7 Probability Distributions

Johann Carl Friedrich Gauss
(April 30, 1777 – Feb. 23, 1855)

Introduction

The frequency distributions are of two types namely Observed frequency distribution and Theoretical frequency distribution. The distributions which are based on actual data or experimentation are called the Observed Frequency distribution. On the other hand, the distributions based on expectations on the basis of past experience are known as Theoretical Frequency distribution or Probability distribution.

Johann Carl Friedrich Gauss (30 April 1777 – 23 February 1855) was a German mathematician and physicist who made significant contributions to many fields in mathematics and sciences. Gauss had an exceptional influence in many fields of mathematics and science, and is ranked among history's most influential mathematicians

In discrete probability distribution we will discuss Binomial and Poisson distribution and the Normal Distribution is a continuous probability distribution

Learning Objectives

After studying this chapter students are able to understand

- Concept of Bernoulli trial
- Binomial , poisson and normal density function
- Mean and variance of binomial and poisson distribution
- Properties of normal probability curve

7.1 Distribution

The following are the two types of Theoretical distributions :

1. Discrete distribution
2. Continous distribution

Discrete distribution

The binomial and Poisson distributions are the most useful theoretical distributions for discrete variables.

7.1.1 Binomial distribution

Binomial distribution was discovered by James Bernoulli(1654-1705) in the year 1700 and was first published posthumously in 1713 , eight years after his death.

A random experiment whose outcomes are of two types namely success S and failure F, occurring with probabilities p and q respectively, is called a Bernoulli trial.

Some examples of Bernoulli trials are :

(i) Tossing of a coin (Head or tail)

(ii) Throwing of a die (getting even or odd number)

Consider a set of n independent Bernoullian trails (n being finite) in which the probability 'p' of success in any trial is constant , then $q = 1-p$, is the probability of failure. The probability of x successes and consequently $(n-x)$ failures in n independent trials, in a specified order (say) SSFSFFFS....FSF is given in the compound probability theorem by the expression

P(SSFSFFFS.....FSF) = P(S)P(S)P(F)P(S)... P(F)P(S)P(F)

$= p.p.q.p \ldots q.p.q$

$= p.p.p.p \ldots q.q.q.q.q.q \ldots$

$= \{x \text{ factors}\}\ \{(n-x) \text{ factors}\}$

$= p^x q^{(n-x)}$

x successes in n trials can occur in ^nC_x ways and the probability for each of these ways is same namely $p^x q^{n-x}$.

The probability distribution of the number of successes, so obtained is called the binomial probability distribution and the binomial expansion is $(q+p)^n$

Definition 7.1

A random variable X is said to follow binomial distribution with parameter n and p, if it assumes only non- negative value and its probability mass function in given by

$$P(X = x) = p(x) = \begin{cases} {}^nC_x p^x q^{n-x}, & x = 0,1,2,\ldots..n; q = 1-p \\ 0, & \text{otherwise} \end{cases}$$

Note

Any random variable which follows binomial distribution is known as binomial variate i.e $X \sim B(n,p)$ is a binomial variate.

The Binomial distribution can be used under the following conditions :

1. The number of trials 'n' finite
2. The trials are independent of each other.
3. The probability of success 'p' is constant for each trial.
4. In every trial there are only two possible outcomes – success or failure.

Derivation of the Mean and Variance of Binomial distribution :

The mean of the binomial distribution

$$E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$= p \sum_{x=1}^{n} x.\left(\frac{n}{x}\right)\binom{n-1}{x-1} p^{x-1} q^{n-x}$$

$= np\,(q+p)^{n-1}$ [since $p+q = 1$]

$= np$

$E(X) = np$

∴ The mean of the binomial distribution is np

$\text{Var}(X) = E(X^2) - \{E(X)\}^2$

Here $E(X^2) = \sum_{x=0}^{n} x^2 \binom{n}{x} p^x q^{n-x}$

$$\sum_{x=0}^{n} \{x(x-1)+x\} \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \{x(x-1)\} \binom{n}{x} p^x q^{n-x} + \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{c=2}^{n} \{x(x-1)\} p^2 \left(\frac{n(n-1)}{x(x-1)}\right) \binom{n-2}{x-2} p^{x-2} q^{n-x}$$

$$+ \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$= n(n-1)\, p^2 \{ \sum_{x=2}^{n} \binom{n-2}{x-2} p^{x-2} q^{n-x} \} \quad + np$$

$$= n(n-1)p^2(q+p)^{(n-2)} + np$$

$$= n(n-1)p^2 + np$$

$\therefore$ Variance $= E(X^2) - \{E(X)\}^2$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np(1-p) = npq$$

Hence, mean of the BD is np and the Variance is npq.

Note

Mean and variance in terms of raw moments and central moments are denoted as μ'_1, and μ_2 respectively.

Properties of Binomial distribution

1. Binomial distribution is symmetrical if $p = q = 0.5$. It is skew symmetric if $p \neq q$. It is positively skewed if $p < 0.5$ and it is negatively skewed if $p > 0.5$

2. For Binomial distribution, variance is less than mean

$$\text{Variance } npq = (np)q < np$$

Example 7.1

A and *B* play a game in which their chance of winning are in the ratio 3:2 Find *A*'s chance of winning atleast three games out of five games played.

Soltion:

Let 'p' be the probability that 'A' wins the game. Then we are given $n = 5$, $p = 3/5$,

$$q = 1- \frac{3}{5} = \frac{2}{5} \text{ (since } q = 1-p)$$

Hence by binomial probability law, the probability that out of the 5 games played, *A* wins 'x' games is given by

$$P(X=x) = p(x) = 5Cx \left(\frac{3}{5}\right)^x \left(\frac{2}{5}\right)^{5-x}$$

The required probability that 'A' wins atleast three games is given by

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$$

$$= 5C3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^2 + 5C4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^1 + 5C5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^o$$

$= 0.6826$

Example 7.2

A fair coin is tossed 6 times. Find the probability that exactly 2 heads occurs.

Solution :

Let X be a random variable follows binomial distribution with probability value $p = 1/2$ and $q = 1/2$

Probability that exactly 2 heads occur are as follows

$$P(X = 2) = \binom{6}{x} p^x q^{n-x}$$

$$= \binom{6}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2}$$

$$= \frac{15}{64}$$

Example 7.3

Verfy the following statement:

The mean of a Binomial distribution is 12 and its standard deviation is 4.

Solution:

Mean: $np = 12$

$$SD = \sqrt{npq} = 4$$

$$npq = 4^2 = 16\ ,\ \frac{np}{npq} = \frac{12}{16} = \frac{3}{4}$$

$$=> q = \frac{4}{3} > 1$$

Since $p + q$ cannot be greater than unity, the Statement is wrong

Example 7.4

The probability that a student get the degree is 0.4 Determine the probability that out of 5 students (i) one will be graduate (ii) atleast one will be graduate

Solution:

Probability of getting a degree $p= 0.4$

$$\therefore\quad q = 1-p$$
$$= 1 - 0.4$$
$$= 0.6$$

(i) P (one will be a graduate) $= P(X = 1)$

$$= 5C_1\,(0.4)(0.6)^4$$
$$= 0.2592$$

(ii) P (atleast one will be a graduate)

$= 1-P$ (none will be a graduate)

$$= 1 - 5C_0(P^0)(q)^{5-0}$$
$$= 1 - 5C_0(0.4)^0\,(0.6)^5$$
$$= 1 - 0.0777$$
$$=0.9222$$

Example 7.5

In tossing of a five fair coin, find the chance of getting exactly 3 heads.

Solution :

Let X be a random variable follows binomial distribution with $p = q = 1/2$

$$P\text{ (3 heads)} = 5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$$
$$= 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$$
$$= 5C_3 \left(\frac{1}{2}\right)^5$$
$$= \frac{5}{16}$$

Example 7.6

The mean of Binomials distribution is 20 and standard deviation is 4. Find the parameters of the distribution.

