UNIT – VI

Mensuration

CHAPTER

12 AREAS RELATED TO CIRCLES

Syllabus

- *Motivate the area of a circle; area of the sectors and the segments of a circle. Problems based on areas and circumference of the plane figures of circles. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only. Plane figures involving triangles, simple quadrilaterals and circle.)*

Chapter Analysis

List of Topics	2016			2017			2018
	Delhi	Outside Delhi	Foreign	Delhi	Outside Delhi	Foreign	Delhi & Outside Delhi
Area of Shaded Region	2 Q (3 M) 1 Q (4 M)	2 Q (3 M) 1 Q (4 M)	2 Q (3 M) 1 Q (4 M)	1 Q (4 M) 1 Q (3 M)	2 Q (3 M) 1 Q (4 M)	1 Q (3 M) 2 Q (4 M)	1 Q (3 M)

Revision Notes

- A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.
- A line segment joining the centre of the circle to a point on the circumference of a circle is called its radius.
- A line segment joining any two points of a circle is called a chord. A chord passing through the centre of circle is called its diameter. A diameter is the largest chord of the circle.
- A part of a circumference of circle is called an arc.
- A diameter of a circle divides a circle into two equal arcs, each known as a semi-circle.
- The region bounded by an arc of a circle and two radii at its end points is called a sector.
- A chord divides the interior of a circle into two parts, each called a segment.
- An arc of a circle whose length is less than that of a semi-circle of the same circle is called a minor arc.
- An arc of a circle whose length is greater than that of a semi-circle of the same circle is called a major arc.
- Circles having the same centre but different radii are called concentric circles.
- Two circles (or arcs) are said to be congruent if on placing one over the other such that they cover each other completely.
- The distance around the circle or the length of a circle is called its circumference or perimeter.
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- Angle subtended at the circumference by a diameter is always a right angle.
- Angle described by minute hand in 60 minutes is 360°.
- Angle described by hour hand in 12 hours is 360°.

Know the Formulae

1. Circumference (perimeter) of a circle $= \pi d$ or $2\pi r$, where d is diameter and r is the radius of the circle.
2. Area of a circle $= \pi r^2$.
3. Area of a semi-circle $= \frac{1}{2}\pi r^2$.
4. Perimeter of a semi-circle $= \pi r + 2r = (\pi + 2)\, r$
5. Area of a ring or an annulus $= \pi(R + r)(R - r)$. where R is the outer radius and r is the inner radius.
6. Length of arc $AB = \frac{2\pi r\theta}{360°}$ or $\frac{\pi r\theta}{180°}$, where θ is the angle subtended at centre by the arc.

7. Area of a sector $= \frac{\pi r^2\theta}{360°}$

   or area of sector $= \frac{1}{2}(l \times r)$, where l is the length of arc.

8. Area of minor segment $= \frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$.

9. Area of major segment = Area of the circle – Area of minor segment
   $= \pi r^2$ – Area of minor segment.

10. If a chord subtends a right angle at the centre, then

$$\text{area of the corresponding segment} = \left[\frac{\pi}{4} - \frac{1}{2}\right]r^2$$

11. If a chord subtends an angle of 60° at the centre, then

$$\text{area of the corresponding segment} = \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2.$$

12. If a chord subtends an angle of 120° at the centre, then

$$\text{area of the corresponding segment} = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2.$$

13. Distance moved by a wheel in 1 revolution = Circumference of the wheel.

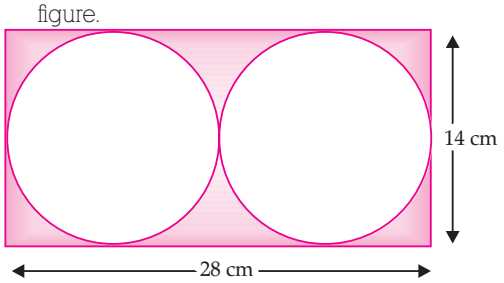
14. Number of revolutions in one minute $= \frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$.

15. Perimeter of a sector $= \frac{\pi r\theta}{180°} + 2r$.

Know the Facts

- An Indian mathematician Srinivas Ramanujan worked out the identity using the value of π correct to million places of decimals.
- The Indian mathematician Aryabhatta gave the value of π as $\frac{62832}{20000}$
- "How I made a greater discovery" this mnemonic help us in getting the value of $\pi = 3.14159$ ------.
- "Can I have a small container of coffee ? " this mnemonic helps us in getting the value of $\pi = 3.1415926$ ------.
- Archimedes calculated the area of a circle by approximating it to a square.
- Area of sector of a circle depends on two parameters-radius and central angle.

Q. Find the area of the shaded region in the given figure. U



Sol. Step I : Length of rectangle = 28 cm
Breadth of rectangle = 14 cm
∴ Diameter of circle = 14 cm
or, radius of circle = 7 cm

Step II : Area of rectangle = 28 x 14 cm^2
= 392 cm^2

Area of each circle = $\frac{22}{7}$ x $(7)^2$
= 154 cm^2

Area of both circles = 2 x 154 cm^2
= 308 cm^2

Step III : Shaded area = Area of rectangle
– Area of both circles
= 392 – 308
= 84 cm^2

Objective Type Questions (1 mark each)

[A] Multiple Choice Questions :

Q. 1. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then :

(a) $R_1 + R_2 = R$ **(b)** $R_1^2 + R_2^2 = R^2$

(c) $R_1 + R_2 < R$ **(d)** $R_1^2 + R_2^2 < R^2$

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : According to the given condition,

Area of circle = Area of first circle + Area of second circle

$$\pi R^2 = \pi R_1^2 + \pi R_2^2$$

$$R^2 = R_1^2 + R_2^2$$

Q. 2. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then :

(a) $R_1 + R_2 = R$

(b) $R_1 + R_2 > R$

(c) $R_1 + R_2 < R$

(d) Nothing definite can be said about the relation among R_1, R_2 and R R [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : According to question,

Circumference of circle = Circumference of first circle + Circumference of second circle

$$2\pi R = 2\pi R_1 + 2\pi R_2$$

$$R = R_1 + R_2$$

Q. 3. If the circumference of a circle and the perimeter of a square are equal, then :

(a) Area of the circle = Area of the square

(b) Area of the circle > Area of the square

(c) Area of the circle < Area of the square

(d) Nothing definite can be said about the relation between the areas of the circle and square.

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : According to question,

Circumference of a circle = Perimeter of square

Let 'r' and 'a' be the radius of circle and side of square.

$$2\pi r = 4a$$

$$\frac{22}{7} r = 2a$$

$$11r = 7a$$

$$r = \frac{7a}{11} \quad ...(i)$$

Area of circle, $A_1 = \pi R^2$

From equation (i), we have

$$A_1 = \pi\left(\frac{7a}{11}\right)^2$$

$$= \frac{22}{7}\left(\frac{49a^2}{121}\right)$$

$$= \frac{14a^2}{11} \quad ...(ii)$$

$$A_2 = a^2 \quad ...(iii)$$

From equation (ii) and (iii), we have

$$A_1 = \frac{14}{11} A_2$$

$$A_1 > A_2$$

Area of circle is greater than the area of square.

Q. 4. Area of the largest triangle that can be inscribed in a semi-circle of radius 'r' units is :

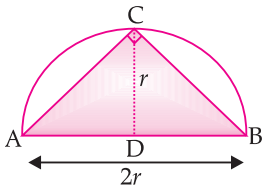
(a) r^2 **sq. units** **(b)** $\frac{1}{2}r^2$ **sq. units**

(c) $2r^2$ **sq. units** **(d)** $\sqrt{2}\, r^2$ **sq. units**

U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Take a point C on the circumference of the semi-circle and join it by the end points of diameter AB.



$\angle C = 90^\circ$ [Angle in a semi-circle is right angle]

So $\Delta ABC = \frac{1}{2} \times AB \times CD$

$$= \frac{1}{2} \times 2r \times r = r^2 \text{ sq. units}$$

Q. 5. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is :

(a) 22 : 7 **(b) 14 : 11**

(c) 7 : 22 **(d) 11 : 14**

U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Let the radius of circle be 'r' and side of square be 'a'.

According to given question,

Perimeter of circle = Perimeter of square

$$2\pi r = 4a$$

$$\therefore a = \frac{\pi r}{2} \quad ...(i)$$

So, $\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2}$ [From equation (i)]

Solving equation (i), we get result as $\frac{14}{11}$.

Q. 6. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be :

(a) 10 m **(b) 15 m**
(c) 20 m **(d) 24 m**

U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Area of first circular park whose diameter is 16 m,

$$= \pi\left(\frac{16}{2}\right)^2$$
$$= \pi(8)^2$$
$$= 64\pi \text{ m}^2$$

Area of second circular park whose diameter is 12 m,

$$= \pi\left(\frac{12}{2}\right)^2$$
$$= \pi(6)^2$$
$$= 36\pi \text{ m}^2$$

According to question,

Area of single circular park = Area of first circular park + Area of second circular park

$$\pi r^2 = 64\pi + 36\pi$$
$$\pi r^2 = 100\pi$$
$$r = 10 \text{ m}$$

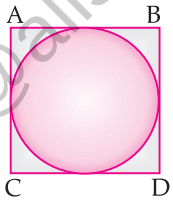
Q. 7. The area of the circle that can be inscribed in a square of side 6 cm is :

(a) 36π cm^2 **(b) 18π cm^2**
(c) 12π cm^2 **(d) 9π cm^2**

R [NCERT Exemp.]

Sol. Correct option : (d)

Explanation :



Given, side of square = 6 cm

Diameter of a circle, (d) = Side of square = 6 cm

Radius of a circle (r) = $\frac{d}{2}$ = 6 = 3 cm.

Area of circle $= \pi r^2$

$\pi(3)^2 = 9\pi$ cm^2

Q. 8. The area of the square that can be inscribed in a circle of radius 8 cm is :

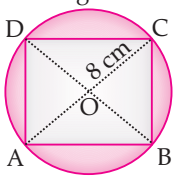
(a) 256 cm^2 **(b) 128 cm^2**
(c) $64\sqrt{2}$ cm^2 **(d) 64 cm^2**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Given, radius of circle, $r = OC = 8$ cm

Diameter of the circle = $AC = 2 \times OC = 2 \times 8 = 16$ cm which is equal to the diagonal of a square.



Let side of square be 'a'.

Using Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$
$$a^2 + a^2 = 16^2$$
$$2a^2 = 256$$
$$a^2 = 128 \text{ cm}^2$$

Q. 9. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is :

(a) 56 cm **(b) 42 cm**
(c) 28 cm **(d) 16 cm**

R [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : Given, $d_1 = 36$ cm, $d_2 = 20$ cm

Circumference of first circle = $2\pi r = \pi d_1 = 36\pi$ cm

And circumference of second circle = $\pi d_2 = 20\pi$ cm

According to question,

Circumference of circle = Circumference of first circle + Circumference of second circle

$$\pi D = 36\pi + 20\pi$$
$$D = 56 \text{ cm}$$

So, diameter of circle is 56 cm

Required radius of circle = $\frac{56}{2} = 28$ cm.

Q. 10. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is :

(a) 31 cm **(b) 25 cm**
(c) 62 cm **(d) 50 cm**

R [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : Let $r_1 = 24$ cm and $r_2 = 7$ cm

Area of first circle = $\pi r_1^2 = \pi(24)^2 = 576\pi$ cm^2

Area of second circle = $\pi r_2^2 = \pi(7)^2 = 49\pi$ cm^2

According to question,

Area of circle = Area of first circle + Area of second circle

$$\pi R^2 = 576\pi + 49\pi$$

[where, R be radius of circle]

$$R^2 = 625 = 25 \text{ cm}$$

Diameter of a circle = $2R = 2 \times 25 = 50$ cm.

Q. 11. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is :

(a) 2 units **(b) π units**
(c) 4 units **(d) 7 units**

R [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Let the radius of the circle be r.

Circumference of circle $= 2\pi r$

Area of circle $= \pi r^2$

Given that, the circumference of the circle and the area of the circle are equal. This implies,

$2\pi r = \pi r^2$

$r = 2$

Therefore, the radius of the circle is 2 units.

Q. 12. Area of a sector of angle p (in degrees) of a circle with radius R is :

(a) $\frac{p}{180} \times 2\pi R$ **(b)** $\frac{p}{180} \times 2\pi R^2$

(c) $\frac{p}{360} \times 2\pi R$ **(d)** $\frac{p}{720} \times 2\pi R^2$

R [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : We know that area of sector of angle θ

$$= \frac{\theta}{360} \times \pi R^2$$

Area of sector of angle, $p = \frac{p}{360} \times \pi R^2 = \frac{p}{720} \times 2\pi R^2$

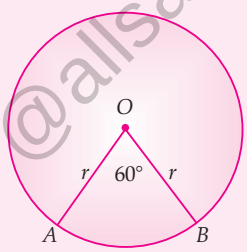
[B] Very Short Answer Type Questions :

Q. 1. What is the perimeter of the sector with radius 10.5 cm and sector angle 60° ?

U [Board Term-2, 2012 Set (40)]

Sol. Perimeter of the sector $= 2r + \frac{2\pi r\theta}{360^\circ}$

$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60^\circ}{360^\circ}$$



$= 21 + 11 = 32$ cm. **1**

[CBSE Marking Scheme, 2012]

Q. 2. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring. U [Delhi 2013]

Sol. $\because$ Circumference of the outer circle, $2\pi r_1 = 88$ cm

$\therefore$ $r_1 = \frac{88 \times 7}{2 \times 22} = 14$ cm.

