CHAPTER 1

Rational Numbers

Learning Objectives

In this chapter you will learn:

- *About number system.*
- *To apply different operations : Addition, Subtraction, Multiplication and Division on rational numbers.*
- *About the properties of rational numbers under different operations.*

1.1 Introduction

In earlier classes, we have studied about **counting numbers** or **natural numbers** i.e. 1, 2, 3, 4 .... By including 0 to natural numbers, we get **whole numbers** i.e. 0, 1, 2, 3, 4 .... The negative of natural numbers and whole numbers, when put together, we get **integers** i.e. .... –4, –3, –2,–1, 0, 1, 2, 3, ...

Fundamental operations i.e. **addition, subtraction, multiplication and division** were defined on integers and various properties of these operations were also discussed in previous class. The concept of rational numbers was introduced and fundamental operations on rational numbers were also discussed. In this chapter, we shall learn about different properties of these operations on rational numbers.

Let us first recall about rational numbers.

1.2 Rational Numbers:

A number of the form $\frac{p}{q}$ or a number which can be expressed in the form $\left(\frac{p}{q}\right)$, where p and q are integers, $q \neq 0$ and p, q are co-prime is called a **rational number** e.g. $\frac{-2}{5}, \frac{10}{7}, \frac{-3}{8}, -3, 0$ etc.

- **All natural numbers are rational numbers.**
- **All whole numbers are rational numbers.**
- **All integers are rational numbers.**

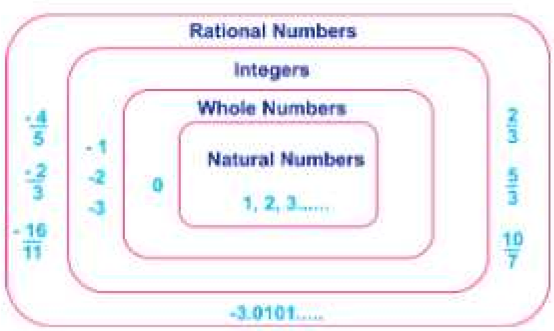

Fig 1.1

1.2.1 Addition of Rational Numbers

In previous class, we have defined the addition of rational numbers. We know if two rational numbers are to be added, then first we have to express each of them as rational number with same and positive denominator (by taking LCM), then we solve.

Let us discuss some examples.

Example 1.1 **Solve the following:**

(i) $\frac{2}{7}+\frac{3}{7}$ (ii) $\frac{5}{9}+\left(\frac{-1}{9}\right)$ (iii) $\frac{-3}{11}+\frac{6}{11}$

(iv) $\frac{5}{-11}+\frac{7}{11}$ (v) $\frac{-4}{11}+\frac{-3}{11}$

Solution : **(i)** $\frac{2}{7}+\frac{3}{7} = \frac{2+3}{7}$

$= \frac{5}{7}$

(ii) $\frac{5}{9}+\left(\frac{-1}{9}\right) = \frac{5}{9}-\frac{1}{9}$

$= \frac{5-1}{9}$

$= \frac{4}{9}$

(iii) $\frac{-3}{11}+\frac{6}{11} = \frac{-3+6}{11}$

$= \frac{3}{11}$

(iv) $\frac{5}{-11}+\frac{7}{11} = \frac{-5}{11}+\frac{7}{11}$

$= \frac{-5+7}{11}$

$= \frac{2}{11}$

(v) $\frac{-4}{11}+\frac{-3}{11} = \frac{-4}{11}-\frac{3}{11}$

$= \frac{-4-3}{11}$

$= \frac{-7}{11}$

Example 1.2 **Solve the following:**

(i) $\frac{5}{12}+\frac{3}{4}$ (ii) $\frac{-3}{8}+\frac{5}{6}$ (iii) $\frac{3}{10}+\frac{-2}{5}$

(iv) $\frac{-5}{8}+\frac{-7}{12}$ (v) $\frac{-7}{15}+\frac{-3}{20}$

Solution : **In these questions, first of all we will make denominator same by taking LCM of denominators.**

LCM of denominators 12, 4

2	12, 4
2	6, 2
	3, 1

LCM of 12 & 4 = 2 × 2 × 3 =12

(i) $\frac{5}{12}+\frac{3}{4}$

Now we express $\frac{5}{12}$ and $\frac{3}{4}$ as rational numbers with same denominators by taking LCM.

Now $\frac{3}{4} = \frac{3\times3}{4\times3}$

$= \frac{9}{12}$

So $\frac{5}{12}+\frac{3}{4} = \frac{5}{12}+\frac{9}{12}$

$= \frac{5+9}{12}$

$= \frac{\cancel{14}^{7}}{\cancel{12}_{6}}$

$= \frac{7}{6}$

Or $\frac{5}{12}+\frac{3}{4} = \frac{(5\times1)+(3\times3)}{12}$

12 ÷ 12 = 1

12 ÷ 4 = 3

$= \frac{5+9}{12}$

$= \frac{14}{12}$

$= \frac{7}{6}$

(ii) **We have,** $\frac{-3}{8}+\frac{5}{6}$

Now, we express $\frac{-3}{8}$ and $\frac{5}{6}$ as rational numbres with same denominators by taking LCM.

LCM of denominators 8, 6

2	8, 6
2	4, 3
	2, 3

LCM of 8 & 6 = 2 × 2 × 2 × 3 = 24

We have, $\frac{-3}{8}=\frac{-3\times3}{8\times3}=\frac{-9}{24}$

and $\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24}$

$$\therefore \quad \frac{-3}{8}+\frac{5}{6} = \frac{-9}{24}+\frac{20}{24}$$

$$= \frac{-9+20}{24}$$

$$= \frac{11}{24}$$

Or

$$\frac{-3}{8}+\frac{5}{6} = \frac{(-3\times 3)+(5\times 4)}{24}$$

$$= \frac{-9+20}{24}$$

$$= \frac{11}{24}$$

$24 \div 8 = 3$

$24 \div 6 = 4$

(iii) We have, $\frac{3}{10}+\frac{-2}{5}$

Now we express $\frac{3}{10}$ **and** $\frac{-2}{5}$ **as rational numbers with same denomiantors by taking LCM.**

LCM of denominators 10, 5

5	10, 5
	2, 1

LCM of 10 & 5 = 5 × 2 = 10

We have, $\frac{3}{10}$ **and** $\frac{-2}{5} = \frac{-2\times 2}{5\times 2} = \frac{-4}{10}$

$$\frac{3}{10}+\frac{-2}{5} = \frac{3}{10}+\frac{-4}{10}$$

$$= \frac{3}{10}-\frac{4}{10}$$

$$= \frac{3-4}{10}$$

$$= \frac{-1}{10}$$

Or

$$\frac{3}{10}+\frac{-2}{5} = \frac{(3\times 1)+(-2\times 2)}{10}$$

$$= \frac{3+(-4)}{10}$$

$$= \frac{3-4}{10}$$

$$= \frac{-1}{10}$$

(iv) We have, $\frac{-5}{8}+\frac{-7}{12}$

Now, we express $\frac{-5}{8}$ **and** $\frac{-7}{12}$ **as rational numbers with same denominantors by taking LCM.**