Solution

The parameters of Binomial distribution are n and p

For Binomial distribution Mean = np =20

$$\text{Standard deviation} = \sqrt{npq} = 4$$

$$\therefore\quad npq = 16$$

$$\Rightarrow\quad npq/np = 16/20 = 4/5$$

$$q = \frac{4}{5}$$

$$\Rightarrow\quad p = 1-q = 1-(4/5) = 1/5$$

Since $\quad np = 20$

$$n = \frac{20}{p}$$

$$n = 100$$

Example 7.7

If x is a binomially distributed random variable with $E(x)$ =2 and van (x) $= \frac{4}{3}$. Find $P(x = 5)$

Solution:

The p.m.f. Binomial distribution is

$p(x) = {}^nC_x p^x \quad q^{n-x}$

Given that $E(x) = 2$

For the Binomial distribution mean is given by $np = 2$ ... (1)

Given that var $(x) = 4/3$

For Binomial distribution variance is given by $npq = 4/3$ ... (2)

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{\left(\frac{4}{3}\right)}{2} = \frac{4}{6} = \frac{2}{3}$$

$q = 2/3$ and $p = 1-2/3 = 1/3$

Substitute in (1) we get

$$n = 6$$

Hence, $P(X=5) = 6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5} = 0.0108$

Example 7.8

If the average rain falls on 9 days in every thirty days, find the probability that rain will fall on atleast two days of a given week.

Solution :

Probability of raining on a particular day is given by $p = 9/30 = 3/10$ and

$q = 1-p = 7/10.$

The *binomial distribution* is

$P(X = x) = {}^nC_x p^x q^{n-x}$

There are 7 days in a week,

$$P(X = x) = \binom{7}{x}\left(\frac{3}{10}\right)^x \left(\frac{7}{10}\right)^{7-x}$$

The probability of raining for atleast 2 days is given by

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - \left[P(X=0) + P(X=1)\right]$$

Here, $P(X=0) = \binom{7}{0}\left(\frac{3}{10}\right)^0 \left(\frac{7}{10}\right)^{7-0}$

$= 0.0823$

and $P(X=1) = \binom{7}{1}\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^{7-1}$

$= 0.2471$

Therefore the required probability

$= 1- [P(x = 0) + P(x = 1)]$

$= 1- \{0.082+ 0.247]$

$= 0.6706$

Example 7.9

What is the probability of guessing correctly atleast six of the ten answers in a TRUE/FALSE objective test?

Solution :

Probability p of guessing an answer correctly is $p = \frac{1}{2}$

$\Rightarrow \quad q = \frac{1}{2}$

Probability of guessing correctly x answers in 10 questions

$$P(X = x) = p(x) = {}^nC_x\, p^x q^{n-x}$$

$$= 10Cx\left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x}$$

The required probability $P(X \geq 6)$

$= P(6) + P(7) + P(8) + P(9) + P(10)$

$$= \left(\frac{1}{2}\right)^{10}\left[10C_6 + 10C_7 + 10C_8 + 10C_9 + 10C_{10}\right]$$

$$= \left[\frac{1}{1024}\right]\left[210 + 120 + 45 + 10 + 1\right]$$

$$= \frac{193}{512}$$

Example 7.10

If the chance of running a bus service according to schedule is 0.8, calculate the probability on a day schedule with 10 services : (i) exactly one is late (ii) atleast one is late

Solution :

Probability of bus running late is denoted as $p = 1-0.8 = 0.2$

Probability of bus running according to the schedule is $q = 0.8$

Also given that $n = 10$

The binomial distribution is

$$p(x) = 10C_x \left(0.2\right)^x \left(0.8\right)^{10-x}$$

(i) probability that exactly one is late

$$P(x=1) = 10C_1\, p q^9$$

$$= 10C_1 \left(0.2\right)\left(0.8\right)^9$$

(ii) probability that at least one is late

= 1 – probability that none is late

$= 1 - p(x=0)$

$= 1 - (0.8)^{10}$

Example 7.11

The sum and product of the mean and variance of a binomial distribution are 24 and 128. Find the distribution.

Solution:

For binomial distribution the mean is np and varaiance is npq

Given values are

$np + npq = 24 \quad np(1 + q) = 24 \ – \ (1)$

Other term

$np \times npq = 128 \quad n^2 p^2 q = 128 \ – \ (2)$

From (1) we get $np = 24/(1+q)$ which implies $n^2p^2 = (24/(1+q))^2$

Substitute this value in equation (2) we get

$$\left(\frac{24}{1+q}\right)^2 q = 128 \text{ which implies } 9q = 2(1+2q+q^2)$$

$(2q - 1)(q - 2) = 0$

Where $q = \frac{1}{2}$ and $p = \frac{1}{2}$

Substitute in (1) we get $n = 32$

Hence the binomial distribution

$$32C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{32-x}$$

Example 7.12

Suppose A and B are two equally strong table tennis players. Which of the following two events is more probable:

(a) A beats B exactly in 3 games out of 4 or

(b) A beats B exactly in 5 games out of 8 ?

Solution :

Here $p = q = 1/2$

(a) probability of A beating B in exactly 3 games out of 4

$$\binom{4}{3}\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3}$$

= ¼ = 25%

(b) probability of A beating B in exactly 5 games out of 8

$$\binom{8}{5}\left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{8-5}$$

$$= \frac{7}{32} = 21.875\%$$

Clearly, the first event is more probable.

Example 7.13

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of 2 successes.

Solution :

In a throw of a pair of dice the doublets are (1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

Probability of getting a doublet

$p = 6/36 = 1/6$

$\Rightarrow q = 1 - p = 5/6$ and also $n = 4$ is given

The probabitliy of successes

$$= \binom{4}{x}\left(\frac{1}{6}\right)^{x}\left(\frac{5}{6}\right)^{4-x}$$

Therefore the probability of 2 successes are

$$P(X=2) = \binom{4}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{4-2}$$

$$= 6 \times \frac{1}{36} \times \frac{25}{36}$$

$$= \frac{25}{216}$$

Exercise 7.1

1. Define Binomial distribution.
2. Define Bernoulli trials.
3. Derive the mean and variance of binomial distribution.
4. Write down the conditions for which the binomial distribution can be used.
5. Mention the properties of binomial distribution.
6. If 5% of the items produced turn out to be defective, then find out the probability that out of 20 items selected at random there are

   (i) exactly three defectives

   (ii) atleast two defectives

   (iii) exactly 4 defectives

   (iv) find the mean and variance
7. In a particular university 40% of the students are having news paper reading habit. Nine university students are selected to find their views on reading habit. Find the probability that

   (i) none of those selected have news paper reading habit

   (ii) all those selected have news paper reading habit

   (iii) atleast two third have news paper reading habit.
8. In a family of 3 children, what is the probability that there will be exactly 2 girls?
9. Defects in yarn manufactured by a local mill can be approximated by a distribution with a mean of 1.2 defects for every 6 metres of length. If lengths of 6 metres are to be inspected, find the probability of less than 2 defects.
10. If 18% of the bolts produced by a machine are defective, determine the probability that out of the 4 bolts chosen at random

    (i) exactly one will be defective

    (ii) none will be defective

    (iii) atmost 2 will be defective
11. If the probability of success is 0.09, how many trials are needed to have a probability of atleast one success as 1/3 or more ?
12. Among 28 professors of a certain department, 18 drive foreign cars and 10 drive local made cars. If 5 of these professors are selected at random, what is the probability that atleast 3 of them drive foreign cars?

13. Out of 750 families with 4 children each, how many families would be expected to have (i) atleast one boy (ii) atmost 2 girls (iii) and children of both sexes? Assume equal probabilities for boys and girls.

14. Forty percent of business travellers carry a laptop. In a sample of 15 business travelers,

    (i) what is the probability that 3 will have a laptop?

    (ii) what is the probability that 12 of the travelers will not have a laptop?

    (iii) what is the probability that atleast three of the travelers have a laptop?

15. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of 2 successes.

16. The mean of a binomial distribution is 5 and standard deviation is 2. Determine the distribution.

17. Determine the binomial distribution for which the mean is 4 and variance 3. Also find P(X=15)

18. Assume that a drug causes a serious side effect at a rate of three patients per one hundred. What is the probability that atleast one person will have side effects in a random sample of ten patients taking the drug?

19. Consider five mice from the same litter, all suffering from Vitamin A deficiency. They are fed a certain dose of carrots. The positive reaction means recovery from the disease. Assume that the probability of recovery is 0.73. What is the probability that atleast 3 of the 5 mice recover.

20. An experiment succeeds twice as often as it fails, what is the probability that in next five trials there will be (i) three successes and (ii) at least three successes

7.1.2 Poisson Distribution

Poisson distribution was derived in 1837 by a French Mathematician Simeon D. Poisson. If n is large, the evaluation of the binomial probabilities can involve complex computations, in such a case, a simple approximation to the binomial probabilities could be use. Such approximation of binomial when n is large and p is close to zero is called the Poisson distribution.

Poisson distribution occurs when there are events which do not occur as a definite number on trials but an events occurs rarely and the following examples may be analysed:

(i) Number of bacteria in one cubic centimeter.

(ii) Number of printing mistakes per page in a text book

(iii) the number of alpha particles emitted by a radioactive substance in a fraction of a second.