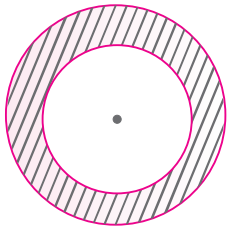
$\because$ Circumference of the inner circle, $2\pi r_2 = 66$ cm

$\therefore$ $r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$ cm $= 10.5$ cm

$\therefore$ Width of the ring $= r_1 - r_2$

$= 14 - 10.5$ cm $= 3.5$ cm. **1**

Q. 3. Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, find the area of the shaded ring shaped region in square cm. A [Board Term-2, 2012, Set (1)]



Sol. $\because$ Area of circle $= \pi r^2$

$\therefore$ Area of the shaded region $= \pi(2)^2 - \pi(1)^2$

$4\pi - \pi = 3\pi$ sq cm **1**

[CBSE Marking Scheme, 2012]

Q. 4. The diameters of two circles with centre A and B are 16 cm and 30 cm respectively . If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C.

U [Board Term-2, 2012 Set (22)]

Sol. Area of circle $= \pi r^2$,

Let the radius of circle with centre $C = R$.

According to question, $\pi(8)^2 + \pi(15)^2$

$= \pi R^2$

or, $64\pi + 225\pi = \pi R^2$

or, $R^2 = 289$ or, $R = 17$ cm

Circumference of circle $= 2\pi R = 2\pi \times 17$

$= 34\pi$ cm **1**

[CBSE Marking Scheme, 2012]

Q. 5. The diameter of a wheel is 1.26 m. What the distance covered in 500 revolutions ?

U [Board Term-2, 2012 Set (50)]

Sol. Distance covered in 1 revolution = circumference of wheel

$= \pi d$

$= \pi \times 1.26$ m.

Distance covered in 500 revolutions

$= 500 \times \pi \times 1.26$

$= 500 \times \frac{22}{7} \times 1.26$

$= 1980$ m. $= 1{\cdot}98$ km **1**

[CBSE Marking Scheme, 2012]

Q. 6. What is the area of the largest square that can be inscribed in a circle of radius 12 cm. ?

U [Board Term-2, 2012 Set (31)]

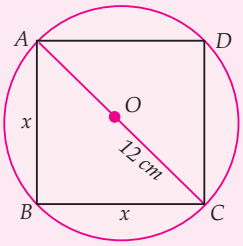
Sol. Radius of the circle $= 12$ cm.

$\therefore$ Diameter of circle $= 24$ cm.

$\therefore$ Diagonal of square $= 24$ cm

Let the side of square $= x$ cm.

$x^2 + x^2 = (24)^2$ (Pythagoras theorem)



or, $2x^2 = 24 \times 24$

or, $x^2 = \frac{24 \times 24}{2} = 288$

Area of square $= x^2 = 288$ cm^2. **1**

[CBSE Marking Scheme, 2012]

Q. 7. What is the name of a line which intersects a circle at two distinct points ? R **[Board Term-2, 2012 (40)]**

Sol. A line intersecting the circle at two distinct points is called a secant. **1**

[CBSE Marking Scheme, 2012]

Q. 8. What is the perimeter of a sector of a circle whose central angle is 90° and radius is 7 cm ?

U **[Board Term-2, 2012 Set (59)]**

Sol. Perimeter of sector $= 2r + \frac{2\pi r\theta}{360°}$

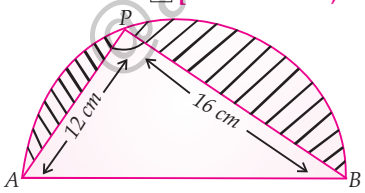
$= 2 \times 7 + 2 \times \frac{22}{7} \times 7 \times \frac{90°}{360°}$

$= 14 + 11 = 25$ cm. **1**

[CBSE Marking Scheme, 2012]

Q. 9. In the given figure, AB is the diameter where $AP = 12$ cm and $PB = 16$ cm. Taking the value of π as 3, find the perimeter of the shaded region.

A **[Board Term-2, 2012 Set (21)]**



Sol. $AB = \sqrt{(16)^2 + (12)^2}$

(From Pythagoras theorem)

$= \sqrt{256 + 144}$

$= \sqrt{400} = 20$ cm

$\therefore$ Radius of circle $= 10$ cm.

Perimeter of shaded region $= \pi r + AP + PB$

$= 3 \times 10 + 12 + 16$

$= 30 + 12 + 16 = 58$ cm. **1**

[CBSE Marking Scheme, 2012]

Q. 10. Find the area of circle that can be inscribed in a square of side 10 cm.

U **[Board Term-2, 2012 Set (44)]**

Sol. Radius of the circle $= \frac{10}{2} = 5$ cm

Area of the circle $= \pi \times r^2$

$= \pi \times (5)^2 = 25\pi$ cm^2 **1**

[CBSE Marking Scheme, 2012]

Q. 11. A thin wire is in the shape of a circle of radius 77 cm. It is bent into a square. Find the side of the square $\left(\textbf{Taking, } \pi = \frac{22}{7}\right)$

U **[Board Term-2, 2012 Set (5)]**

Sol. Perimeter of the circle = Perimeter of square

Let side of square be x cm.

$2\pi r = 4x$

or, $2 \times \frac{22}{7} \times 77 = 4x$

$\therefore$ $x = \frac{2 \times 22 \times 11}{4} = 121$

Side of the square $= 121$ cm. **1**

[CBSE Marking Scheme, 2012]

Q. 12. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm ? U **[Board Term-2, 2012 Set (34)]**

Sol. Area of the circle = sum of areas of two circles

$\pi R^2 = \pi \times (40)^2 + \pi(9)^2$

or, $R^2 = 1600 + 81$

or, $R = \sqrt{1681} = 41$ cm.

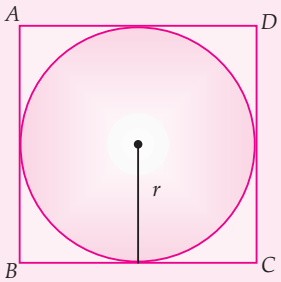
$\therefore$ Diameter of given circle $= 41 \times 2 = 82$ cm. **1**

[CBSE Marking Scheme, 2012]

Q. 13. Find the area (in cm^2) of the circle that can be inscribed in a square of side 8 cm.

U **[Board Term-2, 2012 Set (28, 32, 33)]**

Sol.



Side of square = diameter of circle $= 8$ cm

$\therefore$ Radius of circle, $r = \frac{8}{2} = 4$ cm

Area of circle $= \pi r^2$

$= \pi \times 4 \times 4 = 16\pi$ cm^2 **1**

[CBSE Marking Scheme, 2012]

Q. 14. If the radius of a circle is doubled, what about its area ? A **[Board Term-2, 2012 Set (23)]**

Sol. Let the radius of the circle be r.

$$\text{Area} = \pi r^2$$

Now the radius is doubled

$$\text{Area} = \pi(2r)^2 = 4\pi r^2 = 4 \times \pi r^2$$

The area will be 4 times the area of the first circle. **1**

[CBSE Marking Scheme, 2012]

Q. 15. If the perimeter and the area of the circle are numerically equal, then find the radius of the circle. A **[Board Term-2, 2012 Set (13)]**

Sol. Perimeter of the circle = area of the circle.

$\because \quad 2\pi r = \pi r^2$

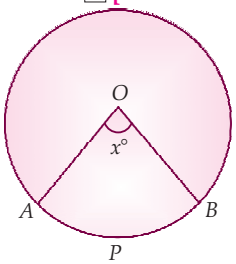
$\therefore \quad r = 2$ units **1**

Hence, radius of the circle = 2 units

[CBSE Marking Scheme, 2012]

Q. 16. In given fig., O is the centre of a circle. If the area of the sector $OAPB$ is $\frac{5}{36}$ times the area of the circle, then find the value of x.

A **[Board Term-2, 2012 Set (12)]**



Sol. Area of sector $OAPB = \frac{5}{36}$ times the area of circle

$$\therefore \quad \pi r^2 \times \frac{x}{360^\circ} = \frac{5}{36}\pi r^2$$

$$\text{or,} \quad \frac{x}{360^\circ} = \frac{5}{36}$$

$$\text{or,} \quad x = 50^\circ \quad \textbf{1}$$

[CBSE Marking Scheme, 2012]

Q. 17. If circumference of a circle is 44 cm, then what will be the area of the circle ?

A **[Board Term-2, 2012 (25)]**

Sol. Circumference of a circle = 44 cm

$$\text{Radius of the circle} = \frac{44}{2 \times \frac{22}{7}} = 7 \text{ cm}$$

$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ cm}^2 \quad \textbf{1}$$

[CBSE Marking Scheme, 2012]

Q. 18. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, then find the circumference of the circle. A **[Board Term-2, 2012 (26)]**

Sol. $\text{Area of square} = (\text{side})^2 = 121 \text{ cm}^2$

$\text{Side of square} = \sqrt{121} = 11$ cm

Perimeter of square $= 4 \times 11 = 44$ cm.

Circumference of the circle = Perimeter of the square

= 44 cm **1**

[CBSE Marking Scheme, 2012]

Q. 19. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of diameter 36 cm and 20 cm.

A **[Board Term-2, 2012, $A1$]**

Sol. $\because$ Circumference of the circle $= 2\pi r$

According to the question,

$$\text{or,} \quad 2\pi r = 2\pi \times 18 + 2\pi \times 10$$

$$\text{or,} \quad 2\pi r = 2\pi (18 + 10)$$

$$\text{or,} \quad r = 28 \text{ cm} \quad \textbf{1}$$

Hence, radius of given circle = 28 cm.

[CBSE Marking Scheme, 2012]

Q. 20. Find the diameter of a circle whose area is equal to the sum of areas of two circles of diameter 16 cm and 12 cm. A **[Board Term-2, 2012, (22)]**

Sol. Let r be the radius of the circle.

Area of the circle = Sum of areas of two circles

$$\pi r^2 = \pi \times (8)^2 + \pi(6)^2$$

$$\text{or,} \quad \pi r^2 = \pi(64 + 36)$$

$$\text{or,} \quad r^2 = 100 \text{ or, } r = 10 \text{ cm}$$

$\therefore$ Diameter of the circle $= 2 \times 10 = 20$ cm. **1**

[CBSE Marking Scheme, 2012]

Q. 21. If the circumference of a circle increases from 4 π to 8π, then what about its area ? A **[Delhi 2013]**

Sol. Circumference of the circle = 4π cm or, $r = 2$ cm.

Increased circumference = 8π cm. or, $r = 4$ cm.

Area of the 1st circle = $\pi \times (2)^2 = 4\pi$ cm^2

Area of the new circle = $\pi (4)^2 = 16\pi = 4 \times 4\pi$

$\therefore$ Area of the new circle = 4 times the area of first circle. **1**

Q. 22. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, find the circumference (in cm) of the circle.

A **[Delhi 2012]**

Sol. Let r be the radius of the circle.

Now, circumference – radius = 37

$$2\pi r - r = 37$$

$$\text{or,} \quad 2 \times \frac{22}{7} r - r = 37$$

$$\text{or,} \quad r\left(\frac{44-7}{7}\right) = 37$$

$$\text{or,} \quad r \times \frac{37}{7} = 37$$

$$\therefore \quad r = \frac{37 \times 7}{37} = 7 \text{ cm.} \quad ½$$

Circumference of the circle $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 7$$

$$= 44 \text{ cm.} \quad ½$$

Short Answer Type Questions-I (2 marks each)

Q. 1. Find the area of the square that can be inscribed in a circle of radius 8 cm. U **[Board Term-2, 2015]**

Sol. Diameter of the circle = diagonal of square
$= 2 \times 8 = 16$ cm
Let x be the side of square.
$\therefore \quad x\sqrt{2} = 16$ or, $x = 8\sqrt{2}$ **1**

Area of square $= x^2 = \left(8\sqrt{2}\right)^2$

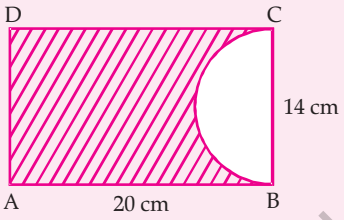
$= 128$ cm^2 **1**

[CBSE Marking Scheme, 2015]

Q. 2. A paper is in the form of a rectangle $ABCD$ in which $AB = 20$ cm and $BC = 14$ cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining part. $\left(\text{Use } \pi = \frac{22}{7}\right)$

A **[Foreign Set I, II, III, 2014]**
[Board Term-2 2012 Set (40)]

Sol. Area of remaining part
= Area of rectangle – Area of semi-circle ½



$= 20 \times 14 - \frac{22 \times 7 \times 7}{7 \times 2}$ **1**

$= 280 - 77$

Hence, area of remaining part = 203 cm^2. ½

[CBSE Marking Scheme, 2012]

Q. 3. If the radius of the circle is 6 cm and the length of an arc is 12 cm. Find the area of the sector.

U **[Board Term-2, 2014]**

Sol. Area of the sector = $\frac{1}{2}$ × (length of the corresponding arc) × radius ½

$= \frac{1}{2} \times l \times r = \frac{1}{2} \times 12 \times 6$

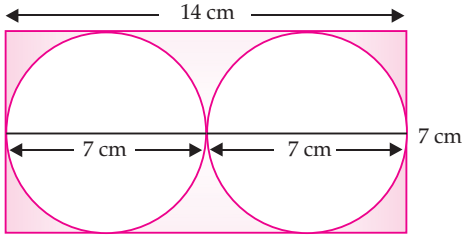
$= 36$ cm^2 **1½**

[CBSE Marking Scheme, 2014]

Q. 4. Two circular pieces of equal radii and maximum areas, touching each other are cut out from a rectangular cardboard of dimensions 14 cm × 7 cm. Find the area of the remaining cardboard. $\left(\text{Use } \pi = \frac{22}{7}\right)$