LCM of denominators 8, 12

2	8, 12
2	4, 6
	2, 3

LCM 8 & 12 = 2 × 2 × 2 × 3 = 24

We have, $\frac{-5}{8} = \frac{-5\times 3}{8\times 3} = \frac{-15}{24}$

and $\frac{-7}{12} = \frac{-7\times 2}{12\times 2} = \frac{-14}{24}$

$\therefore \quad \frac{-5}{8}+\frac{-7}{12} = \frac{-15}{24}+\frac{-14}{24}$

$= \frac{(-15)+(-14)}{24}$

$= \frac{-15-14}{24}$

$= \frac{-29}{24}$

Or $\frac{-5}{8}+\frac{-7}{12} = \frac{-5}{8}+\frac{-7}{12}$

$= \frac{(-5\times3)+(-7\times2)}{24}$

$= \frac{-15-14}{24}$

$= \frac{-29}{24}$

LCM of denominators 15 and 20

5	15, 20
	3, 4

LCM of 15 & 20

$= 5 \times 3 \times 4 = 60$

(iv) We have, $\frac{-7}{15}+\frac{-3}{20}$

Now, we express $\frac{-7}{15}$ **and** $\frac{-3}{20}$ **as rational numbers with same denominators by taking LCM.**

We have, $\frac{-7}{15} = \frac{-7\times4}{15\times4} = \frac{-28}{60}$

and $\frac{-3}{20} = \frac{-3\times3}{20\times3} = \frac{-9}{60}$

$\therefore \quad \frac{-7}{15}+\frac{-3}{20} = \frac{-28}{60}+\frac{-9}{60}$

$= \frac{-28+(-9)}{60}$

$= \frac{-28-9}{60}$

$= \frac{-37}{60}$

Or $\frac{-7}{15}+\frac{-3}{20} = \frac{(-7\times4)+(-3\times3)}{60}$

$= \frac{-28-9}{60}$

$= \frac{-37}{60}$

1.2.2 Properties of Addition of Rational Numbers

In this section, we shall learn some basic properties of rational numbers under addition. These properties are similar to those of addition of integers which we have learnt in the previous classes.

- **Closure Property :** The sum of any two rational numbers is always a rational number. e.g. if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\frac{a}{b}+\frac{c}{d}$ is also a rational number.

For Example:

(i) $\frac{-2}{3}+\frac{4}{5}=\frac{-10+12}{15}=\frac{2}{15}$, which is a rational number.

(ii) $\frac{5}{8}+\frac{-3}{4}=\frac{5+(-6)}{8}=\frac{5-6}{8}=\frac{-1}{8}$, which is a rational number.

- **Commutative Property :** The two rational numbers can be added in any order, the result will be same. We say that addition is commutative for rational numbers.

i.e. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers.

then $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$

e.g. Consider the rational numbers.

$\frac{-3}{8}$ and $\frac{5}{6}$ then

$$\frac{-3}{8}+\frac{5}{6}=\frac{-9+20}{24}=\frac{11}{24}$$

and $\frac{5}{6}+\left(\frac{-3}{8}\right)=\frac{20+(-9)}{24}=\frac{20-9}{24}=\frac{11}{24}$

Thus $\frac{-3}{8}+\frac{5}{6}=\frac{5}{6}+\left(\frac{-3}{8}\right)$

- **Associative Property :** When three rational numbers are to be added by adding the first two rational numbers and then adding the third number in result or by adding the second and third rational numbers and then adding the first number in the result. We get the same result. Then we say that the addition of rational numbers is associative.

i.e. For any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ (b, d and f $\neq 0$)

then $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$

e.g. Consider three rational numbers $\frac{-1}{2}, \frac{3}{4}$ and $\frac{-5}{6}$ then

$$\left(\frac{-1}{2}+\frac{3}{4}\right)+\left(\frac{-5}{6}\right)=\left(\frac{-2+3}{4}\right)+\left(\frac{-5}{6}\right)=\frac{1}{4}+\left(\frac{-5}{6}\right)$$

$$= \frac{3+(-10)}{12} = \frac{3-10}{12} = \frac{-7}{12}$$

and $\frac{-1}{2} + \left[\frac{3}{4} + \left(\frac{-5}{6}\right)\right] = \frac{-1}{2} + \left(\frac{9+(-10)}{12}\right)$

$$= \frac{-1}{2} + \left(\frac{9-10}{12}\right)$$

$$= \frac{-1}{2} + \left(\frac{-1}{12}\right)$$

$$= \frac{(-1\times 6)+(-1\times 1)}{12}$$

$$= \frac{-6+(-1)}{12}$$

$$= \frac{-6-1}{12}$$

$$= \frac{-7}{12}$$

Thus, $\left(\frac{1}{2} + \frac{3}{4}\right) + \left(\frac{-5}{6}\right) = \frac{-1}{2} + \left(\frac{3}{4} + \frac{-5}{6}\right)$

- **Additive Identity :** When we add 0 to any rational number we get the same rational number. i.e. For any rational number $\frac{a}{b}$, $b \neq 0$ there exists a unique rational number 0 such that

  $\frac{a}{b} + 0 = \frac{a}{b} = 0 + \frac{a}{b}$

  then 0 is called the identity for the addition of rational numbers. We say '0' is the additive identity for rational numbers.
- **Additive Inverse:** When two rational numbers are added and give the result zero (0)(additive identity) then both rational numbers are called additive inverse of each other.

  i.e. For any rational number $\frac{a}{b}$ $(b \neq 0)$, there exists $\left(\frac{-a}{b}\right)$

  such that $\frac{a}{b} + \left(\frac{-a}{b}\right) = 0 = \left(\frac{-a}{b}\right) + \frac{a}{b}$

then $\frac{-a}{b}$ is additive inverse of $\frac{a}{b}$ and vice-versa.

e.g. $\frac{2}{3}+\left(\frac{-2}{3}\right) = \frac{2+(-2)}{3} = \frac{2-2}{3} = \frac{0}{3} = 0$

$\therefore \quad \frac{2}{3}+\left(\frac{-2}{3}\right) = 0$

And $\frac{-2}{3}+\frac{2}{3} = \frac{-2+2}{3}$

$= \frac{0}{3}$

$= 0$

So, $\frac{2}{3}+\left(\frac{-2}{3}\right) = 0 = \frac{-2}{3}+\frac{2}{3}$

Thus $\frac{-2}{3}$ is additive inverse of $\frac{2}{3}$ and $\frac{2}{3}$ is additive inverse of $\frac{-2}{3}$.