(iv) Number of road accidents occurring at a particular interval of time per day.

Number of lightnings per second.

Poisson distribution is a limiting case of binomial distribution under the following conditions :

(i) n, the number of trials is indefinitely large i.e $n \to \infty$.

(ii) p, the constant probability of success in each trial is very small, i.e. $p \to 0$

(iii) $np = \lambda$ is finite. Thus $p = \frac{\lambda}{n}$ and $q = 1 - \left(\frac{\lambda}{n}\right)$ where λ is a positive real number.

Definition 7.2

A random variable X is said to follow a Poission distribution with parameter λ if it assumes only non–negative values and its probability mass function is given by

$P(x, \lambda) = P(X=x)$

$$= \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x = 0,1,2\ldots\ldots;\lambda > 0 \\ 0 & otherwise \end{cases}$$

Derivation of Mean and variance of Poisson distribution

$$\text{Mean } E(X) = \sum_{x=0}^{\infty} x\, p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \lambda e^{-\lambda} \left\{ \sum_{x=1}^{\infty} \left(\frac{\lambda^{x-1}}{(x-1)!} \right) \right\}$$

$$= \lambda e^{-\lambda}(1 + \lambda + \lambda^2/2! + \ldots\ldots)$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda$$

$$\text{Variance } (X) = E(X^2) - [E(X)]^2$$

$$\text{Here } E(X^2) = \sum_{x=0}^{\infty} x^2 p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} x^2 p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} \{x(x-1) + x\}\, p(x,\lambda)$$

$$= \sum_{x=0}^{\infty} \{x(x-1) + x\} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= e^{-\lambda}\sum_{x=0}^{\infty} x(x-1)\, \lambda^{x}/x! + \sum_{x=0}^{\infty} x\, e^{-\lambda} \frac{\lambda^{x}}{x!}$$

$$= \lambda^2 e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= \lambda^2 e^{-\lambda} e^{\lambda} + \lambda = \lambda^2 + \lambda$$

$$\text{Variance } (X) = E(X^2) - [E(X)]^2$$

$$= \lambda^2 + \lambda - (\lambda)^2$$

$$= \lambda$$

Properties of Poisson distribution :

1. Poisson distribution is the only distribution in which the mean and variance are equal .

Example 7.14

In a Poisson distribution the first probability term is 0.2725. Find the next Probability term

Solution :

Given that $p(0) = 0.2725$

$$\frac{e^{-\lambda}\lambda^{0}}{0!} = 0.2725$$

$$\Rightarrow e^{-\lambda} = 0.2725$$

(by using exponent table)

$$\lambda = 1.3$$

$$\therefore p(X=1) = e^{-1.3}(1.3)/1!$$

$$= e^{-1.3}(1.3)$$

$$= 0.2725 \text{ x } 1.3$$

$$= 0.3543$$

Example 7.15

In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.

Solution :

The average number of typographical errors per page in the book is given by $\lambda = (390/520) = 0.75$.

Hence using Poisson probability law, the probability of x errors per page is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} = e^{-0.75}\frac{(0.75)^x}{x!}, x = 0,1,2,\ldots$$

The required probability that a random sample of 5 pages will contain no error is given by :

$$[P(X=0)]^5 = \left(e^{-0.75}\right)^5 = e^{-3.75}$$

Example 7.16

An insurance company has discovered that only about 0.1 per cent of the population is involved in a certain type of accident each year. If its 10,000 policy holders were randomly selected from the population, what is the probability that not more than 5 of its clients are involved in such an accident next year? ($e^{-10} = .000045$)

Solution :

p = probability that a person will involve in an accident in a year

$= 0.1/100 = 1/1000$

given $n = 10,000$

so, $\lambda = np = 10000\left(\frac{1}{1000}\right) = 10$

Probability that not more than 5 will involve in such an accident in a year

$$P(X \le 5) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$$

$$= e^{-10}\left[1+\frac{10}{1!}+\frac{10^2}{2!}+\frac{10^3}{3!}+\frac{10^4}{4!}+\frac{10^5}{5!}\right]$$

$= 0.06651$

Example 7.17

One fifth percent of the the blades produced by a blade manufacturing factory turn out to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 1,00,000 packets ($e^{-0.02}$ =.9802).

Solution :

$P = 1/5/100 = 1/500 = 0.002 \quad n = 10 \ . \ \lambda = np = 0.02$

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.02}(0.02)^x}{x!}$$

(i) Number of packets containing no defective

$= N \times p(0) = 1,00,000 \times e^{-0.02}$

$= 98020$

(ii) Number of packets containing one defective

$= N \times p(1) = 1,00,000 \times 0.9802 \times 0.02$

$= 1960$

(iii) Number of packets containing 2 defectives

$= N \times p(2) = 20$

Example 7.18

If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determines the probability that out of 2,000 individuals (a) exactly 3 and (b) more than 2 individuals will suffer a bad reaction.

Solution :

Consider a 2,000 individuals getting injection of a given serum , $n = 2000$

Let X be the number of individuals suffering a bad reaction.

Let p be the probability that an individual suffers a bad reaction = 0.001

and $q = 1 - p = 1 - 0.001 = 0.999$

Since n is large and p is small, Binomial Distribtuion approximated to poisson distribution

So, $\lambda = np = 2000 \times 0.001 = 2$

(i) Probability out of 2000, exactly 3 will suffer a bad reaction is

$$P(X=3) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-2}2^3}{3!} = 0.1804$$

(ii) Probability out of 2000, more than 2 individuals will suffer a bad reaction

$= P(X > 2)$

$= 1 - [(P(X \leq 2)]$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$$= 1 - \left[\frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}\right]$$

$$= 1 - e^{-2}\left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!}\right)$$

$= 0.323$

Example 7.19

When counting red blood cells, a square grid is used, over which a drop of blood is evenly distributed. Under the microscope an average of 8 erythrocytes are observed per single square. What is the probability that exactly 5 erythrocytes are found in one square?

Solution :

Let X be a random variable follows poisson distribution with number of erythrocytes.

Hence, Mean $\lambda = 8$ erythrocytes/single square

P(exactly 5 erythrocytes are in one square)

$$= P(X=5) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-8}8^5}{5!}$$

$$= \frac{0.000335 \times 32768}{120}$$

$= 0.0916$

The probability that exactly 5 erythrocytes are found in one square is 0.0916. i.e there are 9.16% chances that exactly 5 erythrocytes are found in one square.

Example 7.20

Assuming one in 80 births is a case of twins, calculate the probability of 2 or more sets of twins on a day when 30 births occur.

Solution :

Let x denotes the set of twins on a day

P(twin birth) $= p = 1/80 = 0.0125$ and $n = 30$

The value of mean $\lambda = np = 30 \times 0.0125 = 0.375$

Hence, X follows poisson distribution with

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

The probability is

P(2 or more) $= 1 - [p\ (x=0) + p\ (x=1)]$

$$= 1 - [\frac{e^{-0.375}(0.375)^0}{0!} + \frac{e^{-0.375}(0.375)^1}{1!}]$$

$= 1 - e^{-0.375}\ [1+0.375]$

$= 1 - (0.6873 \times 1.375)$

$= 0.055$

Exercise 7.2

1. Define Poisson distribution.
2. Write any 2 examples for Poisson distribution.
3. Write the conditions for which the poisson distribution is a limiting case of binomial distribution.
4. Derive the mean and variance of poisson distribution.
5. Mention the properties of poisson distribution.
6. The mortality rate for a certain disease is 7 in 1000. What is the probability for just 2 deaths on account of this disease in a group of 400? [Given $e^{(-2.8)} = 0.06$]

7. It is given that 5% of the electric bulbs manufactured by a company are defective. Using poisson distribution find the probability that a sample of 120 bulbs will contain no defective bulb.

8. A car hiring firm has two cars. The demand for cars on each day is distributed as a Poisson variate, with mean 1.5. Calculate the proportion of days on which

   (i) Neither car is used

   (ii) Some demand is refused

9. The average number of phone calls per minute into the switch board of a company between 10.00 am and 2.30 pm is 2.5. Find the probability that during one particular minute there will be (i) no phone at all (ii) exactly 3 calls (iii) atleast 5 calls

10. The distribution of the number of road accidents per day in a city is poisson with mean 4. Find the number of days out of 100 days when there will be (i) no accident (ii) atleast 2 accidents and (iii) at most 3 accidents.

11. Assuming that a fatal accident in a factory during the year is 1/1200, calculate the probability that in a factory employing 300 workers there will be atleast two fatal accidents in a year. (given $e^{-0.25} = 0.7788$)

12. The average number of customers, who appear in a counter of a certain bank per minute is two. Find the probability that during a given minute (i) No customer appears (ii) three or more customers appear .