A **[Delhi 2013]**

Sol. Area of the remaining cardboard
= Area of rectangular cardboard
– 2 × Area of circle **1**



$= 14 \times 7 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= 98 - \frac{44}{7} \times \frac{49}{4}$

$= 98 - 77 = 21$

Hence, area of remaining card board = 21 cm^2. **1**

Q. 5. If the perimeter of a semi-circular protractor is 36 cm, find its diameter. $\left(\text{Use } \pi = \frac{22}{7}\right)$

U **[Board Term-2, 2012 Set (59)]**

Sol. Perimeter = $\pi r + 2r = (\pi + 2)r = 36$ ½

or, $\left(\frac{22}{7} + 2\right) r = 36$ or, $r = 7$ **1**

$\therefore$ Diameter = 14 cm. ½

[CBSE Marking Scheme, 2012]

Q. 6. If the perimeter of a protractor is 72 cm, calculate its area. $\left(\text{Use } \pi = \frac{22}{7}\right)$

U **[Board Term-2, 2012 Set (22)]**

Sol. Perimeter of semi-circle, **1**

$(\pi + 2)r = 72$ cm

or, $r\left[\frac{22}{7} + 2\right] = 72$ cm

or, $r\left[\frac{22 + 14}{7}\right] = 72$

or, $\frac{36}{7} r = 72$

$r = 14$ cm ½

$\therefore$ Area of protractor $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$

$= 308$ cm^2. ½

[CBSE Marking Scheme, 2012]

Q. 7. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. (Use $\pi = 3.14$) U **[Board Term-2, 2012 Set (5)]**

Sol. Given, Radius of circle (r) = 10 cm, central angle = 90° ½

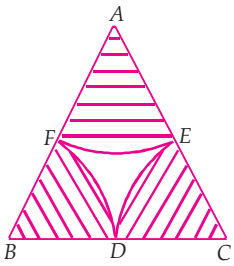
$\therefore$ Area of minor segment

$$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90°\right] \quad ½$$

$$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5 \text{ cm}^2. \quad 1$$

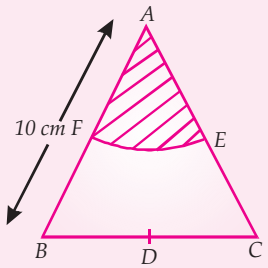
[CBSE Marking Scheme, 2012]

Q. 8. In fig., arcs are drawn by taking vertices *A, B* and *C* of an equilateral triangle of side 10 cm, to intersect the sides *BC, CA* and *AB* at their respective mid-points *D, E* and *F*. Find the area of the shaded region. (Use $\pi = 3.14$).



A **[Board Term-2, 2011 Set (34)]**

Sol. Since, ΔABC is an equilateral triangle.



$\therefore \quad \angle A = \angle B = \angle C = 60°$

Area of sector, $AFEA = \frac{\theta}{360} \times \pi r^2 \text{ cm}^2$

$$= \left(\frac{60°}{360°} \times \pi (5)^2\right) \text{ cm}^2 \quad 1$$

$$= \frac{25}{6}\pi \text{ cm}^2$$

$\because$ Areas of all three sectors are equal.

$\therefore$ Total area of shaded region $= 3\left(\frac{25}{6}\pi\right) \text{ cm}^2$ 1

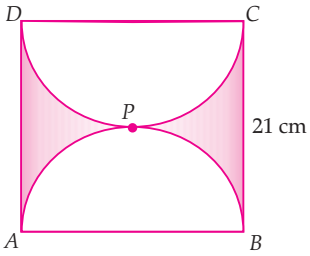
$$= \frac{25 \times 3.14}{2}$$

$$= 39.25 \text{ cm}^2.$$

[CBSE Marking Scheme, 2011]

Q. 9. Find the perimeter of the shaded region if *ABCD* is a square of side 21 cm and *APB* and *CPD* are semicircles. $\left(\text{Use } \pi = \frac{22}{7}\right)$

U **[Board Sample paper 2016]**



Sol. Perimeter of the shaded region

$= AD + BC +$ lengths of the arcs of semi circles *APB* and *CPD* **1**

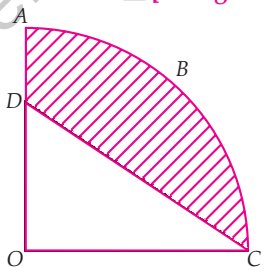
$$= 21 + 21 + 2\left(\frac{22}{7} \times \frac{21}{2}\right) \quad ½$$

$$= 42 + 66$$

$$= 108 \text{ cm.} \quad ½$$

[CBSE Marking Scheme, 2016]

Q. 10. In the figure *OABC* is a quadrant of a circle of radius 7 cm. If *OD* = 4 cm, find the area of shaded region. A **[Foreign Set I, II, III, 2014]**



Sol. Area of shaded region = Area of sector *OCBAD* – Area of ΔODC

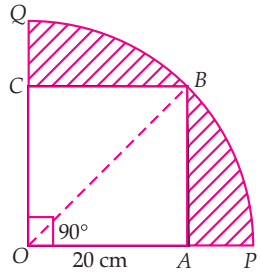
$$= \frac{90°}{360°} \times \pi \times (7)^2 - \frac{1}{2} \times 7 \times 4$$

$$= \frac{49\pi}{4} - 14 \quad 1$$

$$= 24.5 \text{ cm}^2. \quad 2$$

Q. 11. A square *OABC* is inscribed in a quadrant *OPBQ* of a circle. If *OA* = 20 cm, find the area of the shaded region. [Use $\pi = 3.14$]

A **[Delhi CBSE, Term-2, 2014]**



Sol. $OB = \sqrt{OA^2 + AB^2}$

$$= \sqrt{20^2 + 20^2}$$

$$= \sqrt{800}$$

$OB = 20\sqrt{2}$ cm or, radius $= 20\sqrt{2}$ cm

$\therefore$ Area of shaded region

= Area of sector $OQBPO$ – Area of square $OABC$ **1**

$$= \frac{90°}{360°} \times 3.14 \times 20\sqrt{2} \cdot 20\sqrt{2} - (20)^2$$

$$= \frac{1}{4} \times 3{\cdot}14 \times 800 - 400$$

$$= 2(314) - 400 = 628 - 400$$

$$= 628 - 400 = 228 \text{ cm}^2$$

Thus, the required area = 228 cm^2. **1**

Q. 12. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand from 9 a.m. to 9.35 a.m. U [Board Term-2, 2012 Set (13)]

Sol. Angle subtended in 1 minute = 6°

θ = angle subtended in 35 minutes

$= 35 \times 6° = 210°$ ½

$\therefore$ Area swept by the minute hand

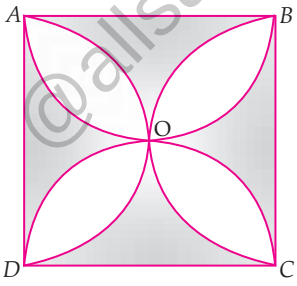
= Area of the sector ½

$$= \frac{\pi r^2 \theta}{360°} = \frac{22}{7} \times \frac{14 \times 14 \times 210}{360}$$ ½

$$= \frac{1078}{3} = 359{\cdot}33 \text{ cm}^2.$$ ½

[CBSE Marking Scheme, 2012]

Q. 13. In Fig., *ABCD* is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



A [Delhi Set I, II, III, 2016]

Sol. Area of square = 196 cm^2

Area of semicircles = area of AOB + area of DOC

$$= \frac{22}{7} \times 49 = 154 \text{ cm}^2$$ ½

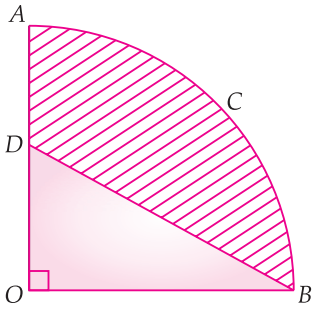
So, area of each of the two shaded parts

$= 196 - 154 = 42$ cm^2 ½

Hence, area of four shaded parts = 84 cm^2. **1**

[CBSE Marking Scheme, 2016]

Q. 14. In the given figure, DACB is a quadrant of a circle with centre *O* and radius 3·5 cm. If *OD* = 2 cm, find the area of the shaded region.



A [Delhi Set-I, II, 2017]

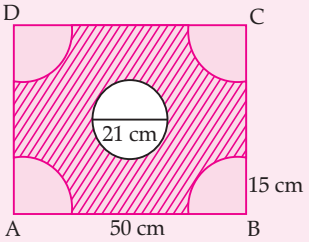
Sol. Try yourself, similar to Q No. 10 in SATQ-I.

Q. 15. A child prepares a poster on "save water" on a square sheet whose each side measures 50 cm. At each corner of the sheet, she draws a quadrant of radius 15 cm in which she shows the ways to save water. At the centre, she draws a circle of diameter 21 cm and writes a slogan save water in it. Find the area of the remaining sheet.

AE [Board Term-2, 2015]

Sol. Side of square = 50 cm

$\therefore$ Area of square $= 50 \times 50 = 2500$ cm^2 ½



Radius of quadrant = 15 cm.

Area of 4 quadrants $= 4 \times \frac{1}{4} \pi r^2 = \pi r^2$

$= \pi \times 15 \times 15$

$= \frac{22}{7} \times 225$

$= 707.14$ cm^2 ½

Area of circle $= \pi r^2$

$$= \frac{22}{7} \times \left(\frac{21}{2}\right)^2$$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$

$= 346.5$ cm^2

Area of remaining sheet = Area of square

– 4(area of quadrant) – Area of circle ½

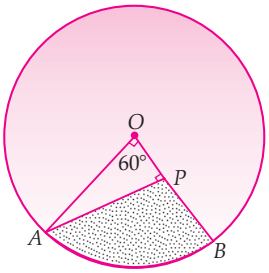
$= 2500 - 707.14 - 346.5$

$= 1446.36$ sq. cm **1**

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II (3 marks each)

Q. 1. In the given figure, *AOB* is a sector of angle 60° of a circle with centre O and radius 17 cm. If *AP* ⊥ *OB* and *AP* = 15 cm, find the area of the shaded region.



A [CBSE S.A.2 2016 Set-HODM4OL]

Sol. As $OA = 17$ cm, $AP = 15$ cm and ΔOPA is right triangle.

$\therefore$ Using Pythagoras theorem,

$OP = 8$ cm **1**

Area of the shaded region

= Area of the sector $AOBA$

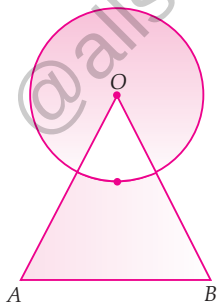
– Area of ΔOPA **1**

$$= \frac{60°}{360°} \times \pi r^2 - \frac{1}{2} \times b \times h$$

$$= \frac{60°}{360°} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15$$

$= 151.38 - 60 = 91.38$ cm^2 **1**

Q. 2. Find the area of shaded region shown in the given figure, where a circular arc of radius 6 cm has been drawn with vertex *O* of an equilateral triangle *OAB* of side 12 cm as centre.



A [Board Sample Paper 2016]
[Foreign Set I, II, III, 2016]

Sol. $\angle AOB = 60°$

Area of shaded region = Area of major sector + (Area of ΔAOB – Area of minor sector)

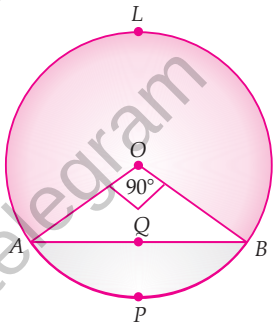
$$= \frac{300}{360} \times \frac{22}{7} \times (6)^2 + \left(\frac{\sqrt{3}}{4}(12)^2 - \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6^2 \right)$$ **1**

$$= \frac{660}{7} + 36\sqrt{3} - \frac{132}{7}$$

$$= \left(36\sqrt{3} + \frac{528}{7} \right) \text{ cm}^2$$ **1**

[CBSE Marking Scheme 2016]

Q. 3. In the given figure, a chord *AB* of the circle with centre *O* and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment *AQBP*. Hence find the area of major segment *ALBQA*. (Use $\pi = 3.14$)



A [Foreign Set I, II, III, 2016]

Sol. Here, the Area of sector

$$OAPB = \frac{90°}{360°} \pi (10)^2 = 25\pi \text{ cm}^2$$

Area of $\Delta AOB = \frac{1}{2} \times 10 \times 10 = 50$ cm^2 ½

$\therefore$ Area of minor segment $AQBP = (25\pi - 50)$ cm^2

$= 25 \times 3.14 - 50$

$= 78.5 - 50$

$= 28.5$ cm^2 ½

Also, area of circle $= \pi(10)^2$

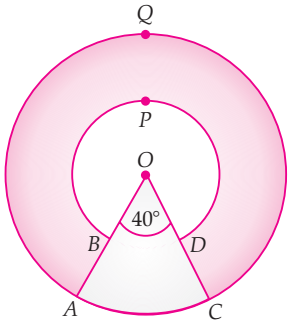
$= 3.14 \times 100 = 314$ cm^2 **1**

Area of major segment $ALBQA = (314 - 28.5)$

$= 285.5$ cm^2 **1**

Q. 4. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$. $\left(\text{Use } \pi = \frac{22}{7} \right)$

A [O.D. Set I, II, III, 2016]



Sol.