Example 1.3 **Verify commutative property of addition of rational numbers for the following:**

(i) $\mathbf{\frac{-5}{12}}$ **and** $\mathbf{\frac{3}{8}}$ (ii) $\mathbf{\frac{2}{-7}}$ **and** $\mathbf{\frac{-11}{21}}$

Solution : (i) We have, $\frac{-5}{12}$ and $\frac{3}{8}$

Firstly $\frac{-5}{12}+\frac{3}{8} = \frac{(-5\times 2)+(3\times 3)}{24}$

$= \frac{-10+9}{24} = \frac{-1}{24}$

and $\frac{3}{8}+\left(\frac{-5}{12}\right) = \frac{(3\times 3)+(-5\times 2)}{24} = \frac{9+(-10)}{24} = \frac{9-10}{24}$

$= \frac{-1}{24}$

$\therefore \quad \frac{-5}{12}+\frac{3}{8} = \frac{3}{8}+\left(\frac{-5}{12}\right)$

Thus, commutative property under addition holds.

LCM of denominators 12, 8

2	12, 8
2	6, 4
	3, 2

LCM of 12 & 8

$= 2 \times 2 \times 2 \times 3 = 24$

(ii) We have, $\frac{2}{-7}$ and $\frac{-11}{21}$

Firstly $\frac{2}{-7}+\left(\frac{-11}{21}\right) = \frac{-2}{7}+\left(\frac{-11}{21}\right)$

$= \frac{(-2\times3)+(-11\times1)}{21} = \frac{-6+(-11)}{21}$

$= \frac{-6-11}{21} = \frac{-17}{21}$

LCM of denominators 7, 21

7	7, 21
	1, 3

LCM of 7 & 21
$= 7 \times 3 = 21$

and $\frac{-11}{21}+\frac{2}{-7} = \frac{-11}{21}+\frac{(-2)}{7}$

$= \frac{(-11\times1)+(-2\times3)}{21} = \frac{-11+(-6)}{21} = \frac{-11-6}{21} = \frac{-17}{21}$

$\therefore \quad \frac{2}{-7}+\left(\frac{-11}{21}\right) = \frac{-11}{21}+\frac{2}{-7}$

Thus, commutative property under addition holds.

Example 1.4 Verify associative property of addition of rational numbers for the following:

(i) $\frac{5}{3},\frac{1}{6}$ **and** $\frac{-3}{5}$ **(ii)** $-4, \frac{3}{7}$ **and** $\frac{-4}{5}$

Solution : (i) We have, $\frac{5}{3},\frac{1}{6}$ and $\frac{-3}{5}$

Firstly $\left(\frac{5}{3}+\frac{1}{6}\right)+\left(\frac{-3}{5}\right) = \left[\frac{(5\times2)+(1\times1)}{6}\right]+\left(\frac{-3}{5}\right)$

3	3, 6
	1, 2

LCM of 3 & 6 = 3 × 2 = 6

$= \left(\frac{10+1}{6}\right)+\left(\frac{-3}{5}\right)$

$= \frac{11}{6}+\left(\frac{-3}{5}\right)$

2	6, 5
	3, 5

LCM of 6 & 5 = 2 × 3 × 5
= 30

$= \frac{(11\times5)+(-3\times6)}{30}$

$= \frac{55+(-18)}{30} = \frac{55-18}{30} = \frac{37}{30}$

and $\frac{5}{3}+\left[\frac{1}{6}+\left(\frac{-3}{5}\right)\right]$ $=\frac{5}{3}+\left[\frac{(1\times5)+(-3\times6)}{30}\right]$

$=\frac{5}{3}+\left(\frac{5+(-18)}{30}\right)$

$=\frac{5}{3}+\left(\frac{5-18}{30}\right)$

$=\frac{5}{3}+\left(\frac{-13}{30}\right)$

3	3, 30
	1, 10

LCM of 3 & 30 = 3 × 10 = 30

$=\frac{(5\times10)+(-13\times1)}{30}$

$=\frac{50+(-13)}{30}$

$=\frac{50-13}{30}$

$=\frac{37}{30}$

$\therefore \quad \left(\frac{5}{3}+\frac{1}{6}\right)+\left(\frac{-3}{5}\right)=\frac{5}{3}+\left(\frac{1}{6}+\left(\frac{-3}{5}\right)\right)$

Thus, associative property under addition holds.

(ii) We have, –4, $\frac{3}{7}$ and $\frac{-4}{5}$

Firstly $\left(\frac{-4}{1}+\frac{3}{7}\right)+\left(\frac{-4}{5}\right)=\left(\frac{-28+3}{7}\right)+\left(\frac{-4}{5}\right)$ (LCM of 1 & 7 is 7)

$=\frac{-25}{7}+\left(\frac{-4}{5}\right)=\frac{-125+(-28)}{35}$ (LCM of 7 & 5 is 35)

$=\frac{-125+(-28)}{35}=\frac{-153}{35}$

and $-4+\left(\frac{3}{7}+\left(\frac{-4}{5}\right)\right)=-4+\left(\frac{15+(-28)}{35}\right)$ (LCM of 7 & 5 is 35)

$=-4+\left(\frac{15-28}{35}\right)=-4+\left(\frac{-13}{35}\right)$

$=\frac{-140+(-13)}{35}=\frac{-140-13}{35}=\frac{-153}{35}$

$$\therefore \quad \left(-4+\frac{3}{7}\right)+\left(\frac{-4}{5}\right) = -4+\left[\frac{3}{7}+\left(\frac{-4}{5}\right)\right]$$

Thus, associative property under addition holds.

Example 1.5 **Solve the following by rearranging and grouping rational numbers.**

(i) $\frac{-3}{5}+\frac{7}{6}+\frac{-2}{5}+\frac{5}{6}$ **(ii)** $\frac{-2}{3}+\frac{7}{5}+\frac{4}{3}+\frac{-4}{5}+(-3)$

Solution : (i) Re-arranging and grouping the numbers in pairs in such a way that each group contain rational numbers with equal denominators.

$$\frac{-3}{5}+\frac{7}{6}+\frac{-2}{5}+\frac{5}{6} = \left(\frac{-3}{5}+\left(\frac{-2}{5}\right)\right)+\left(\frac{7}{6}+\frac{5}{6}\right)$$

$$= \frac{-3+(-2)}{5}+\frac{7+5}{6} = \frac{-5}{5}+\frac{12}{6}$$

$$= -1+2 = 1$$

(ii) Re-arranging and grouping the numbers in pairs in such a way that each group contain rational numbers with equal denominators.

$$\frac{-2}{3}+\frac{7}{5}+\frac{4}{3}+\frac{-4}{5}+(-3) = \left(\frac{-2}{3}+\frac{4}{3}\right)+\left(\frac{7}{5}+\frac{-4}{5}\right)+(-3)$$

$$= \frac{-2+4}{3}+\frac{7+(-4)}{5}+(-3)$$

$$= \frac{2}{3}+\frac{3}{5}+(-3) = \frac{10+9+(-45)}{15} \quad \text{[LCM of 3 \& 5 is 15]}$$

$$= \frac{10+9-45}{15} = \frac{-26}{15}$$

Example 1.6 **Write the additive inverse of each of the following:**

(i) $\frac{-5}{13}$ **(ii)** $\frac{3}{-10}$ **(iii)** $\frac{-7}{-9}$

Solution : (i) Additive inverse of $\frac{-5}{13} = -\left(\frac{-5}{13}\right) = \frac{5}{13}$