Continuous distribtuion

The binomial and Poisson distributions discussed in the previous chapters are the most useful theoretical distributions for discrete variables. In order to have mathematical distributions suitable for dealing with quantities whose magnitudes vary continuously like weight, heights of individual, a continuous distribution is needed. Normal distribution is one of the most widely used continuous distribution.

Normal distribution is the most important and powerful of all the distribution in statistics. It was first introduced by De Moivre in 1733 in the development of probability. Laplace (1749-1827) and Gauss (1827-1855) were also associated with the development of Normal distribution.

7.1.3 Normal Ditribution

Definition 7.3

A random variable X is said to follow a normal distribution with parameters mean μ and variance σ^2 , if its probability density function is given by

$$f(x:\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

$$\left.\begin{array}{r} -\infty < x < \infty, \\ -\infty < \alpha < \infty, \\ \sigma > 0 \end{array}\right\}$$

Normal distribution is diagrammatically represented as follows :

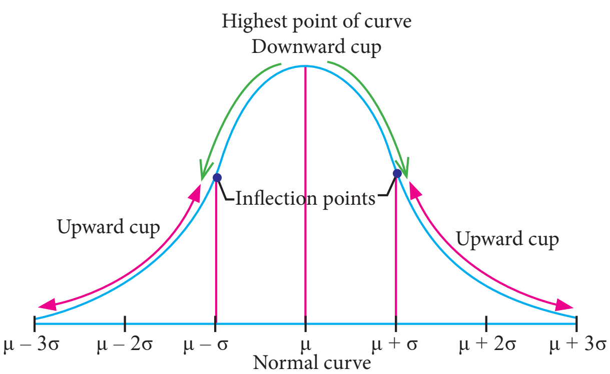


Fig 7.1

Normal distribution is a limiting case of binomial distribution under the following conditions:

(i) n, the number of trials is infinitely large, i.e. $n \to \infty$

(ii) neither p (nor q) is very small,

The normal distribution of a variable when represented graphically, takes the shape of a symmetrical curve, known as the Normal Curve. The curve is asymptotic to x-axis on its either side.

Chief Characterisitics or Properties of Normal Probabilty distribution and Normal probability Curve .

The normal probability curve with mean μ and standard deviation σ has the following properties :

(i) the curve is bell- shaped and symmetrical about the line $x=\mu$

(ii) Mean, median and mode of the distribution coincide.

(iii) x – axis is an asymptote to the curve. (tails of the cuve never touches the horizontal (x) axis)

(iv) No portion of the curve lies below the x-axis as $f(x)$ being the probability function can never be negative.

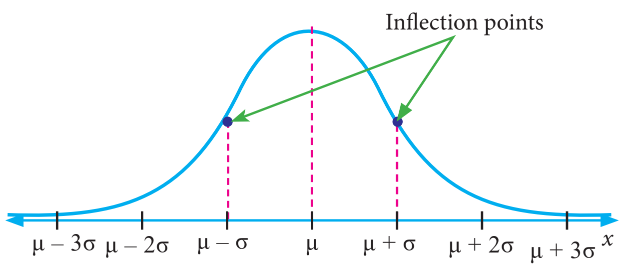


Fig 7.2

(v) The Points of inflexion of the curve are $x = \mu \pm \sigma$

(vi) The curve of a normal distribution has a single peak i.e it is a unimodal.

(vii) As x increases numerically, $f(x)$ decreases rapidly, the maximum probability occurring at the point x = μ and is given by $[p(x)]\max = 1/\sigma\sqrt{2\pi}$

(viii) The total area under the normal curve is equal to unity and the percentage distribution of area under the normal curve is given below

(a) About 68.27% of the area falls between μ – σ and μ + σ

$P(\mu - \sigma < X < \mu + \sigma) = 0.6826$

(b) About 95.5% of the area falls between μ – 2σ and μ + 2σ

$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$

(c) About 99.7% of the area falls between μ – 3σ and μ + 3σ

$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$

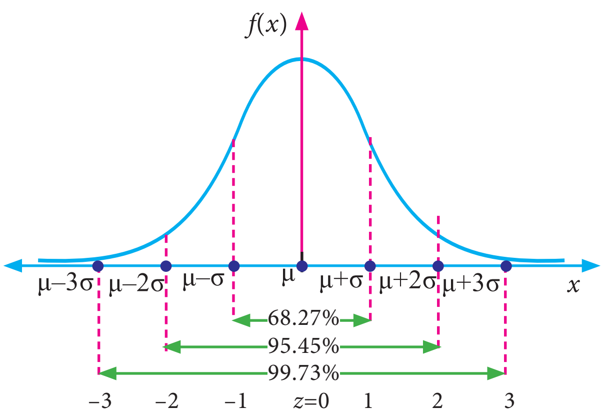


Fig 7.3

Standard Normal Distribution

A random variable Z = (X–μ)/σ follows the standard normal distribution. Z is called the standard normal variate with mean 0 and standard deviation 1 i.e Z ~ N(0,1). Its Probability density function is given by :

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad -\infty < z < \infty$$

1. The area under the standard normal curve is equal to 1.

2. 68.26% of the area under the standard normal curve lies between $z = -1$ and $Z = 1$

3. 95.44% of the area lies between $Z = -2$ and $Z = 2$

4. 99.74% of the area lies between $Z = -3$ and $Z = 3$

Example 7.21

What is the probability that a standard normal variate Z will be

(i) greater than 1.09

(ii) less than −1.65

(iii) lying between −1.00 and 1.96

(iv) lying between 1.25 and 2.75

Solution :

(i) greater than 1.09

The total area under the curve is equal to 1 , so that the total area to the right $Z = 0$ is 0.5 (since the curve is symmetrical). The area between $Z = 0$ and 1.09 (from tables) is 0.3621

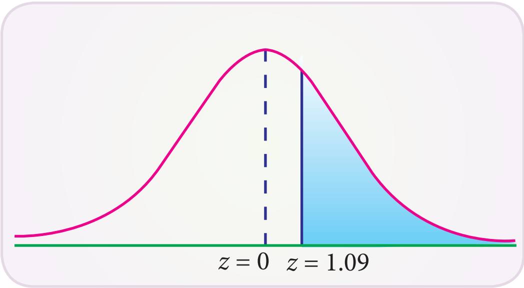


Fig 7.4

$P(Z > 1.09) = 0.5000 - 0.3621 = 0.1379$

The shaded area to the right of $Z = 1.09$ is the probability that Z will be greater than 1.09

(ii) less than −1.65

The area between −1.65 and 0 is the same as area between 0 and 1.65. In the table the area between zero and 1.65 is 0.4505 (from the table). Since the area to the left of zero is 0.5, $P(Z< 1.65) = 0.5000 - 0.4505 = 0.0495$.

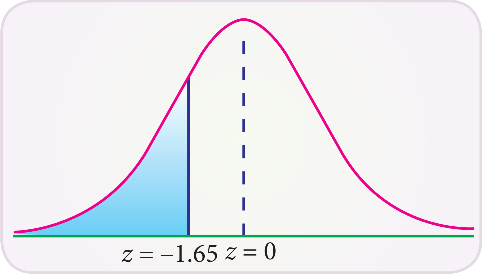


Fig 7.5

(iii) lying between −1.00 and 1.96

The probability that the random variable Z in between −1.00 and 1.96 is found by adding the corresponding areas :

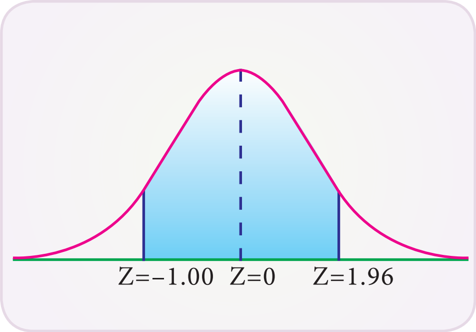


Fig 7.6

Area between −1.00 and 1.96 = area between (−1.00 and 0) + area betwn (0 and 1.96)

$P(-1.00 < Z < 1.96) = P(-1.00 < Z < 0) + P(0 < Z < 1.96)$

$= 0.3413 + 0.4750$ (by tables)

$= 0.8163$

(iv) lying between 1.25 and 2.75

Area between $Z = 1.25$ and 2.75
= area between ($z = 0$ and $z = 2.75$)
− area between ($z=0$ and $z = 1.25$)