Radii of two concentric circle = 7cm & 14cm

and $\angle AOC = 40^\circ$

$m\angle AOC = 360^\circ - 40^\circ = 320^\circ$

Area of shaded region

$$\frac{\theta}{360}\left[R^2 - r^2\right]$$

$$\Rightarrow \frac{320}{360} \times \frac{22}{7}\left[14^2 - 7^2\right] \Rightarrow \frac{8}{9} \times \frac{22}{7} \times 7 \times 21 \times \frac{7}{3}$$

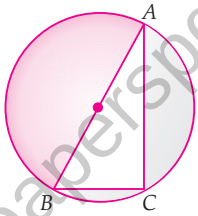
$$\Rightarrow \frac{8 \times 154\ cm^2}{3}$$

Required area = $\frac{1232\ cm^2}{3}$

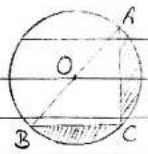
$= 410.6\ cm^2$

[Topper Answer, 2016] 3

Q. 5. In the given figure, *O* is the centre of circle such that diameter *AB* = 13 cm and *AC* = 12 cm. *BC* is joined. Find the area of the shaded region. ($\pi = 3.14$) A [O.D. Set I, II, III, 2016]



Sol.



In –

Radius of semicircle ACB = $\frac{13}{2}$ cm.

Area of semicircle = $\frac{\pi r^2 \times 1}{2}$

Its area = $\frac{1}{2} \times 3.14 \times \frac{13}{2} \times \frac{13}{2} cm^2$

$$\frac{3.14 \times 169\ cm^2}{8} = \frac{530.66\ cm^2}{8}$$

Semicircle subtend 90° at circle, $\angle ACB = 90^\circ$

In ΔABC –

$AC^2 + BC^2 = AB^2 \Rightarrow 12^2 + BC^2 = 169\ cm^2$

$\Rightarrow BC^2 = (169 - 144)\ cm^2$ $BC^2 = 25\ cm$

$BC = 5\ cm$.

Area of $\Delta = \frac{1}{2} \times$ Base $\times$ Height

Area of $\Delta ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times 12 \times 5 = 30\ cm^2$

Area of shaded region = $\frac{530.66\ cm^2}{8} - 30\ cm^2$

$\Rightarrow (66.3325 - 30)\ cm^2$

$\Rightarrow 36.3325\ cm^2$

[Topper Answer, 2016]

Q. 6. Find the area of minor segment of a circle of radius 14 cm, when its central angle is 60°. Also, find the area of corresponding major segment.

[Use $\pi = \frac{22}{7}$] A **[Outside Delhi Set I, II, II, 2015]**

Sol. Here, $r = 14$ cm, $\theta = 60°$

Then, the area of minor segment

$= \pi r^2 \frac{\theta}{360°} - \frac{1}{2} r^2 \sin\theta$ ½

$= \frac{22}{7} \times 14 \times 14 \times \frac{60°}{360°} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$ ½

$= (\frac{308}{3} - 49\sqrt{3})$ cm^2 = 17.89 cm^2 = 17.9 approx. **1**

and the area of major segment $= \frac{22}{7} \times 14 \times 14$

$- (\frac{308}{3} - 49\sqrt{3})$

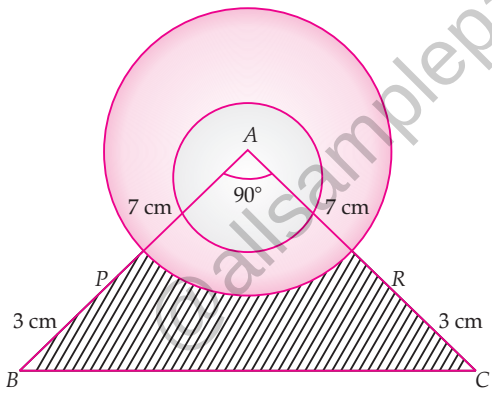
$= \frac{1540}{3} + 49\sqrt{3}$ = 598.10 cm^2

= 598 cm^2 approx. **1**

[CBSE Marking Scheme, 2015]

Q. 7. A momento is made as shown in the figure. Its base *PBCR* is silver plated from the front side. Find the area which is silver plated.

(Use $\pi = \frac{22}{7}$)



A **[Board Term-2, 2015]**

Sol. From the given figure

Area of right-angled $\Delta ABC = \frac{1}{2} \times 10 \times 10$

= 50 cm^2 **1**

Area of quadrant *APR* of the circle of radius 7 cm

$= \frac{1}{4} \times \pi \times (7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 49$

= 38.5 cm^2 **1**

$\therefore$ Area of base *PBCR* = Area of ΔABC

– Area of quadrant *APR*

= 50 – 38.5 = 11.5 cm^2.**1**

[CBSE Marking Scheme, 2015]

Q. 8. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle. (Use $\pi = \frac{22}{7}$) U **[Board Term-2, 2015]**

Sol. Let radius of the circle be *r* cm.

Diameter = 2*r* cm

Circumference = $2\pi r$ ½

Circumference = Diameter + 16.8 ½

or, $2\pi r = 2r + 16.8$

or, $2\left(\frac{22}{7}\right) r = 2r + 16.8$

or, $\frac{44}{7} r = 2r + 16.8$

or, $44r = 14r + 16.8 \times 7$

or, $30r = 117.6$

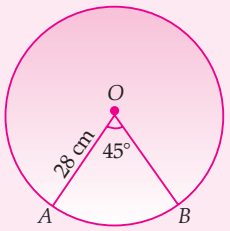
or, $r = \frac{117.6}{30}$ **1**

$\therefore$ r = 3.92 cm **1**

[CBSE Marking Scheme, 2015]

Q. 9. Find the area of the corresponding major sector of a circle of radius 28 cm and the central angle 45°. A **[Board Term-2, 2015]**

Sol. Area of major sector = area of circle – area of minor sector



$= \pi r^2 \left(1 - \frac{\theta}{360°}\right)$ **1**

$= \frac{22}{7} \times 28 \times 28 \left(1 - \frac{45°}{360°}\right)$ **1**

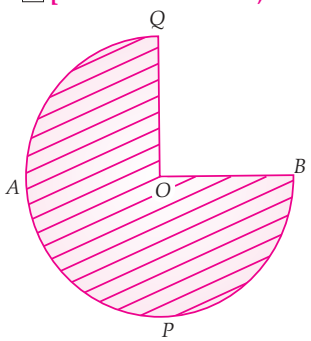
$= \frac{22}{7} \times 28 \times 28 \times \frac{7}{8}$

= 2156 cm^2 **1**

[CBSE Marking Scheme, 2015]

Q. 10. In fig., *APB* and *AQP* are semi-circles, and *AO* = *OB*. If the perimeter of the figure is 47 cm, find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$

A **[Delhi CBSE Board, 2015, Set I, II, III]**



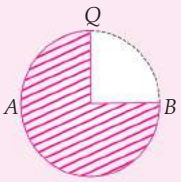
Sol. Let 'r' be the radius of given circle.
Perimeter of given figure = 47 cm

Perimeter of full circle – perimeter of $\left(\frac{1}{4}\right)^{th}$ circle

$= 47 - 2r$ ½

or, $2\pi r - \frac{1}{4}(2\pi r) = 47 \text{ cm} - 2r$ ½

or, $\frac{3\pi r}{2} + 2r = 47 \text{ cm}$



or, $r\left(\frac{3}{2}\times\frac{22}{7}+2\right) = 47 \text{ cm}$

or, $r\left(\frac{33}{7}+2\right) = 47 \text{ cm}$

or, $r = \frac{47\times7}{47} \text{ cm}$

or, $r = 7 \text{ cm}$ **1**

Now, area of shaded region

A = area of circle – $\frac{1}{4}$ area of circle

$= \frac{3}{4}$ area of circle

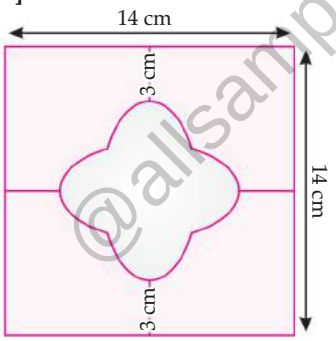
$= \frac{3}{4}\times\pi r^2 = \frac{3}{4}\times\frac{22}{7}\times7\times7 \text{ cm}^2$

$= \frac{3}{2}\times77 \text{ cm}^2$

$= 115{\cdot}5 \text{ cm}^2$ **1**

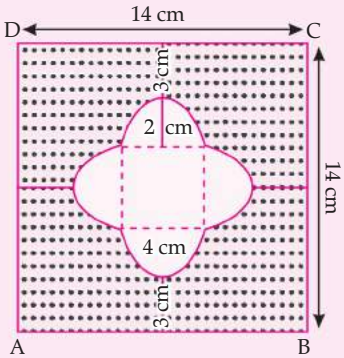
[CBSE Marking Scheme, 2015]

Q. 11. In fig., find the area of the shaded region [Use $\pi = 3.14$]



A **[Delhi Set I, II, III 2015]**
[Board Term-2, 2011 Set–B1]

Sol.



Area of square $ABCD = 14 \times 14 = 196 \text{ cm}^2$ ½
Radius of the semi-circle formed inside = 2 cm

Area of 4 semi-circles $= 4 \times \frac{1}{2}\pi r^2$

$= 2 \times 3.14 \times 2 \times 2$

$= 25.12 \text{ cm}^2$ ½

Length of the side of square formed inside the semi-circles = 4 cm.

Area of the square $= 4 \times 4 = 16 \text{ cm}^2$ ½

Area of the shaded region = Area of square $ABCD$ – (Area of 4 semi-circles + Area of square)

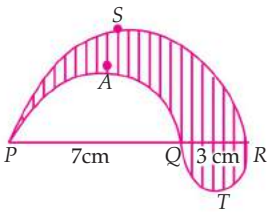
$= 196 - (25.12 + 16)$ ½

$= 196 - 41.12$

$= 154.88 \text{ cm}^2$ **1**

[CBSE Marking Scheme, 2011]

Q. 12. In the fig., *PSR*, *RTQ* and *PAQ* are three semi-circles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$ A **[Delhi CBSE, Term II 2014]**



Sol. Perimeter of shaded region = Perimeter of semi-circles *PSR* + Perimeter of semi circle *RTQ* + Perimeter of semi circle *PAQ* **1**

$= \pi \times 5 + \pi(1.5) + \pi(3.5)$ **1**

$= (5 + 5)\pi$

$= 10\pi$

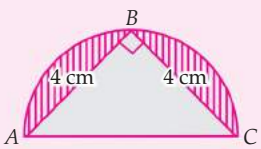
$= 3.14 \times 10$

$= 31.4 \text{ cm}$ **1**

Q. 13. In the figure, ΔABC is in the semi-circle, find the area of the shaded region given that $AB = BC = 4$ cm. (use $\pi = 3.14$) A **[Board Term-2, 2014]**

Sol. $AC = \sqrt{4^2+4^2} = 4\sqrt{2}$ cm. (As ΔABC is a triangle in semi-circle)

$\therefore$ Radius of circle $= \frac{4\sqrt{2}}{2} = 2\sqrt{2}$ cm.



Area of shaded portion = Area of the semi-circle – (Area of ΔABC) **1**

$= \left\{\frac{1}{2}\pi\times\left(2\sqrt{2}\right)^2\right\} - \left\{\frac{1}{2}\times4\times4\right\}$

$$= \left\{\frac{1}{2}\times 3.14\times 8\right\} - 8$$

$= 12.56 - 8$

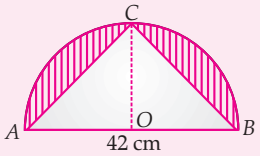
$= 4.56\ \text{cm}^2$. **2**

[CBSE Marking Scheme, 2014]

Q. 14. In the figure, ΔACB is in the semi-circle. Find the area of shaded region given that $AB = 42$ cm.

A **[Board Term-2, 2014]**

Sol.



