times 931)$ MeV = 5.136 MeV
Similarly, $E_{\text{fission}} = 200$ MeV (given)

Mean life is given as $t_{\text{mean}} = 10^{13}\text{s} = \frac{1}{\lambda}$

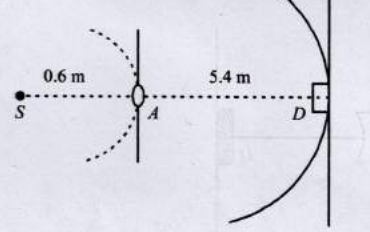
$\therefore$ Disintegration constant $\lambda = 10^{-13}\ \text{s}^{-1}$
Rate of decay at the moment when number of nuclei are 10^{20}

$$\frac{dN}{dt} = \lambda N = (10^{-13})(10^{20}) = 10^7 \text{ dps}$$

8% disintegrations are in fission and 92% are in α-decay.
$\therefore$ Energy released per second
$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136)$ MeV
$= 2.074 \times 10^8$ MeV
$\therefore$ Power output (in watt) = energy released per second (J/s) $= (2.074 \times 10^8)(1.6 \times 10^{-13})$
$\therefore$ Power output, $P_{\text{output}} = 3.32 \times 10^{-5}$ watt.

20. Energy of one photon, $E = \frac{hc}{\lambda} = \frac{(6.6 \times 10^{-34})(3.0 \times 10^8)}{6000 \times 10^{-10}}$

$= 3.3 \times 10^{-19}$ J



Power of the source is 2 W or 2 J/s. Therefore, number of photons emitting per second,

$$n_1 = \frac{2}{3.3 \times 10^{-19}} = 6.06 \times 10^{18} / \text{s}$$

At distance 0.6 m, number of photons incident per unit area per unit time :

$$n_2 = \frac{n_1}{4\pi(0.6)^2} = 1.34 \times 10^{18} / \text{m}^2 / \text{s}$$

Area of aperture $S_1 = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.1)^2 = 7.85 \times 10^{-3}\ \text{m}^2$

$\therefore$ Total number of photons incident per unit time on the aperture,

$n_3 = n_2 s_1 = (1.34 \times 10^{18})(7.85 \times 10^{-3}) / \text{s} = 1.052 \times 10^{16}/\text{s}$

The aperture will become new source of light.
Now these photons are further distributed in all directions. Hence, at the location of detector, photons incident per unit area per unit time :

$$n_4 = \frac{n_3}{4\pi(6-0.6)^2} = \frac{1.052\times10^{16}}{4\pi(5.4)^2} = 2.87\times10^{13}\,s^{-1}\,m^{-2}$$

This is the photon flux at the centre of the screen. Area of detector is 0.5 cm^2 or 0.5×10^{-4} m^2.

Therefore, total number of photons incident on the detector per unit time :

$$n_5 = (0.5\times10^{-4})(2.87\times10^{13}d) = 1.435\times10^9\,s^{-1}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time

$$n_6 = 0.9n_5 = 1.2915\times10^9\,s^{-1}$$

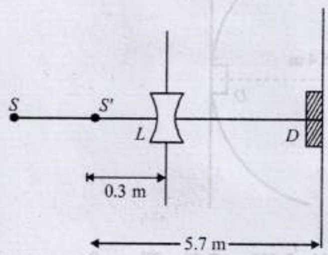
or, photocurrent in the detector

$$i = (e)n_6 = (1.6\times10^{-19})(1.2915\times10^9) = 2.07\times10^{-10}\,A$$

(b) Using the lens formula : $\frac{1}{V}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}-\frac{1}{-0.6}=\frac{1}{-0.6} \text{ or } v=-0.3\text{ m}$$

i.e., Image of source (say S') is formed at 0.3 m from the lens.



Total number of photons incident per unit time on the lens are still n_3 or 1.052×10^{16}/s. 80% of it transmits to second medium. Therefore, at a distance of 5.7 m from S' number of photons incident per unit area per unit time

$$n_7 = \frac{(80/100)(1.05\times10^{16})}{(4\pi)(5.7)^2} = 2.06\times10^{13}\,s^{-1}\,m^{-2}$$

This is the photon flux at the detector.

New value of photocurrent

$$i = (2.06\times10^{13})(0.5\times10^{-4})(0.9)(1.6\times10^{-19})$$
$$= 1.483\times10^{-10}\,A$$

(c) The stopping potential depends on incident frequency, therefore it remains same with or without lens

$$\frac{hc}{\lambda} = (E_K)_{max} + \phi = eV_0 + \phi$$

$$\therefore eV_0 = \frac{hc}{\lambda} - \phi = \frac{3.315\times10^{-19}}{1.6\times10^{-19}} - 1 = 1.07\text{eV}$$

($\because \phi = 1$ ev given)

or, stopping potential, $V_0 = 1.07$ Volt

21. (i) In a nucleus, number of electrons = 0 ($\because$ electrons don't reside in the nucleus of atom, electron revolves round the nucleus in its permissible orbit).

(ii) number of protons = atomic number = 11

(iii) number of neutrons = mass number – atomic number = 24 – 11 = 13