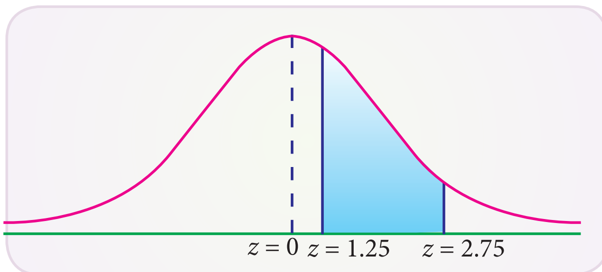


Fig 7.7

$P(1.25 < Z < 2.75) = P(0 < Z < 2.75) - P(0 < Z < 1.25)$

$= 0.4970 - 0.3944 = 0.1026$

Example 7.22

If X is a normal variate with mean 30 and SD 5. Find the probabilities that (i) $26 \leq X \leq 40$ (ii) $X > 45$

Solution :

Here mean μ= 30 and
standard deviation σ = 5

(i) When $X = 26$ $\quad Z = (X-\mu)/\sigma$

$= (26 - 30)/5 = -0.8$

And when $X = 40$, $\quad Z = \dfrac{40-30}{5} = 2$

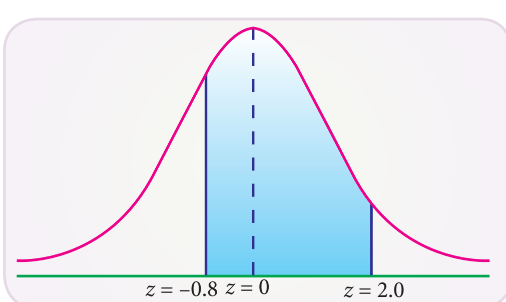


Fig 7.8

Therefore,

$P(26<X<40)) = P(-0.8 \leq Z < 2)$

$= P(-0.8 \leq Z \leq 0) + p(0 \leq Z \leq 2)$

$= P(0 \leq Z \leq 0.8) + P(0 \leq Z \leq 2)$

$= 0.2881 + 0.4772$ (By tables)

$= 0.7653$

(ii) The probability that $X \geq 45$

When $X = 45$

$$Z = \frac{X-\mu}{\sigma} = \frac{45-30}{5} = 3$$

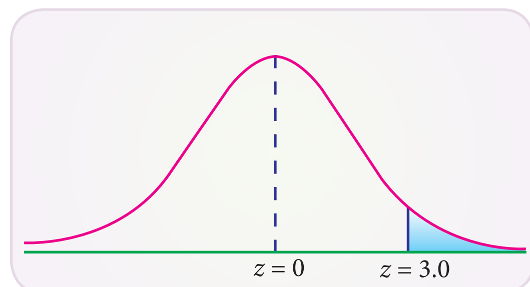


Fig 7.9

$P(X \geq 45) = P(Z \geq 3)$

$= 0.5 - 0.49865$

$= 0.00135$

Example 7.23

The average daily sale of 550 branch offices was ₹ 150 thousand and standard deviation is ₹ 15 thousand. Assuming the distribution to be normal, indicate how many branches have sales between

(i) ₹ 1,25,000 and ₹ 1, 45, 000

(ii) ₹ 1,40,000 and ₹ 1,60,000

Solution :

Given that mean μ= 150 and standard deviation σ= 15

(i) when $X = 125$ thousand

$$Z = \frac{X-\mu}{\sigma} = \frac{125-150}{15} = -1.667$$

When $X = 145$ thousand

$$Z = \frac{X-\mu}{\sigma} = \frac{145-150}{15} = -0.33$$

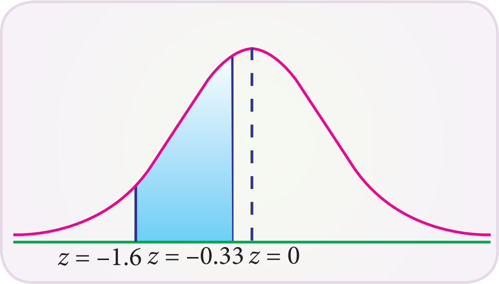


Fig 7.10

Area between $Z = 0$ and $Z = -1.67$ is 0.4525

Area between $Z = 0$ and $Z = -0.33$ is 0.1293

$$P(-1.667 \le Z \le -0.33) = 0.4525 - 0.1293$$

$$= 0.3232$$

Therefore the number of branches having sales between ₹ 1,25,000 and ₹ 1,45,000 is $550 \times 0.3232 = 178$

(ii) When $X = 140$ thousand

$$Z = \frac{X-\mu}{\sigma} = \frac{140-150}{15} = -0.67$$

When $X = 160$ thousand

$$Z = \frac{X-\mu}{\sigma} = \frac{160-150}{15} = 0.67$$

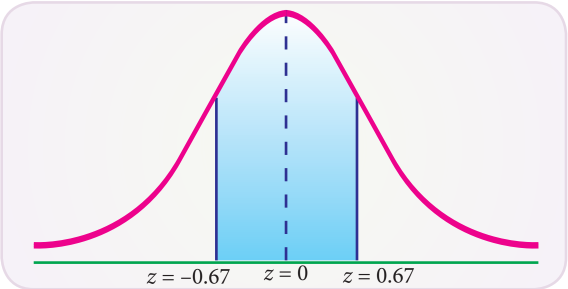


Fig 7.11

$P(-0.67 < Z < 0.67)$

$= P(-0.67 < Z < 0) + P(0 < Z < 0.67)$

$= P(0 < Z < 0.67) + P(0 < Z < 0.67)$

$= 2\ P(0 < Z < < 0.67)$

$= 2 \times 0.2486$

$= 0.4972$

Therefore, the number of branches having sales between ₹ 1,40,000 and ₹ 1,60,000

$= 550 \times 0.4972 = 273$

Example 7.24

Assume the mean height of children to be 69.25 cm with a variance of 10.8 cm. How many children in a school of 1,200 would you expect to be over 74 cm tall?

Solution

Let the distribution of heights be normally distributed with mean 68.22 and standard deviation $= 3.286$

$$Z = \frac{X-\mu}{\sigma} = \frac{X-69.25}{3.286}$$

When $X = 74$

$$Z = \frac{X-\mu}{\sigma} = \frac{74-69.25}{3.286} = 1.4455$$

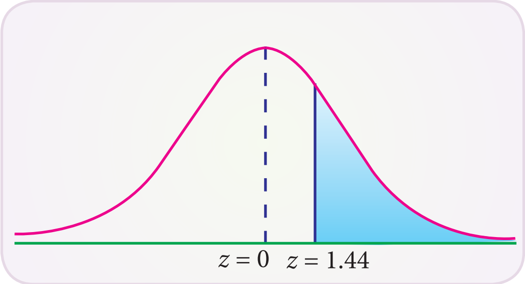


Fig 7.12

Now $P(Z > 74) = P(Z > 1.44)$

$= 0.5 - 0.4251$

$= 0.0749$

Expected number of children to be over 74 cm out of 1200 children

$= 1200 \times 0.0749 \approx 90$ children

Example 7.25

The marks obtained in a certain exam follow normal distribution with mean 45 and SD 10. If 1,300 students appeared at the examination, calculate the number of students scoring (i) less than 35 marks and (ii) more than 65 marks.

Solution :

Let X be the normal variate showing the score of the candidate with mean 45 and standard deviation 10.

(i) less than 35 marks

When $X = 35$

$$Z = \frac{X-\mu}{\sigma} = \frac{35-45}{10} = -1$$

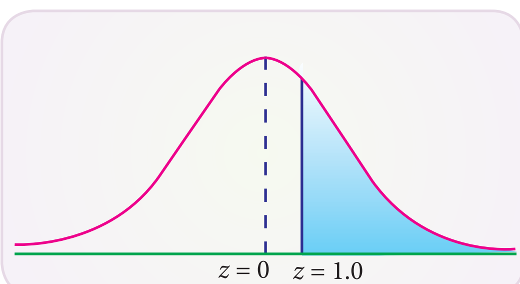


Fig 7.13

$P(X < 35) = P(Z < -1)$

$P(Z > 1) = 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

Expected number of students scoring less than 35 marks are $0.1587 \times 1300 = 206$

(ii) more than 65 marks

When $X = 65$

$$Z = \frac{X-\mu}{\sigma} = \frac{65-45}{10} = 2.0$$

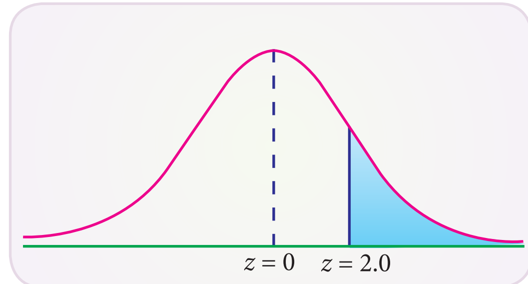


Fig 7.14

$P(X > 65) = P(Z > 2.0)$

$0.5 - P(0 < Z < 2.0)$

$0.5 - 0.4772$

$= 0.0228$

Expected number of students scoring more than 65 marks are 0.0228 x 1300 = 30

Example 7.26

900 light bulbs with a mean life of 125 days are installed in a new factory. Their length of life is normally distributed with a standard deviation of 18 days. What is the expected number of bulbs expire in less than 95 days?