½

Base of triangle = diameter of semicircle

= 42 cm

and its height = radius of semicircle

$= \frac{42}{2} = 21$ cm ½

Area of shaded portion = Area of semicircle – area of ΔABC

$= \frac{1}{2}\pi r^2 - \frac{1}{2}\times \text{base} \times \text{height}$ ½

$= \frac{1}{2}\times\frac{22}{7}\times 21\times 21 - \frac{1}{2}\times 42\times 21$ **1**

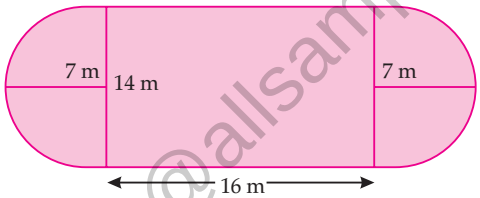
$= 693 - 441 = 252$

Hence, the area of shaded portion = 252 cm^2. ½

[CBSE Marking Scheme, 2014]

Q. 15. Find the area of the adjoining diagram.

A **[Board Term-2, 2014]**



Sol. Required area = area of two semi-circles of same radii + area of rectangle

= area of one circle + area of rectangle **1**

$= \pi r^2 + (l \times b)$ (where r is radius of circle and l and b are length and breadth of rectangle) **1**

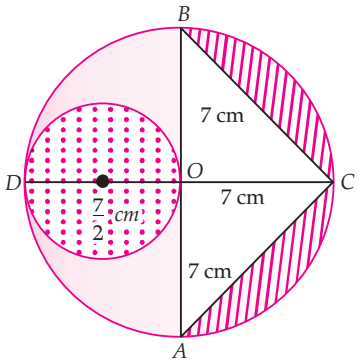
$= \frac{22}{7}\times 7\times 7 + (16\times 14)$

$= 154 + 224$

$= 378\ \text{m}^2$ **1**

[CBSE Marking Scheme, 2014]

Q. 16. AB and CD are two diameters of a circle perpendicular to each other and OD is the diameter of the smaller circle. If $OA = 7$ cm, find the area of the shaded region. A **[Board Term-2, 2012 Set (13)]**



Sol. Area of a circle with DO as diameter

$= \pi r^2 = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = \frac{77}{2}$ sq. cm ½

Area of semi-circle with AB as diameter

$= \frac{\pi r^2}{2} = \frac{22\times 7\times 7}{7\times 2} = 77$ sq. cm ½

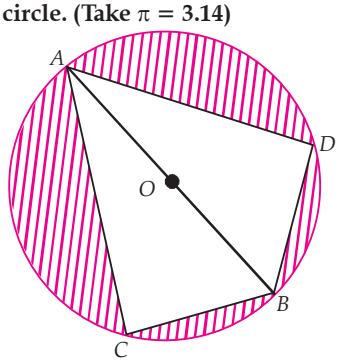
Area of $\Delta ABC = \frac{1}{2}\times 14\times 7 = 49$ sq. cm **1**

Area of shaded region = Area of circle + Area of semi-circle – Area of ΔABC

$= \frac{77}{2} + 77 - 49 = 66.5$ sq. cm. **1**

[CBSE Marking Scheme, 2012]

Q. 17. Find the area of the shaded region in figure, if $BC = BD = 8$ cm, $AC = AD = 15$ cm and O is the centre of the circle. (Take $\pi = 3.14$)



A **[Board Term-2, 2012 Set (34)]**

Sol. Given, $\angle ADB = \angle ACB = 90°$ (Angle in a semicircle)

Since $\Delta ADB \cong \Delta ACB$

$\therefore$ ar ΔADB = ar ΔACB

$= \frac{1}{2}\times 15\times 8$

$= 60\ \text{cm}^2$ **1**

and ar ΔADB + ar ΔACB

$= 2 \times 60 = 120\ \text{cm}^2$

Now in ΔABC $AB = \sqrt{AC^2 + BC^2}$

$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$

$= 17$ cm

Area of circle $= \pi r^2 = 3.14\times\frac{17}{2}\times\frac{17}{2}$

$= 226{\cdot}87\ \text{cm}^2$ **1**

Area of shaded portion = area of circle – area of sum of ΔACB and ΔADB.

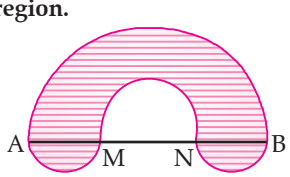
$= 226{\cdot}87 - 120$

$= 106{\cdot}87$

Hence, area of shaded region = $106{\cdot}87\ cm^2$ **1**

[CBSE Marking Scheme, 2012]

Q. 18. In the given figure, *AB* is the diameter of the largest semi-circle. *AB* = 21 cm, *AM* = *MN* = *NB*. Semi-circles are drawn with *AM*, *MN* and *NB* as shown. Using $\pi = \frac{22}{7}$, calculate the area of the shaded region.



A **[Board Term-2, 2012 Set (21)]**

Sol. Area of semi-circle with diameter 21 cm,

$r = \frac{21}{2}\text{cm}, A = \frac{1}{2}\pi r^2 = \frac{1}{2}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2} = \frac{693}{4}\ cm^2$

1

$AM = MN = NB$

$= \frac{21}{3} = 7\ cm$

$\therefore$ Radii smaller semi circles $= \frac{7}{2}\ cm$

Area of semi-circle with diameter *AM*

= Area of semi-circle with diameter *MN*

= Area of semi-circle with diameter *NB*

Radius $= \frac{7}{2}\ cm \therefore$ Area $= \frac{1}{2}\pi r^2$

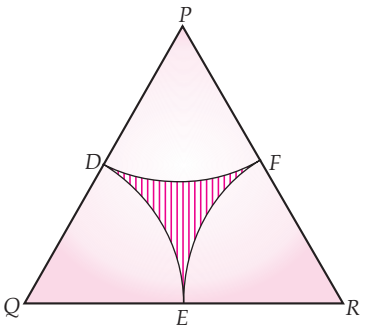
$= \frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = \frac{77}{4}\ cm^2$ **1**

Area of shaded region = Area of larger semicircle + Area of smaller semicircle

$= \frac{693}{4} + \frac{77}{4} = \frac{770}{4} = 192.5\ cm^2.$ **1**

[CBSE Marking Scheme, 2012]

Q. 19. In the given figure, ΔPQR is an equilateral triangle of side 8 cm and *D*, *E*, *F* are centres of circular arcs, each of radius 4 cm. Find the area of shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.732$)



A **[Board Term-2, 2012, Set (28)]**

Sol. Area of shaded region

= Area of ΔPQR – 3 (area of sector)

$= \frac{\sqrt{3}}{4}(\text{side})^2 - 3\left[\frac{\theta}{360^\circ}\times\pi r^2\right]$ ½

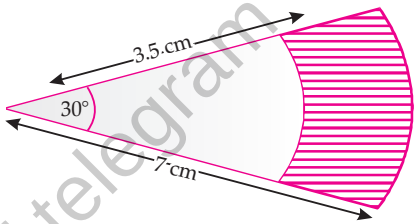
$= \frac{\sqrt{3}}{4}\times 8\times 8 - 3\left[\frac{60^\circ}{360^\circ}\times 3.14\times 4\times 4\right]$ ½

$= 16\sqrt{3} - 3.14\times 8 = 16\times 1.732 - 25.12$ **1**

$= 27.712 - 25.12 = 2.59\ cm^2.$ **1**

[CBSE Marking Scheme, 2012]

Q. 20. In fig., sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



[Board Term-2, 2012, Set B1]

Sol. Area of shaded region $= \pi[R^2 - r^2]\frac{\theta}{360^\circ}$

$= \frac{22}{7}[7^2 - (3.5)^2]\frac{30^\circ}{360^\circ}$ **1**

$= \frac{22}{7}(7 + 3{\cdot}5)(7 - 3{\cdot}5)\times\frac{1}{12}$

$= \frac{22}{7}\times 10.5\times 3.5\times\frac{1}{12}$ **1**

$= 9.62\ cm^2.$ **1**

[CBSE Marking Scheme, 2012]

Q. 21. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of sector formed by the arc. A **[Delhi Set Compt. Set-I, II, III 2017]**

Sol. Given, $r = 21$ cm and $\theta = 60^\circ$

$\therefore$ Area formed by the sector

$= \frac{\theta}{360^\circ}\times\pi r^2$ **1**

$= \frac{60^\circ}{360^\circ}\times\frac{22}{7}\times 21\times 21$ **1**

$= \frac{1}{6}\times 22\times 3\times 21$

$= 11\times 21 = 231\ cm^2$ **1**

Hence, area of the sector = $231\ cm^2$

[CBSE Marking Scheme, 2017]

Q. 22. A wire, when bent in the form of an equilateral triangle, encloses an area of $121\sqrt{3}\ cm^2$. If the wire is bent in the form of a circle, find the area enclosed by the circle, $\left(\text{Use } \pi = \frac{22}{7}\right)$

A **[Outside Delhi Set-I, II, III 2017]**

Sol. Let a be the side of triangle.

$\therefore$ Area enclosed by the triangle $= \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 121\sqrt{3}$

$\Rightarrow \quad a^2 = \frac{121\sqrt{3}\times 4}{\sqrt{3}}$

$a = 22$ cm ½

Perimeter of triangle = circumference of circle formed.

$\therefore \quad 2\pi r = 22 \times 3$ **1**

$\Rightarrow \quad 2\times\frac{22}{7}\times r = 22\times 3$

$\Rightarrow \quad r = \frac{22\times 3\times 7}{22\times 2} = \frac{21}{2}$ cm ½

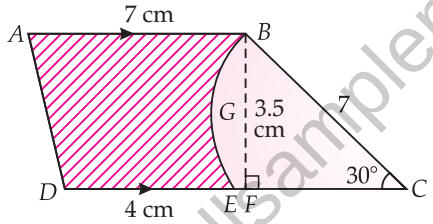
Area enclosed by the circle $= \pi r^2$

$= \frac{22}{7}\times\frac{21}{2}\times\frac{21}{2} = \frac{693}{2}$

$= 346{\cdot}5$ cm^2 **1**

[CBSE Marking Scheme, 2017]

Q. 23. In adjoining fig, *ABCD* is a trapezium with *AB* ‖ *DC* and $\angle BCD = 30°$. Fig. *BGEC* is a sector of a circle with centre *C* and *AB* = *BC* = 7 cm, *DE* = 4 cm and *BF* = 3·5 cm, then find the the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



A **[Outside Delhi Compt. Set-I, II, III 2017]**

Sol. Given, *AB* = 7 cm, *DE* = 4 cm, and *BF* = 3·5 cm

DC = *DE* + *EC* = 4 + 7 = 11 cm

Area of trapezium ABCD

$= \frac{1}{2}$ (Sum of ‖ lines) × (distance between them)

$= \frac{1}{2}(11+7)\times 3{\cdot}5 = \frac{1}{2}\times 18\times 3{\cdot}5$

$= 31{\cdot}5$ cm^2 **1**

Area of the sector *BGEC*

$= \frac{30°}{360°}\times\frac{22}{7}\times 7\times 7 = \frac{1}{12}\times 22\times 7 = 12{\cdot}83$ cm^2 **1**

Area of shaded region

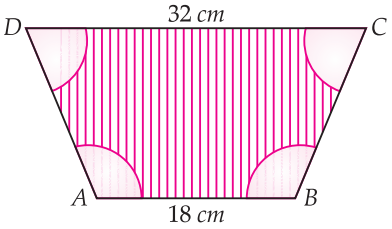
= Area of trapezium – Area of sector

$= 31{\cdot}5 - 12{\cdot}83$

$= 18{\cdot}67$ cm^2 **1**

[CBSE Marking Scheme, 2017]

Q. 24. In the given figure *ABCD* is a trapezium with *AB* ‖ *DC*, *AB* = 18 cm and *DC* = 32 cm and the distance between *AB* and *AC* is 14 cm. If arcs of equal radii 7 cm taking *A*, *B*, *C* and *D* have been drawn, then find the area of the shaded region.



A **[Foreign Set-I, II, III 2017]**

Sol. Given, in trapezium *ABCD*, AB = 18 cm, *CD* = 32 cm, *AB* ‖ *CD* and distance between ‖ lines = 14 cm and the radius of each sector = 7 cm.

Area of trapezium *ABCD* $= \frac{1}{2}(18+32)\times 14$

$= \frac{1}{2}\times 50\times 14$

$= 350$ cm^2 **1**

Let, $\angle A = \theta_1$, $\angle B = \theta_2$, $\angle C = \theta_3$

and $\angle D = \theta_4$

ar of sector $A = \frac{\theta_1}{360°}\pi r^2 = \frac{\theta_1}{360°}\times\frac{22}{7}\times 7\times 7$

$= \frac{\theta_1}{360°}\times 154$ cm^2

ar of sector $B = \frac{\theta_2}{360°}\times 154$ cm^2

ar of sector $C = \frac{\theta_3}{360°}\times 154$ cm^2

ar of sector $D = \frac{\theta_4}{360°}\times 154$

ar of 4 sectors $= \frac{\theta_1+\theta_2+\theta_3+\theta_4}{360°}\times 154$

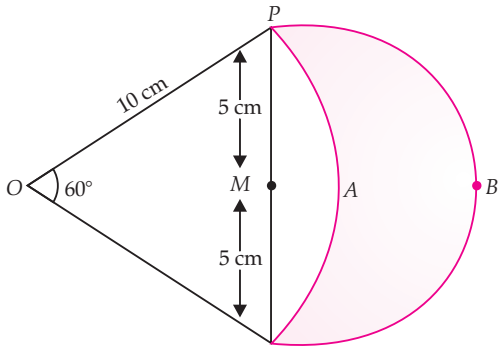
$= \frac{360°}{360°}\times 154$

$= 154$ cm^2 **1**

$\therefore$ Area of shaded region = 350 – 154 = 196 cm^2 **1**

[CBSE Marking Scheme, 2017]

Q. 25. Figure shows two arcs *PAQ* and *PBQ*. Arc *PAQ* is a part of circle with centre *O* and radius *OP* while arc *PBQ* is a semi-circle drawn on *PQ* as diameter with centre *M*. If *OP* = *PQ* = 10 cm, show that area of shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)$ cm^2.



A [Delhi Set I, II, III, 2016]

Sol. Given, $OP = OQ = PQ = 10$ cm

and $\angle POQ = 60°$ ½

Area of segment $PAQM$

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{cm}^2. \quad \mathbf{1}$$

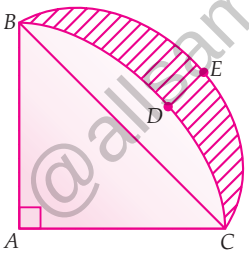
Area of semicircle $= \frac{25\pi}{2}$ cm^2 ½

Area of shaded region $= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$.

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{cm}^2. \quad \mathbf{1}$$

[CBSE Marking Scheme, 2016]

Q. 26. ***ABDC* is a quadrant of a circle of radius 28 cm and a semi-circle *BEC* is drawn with *BC* as diameter. Find the area of the shaded region.** $\left(\text{Use } \pi = \frac{22}{7}\right)$



A [Sample Question Paper 2017]

[Foreign Set I, II, III, 2014][Delhi CBSE, Term-2 2014]

Sol. As ABC is a quadrant of the circle, $\angle BAC$ will be measured 90°.