(ii) Additive inverse of $\frac{3}{-10} = -\left[\frac{3}{-10}\right] = -\left(\frac{-3}{10}\right) = \frac{3}{10}$

(iii) Additive inverse of $\frac{-7}{-9} = -\left[\frac{-7}{-9}\right] = -\left[\frac{7}{9}\right] = \frac{-7}{9}$

Exercise 1.1

1. Solve the following:-

(i) $\frac{-5}{6}+\frac{3}{4}$ (ii) $\frac{6}{11}+\left(\frac{-2}{3}\right)$ (iii) $\frac{-5}{24}+\frac{7}{12}$ (iv) $\frac{-11}{12}+\frac{7}{8}$

(v) $\frac{-3}{10}+\left(\frac{-7}{15}\right)$ (vi) $\frac{-5}{7}+\frac{3}{14}$ (vii) $\frac{7}{6}+\left(\frac{-5}{9}\right)$ (viii) $\frac{-11}{15}+\frac{21}{25}$

2. Verify commutative property of addition of rational numbers for each of the following.

(i) $\frac{-5}{8}$ and $\frac{3}{4}$ (ii) $\frac{-2}{5}$ and $\frac{-3}{15}$ (iii) $\frac{-7}{10}$ and $\frac{8}{15}$

(iv) $\frac{-11}{14}$ and $\frac{17}{21}$ (v) -5 and $\frac{2}{3}$

3. Verify associative property of addition of rational numbers i.e. (x+y) +z = x +(y + z)

(i) $x=\frac{-2}{3}, y=\frac{1}{2}, z=\frac{5}{6}$ (ii) $x=\frac{-3}{4}$ $y=\frac{1}{6}, z=\frac{5}{8}$

(iii) $x=2, y=\frac{-5}{12}, z=\frac{-3}{8}$

4. Write the additive inverse of the following:

(i) $\frac{-5}{11}$ (ii) $\frac{8}{9}$ (iii) $\frac{-15}{13}$ (iv) $\frac{-2}{-9}$

(v) $\frac{3}{-8}$ (vi) $\frac{2}{-7}$ (vii) $\frac{-18}{-11}$ (viii) 0

5. Rearrange and Regroup the rational numbers and solve :

(i) $\frac{2}{5}+\left(\frac{-7}{3}\right)+\frac{4}{5}+\frac{1}{3}$ (ii) $\left(\frac{-3}{8}\right)+\frac{4}{7}+\frac{2}{8}+\left(\frac{-3}{7}\right)$

(iii) $\left(\frac{-6}{7}\right)+\left(\frac{-4}{9}\right)+\left(\frac{-15}{7}\right)+\left(\frac{-5}{6}\right)$ (iv) $\frac{2}{3}+\left(\frac{-4}{5}\right)+\frac{3}{10}+\frac{1}{3}$

(v) $\left(\frac{-1}{8}\right)+\frac{5}{12}+\frac{2}{7}+\frac{5}{7}+\left(\frac{-5}{16}\right)$

6. **Multiple Choice Questions :**

(i) Which of the following is commutative property for addition?

(a) $x \times y = y \times x$ (b) $(x+y) = (y+x)$

(c) $(x+y)+z = x+(y+z)$ (d) $(x-y) = (y-x)$

(ii) Which of the following is associative property for addition?

(a) $x \times y = y \times x$ (b) $x + y = y + x$

(c) $(x+y) + z = x + (y + z)$ (d) $x - y = y - x$

(iii) The additive inverse of $\frac{-5}{-9}$ is

(a) $\frac{5}{9}$ (b) $\frac{5}{-9}$ (c) 0 (d) $\frac{2}{-3}$

(iv) The additive identity of $\frac{2}{3}$ is

(a) 0 (b) $\frac{-2}{3}$ (c) $\frac{-2}{-3}$ (d) $\frac{3}{2}$

1.3 Subtraction of Rational Numbers:

In previous class, we have defined the subtraction of rational numbers. We know if two rational numbers are to be subtracted, firstly we express each one of them as rational number with same denominators if required (by taking LCM) and then we solve.

Example 1.7 **Subtract:**

(i) $\frac{2}{7}$ **from** $\frac{5}{7}$ **(ii)** $\frac{5}{8}$ **from** $\frac{-3}{8}$ **(iii)** $\frac{-3}{10}$ **from** $\frac{2}{5}$

(iv) $\frac{-5}{6}$ **from** $\frac{-3}{4}$ **(v)** $\frac{7}{15}$ **from** $\frac{-7}{10}$

Solution : (i) $\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$

(ii) $\frac{-3}{8} - \frac{5}{8} = \frac{-3-5}{8} = \frac{-8}{8} = -1$

(iii) $\frac{2}{5} - \left(\frac{-3}{10}\right) = \frac{4-(-3)}{10} = \frac{4+3}{10} = \frac{7}{10}$

(iv) $\frac{-3}{4} - \left(\frac{-5}{6}\right) = \frac{-9-(-10)}{12} = \frac{-9+10}{12} = \frac{1}{12}$

(v) $\frac{-7}{10} - \frac{7}{15} = \frac{-21-14}{30} = \frac{-35}{30} = \frac{-7}{6}$

1.3.1 Properties of Subtraction of Rational Numbers:

In this section, we shall learn some basic properties of subtraction of rational numbers. These properties are similar to those subtraction of integers which we have learnt in previous classes.

- **Closure Property :** The subtraction or difference of any two rational numbers is also a rational number.

  i.e. For any two rational number $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq 0$)

  $\frac{a}{b}-\frac{c}{d}$ is also a rational number.

  (i) $\frac{5}{6}-\frac{2}{3}=\frac{5-4}{6}=\frac{1}{6}$, rational number.

  (ii) $\frac{-3}{8}-\left(\frac{-5}{6}\right)=\frac{-9-(-20)}{24}=\frac{-9+20}{24}=\frac{11}{24}$, rational number.

  (iii) $\frac{7}{10}-\left(\frac{-2}{5}\right)=\frac{7-(-4)}{10}=\frac{7+4}{10}=\frac{11}{10}$, rational number.

- **Commutative Property :** The subtraction of rational numbers is not commutative i.e. For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$.