Solution :

Let X be the normal variate of life of light bulbs with mean 125 and standard deviation 18.

(i) less than 95 days

When $X = 95$

$$Z = \frac{X-\mu}{\sigma} = \frac{95-125}{18} = -1.667$$

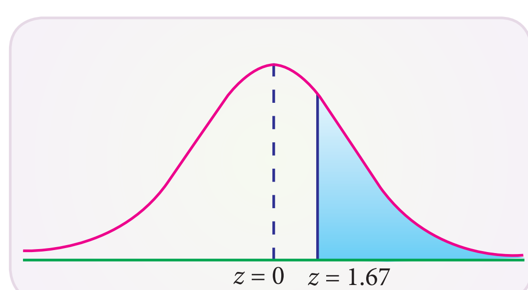


Fig 7.15

$$P(X < 95) = P(Z < -1.667)$$

$$= P(Z > 1.667)$$

$$= 0.5 - P(0 < Z < 1.67)$$

$$= 0.5 - 0.4525$$

$$= 0.0475$$

No. of bulbs expected to expire in less than 95 days out of 900 bulbs is $900 \times .0475 = 43$ bulbs

Example 7.27

Assume that the mean height of soldiers is 69.25 inches with a variance of 9.8 inches. How many soldiers in a regiment of 6,000 would you expect to be over 6 feet tall?

Solution :

Let X be the height of soldiers follows normal distribution with mean 69.25 inches and standard deviation 3.13. then the soldiers over 6 feet tall (6ft × 12= 72 inches)

The standard normal variate

$$Z = \frac{X-\mu}{\sigma} = \frac{72-69.25}{3.13} = 0.8786$$

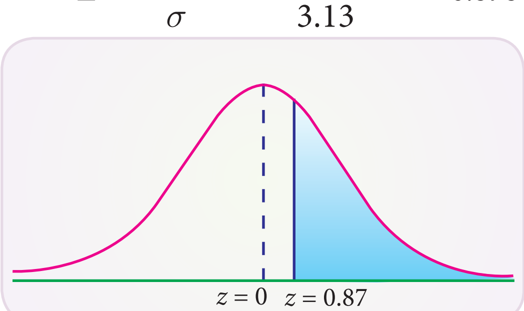


Fig 7.16

$$P(X > 72) = P(Z > 0.8786)$$

$$= 0.5 - P(0 < Z < 0.88)$$

$$= 0.5 - 0.3106 = 0.1894$$

Number of soldiers expected to be over 6 feet tall in 6000 are

$$6000 \times 0.1894 = 1136$$

Example 7.28

A bank manager has observed that the length of time the customers have to wait for being attended by the teller is normally distributed with mean time of 5 minutes and standard deviation of 0.7 minutes. Find the probability that a customer has to wait

(i) for less than 6 minutes

(ii) between 3.5 and 6.5 minutes

Solution :

Let X be the waiting time of a customer in the queue and it is normally distributed with mean 5 and SD 0.7.

(i) for less than 6 minutes

$$Z = \frac{X-\mu}{\sigma} = \frac{6-5}{0.7} = 1.4285$$

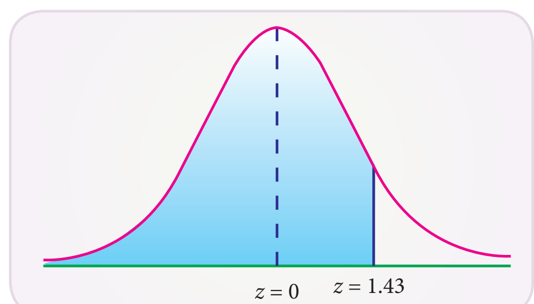


Fig 7.17

$$P(X < 6) = P(Z < 1.43)$$

$$= 0.5 + 0.4236$$

$$= 0.9236$$

(ii) between 3.5 and 6.5 minutes

When $X = 3.5$

$$Z = \frac{X-\mu}{\sigma} = \frac{3.5-5}{0.7} = -2.1429$$

When $X = 6.5$

$$Z = \frac{X-\mu}{\sigma} = \frac{6.5-5}{0.7} = 2.1429$$

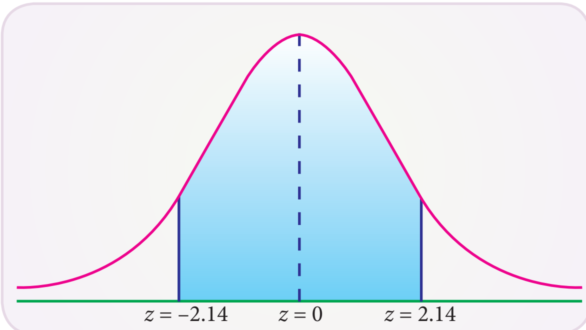


Fig 7.18

$P(3.5 < X < 6.5)$

$= P(-2.1429 < Z < 2.1429)$

$= \text{P}(0 < \text{Z} < 2.1429) + \text{P}(0 < \text{Z} < 2.1429)$

$= 2\ \text{P}(0 < \text{Z} < 2.1429)$

$= 2 \times .4838$

$= 0.9676$

Example 7.29

A sample of 125 dry battery cells tested to find the length of life produced the following resultd with mean 12 and SD 3 hours. Assuming that the data to be normal distributed , what percentage of battery cells are expected to have life

(i) more than 13 hours

(ii) less than 5 hours

(iii) between 9 and 14 hours

Solution :

Let X denote the length of life of dry battery cells follows normal distribution with mean 12 and SD 3 hours

(i) more than 13 hours

$P(X > 13)$

When $X = 13$

$$Z = \frac{X-\mu}{\sigma} = \frac{13-12}{3} = 0.333$$

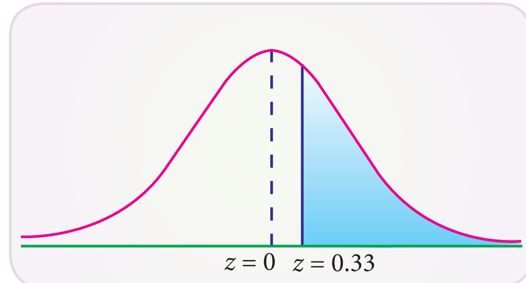


Fig 7.19

$P(X > 13) = P(Z > 0.333) = 0.5 - 0.1293 = 0.3707$

The expected battery cells life to have more than 13 hours is $125 \times 0.3707 = 46.34\%$

(ii) less than 5 hours

$P(X < 5)$

When $X = 5$ $Z = \dfrac{X-\mu}{\sigma} = \dfrac{5-12}{3} = -2.333$

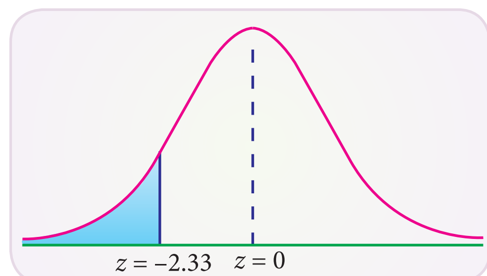


Fig 7.20

$P(X < 5) = P(Z < -2.333) = P(Z > 2.333)$

$= 0.5 - 0.4901 = 0.0099$

The expected battery cells life to have more than 13 hours is $125 \times 0.0099 = 1.23\%$

(iii) between 9 and 14 hours

When $X = 9$

$$Z = \frac{X-\mu}{\sigma} = \frac{9-12}{3} = -1$$

When $X = 14$

$$Z = \frac{X-\mu}{\sigma} = \frac{14-12}{3} = 0.667$$

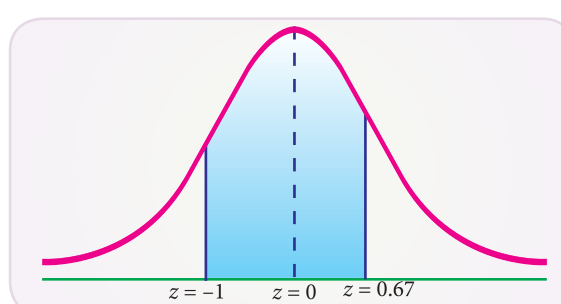


Fig 7.21

$P(9 < X < 14) = P(-1 < Z < 0.667)$

$= P(0 < Z < 1) + P(0 < Z < 0.667)$

$= 0.3413 + 0.2486$

$= 0.5899$

The expected battery cells life to have more than 13 hours is 125 x 0.5899 =73.73%

Example 7.30

Weights of fish caught by a traveler are approximately normally distributed with a mean weight of 2.25 kg and a standard deviation of 0.25 kg. What percentage of fish weigh less than 2 kg?