In ΔABC, $BC^2 = AC^2 + AB^2$

$= (28)^2 + (28)^2$

$= 2(28)^2$

$\therefore$ $BC = 28\sqrt{2}$ cm

Radius of semi-circle drawn on $BC = \frac{28\sqrt{2}}{2}$

$= 14\sqrt{2}$ cm

Area of semi-circle $= \frac{1}{2} \times \frac{22}{7} \times (14\sqrt{2})^2$

$= 616$ cm^2 **1**

Area of $\Delta ABC = \frac{1}{2} \times 28 \times 28 = 392$ cm^2 ½

Area of of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

$= 616$ cm^2 **1**

Area of the shaded region

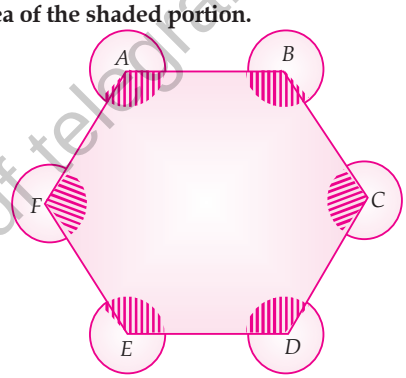
= Area of semi-circle + Area of Δ – Area of quadrant

$= 616 + 392 - 616$

$= 392$ cm^2. ½

[CBSE Marking Scheme, 2014]

Q. 27. In fig., *ABCDEF* is any regular hexagon with different vertices *A, B, C, D, E* and *F* as the centres of circles with same radius '*r*' are drawn. Find the area of the shaded portion.



A [Board Term-2, 2011, B1]

Sol. Let number of sides is n.

$n \times$ each angle $= (n-2) \times 180°$ **1**

$6 \times$ each angle $= 4 \times 180°$

each angle $= 120°$ **1**

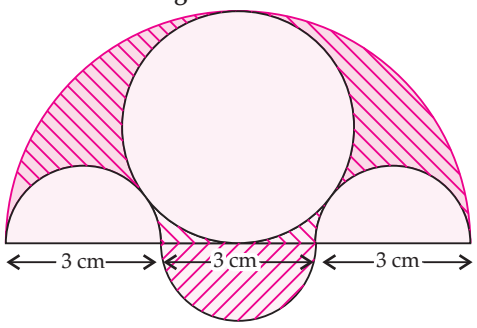
Then, the area of a sector $= \frac{120°}{360°} \times \pi r^2$

Area of 6 shaded regions $= 6 \times \frac{120°}{360°} \times \pi r^2$ **1**

$= 2\pi r^2$.

[CBSE Marking Scheme, 2011]

Q. 28. Three semicircles each of diameter are 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region. **3**



A [OD Set II, 2017]

Sol. 13. Area of shaded region = Area of semicircle with d = 4.5 cm
+ Area of semicircle with d = 3 cm
− 2× area of semicircle with d = 3 cm
− area of circle with d = 4.5 cm.

$= \frac{1}{2} \times \pi \times (4.5)^2 + \left(\frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2\right) - 2\times\left(\frac{1}{2} \times \pi \left(\frac{3}{2}\right)^2\right) - \pi\left(\frac{4.5}{2}\right)^2$

$= \frac{2\times 1}{4} \times \pi \times \frac{20.25}{4} + \frac{\pi}{2} \times \frac{9}{4} - \pi \times \frac{20.25}{4}$

$= \frac{\pi}{4}\left[\frac{2\times 20.25}{2} - \frac{9}{2} - 20.25\right]$

$= \frac{\pi}{4}\left[40.5 - 4.5 - 20.25\right]$

$= \frac{\pi}{4}\left[20.25 - 4.5\right]$

$= \frac{\pi}{4}(15.75)$

$= \frac{22}{7\times 4_2} \times 15.75$

$= \frac{11 \times 2.25}{2}$

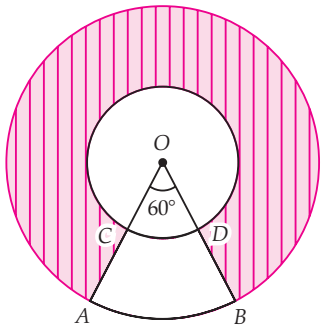
$= \frac{24.75}{2}$

$= 12.375 \text{ cm}^2$.

∴ area of shaded region is 12.375 cm²

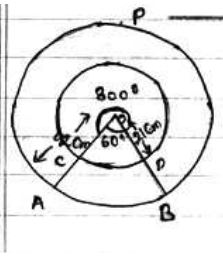
[Topper Answer, 2017] 3

Q. 29. In the given figure, two concentric circles with centre *O* have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



A [OD Set III, 2017]

Sol. 11.



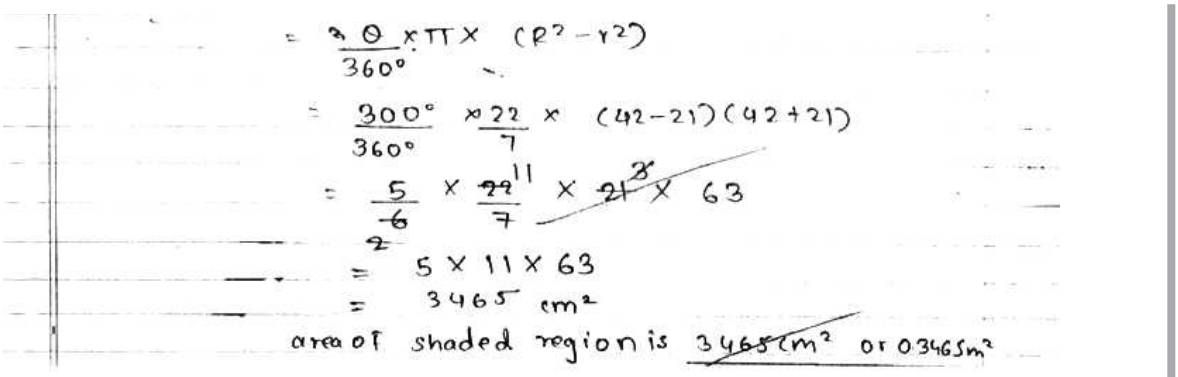
$\angle AOB = \angle COD = 60°$ R = 42 cm, r = 21 cm.

∴ reflex of $\angle AOB = 300° = \theta\ (360° - 60°)$

Now,

area of shaded region

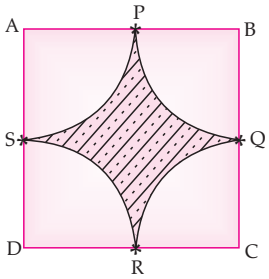
$= \frac{\theta}{360°} \times \pi R^2 - \frac{\theta}{360°} \times \pi r^2$



[Topper Answer, 2017] 3

Q. 30. Find the area of the shaded region in given figure, where arcs drawn with centres *A*, B, C and *D* intersect in pairs at mid-points *P*, *Q*, *R* and *S* of the sides *AB*, *BC*, *CD* and *DA* respectively of a square *ABCD* of side 12 cm. [Use π = 3.14]

A [CBSE Delhi/OD Set, 2018]



Sol. Radius of each arc drawn = 6 cm ½

Area of one quadrant = $(3.14) \times \frac{36}{4}$

Area of four quadrants = 3.14 × 36 **1**

= 113.04 cm² **1**

Area of square ABCD = 12 × 12 = 144 cm²

Hence, Area of shaded region = 144 – 113.04

= 30.96 cm² ½

[CBSE Marking Scheme, 2018]

Q. 31. Find the area of the minor segment of a circle of radius 42 cm, if length of the corresponding arc is 44 cm. A [CBSE SQP, 2017]

Sol. Here, $r = 42$ cm

$$\frac{2\pi r\theta}{360^\circ} = 44$$

$$\theta = \frac{44 \times 360^\circ \times 7}{2 \times 22 \times 42} = 60^\circ \quad \textbf{1}$$

Area of minor segment = area of sector – area of corresponding triangle

$$= \frac{\pi r^2\theta}{360^\circ} - \frac{\sqrt{3}}{4}r^2 \quad ½$$

$$= r^2\left[\frac{22}{7} \times \frac{60^\circ}{360^\circ} - \frac{\sqrt{3}}{4}\right]$$

$$= 42 \times 42 \times \left[\frac{11}{21} - \frac{\sqrt{3}}{4}\right] \quad \textbf{1}$$

$$= 42 \times 42 \times \left[\frac{44 - 21\sqrt{3}}{84}\right] \quad ½$$

$= 21(44 - 21\sqrt{3})$ cm².

$= 21(44 - 36.37)$ cm²

$= 21 \times 7.63$ cm²

$= 160.23$ cm² ½

[CBSE Marking Scheme, 2017-18]

Q. 32. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 48 hours.

C + A [CBSE Compt Set, I, II, III, 2018]

[Foreign Set-I, II, III, 2015]

Sol. Distance travelled by short hand in 48 hours

= 4 × 2π × 4 cm = 32π cm. **1**

Distance travelled by long hand in 48 hours

= 48 × 2π × 6 cm = 576π cm **1**

Total distance travelled = (32π + 576π) cm

= 608 cm **1**

[CBSE Marking Scheme, 2018]

Detailed Answer :

Short hand makes 4 rounds in 48 hours

Long hand makes 48 rounds in 48 hours ½

Radius of the circle formed by short hand = 4 cm

and radius of the circle formed by long hand

= 6 cm

Distance travelled by short hand in one round

= circumference of the circle

= 2 × 4 × π = 8π cm ½

Distance travelled by short hand in 4 rounds

= 2 × 4 × 4π = 32π cm ½

Distance travelled by long hand in one round

= 2 × π × 6 = 12π cm ½

Distance travelled by long hand in 48 rounds

= 48 × 12π = 576π cm ½

Sum of the distance = 32π + 576π = 608π cm ½

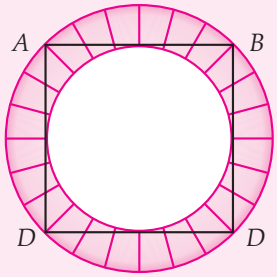
Q. 33. The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square.

A [CBSE Comptt Set-I, II, III, 2018]

Sol. Radius of inner circle = 5 cm ½

Radius of outer circle = $5\sqrt{2}$ cm **1**

Required area = Area of outer circle – Area of inner circle **1**



$= \pi[(5\sqrt{2})^2 - 5^2] = 25\pi$ cm² ½

[CBSE Marking Scheme, 2018]

Detailed Answer :

Here, diameter of inner circle = side of the square
= 10 cm

$\therefore$ Radius of inner circle $= \frac{10}{2} = 5$ cm **1**

Diameter of outer circle = Diagonal of square
$= 10\sqrt{2}$ cm

$\therefore$ Radius of outer circle $= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ cm **1**

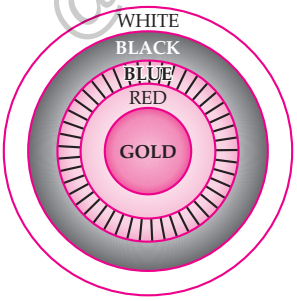
Then, the required Area = Area of outer circle – Area of inner circle

$= \pi[(5\sqrt{2})^2 - (5)^2] = 25\pi$ cm² **1**

Q. 34. The adjoining figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide.

(i) Find the area of each of the five scoring regions.

(ii) Which mathematical concept is used in the above problem ? AE



Sol. (i) Radius of the Gold scoring area

$= \frac{21}{2} = 10.5$ cm

$\therefore$ Area of the Gold scoring region

$= \frac{22}{7} \times 10.5 \times 10.5$ $(A = \pi r^2)$

$= 346.5$ cm²

Radius of combined Gold and Red region

$= 10.5 + 10.5$

$= 21$ cm

$\therefore$ Area of Red scoring region = Area of combined Gold and Red regions – Area of the Gold region

$= \frac{22}{7} \times 21 \times 21 - 346.5$

$= 1386 - 346.5 = 1039.5$ cm² ½

Radius of combined Gold, Red and Blue regions

$= 21 + 10.5 = 31.5$ cm

$\therefore$ Area of Blue scoring region = Area of combined Gold, Red and Blue region – Area of combined Gold and Red region

$= \frac{22}{7} \times 31.5 \times 31.5 - 1386$

$= 3118.5 - 1386 = 1732.5$ cm² ½

Radius of combined Gold, Red, Blue and Black region

$= 31.5 + 10.5 = 42$ cm

$\therefore$ Area of black scoring regions = Area of combined Gold, Red, Blue and Black regions – Area of combined Gold, Red and Blue regions

$= \frac{22}{7} \times 42 \times 42 - 3118.5$

$= 5544 - 3118.5$

$= 2425.5$ cm²

Radius of combined Gold, Red, Blue, Black and White regions

$= 42 + 10.5 = 52.5$ cm

$\therefore$ Area of white scoring region = Area of combined Gold, Red, Blue, Black and White regions – Area of combined Gold, Red, Blue and Black regions.

$= \frac{22}{7} \times 52.5 \times 52.5 - 5544$

$= 8662.5 - 5544 = 3118.5$ cm² **1**

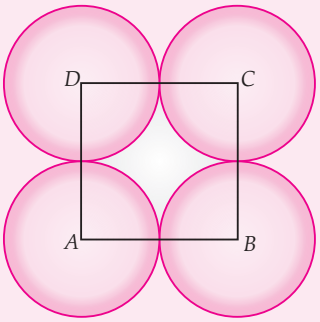
(ii) Areas related to circle. **1**

Long Answer Type Questions (4 marks each)

Q. 1. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}$ cm². Find the radius of each circle.

C + A **[Board Sample Paper, 2016]**

Sol.



1

Let r cm be the radius of each circle.