  We have $\frac{a}{b}-\frac{c}{d}\neq\frac{c}{d}-\frac{a}{b}$

  e.g. $\frac{5}{4}-\frac{3}{5}=\frac{25-12}{20}=\frac{13}{20}$

  and $\frac{3}{5}-\frac{5}{4}=\frac{12-25}{20}=\frac{-13}{20}$

  $\therefore$ $\frac{5}{4}-\frac{3}{5}\neq\frac{3}{5}-\frac{5}{4}$

- **Associative Property :** The subtraction of rational numbers is not associative i.e. For any rational numbers $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$; b, d, f $\neq 0$, We have

  $\left(\frac{a}{b}-\frac{c}{d}\right)-\frac{e}{f}\neq\frac{a}{b}-\left(\frac{c}{d}-\frac{e}{f}\right)$

  e.g. $\left(\frac{3}{4}-\frac{2}{3}\right)-\frac{1}{2}=\left(\frac{9-8}{12}\right)-\frac{1}{2}=\frac{1}{12}-\frac{1}{2}=\frac{1-6}{12}=\frac{-5}{12}$

and $\frac{3}{4}-\left(\frac{2}{3}-\frac{1}{2}\right) = \frac{3}{4}-\left(\frac{4-3}{6}\right) = \frac{3}{4}-\frac{1}{6} = \frac{9-2}{12} = \frac{7}{12}$

$\therefore \quad \left(\frac{3}{4}-\frac{2}{3}\right)-\frac{1}{2} \neq \frac{3}{4}-\left(\frac{2}{3}-\frac{1}{2}\right)$

- **Existence of Identity :** Identity does not exist in subtraction as commutative property does not hold in subtraction. Because for any rational number a, a – 0 = a but 0 – a ≠ a, so in case of subtraction identity does not exist.

Example 1.8 **Verify that x – y ≠ y – x when**

(i) $x = \frac{-2}{5}, y = \frac{-3}{4}$ **(ii)** $x = \frac{5}{12}, y = \frac{-7}{8}$

Solution : (i) LHS : $x - y = \frac{-2}{5}-\left(\frac{-3}{4}\right) = \frac{-2}{5}+\frac{3}{4}$

$= \frac{-8+15}{20} = \frac{7}{20}$

RHS : $y - x = \frac{-3}{4}-\left(\frac{-2}{5}\right) = \frac{-15-(-8)}{20} = \frac{-15+8}{20}$

$= \frac{-7}{20}$

$\therefore$ LHS ≠ RHS thus, x–y ≠ y–x

(ii) LHS : $x - y = \frac{5}{12}-\left(\frac{-7}{8}\right) = \frac{10-(-21)}{24}$

$= \frac{10+21}{24} = \frac{31}{24}$

RHS : $y - x = \frac{-7}{8}-\frac{5}{12} = \frac{-21-10}{24} = \frac{-31}{24}$

$\therefore$ LHS ≠ RHS

Thus, x – y ≠ y – x

Example 1.9 **Verify (x – y) – z ≠ x – (y – z) when**

(i) $x = \frac{-2}{3}, y = \frac{5}{8}, z = \frac{-7}{12}$ **(ii)** $x = \frac{1}{2}, y = \frac{-2}{5}, z = \frac{3}{10}$

Solution : (i) LHS : $(x-y) - z = \left(\frac{-2}{3}-\frac{5}{8}\right)-\left(\frac{-7}{12}\right)$

$$= \frac{-16-15}{24} - \left(\frac{-7}{12}\right) = \frac{-31}{24} - \left(\frac{-7}{12}\right)$$

$$= \frac{-31-(-14)}{24} = \frac{-31+14}{24} = \frac{-17}{24}$$

RHS : $x - (y - z) = \frac{-2}{3} - \left(\frac{5}{8} - \left(\frac{-7}{12}\right)\right)$

$$= \frac{-2}{3} - \left(\frac{15-(-14)}{24}\right) = \frac{-2}{3} - \left(\frac{15+14}{24}\right)$$

$$= \frac{-2}{3} - \frac{29}{24} = \frac{-16-29}{24} = \frac{-45}{24}$$

$\therefore$ LHS $\neq$ RHS

Thus, $(x - y) - z \neq x - (y - z)$

(ii) LHS : $(x-y) - z = \left(\frac{1}{2} - \left(\frac{-2}{5}\right)\right) - \frac{3}{10}$

$$= \frac{5-(-4)}{10} - \frac{3}{10} = \frac{5+4}{10} - \frac{3}{10} = \frac{9}{10} - \frac{3}{10}$$

$$= \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5}$$

RHS : $x - (y-z) = \frac{1}{2} - \left(\frac{-2}{5} - \frac{3}{10}\right)$

$$= \frac{1}{2} - \left(\frac{-4-3}{10}\right) = \frac{1}{2} - \left(\frac{-7}{10}\right) = \frac{5-(-7)}{10} = \frac{5+7}{10}$$

$$= \frac{12}{10} = \frac{6}{5}$$

$\therefore$ LHS $\neq$ RHS

Thus, $(x-y) - z \neq x - (y-z)$

Exercise 1.2

1. **Subtract:-**

(i) $\frac{2}{5}$ from $\frac{4}{5}$ (ii) $\frac{-3}{7}$ from $\frac{4}{7}$ (iii) $\frac{-5}{8}$ from $\frac{3}{4}$ (iv) $\frac{-8}{21}$ from $\frac{5}{14}$

(v) $\frac{-7}{10}$ from $\frac{-8}{15}$ (vi) $\frac{6}{11}$ from $\frac{5}{6}$ (vii) $\frac{-3}{4}$ from $\frac{-5}{12}$ (viii) $\frac{13}{10}$ from $\frac{-8}{25}$

2. **Verify that x – y ≠ y – x when**

(i) $x = \frac{-5}{12}, y = \frac{-3}{8}$ (ii) $x = \frac{7}{15}, y = \frac{-3}{10}$

(iii) $x = \frac{-15}{16}, y = \frac{7}{12}$ (iv) $x = \frac{-3}{4}, y = \frac{-5}{6}$

3. **Verify that (x–y) – z ≠ x – (y – z) when**

(i) $x = \frac{-7}{12}, y = \frac{-3}{4}, z = \frac{2}{3}$ (ii) $x = \frac{3}{8}, y = \frac{-2}{5}, z = \frac{-7}{10}$

(iii) $x = \frac{-1}{2}, y = \frac{-5}{4}, z = \frac{3}{8}$

4. **Solve the following:-**

(i) $\frac{3}{4} + \frac{5}{6} - \frac{7}{8}$ (ii) $\frac{-11}{2} + \frac{7}{6} - \frac{5}{8}$ (iii) $\frac{-4}{5} - \left(\frac{-7}{10}\right) + \left(\frac{-8}{15}\right)$

(iv) $\frac{-2}{5} - \left[\frac{-3}{10} - \left(\frac{-4}{15}\right)\right]$ (v) $\frac{3}{8} - \left(\frac{-2}{9}\right) + \left(\frac{5}{-36}\right)$

1.4 Multiplication of Rational Numbers:

In earlier classes, we have learnt the multiplication of two fractions. The product of two given fractions is a fraction whose numerator is the product of the numerators of the given fractions and whose denominator is the product of the denominators of the given fractions.

i.e.

$$\text{Product of Fractions} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

This same rule is applicable to the product of rational numbers.

$$\therefore \quad \text{Product of Rational Numbers} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

i.e. For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ $(b, d \neq 0)$

then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Let's discuss some examples.