Solution :

We are given mean $\mu = 2.25$ and standard deviation $\sigma = 0.25$. Probability that weight of fish is less than 2 kg is $P(X < 2.0)$

When $x = 20$ $\quad Z = \dfrac{X-\mu}{\sigma} = \dfrac{2.0-2.25}{0.25} = -1.0$

$P(Z < -1.0) = P(Z > 1.0) = 0.5 - 0.3413 = 0.1587$

Therefore 15.87% of fishes weigh less than 2 kg.

Example 7.31

The average daily procurement of milk by village society in 800 litres with a standard deviation of 100 litres. Find out proportion of societies procuring milk between 800 litres to 1000 litres per day.

Solution :

We are given mean $\mu = 800$ and standard deviation $\sigma = 100$. Probability that the procurement of milk between 800 litres to 1000 litres per day is

$P(800 < X < 1000) = P\left(\dfrac{800-800}{100} < z < \dfrac{1000-800}{100}\right)$

$= P(0 < Z < 2) = 0.4772$ (table value)

Therefore 47.75 percent of societies procure milk between 800 litres to 1000 litres per day.

Exercise 7.3

1. Define Normal distribution.
2. Define Standard normal variate.
3. Write down the conditions in which the Normal distribution is a limiting case of binomial distribution.
4. Write down any five chief characteristics of Normal probability curve.
5. In a test on 2,000 electric bulbs, it was found that bulbs of a particular make, was normally distributed with an average life of 2,040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for (i) more than 2,150 hours (ii) less than 1,950 hours (iii) more 1,920 hours but less than 2,100 hours.
6. In a distribution 30% of the items are under 50 and 10% are over 86. Find the mean and standard deviation of the distribution.
7. X is normally distributed with mean 12 and SD 4. Find $P(X \leq 20)$ and $P(0 \leq X \leq 12)$
8. If the heights of 500 students are normally distributed with mean 68.0 inches and standard deviation 3.0 inches , how many students have height (a) greater than 72 inches (b) less than or equal to 64 inches (c) between 65 and 71 inches.
9. In a photographic process, the developing time of prints may be looked upon as a random variable having the normal distribution with a mean of 16.28 seconds and a standard deviation of 0.12 second. Find the probability that it will take less than 16.35 seconds to develop prints.
10. Time taken by a construction company to construct a flyover is a normal variate with mean 400 labour days and standard deviation of 100 labour days. If the company

promises to construct the flyover in 450 days or less and agree to pay a penalty of ₹10,000 for each labour day spent in excess of 450 days. What is the probability that

(i) the company pays a penalty of atleast ₹2,00,000?

(ii) the company takes at most 500 days to complete the flyover?

Exercise 7.4

Choose the correct Answer

1. Normal distribution was invented by

   (a) Laplace (b) De-Moivre

   (c) Gauss (d) all the above

2. If X ~N(9,81) the standard normal variate Z will be

   (a) $Z=\frac{X-81}{9}$ (b) $Z=\frac{X-9}{81}$

   (c) $Z=\frac{X-9}{9}$ (d) $Z=\frac{9-X}{9}$

3. If Z is a standard normal variate, the proportion of items lying between $Z = -0.5$ and $Z = -3.0$ is

   (a) 0.4987 (b) 0.1915

   (c) 0.3072 (d) 0.3098

4. If X ~N(μ, σ^2), the maximum probability at the point of inflexion of normal distribution is

   (a) $\left(\frac{1}{\sqrt{2\pi}}\right)e^{\frac{1}{2}}$

   (b) $\left(\frac{1}{\sqrt{2\pi}}\right)e^{\left(-\frac{1}{2}\right)}$

   (c) $\left(\frac{1}{\sigma\sqrt{2\pi}}\right)e^{\left(-\frac{1}{2}\right)}$

   (d) $\left(\frac{1}{\sqrt{2\pi}}\right)$

5. In a parametric distribution the mean is equal to variance is

   (a) binomial (b) normal

   (c) poisson (d) all the above

6. In turning out certain toys in a manufacturing company, the average number of defectives is 1%. The probability that the sample of 100 toys there will be 3 defectives is

   (a) 0.0613 (b) 0.613

   (c) 0.00613 (d) 0.3913

7. The parameters of the normal distribution

   $$f(x)=\left(\frac{1}{\sqrt{72\pi}}\right)\frac{e^{-(x-10)^2}}{72} \quad -\infty < x < \infty$$

   (a) (10,6) (b) (10,36)

   (c) (6,10) (d) (36,10)

8. A manufacturer produces switches and experiences that 2 per cent switches are defective. The probability that in a box of 50 switches, there are atmost two defective is :

   (a) 2.5 e^{-1}

   (b) e^{-1}

   (c) 2 e^{-1}

   (d) none of the above

   E63WR

9. An experiment succeeds twice as often as it fails. The chance that in the next six trials, there shall be at least four successes is

   (a) 240/729 (b) 489/729

   (c) 496/729 (d) 251/729

10. If for a binomial distribution B(n,p) mean = 4 and variance = 4/3, the probability, $P(X \geq 5)$ is equal to

    (a) $(2/3)^6$ (b) $(2/3)^5(1/3)$

    (c) $(1/3)^6$ (d) $4(2/3)^6$

11. The average percentage of failure in a certain examination is 40. The probability that out of a group of 6 candidates atleast 4 passed in the examination are

(a) 0.5443 (b) 0.4543

(c) 0.5543 (d) 0.4573

12. Forty percent of the passengers who fly on a certain route do not check in any luggage. The planes on this route seat 15 passengers. For a full flight, what is the mean of the number of passengers who do not check in any luggage?

(a) 6.00 (b) 6.45

(c) 7.20 (d) 7.50

13. Which of the following statements is/are true regarding the normal distribution curve?

(a) it is symmetrical and bell shaped curve

(b) it is asymptotic in that each end approaches the horizontal axis but never reaches it

(c) its mean, median and mode are located at the same point

(d) all of the above statements are true.

14. Which of the following cannot generate a Poisson distribution?

(a) The number of telephone calls received in a ten-minute interval

(b) The number of customers arriving at a petrol station

(c) The number of bacteria found in a cubic feet of soil

(d) The number of misprints per page

15. The random variable X is normally distributed with a mean of 70 and a standard deviation of 10. What is the probability that X is between 72 and 84?

(a) 0.683 (b) 0.954

(c) 0.271 (d) 0.340

16. The starting annual salaries of newly qualified chartered accountants (CA's) in South Africa follow a normal distribution with a mean of ₹1,80,000 and a standard deviation of ₹10,000. What is the probability that a randomly selected newly qualified CA will earn between ₹1,65,000 and ₹1,75,000 per annum?

(a) 0.819 (b) 0.242

(c) 0.286 (d) 0.533

17. In a large statistics class the heights of the students are normally distributed with a mean of 172cm and a variance of 25cm. What proportion of students are between 165cm and 181cm in height?

(a) 0.954 (b) 0.601

(c) 0.718 (d) 0.883

18. A statistical analysis of long-distance telephone calls indicates that the length of these calls is normally distributed with a mean of 240 seconds and a standard deviation of 40 seconds. What proportion of calls lasts less than 180 seconds?

(a) 0.214 (b) 0.094

(c) 0933 (d) 0.067

19. Cape town is estimated to have 21% of homes whose owners subscribe to the satelite service, DSTV. If a random sample of your home in taken, what is the probability that all four home subscribe to DSTV?

(a) 0.2100 (b) 0.5000

(c) 0.8791 (d) 0.0019

20. Using the standard normal table, the sum of the probabilities to the right of $z = 2.18$ and to the left of $z = -1.75$ is

(a) 0.4854 (b) 0.4599

(c) 0.0146 (d) 0.0547

21. The time until first failure of a brand of inkjet printers is normally distributed with a mean of 1,500 hours and a standard deviation of 200 hours. What proportion of printers fails before 1,000 hours?