Area of square – Area of 4 sectors $= \frac{24}{7}$ cm^2 ½

or, $(2r)^2 - 4\left(\frac{90°}{360°} \times \pi r^2\right) = \frac{24}{7}$ 1

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$ ½

or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

or, $6r^2 = 24$

or, $r^2 = 4$ 1

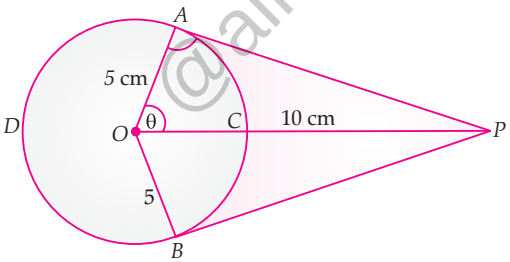
or, $r = \pm 2$

Radius of each circle is 2 cm (r cannot be negative)

[CBSE Marking Scheme, 2016]

Q. 2. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



C + A **[Delhi Set I, II, III, 2016]**

Sol. $\cos\theta = \frac{1}{2}$ or, $\theta = 60°$ ½

Reflex $\angle AOB = 240°$ ½

$\therefore ADB = \frac{2 \times 3.14 \times 5 \times 240°}{360°} = 20.93$ cm 1

Hence length of elastic in contact = 20.93 cm

Now, $AP = 5\sqrt{3}$ cm

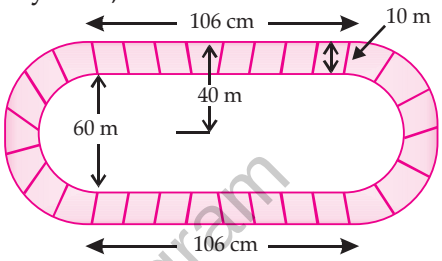
Area ($\Delta OAP + \Delta OBP$) $= 25\sqrt{3} = 43.25$ cm^2 ½

Area of sector $OACB = \frac{25 \times 3.14 \times 120°}{360°} = 26.16$ cm^2

½

Shaded Area $= 43.25 - 26.16 = 17.09$ cm^2 1

Q. 3. Fig. depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide everywhere, find the area of the track.



C + A **[Board Term-2, 2011, Set B1]**

Sol. Width of the inner parallel lines = 60 m

And the width of the outer lines = 40 × 2 = 80 m

Radius of the inner semicircles

$= \frac{60}{2} = 30$ m

Radius of the outer semicircles $= \frac{80}{2} = 40$ m

Area of inner rectangle $= 106 \times 30 = 3180$ m^2 ½

Area of outer rectangle $= 106 \times 40 = 4240$ m^2 ½

Area of the inner semicircle

$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 30 \times 30$

$= \frac{19800}{7}$ m^2 1

Area of outer semicircles

$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 40 \times 40$

$= \frac{35200}{7}$ m^2 1

Area of racing track = (area of outer rectangle + area of outer semicircles) – (area of inner rectangle + area of inner semicircles.)

$= 4240 + \frac{35200}{7} - \left(3180 + \frac{19800}{7}\right)$

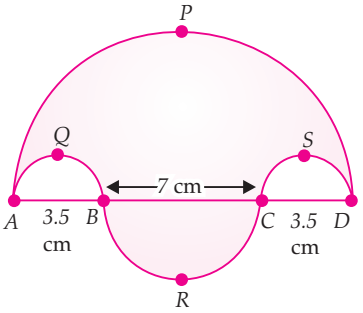
$= 1060 + \frac{15400}{7} = \frac{7420 + 15400}{7}$

$= \frac{22820}{7} = 3260$ m^2

Hence, area of track = 3260 m^2. 1

[CBSE Marking Scheme, 2011]

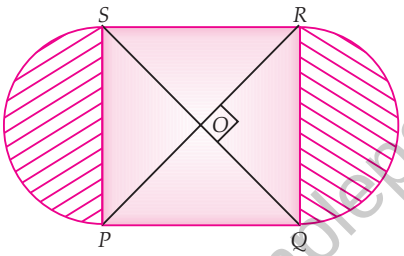
Q. 4. Find the area of the shaded region in figure, $\overset{\frown}{APD}$, $\overset{\frown}{AQB}$, $\overset{\frown}{BRC}$ and $\overset{\frown}{CSD}$, are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. $\left(\text{Use } \pi = \frac{22}{7}\right)$.



C + A **[Foreign Set I, II, III, 2016]**

Sol. Try yourself, similar to Q No. 28 in SATQ-II.

Q. 5. In figure, *PQRS* is square lawn with side *PQ* = 42 metre. Two circular flower beds are there on the sides *PS* and *QR* with centre at *O*, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



C + A **[Outside Delhi Set I, II, III, 2015]**

Sol. Radius of circle with centre *O* is *OR*.

Let $OR = x$

$\therefore \quad x^2 + x^2 = (42)^2$ or, $x = 21\sqrt{2}$ m **1**

(Using pythagoras theorem)

Area of the flower bed = Area of segment of circle with centre angle = 90°

$$= \frac{22}{7} \times 21\sqrt{2} \times 21\sqrt{2} \times \frac{90^\circ}{360^\circ}$$

$$- \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2} \quad \mathbf{1}$$

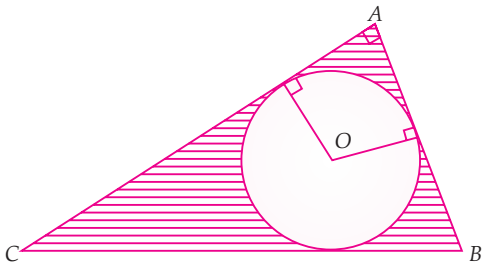
$$= 693 - 441 = 252 \text{ m}^2$$

½ + ½ + ½

$\therefore$ Area of flower beds $= 2 \times 252 = 504 \text{ m}^2$. ½

[CBSE Marking Scheme, 2015]

Q. 6. In the figure, *ABC* is a right angled triangle right angled at $\angle A$. Find the area of the shaded region, if *AB* = 6 cm, *BC* = 10 cm and *O* is the centre of the incircle of the triangle *ABC*.



C + A **[Board Term-2, 2015]**

Sol. Let r be the radius of incircle.

Then, $BC = 10 = 8 - r + 6 - r$ **1**

(By using the tangent properties)

or, $2r = 8 + 6 - 10$

or, $2r = 4$ or, $r = 2$ cm

$\therefore$ Area of circle $= \pi r^2 = \frac{22}{7} \times 2 \times 2$

$$= \frac{88}{7} = 12.57 \text{ cm}^2 \quad \mathbf{1}$$

Now, area of $\Delta ABC = \frac{1}{2} \times 8 \times 6$

$= 24 \text{ cm}^2$ **1**

$\therefore$ Area of shaded region = Area of ΔABC – Area of circle

$= 24 - 12.57$

$= 11.43 \text{ cm}^2$ **1**

[CBSE Marking Scheme, 2015]

Q. 7. Two circular beads of different sizes are joined together such that the distance between their centres is 14 cm. The sum of their areas is 130 π cm². Find the radius each bead.

C + A **[Board Term-2, 2015]**

Sol. Let the radii of the circles are r_1 cm and r_2 cm.

$\therefore \quad r_1 + r_2 = 14$ ...(i)

and sum of their areas $= \pi r_1^2 + \pi r_2^2$

$130\,\pi = \pi(r_1^2 + r_2^2)$

or, $130\,\pi = \pi(r_1^2 + r_2^2)$

$\therefore \quad r_1^2 + r_2^2 = 130$ ...(ii) **1**

$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

or, $(14)^2 = 130 + 2r_1r_2$

or, $2r_1r_2 = 196 - 130$

$= 66$ **1**

$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$= 130 - 66 = 64$

or, $r_1 - r_2 = 8$ ...(iii) **1**

From (i) and (iii), $2r_1 = 22$

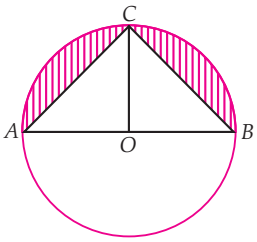
or, $r_1 = 11$ cm

and $r_2 = 14 - 11$

$= 3$ cm. **1**

[CBSE Marking Scheme, 2015]

Q. 8. A round thali has 2 inbuilt triangular for serving vegetables and a separate semi-circular area for keeping rice or chapati. If radius of thali is 21 cm, find the area of the thali that is shaded in the figure.



C + A [Board Term-2, 2014]

Sol. Since, AOB is the diameter of the circle. So,
Area of shaded region = (Area of semi-circle – Area of ΔABC) **1**

$$\text{Area of semi-circle} = \frac{\pi r^2}{2} = \frac{1}{2}\times\frac{22}{7}\times 21\times 21 \text{ cm}^2$$

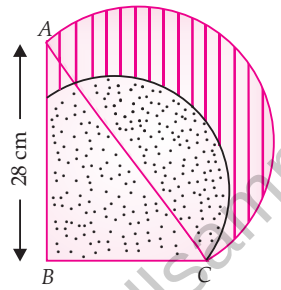
$$= \frac{1386}{2} = 693\,\text{cm}^2$$

$$\text{Area of triangle} = \frac{1}{2}\times 42\times 21 = 441 \text{ cm}^2 \quad \mathbf{1}$$

Area of shaded region = 693 – 441
= 252 sq. cm. **1**

[CBSE Marking Scheme, 2011]

Q. 9. In the fig., ABC is a right-angled triangle, $\angle B = 90°$, $AB = 28$ cm and $BC = 21$ cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region.



C + A [Foreign Set I, II, III, 2014]
[Board Term-2, 2011, A1]

Sol. In right ΔABC, right angled at B,

$$AC^2 = AB^2 + BC^2$$

(Pythagoras theorem)

$$= 28^2 + 21^2$$

$$= 784 + 441 = 1225$$

$$AC = 35 \text{ cm} \quad \mathbf{1}$$

Area of shaded region = area of ΔABC + area of semi-circle with diameter AC – area of quadrant with radius BC

$$= \frac{1}{2}(21\times 28) + \frac{1}{2}\times\frac{22}{7}\times\frac{35}{2}\times\frac{35}{2} - \frac{1}{4}\times\frac{22}{7}\times 21\times 21$$

= 294 + 481.25 – 346.5 **1½**
= 775.25 – 346.5 **1**
= 428.75 cm². **½**

[CBSE Marking Scheme, 2011]

Q. 10. The diameters of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. Find the number of revolutions of rear wheel to cover the distance which the front wheel covers in 800 revolutions. C + U [Delhi 2013]

Sol. Here, circumference of front wheel

$$= \pi d = \frac{22}{7}\times 80 = \frac{1760}{7} \text{ cm} \quad \mathbf{1}$$

Distance covered by front wheel in 800 revolutions

$$= \frac{1760}{7}\times 800 \quad \mathbf{1}$$

Circumference of rear wheel

$$= \frac{22}{7}\times 200 = \frac{4400}{7} \text{ cm} \quad \mathbf{1}$$

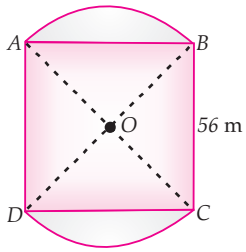
No. of revolutions made by rear wheel

$$\frac{\frac{1760}{7}\times 800}{\frac{4400}{7}} = \frac{1760\times 800}{4400}$$

$$= 320 \quad \mathbf{1}$$

[CBSE Marking Scheme, 2011]

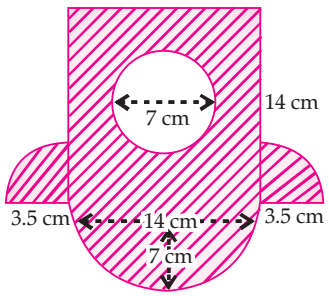
Q. 11. In fig., two circular flower beds have been shown on two sides of a square lawn $ABCD$ of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and flower beds.



C + A [Board Term-2, 2011, Set A1]

Sol. Try yourself, similar to Q. No. 5 in LATQ.

Q. 12. In fig., find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$ C + A [Board Term-2, 2011, Set B1]



Sol. Area of square = $(14)^2$ cm² = 196 cm² **1**

Area of internal circle = $\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$ cm²

$$= \frac{77}{2} = 38.5 \text{ cm}^2 \quad \mathbf{1}$$

Area of semi-circle with 14 cm diameter

$= \frac{1}{2} \times \frac{22}{7} \times 7^2 \text{ cm}^2$ ½

$= 77 \text{ cm}^2$ ½

Area of two quarter circles of radius $\frac{7}{2}$ cm

$= 2 \times \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{4} = 19.25 \text{ cm}^2$

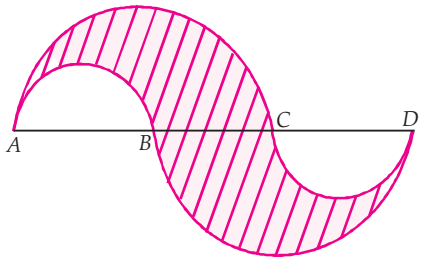
$\therefore$ Shaded area $= 196 - 38.5 + 77 + 19.25$

$= 292.25 - 38.5$ **1**

$= 253.75 \text{ cm}^2$.