Example 1.10 **Multiply :**

(i) $\frac{3}{4}$ **by** $\frac{5}{11}$ **(ii)** $\frac{-2}{3}$ **by** $\frac{5}{9}$ **(iii)** $\left(\frac{-7}{8}\right)$ **by 5**

(iv) $\left(\frac{-5}{8}\right)$ by $\frac{4}{3}$ (v) $\left(\frac{-10}{7}\right)$ by $\left(\frac{-14}{15}\right)$

Solution : (i) $\frac{3}{4}\times\frac{5}{11}=\frac{3\times5}{4\times11}=\frac{15}{44}$

(ii) $\left(\frac{-2}{3}\right)\times\frac{5}{9}=\frac{(-2)\times5}{3\times9}=\frac{-10}{27}$

(iii) $\left(\frac{-7}{8}\right)\times5=\frac{-7}{8}\times\frac{5}{1}=\frac{-7\times5}{8\times1}=\frac{-35}{8}$

(iv) $\left(\frac{-5}{8}\right)\times\frac{4}{3}=\frac{(-5)\times\overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}}\times3}=\frac{-5}{6}$

(v) $\left(\frac{-10}{7}\right)\times\left(\frac{-14}{15}\right)=\frac{\left(-\overset{2}{\cancel{10}}\right)\times\left(-\overset{2}{\cancel{14}}\right)}{\underset{1}{\cancel{7}}\times\underset{3}{\cancel{15}}}=\frac{4}{3}$

Example 1.11 **Simplify :**

(i) $\left(\frac{-8}{9}\right)\times\frac{3}{64}$ (ii) $\left(\frac{-9}{16}\right)\times\left(\frac{-64}{27}\right)$ (iii) $\left(\frac{-10}{9}\right)\times\left(\frac{36}{-25}\right)$

(iv) $\frac{15}{32}\times\left(\frac{-18}{25}\right)$ (v) $\frac{13}{20}\times\left(\frac{25}{-26}\right)$

Solution : (i) $\left(\frac{-8}{9}\right)\times\frac{3}{64}=\frac{\left(\left(-\overset{1}{\cancel{8}}\right)\times\overset{1}{\cancel{3}}\right)}{\underset{3}{\cancel{9}}\times\underset{8}{\cancel{64}}}=\frac{-1}{24}$

(ii) $\left(\frac{-9}{16}\right)\times\left(\frac{-64}{27}\right)=\frac{\left(-\overset{1}{\cancel{9}}\right)\times\left(-\overset{4}{\cancel{64}}\right)}{\underset{1}{\cancel{16}}\times\underset{3}{\cancel{27}}}=\frac{4}{3}$

(iii) $\left(\frac{-10}{9}\right)\times\left(\frac{36}{-25}\right)=\frac{\left(-\overset{2}{\cancel{10}}\right)\times\overset{4}{\cancel{36}}}{\underset{1}{\cancel{9}}\times\left(-\underset{5}{\cancel{25}}\right)}=\frac{8}{5}$

(iv) $\frac{15}{32}\times\left(\frac{-18}{25}\right) = \frac{\overset{3}{\cancel{15}}\times\left(-\overset{9}{\cancel{18}}\right)}{\underset{16}{\cancel{32}}\times\underset{5}{\cancel{25}}} = \frac{-27}{80}$

(v) $\frac{13}{20}\times\left(\frac{25}{-26}\right) = \frac{\overset{1}{\cancel{13}}\times\overset{5}{\cancel{25}}}{\underset{4}{\cancel{20}}\times\left(-\underset{2}{\cancel{26}}\right)} = \frac{-5}{8}$

1.4.1 Properties of Multiplication of Rational Numbers:

In this section, we shall learn some basic properties of multiplication of rational numbers. These properties are similar to those of multiplication of integers which we have learnt in previous classes.

- **Closure Property :** The product of any two rational numbers is always a rational number.

i.e.

if $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq$ 0) are any two rational numbers.

then $\frac{a}{b}\times\frac{c}{d}$ is also a rational number.

e.g. (i) $\frac{-5}{8}\times\frac{3}{4} = \frac{-5\times3}{8\times4} = \frac{-15}{32}$ is a rational number.

(ii) $\frac{-10}{9}\times\frac{3}{5} = \frac{-\overset{2}{\cancel{10}}\times\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}\times\underset{1}{\cancel{5}}} = \frac{-2}{3}$ is a rational number.

- **Commutative Property :** When two rational numbers can be multiplied in any order then we say that multiplication is commutative for rational numbers.

i.e. If $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq$ 0) are any two rational numbers then $\frac{a}{b}\times\frac{c}{d} = \frac{c}{d}\times\frac{a}{b}$

e.g. Consider two rational numbers $\frac{-5}{6}$ and $\frac{2}{3}$

then $\left(\frac{-5}{6}\right)\times\frac{2}{3} = \frac{(-5)\times\overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}}\times3} = \frac{-5}{9}$

and $\frac{2}{3}\times\left(\frac{-5}{6}\right) = \frac{\overset{1}{\cancel{2}}\times(-5)}{3\times\underset{3}{\cancel{6}}} = \frac{-5}{9}$

$\therefore$ $\left(\frac{-5}{6}\right)\times\frac{2}{3} = \frac{2}{3}\times\left(\frac{-5}{6}\right)$

- **Associatve Property :** When three rational numbers are to be multiplied, by multiplying the first two rational numbers and then multiply the third number or by multiplying the second and third rational numbers and then multiplying the first number If we get the same result, then we say that the multiplication of rational numbers is associative.

i.e. For any three rational numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$ (b, d, f $\neq 0$)

We have $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{c}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$

e.g. Consider three rational numbers $\frac{-5}{8}, \frac{4}{9}$ and $\frac{-3}{10}$ then

$$\left(\frac{-5}{8} \times \frac{4}{9}\right) \times \left(\frac{-3}{10}\right) = \frac{(-5) \times 4}{8 \times 9} \times \left(\frac{-3}{10}\right) = \frac{-5}{18} \times \left(\frac{-3}{10}\right)$$

$$= \frac{(-5) \times (-3)}{18 \times 10} = \frac{1}{12}$$

and $$\left(\frac{-5}{8}\right) \times \left(\frac{4}{9} \times \left(\frac{-3}{10}\right)\right) = \left(\frac{-5}{8}\right) \times \frac{4 \times (-3)}{9 \times 10}$$

$$= \left(\frac{-5}{8}\right) \times \left(\frac{-2}{15}\right) = \frac{(-5) \times (-2)}{8 \times 15} = \frac{1}{12}$$

$$\therefore \left(\frac{-5}{8} \times \frac{4}{9}\right) \times \left(\frac{-3}{10}\right) = \left(\frac{-5}{8}\right) \times \left(\frac{4}{9} \times \left(\frac{-3}{10}\right)\right)$$

- **Multiplicative Identity :** When we multiply 1 by any rational number then the product is the same rational number.

i.e. For any rational number $\frac{a}{b}$, (b $\neq 0$) there exists a unique natural number 1 such that $1 \times \frac{a}{b} = \frac{a}{b} = \frac{a}{b} \times 1$

So 1 is the multiplicative identity for rational numbers.