(a) 0.0062 (b) 0.0668

(c) 0.8413 (d) 0.0228

22. The weights of newborn human babies are normally distributed with a mean of 3.2kg and a standard deviation of 1.1kg. What is the probability that a randomly selected newborn baby weighs less than 2.0kg?

(a) 0.138 (b) 0.428

(c) 0.766 (d) 0.262

23. Monthly expenditure on their credit cards, by credit card holders from a certain bank, follows a normal distribution with a mean of ₹1,295.00 and a standard deviation of ₹750.00. What proportion of credit card holders spend more than ₹1,500.00 on their credit cards per month?

(a) 0.487 (b) 0.392

(c) 0.500 (d) 0.791

24. Let z be a standard normal variable. If the area to the right of z is 0.8413, then the value of z must be:

(a) 1.00 (b) −1.00

(c) 0.00 (d) −0.41

25. If the area to the left of a value of z (z has a standard normal distribution) is 0.0793, what is the value of z?

(a) −1.41 (b) 1.41

(c) −2.25 (d) 2.25

26. If $P(Z > z) = 0.8508$ what is the value of z (z has a standard normal distribution)?

(a) −0.48 (b) 0.48

(c) −1.04 (d) 1.04

27. If $P(Z > z) = 0.5832$ what is the value of z (z has a standard normal distribution)?

(a) −0.48 (b) 0.48

(c) 1.04 (d) −0.21

28. In a binomial distribution, the probability of success is twice as that of failure. Then out of 4 trials, the probability of no success is

(a) 16/81 (b) 1/16

(c) 2/27 (d) 1/81

Miscellaneous Problems

1. A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain

(a) no more than 2 rejects?

(b) at least 2 rejects?

2. Hospital records show that of patients suffering from a certain disease 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

3. If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that there will not be more than one failure during a particular week.

4. Vehicles pass through a junction on a busy road at an average rate of 300 per hour.

   1. Find the probability that none passes in a given minute.

   2. What is the expected number passing in two minutes?

5. Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Raghul

wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Raghul takes the test and scores 585. Will he be admitted to this university?

6. The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant in a period of time .

   a) less than 19.5 hours?

   b) between 20 and 22 hours?

7. The annual salaries of employees in a large company are approximately normally distributed with a mean of $50,000 and a standard deviation of $20,000.

   (a) What percent of people earn less than $40,000?

   (b) What percent of people earn between $45,000 and $65,000?

   (c) What percent of people earn more than $70,00

8. X is a normally normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

   (a) $P(x < 40)$

   (b) $P(x > 21)$

   (c) $P(30 < x < 35)$

9. The birth weight of babies is Normally distributed with mean 3,500g and standard deviation 500g. What is the probability that a baby is born that weighs less than 3,100g?

10. People's monthly electric bills in chennai are normally distributed with a mean of ₹ 225 and a standard deviation of ₹ 55. Those people spend a lot of time online. In a group of 500 customers, how many would we expect to have a bill that is ₹ 100 or less?

Summary

- Conditions for the binomial probability distribution are

  (i) the trials are independent

  (ii) the number of trials is finite

  (iii)each trial has only two possible outcomes called success and failure.

  (iv) the probability of success in each trial is a constant.

- The probability for exactly x success in n independent trials is given by

  $p(x) = \binom{n}{x} p^x q^{n-x}$ where $x = 0,1,2,3,\ldots,n$ and $q = 1 - p$

- The parameters of the binomial distributions are n and p
- The mean of the binomial distribution is np and variance are npq
- Poisson distribution as limiting form of binomial distribution when n is large, p is small and np is finite.
- The Poisson probability distribution is $p(x) = e^{-\lambda}\dfrac{\lambda^x}{x!}$ $x = 0,1,2,3,\ldots$

Where $\lambda = np$

- The mean and variance of the poisson distribtution is λ.
- The λ is the only parameter of poisson distribution.
- Poisson distribution can never be symmetrical.
- It is a distribution for rare events.
- Normal distribution is the limiting form of binomial distribution when n is large and neither p nor q is small
- The normal probability distribution is given by $f(x)\ \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$
- The mean of the distribution is μ
- The SD of the distribution is σ.
- It is a symmetrical distribution
- The graph of the distribution is bell shaped
- In normal distribution the mean, median and mode are equal
- The points of inflexion are $\mu - \sigma$ and $\mu + \sigma$
- The normal curve approaches the horizontal axis asymptotically
- Area Property : In a normal distribution about 68% of the item will lie between $\mu - \sigma$ and $\mu + \sigma$. About 95% will lie between are $\mu - 2\sigma$ and $\mu + 2\sigma$. About 99% will lie between $\mu - 3\sigma$ and $\mu + 3\sigma$.
- Standard normal random variate is denoted as $Z = (X - \mu)/\sigma$
- The standard normal probability distribution is $1/\sqrt{2\pi}(e^{\frac{-z^2}{2}})$
- The mean of the distribution is zero and SD is unity
- The points of inflexion are at $z = -1$ and $z = +1$

GLOSSARY (கலைச்சொற்கள்)

bell shaped curve	மணி வடிவ வளைவரை
Binomial	ஈருறுப்பு
continuous distribution	தொடர்ச்சியான பரவல்
discrete distribution	தனிநிலைப் பரவல் / தனித்த பரவல்
Distribution	பரவல்
Independent	சார்பற்ற
Normal	இயல்நிலை
parameter	பண்பளவை
point of inflexion	வளைவு மாற்றுப்புள்ளி
random experiment	சமவாய்ப்பு சோதனை
Random variable	இயைபிலா மாறி / சமவாய்ப்பு மாறி
Sample space	கூறுவெளி
Skewness	கோட்ட அளவை
Standard deviation	திட்ட விலக்கம்
Symmetry	சமச்சீர்

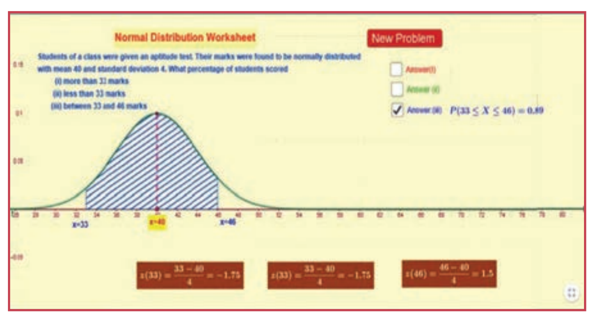
ICT Corner

Step – 1 : Open the Browser, type the URL Link given below (or) Scan the QR Code. GeoGebra work Book named "12th Standard Business Mathematics and Statistics " will open. In the work book there are two Volumes. Select "Volume-2".

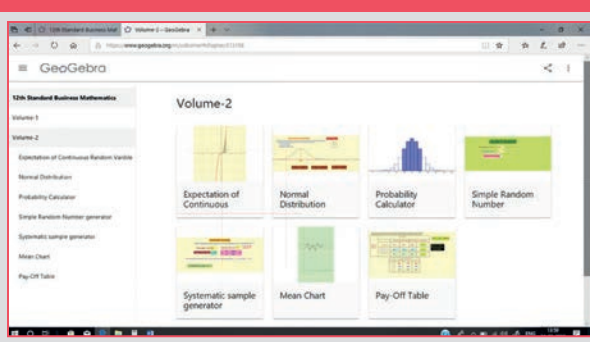
Step - 2 : Select the worksheet named "Normal Distribution"

Click on "New Problem" and click on Answer-I, Answer-ii and Answer -iii to see the answer for respective questions. Though the Normal Curve seems touching the bottom line, it don't touch when we zoom. Work out the answer and check yourself and analyse the shaded region.

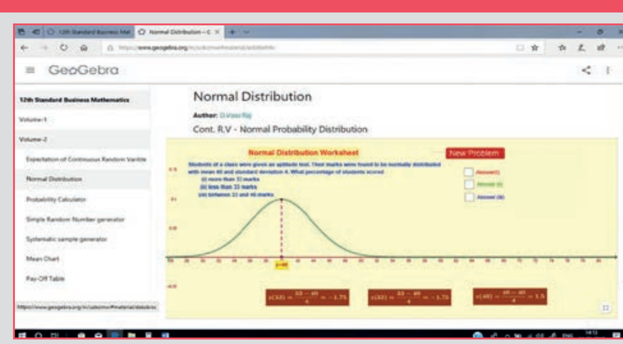
Expected Result is shown in this picture



Step 1



Step 2



Browse in the link

12th standard Business Mathematics and Statistics : https://ggbm.at/uzkcrnwr (or) Scan the QR Code.