[CBSE Marking Scheme, 2011]

Q. 13. In fig., $AC = BD = 7$ cm and $AB = CD = 1.75$ cm. Semi-circles are drawn as shown in the figure. Find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



C + A **[Board Term-2, 2011, Set B1]**

Sol. Area of shaded region

$= 2\left(\text{Area of semi-circle of radius } \frac{7}{2} \text{cm}\right)$

$-2\left(\text{Area of semi-circle of radius } \frac{7}{8} \text{cm}\right)$ 1½

$= 2\left[\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right] - 2\left[\frac{1}{2} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\right]$

$= \left(\frac{77}{2} - \frac{77}{8}\right) \text{cm}^2$ 1½

$= \frac{77}{2}\left[1 - \frac{1}{4}\right]$

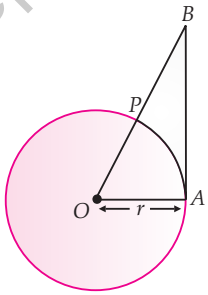
$= \frac{77}{2} \times \frac{3}{4} \text{ cm}^2$

$= \frac{231}{8} \text{cm}^2$

$= 28.87 \text{cm}^2$ **1**

[CBSE Marking Scheme, 2011]

Q. 14. The given fig. shows a sector OAP of a circle with centre O, containing $\angle \theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right]$



C + A **[O.D. Set I,II, III,2015, 16]**

Sol.

Given – OAP is sector of circle with centre O, $\angle POA = \theta$ and $OA \perp AB$

To prove –

Perimeter of shaded region $= r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$

Proof –

Perimeter of shaded region $= BP + AB + \text{arc } \hat{AP}$ — (IV)

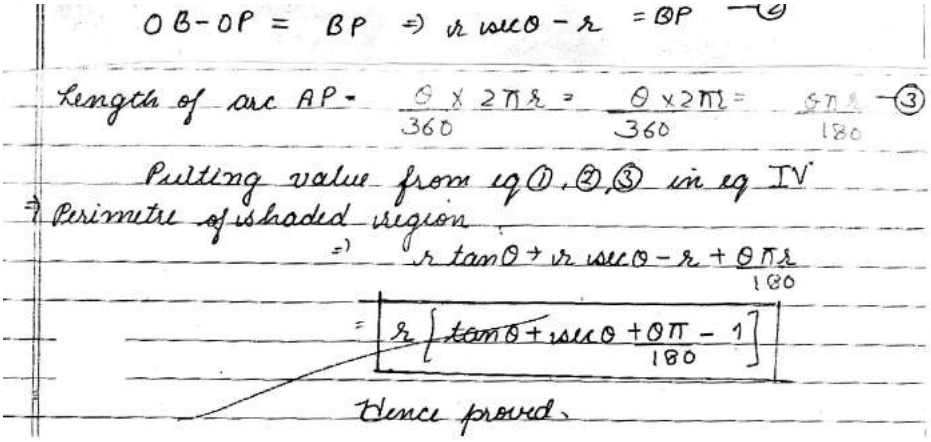
$= BA$

Now –

$\tan\theta = \frac{AB}{r} \Rightarrow r\tan\theta = AB$ — (1)

$\sec\theta = \frac{OB}{r} \Rightarrow r\sec\theta = OB$ ✓

(2)



[Topper Answer, 2016] **4**

Q. 15. Two circles touch internally. The sum of their areas is 116π and the difference between their centres is 6 cm. Find the radii of the circles.

C + A **[Foreign Set-I, II, III 2017]**

Sol. Let the radius of larger circle be x and the radius of smaller circle be y. **1**

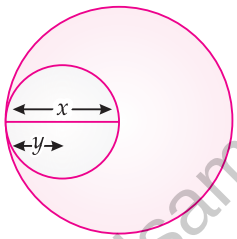
$\therefore \quad x - y = 6 \quad ...(i)$

and $\pi x^2 + \pi y^2 = 116\pi$

$\pi(x^2 + y^2) = 116\pi$

$x^2 + y^2 = 116 \quad ...(ii)$ **1**

from (i) and (ii),



$x^2 + (x - 6)^2 = 116$

$x^2 + x^2 - 12x + 36 = 116$

$\Rightarrow \quad x^2 - 6x - 40 = 0$

$\Rightarrow \quad x^2 - 10x + 4x - 40 = 0$ **1**

$\Rightarrow x(x - 10) + 4(x - 10) = 0$

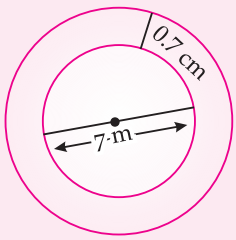
$\Rightarrow \quad (x - 10)(x + 4) = 0$

$\Rightarrow \quad x = 10$, and $y = 10 - 6 = 4$

Hence, radii of the circles = 10 cm and 4 cm. **1**

Q. 16. A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0·7 m. Find the expenditure of cementing the path. If its cost is ₹ 110 per sq. m. C + A **[Foreign Set-I, II, III 2017]**

Sol.



Given, the diameter of park = 7 m ½

$\therefore \quad$ Radius $= \frac{7}{2} = 3{\cdot}5$ m

The width of path = 0·7 m

$\therefore$ Radius of park with path

$= 3{\cdot}5 + 0{\cdot}7 = 4{\cdot}2$ m ½

Area of the path $= \pi(4{\cdot}2)^2 - \pi(3{\cdot}5)^2$

$= \frac{22}{7}(17{\cdot}64 - 12{\cdot}25)$ ½

$= \frac{22}{7} \times 5{\cdot}39 = 22 \times 0{\cdot}77$

$= 16{\cdot}94$ m^2 **1½**

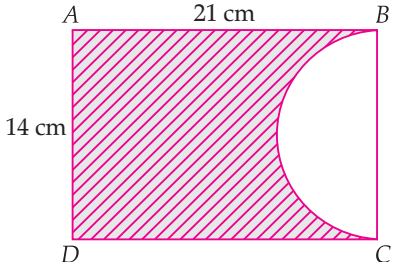
Cost of the cementing the path

$= 16{\cdot}94 \times 110$

$=$ ₹ 1863·40 **1**

[CBSE Marking Scheme, 2017]

Q. 17. In the given figure, *ABCD* is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with *BC* as diameter. Find the area and the perimeter of the shaded region in the figure.



C + A **[Outside Delhi Set-I, 2017]**

Sol. Area of shaded region = Area of rectangle *ABCD* – Area of semicircle **1**

$= 21 \times 14 - \frac{1}{2} \times \pi \times 7 \times 7$

$= 294 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 294 - 77$ **1**

$= 217 \text{ cm}^2$

Perimeter of shaded area

$= AB + AD + CD +$ circumference of semicircle

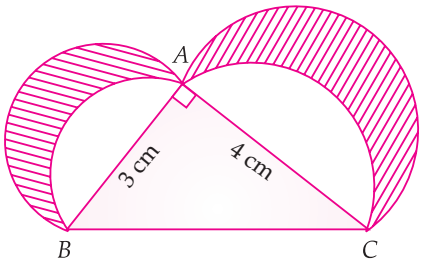
$= 21 + 14 + 21 + \frac{22}{7} \times 7$

$= 21 + 14 + 21 + 22$

$= 78$ cm **1**

Hence, area of shaded region $= 217 \text{ cm}^2$ and perimeter $= 78$ cm. **1**

Q. 18. In the given figure, ΔABC is a right angled triangle in which $\angle A = 90°$. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. C + A **[Outside Delhi Set-II 2017]**



Sol. In ΔABC, $\angle A = 90°$, $AB = 3$ cm, and $AC = 4$ cm

$\therefore$ $BC = \sqrt{AB^2 + AC^2} = \sqrt{3^2 + 4^2}$

$= 5$ cm. ½

Area of semicircle with radius $\frac{3}{2}$ cm + Area of semi circle with radius $\frac{4}{2}$ cm $= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}\left(\frac{4}{2}\right)^2$

Area of semicircle with radius $\frac{5}{2}$ cm – Area of $\Delta ABC = \frac{\pi}{2}\left(\frac{5}{2}\right)^2 - \frac{1}{2} \times 3 \times 4$

$= \left(\frac{25}{8}\pi - 6\right)\text{cm}^2$ ...(i) **1**

Area of shaded region

$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}(2)^2 - \left[\frac{25}{8}\pi - 6\right]\text{cm}^2$ **1**

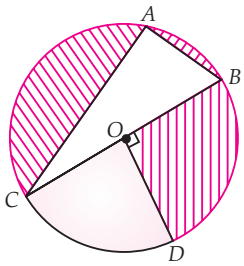
$= \frac{\pi}{2}\left[\frac{9}{4} + 4 - \frac{25}{4}\right] + 6$

$= \frac{\pi}{2}\left[\frac{9}{4} + \frac{16 - 25}{4}\right] + 6$

$= \frac{\pi}{2}\left[\frac{9}{4} - \frac{9}{4}\right] + 6$ **1**

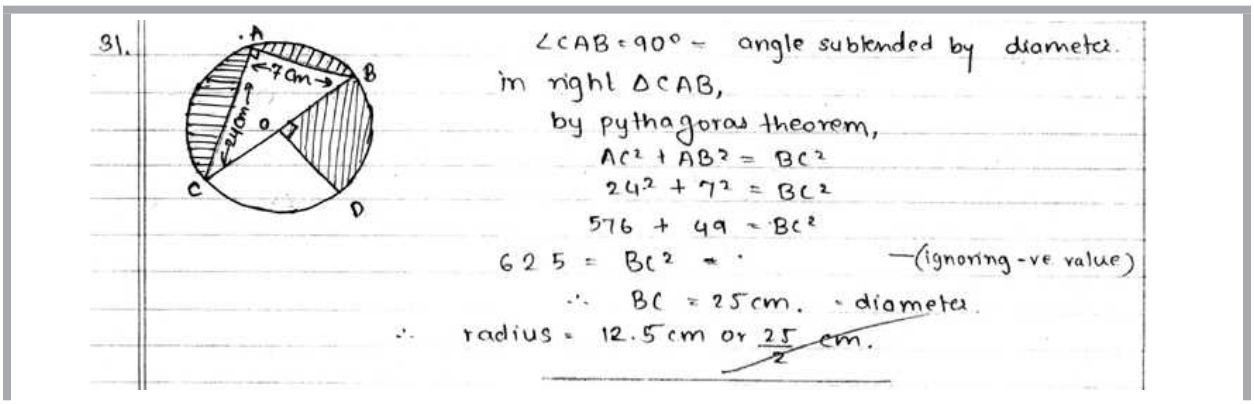
$= 6 \text{ cm}^2$ **[CBSE Marking Scheme, 2017]** ½

Q. 19. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$. Find the area of the shaded region.



C + A **[OD Set-III, 2017]**

Sol.



area of shaded region = area of semicircle + area of quadrant − area of ΔACB

$= \frac{1}{2} \times \pi r^2 + \frac{1}{4} \times \pi r^2 - \frac{1}{2} \times AB \times AC$

$= \frac{3}{4} \pi r^2 - \frac{1}{2} \times 7 \times 24$

$= \frac{3}{4} \times \frac{22}{7} \times \frac{625}{4} - 7 \times 12$

$= 368.3035 - 84$

$= 284.3035$

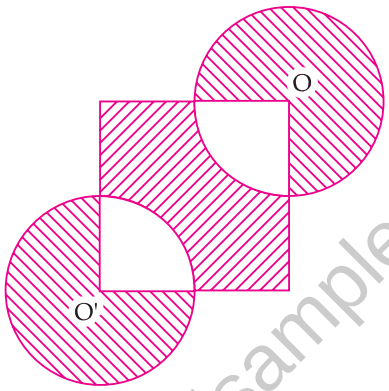
$\approx 284.3 \text{ cm}^2$

∴ The area of shaded region is 284.3035 cm^2

[Topper Answer, 2017] 4

Q. 20. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square, where O and O′ are centres of the circle. Find the area of shaded area.

C + A [Delhi Set-I, II 2017]



Sol. Given, the side of the square = 28 cm.

Area of the square = $28 \times 28 = 784$ cm ½

Radius of each circle = $\frac{28}{2} = 14$ cm ½

∴ Area of two circles = $2 \times \frac{22}{7} \times 14 \times 14$

$= 1232 \text{ cm}^2$ ½

Area of the 2 quadrant = $\frac{90^\circ}{360^\circ} \times 14 \times 14 \times 2$

$= 98 \text{ cm}^2$ ½

Area the shaded region

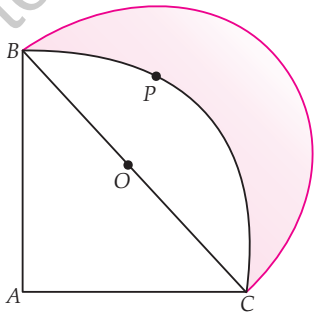
= Area of square + Area of two circles – area of two quadrants

$= 784 + 1232 - 98 = 1918 \text{ cm}^2$

Hence, the area of shaded region = 1918 cm^2 **2**

[CBSE Marking Scheme, 2017]

Q. 21. In given figure, *ABPC* is a quadrant of a circle of radius 14 cm and a semicircle is drawn with *BC* as diameter. Find the area of the shaded region.



C + A [Sample Question Paper 2017]

Sol. Given, radius of the quadrant $AB = AC = 14$ cm $= R$.

∴ $BC = \sqrt{14^2 + 14^2} = 14\sqrt{2}$ cm ½

∴ Radius of semicircle $= 7\sqrt{2}$ cm ½

Area of semicircle $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}$

$= 154 \text{ cm}^2$ **1**

Area of segment $BPCO = \frac{\pi R^2 \theta}{360^\circ} - \frac{1}{2} \sin\theta R^2$ **1**

$\Rightarrow R^2 \left(\frac{\pi\theta^\circ}{360^\circ} - \frac{1}{2} \sin\theta \right)$

$= 14 \times 14 \left(\frac{22}{7} \times \frac{90^\circ}{360^\circ} - \frac{1}{2} \sin 90^\circ \right)$

$= 14 \times 14 \times \frac{2}{7} = 56 \text{ cm}^2$ **1**

Hence, area of shaded region = 56 cm^2.

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