- **Multiplicative Inverse:** When two rational numbers are multiplied and give result 1 (multiplicative identity) then one rational number is called the multiplicative inverse of other i.e. For any rational number $\frac{a}{b}$ (b $\neq 0$) there exist $\frac{b}{a}$ (a $\neq 0$) such that

$$\frac{a}{b}\times\frac{b}{a}=1=\frac{b}{a}\times\frac{a}{b}$$

Note :- 0 has no multiplicative inverse. As reciprocal of 0 is $\frac{1}{0}$, which does not exists.

e.g. $\frac{5}{6}\times\frac{6}{5} = \frac{\cancel{5}\times\cancel{6}}{\cancel{6}\times\cancel{5}} = 1$

and $\frac{6}{5}\times\frac{5}{6} = \frac{\cancel{6}\times\cancel{5}}{5\times\cancel{6}} = 1$

∴ $\frac{5}{6}\times\frac{6}{5} = \mathbf{1} = \frac{6}{5}\times\frac{5}{6}$

Thus $\frac{5}{6}$ is the multiplicative inverse (reciprocal) of $\frac{6}{5}$ and vice versa.

1.4.2 Distributive Property of Multiplication over Addition and Subtraction

The multiplication of rational numbers is distributive over their addition and subtraction.

i.e. For any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$, (b, d, f ≠ 0)

$$\frac{a}{b}\times\left[\frac{c}{d}+\frac{e}{f}\right] = \frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$$

e.g. Consider three rational numbers $\frac{-2}{3}, \frac{5}{6}$ and $\frac{-3}{4}$

$$\frac{-2}{3}\times\left(\frac{5}{6}+\left(\frac{-3}{4}\right)\right) = \frac{-2}{3}\times\left(\frac{10+(-9)}{12}\right)$$

$$= \frac{-2}{3}\times\frac{1}{12} = \frac{-\cancel{2}\times 1}{3\times\cancel{12}} = \frac{-1}{18}$$

$$\text{and}\left(\frac{-2}{3}\right)\times\frac{5}{6}+\left(\frac{-2}{3}\right)\times\left(\frac{-3}{4}\right) = \frac{\left(-\cancel{2}\right)\times 5}{3\times\cancel{6}} + \frac{\left(-\cancel{2}\right)\times\left(-\cancel{3}\right)}{\cancel{3}\times\cancel{4}}$$

$$= \frac{-5}{9}+\frac{1}{2} = \frac{-10+9}{18} = \frac{-1}{18}$$

$$\therefore \quad \frac{-2}{3}\times\left(\frac{5}{6}+\left(\frac{-3}{4}\right)\right) = \left(\frac{-2}{3}\right)\times\frac{5}{6}+\left(\frac{-2}{3}\right)\times\left(\frac{-3}{4}\right)$$

Example 1.12 **Verify x × y = y × x when**

(i) $x = \frac{-5}{12}$, $y = \frac{4}{15}$ **(ii)** $x = \frac{-6}{7}$, $y = \frac{-14}{9}$

Solution : (i) LHS : $x \times y = \frac{-5}{12}\times\frac{4}{15} = \frac{(-5)\times 4}{12\times 15} = \frac{-1}{9}$

RHS : $y \times x = \frac{4}{15}\times\left(\frac{-5}{12}\right) = \frac{4\times(-5)}{15\times 12} = \frac{-1}{9}$

$\therefore$ LHS = RHS

Thus, x × y = y × x

(ii) LHS : $x \times y = \left(\frac{-6}{7}\right)\times\left(\frac{-14}{9}\right) = \frac{(-6)\times(-14)}{7\times 9} = \frac{4}{3}$

RHS : $y \times x = \left(\frac{-14}{9}\right)\times\left(\frac{-6}{7}\right) = \frac{(-14)\times(-6)}{9\times 7} = \frac{4}{3}$

$\therefore$ LHS = RHS

Thus, x × y = y × x

Example 1.13 **Verify x × (y × z) = (x×y)×z when**

(i) $x = \frac{-7}{3}$, $y = \frac{12}{5}$, $z = \frac{4}{9}$ **(ii)** $x = \frac{-1}{2}$, $y = \frac{5}{4}$, $z = \frac{-7}{5}$

Solution : (i) LHS : $x \times (y\times z) = \frac{-7}{3}\times\left(\frac{12}{5}\times\frac{4}{9}\right)$

$$= \frac{-7}{3}\times\frac{12\times 4}{5\times 9} = \frac{-7}{3}\times\frac{16}{15} = \frac{-7\times 16}{3\times 15} = \frac{-112}{45}$$

RHS : $(x \times y)\times z = \left(\frac{-7}{3}\times\frac{12}{5}\right)\times\frac{4}{9}$

$$= \frac{-7\times 12}{3\times 5}\times\frac{4}{9} = \frac{-28}{5}\times\frac{4}{9} = \frac{-28\times 4}{5\times 9} = \frac{-112}{45}$$

$\therefore$ LHS = RHS

Thus, $x\times(y\times z) = (x\times y)\times z$

(ii) LHS : $x\times(y\times z) = \frac{-1}{2}\times\left(\frac{5}{4}\times\left(\frac{-7}{5}\right)\right)$

$$= \frac{-1}{2}\times\frac{\cancel{5}^{1}\times(-7)}{4\times\cancel{5}_{1}} = \frac{-1}{2}\times\left(\frac{-7}{4}\right) = \frac{(-1)\times(-7)}{2\times4} = \frac{7}{8}$$

RHS : $(x\times y)\times z = \left(\left(\frac{-1}{2}\right)\times\frac{5}{4}\right)\times\left(\frac{-7}{5}\right)$

$$= \frac{(-1)\times5}{2\times4}\times\left(\frac{-7}{5}\right) = \frac{-5}{8}\times\left(\frac{-7}{5}\right) = \frac{\left(-\cancel{5}^{1}\right)\times(-7)}{8\times\cancel{5}_{1}} = \frac{7}{8}$$

$\therefore$ LHS = RHS

Thus, $x\times(y\times z) = (x\times y)\times z$

Example 1.14 Write the reciprocal (multiplicative inverse) of each of the following rational numbers :

(i) –5 (ii) $\frac{-2}{3}$ (iii) $\frac{7}{15}$ (iv) $\frac{-2}{5}\times\frac{3}{7}$ (v) $\frac{-5}{8}\times\frac{4}{3}$

Solution : (i) Reciprocal of –5 i.e. $\frac{-5}{1} = \frac{1}{-5} = \frac{-1}{5}$

(ii) Reciprocal of $\frac{-2}{3} = \frac{3}{-2} = \frac{-3}{2}$

(iii) Reciprocal of $\frac{7}{15} = \frac{15}{7}$

(iv) We have, $\frac{-2}{5}\times\frac{3}{7} = \frac{(-2)\times3}{5\times7} = \frac{-6}{35}$

$\therefore$ Reciprocal of $\frac{-6}{35} = \frac{35}{-6} = \frac{-35}{6}$

(v) We have, $\frac{-5}{8}\times\frac{4}{3} = \frac{-5\times\cancel{4}^{1}}{\cancel{8}_{2}\times3} = \frac{-5}{6}$

$\therefore$ Reciprocal of $\frac{-5}{6} = \frac{6}{-5} = \frac{-6}{5}$

Example 1.15 **Verify the property : x × (y + z) = x × y + x × z when**

(i) $x = \frac{-3}{7}, y = \frac{12}{13}, z = \frac{-5}{6}$ **(ii)** $x = \frac{-3}{4}, y = \frac{5}{2}, z = \frac{-7}{6}$

Solution : (i) LHS : x×(y+z) $= \left(\frac{-3}{7}\right)\times\left(\frac{12}{13}+\left(\frac{-5}{6}\right)\right)$

$$= \left(\frac{-3}{7}\right)\times\left(\frac{72+(-65)}{78}\right) = \left(\frac{-3}{7}\right)\times\frac{7}{78} = \frac{(-3)\times 7}{7\times 78} = \frac{-1}{26}$$

(ii) RHS : x×y +x×z $= \left(\frac{-3}{7}\right)\times\left(\frac{12}{13}\right)+\left(\frac{-3}{7}\right)\times\left(\frac{-5}{6}\right)$

$$= \frac{(-3)\times 12}{7\times 13}+\frac{(-3)\times(-5)}{7\times 6} = \frac{-36}{91}+\frac{15}{14}$$

$$= \frac{-72+65}{182} = \frac{-7}{182} = \frac{-1}{26}$$

∴ LHS = RHS

Thus, x×(y+z) = x × y + x × z

(ii) LHS : x×(y+z) $= \left(\frac{-3}{4}\right)\times\left(\frac{5}{2}+\left(\frac{-7}{6}\right)\right)$

$$= \left(\frac{-3}{4}\right)\times\left(\frac{15+(-7)}{6}\right) = \left(\frac{-3}{4}\right)\times\frac{8}{6} = \frac{(-3)\times 8}{4\times 6} = -1$$

RHS : x×y +x×z $= \left(\frac{-3}{4}\right)\times\frac{5}{2}+\left(\frac{-3}{4}\right)\times\left(\frac{-7}{6}\right)$

$$= \frac{(-3)\times 5}{4\times 2}+\frac{(-3)\times(-7)}{4\times 6} = \frac{-15}{8}+\frac{7}{8} = \frac{-15+7}{8}$$

$$= \frac{-8}{8} = -1$$

∴ LHS = RHS

Thus, x × (y + z) = x × y + x × z

Example 1.16 **Using distributive property, simplify :**

(i) $\left(\frac{-5}{4}\right)\times\frac{8}{5}+\left(\frac{-5}{4}\right)\times\frac{16}{5}$ (ii) $\frac{2}{7}\times\frac{7}{16}-\frac{-2}{7}\times\frac{21}{4}$

Solution : (i) We have, $\left(\frac{-5}{4}\right)\times\frac{8}{5}+\left(\frac{-5}{4}\right)\times\frac{16}{5}$

$\left(\text{By using } x\times y - x\times z = x\times(y-z) \text{ where } x=\frac{2}{7},\ y=\frac{7}{16},\ z=\frac{21}{4}\right)$

$=\left(\frac{-5}{4}\right)\times\left(\frac{8}{5}+\frac{16}{5}\right)$ (Taking $\frac{-5}{4}$ Common)

$=\left(\frac{-5}{4}\right)\times\left(\frac{8+16}{5}\right)=\frac{-5}{4}\times\frac{24}{5}=\frac{-\cancel{5}\times\cancel{24}}{\cancel{4}\times\cancel{5}}=-6$

(ii) We have, $\frac{2}{7}\times\frac{7}{16}-\frac{2}{7}\times\frac{21}{4}$

$\left(\text{By using } x\times y - x\times z = x\times(y-z) \text{ where } x=\frac{2}{7},\ y=\frac{7}{16},\ z=\frac{21}{4}\right)$

$=\frac{2}{7}\times\left(\frac{7}{16}-\frac{21}{4}\right)$ (Taking $\frac{2}{7}$ common)

$=\frac{2}{7}\times\left(\frac{7-84}{16}\right)=\frac{2}{7}\times\left(\frac{-77}{16}\right)=\frac{\cancel{2}\times\left(-\cancel{77}\right)}{\cancel{7}\times\cancel{16}}=\frac{-11}{8}$

Exercise 1.3

1. Solve the following:-

(i) $\frac{7}{11}\times\frac{5}{4}$ (ii) $\frac{5}{7}\times\left(\frac{-3}{4}\right)$ (iii) $\frac{2}{9}\times\frac{-5}{11}$

(iv) $\frac{-3}{5}\times\frac{4}{7}$ (v) $\left(\frac{-8}{7}\right)\times\left(\frac{-14}{5}\right)$ (vi) $\left(\frac{-5}{9}\right)\times\left(\frac{36}{-25}\right)$

(vii) $\left(\frac{-8}{25}\right)\times\left(\frac{-15}{16}\right)$ (viii) $\left(\frac{-6}{11}\right)\times\left(\frac{-44}{30}\right)$ (ix) $\frac{5}{17}\times\left(\frac{-51}{30}\right)$

(x) $\left(\frac{-7}{18}\right)\times\left(\frac{15}{-7}\right)$ (xi) $\left(\frac{-16}{5}\right)\times\frac{20}{9}\times\left(\frac{-3}{4}\right)$ (xii) $\frac{9}{10}\times\left(\frac{-15}{27}\right)\times\frac{18}{5}$

2. Verify that x× y = y×x for the following when

(i) $x = \frac{-5}{7}$ $y = \frac{9}{13}$ (ii) $x = \frac{3}{10}, y = \frac{-15}{8}$ (iii) $x = \frac{-7}{8}, y = \frac{-4}{9}$

(iv) $x = 5, y = \frac{-9}{10}$

3. Verify that x × (y × z) = (x×y) × z for the following when

(i) $x = \frac{-7}{6}, y = \frac{12}{5}, z = \frac{-2}{9}$ (ii) $x = \frac{1}{2}, y = \frac{-5}{8}, z = \frac{-3}{5}$

(iii) $x = \frac{5}{7}, y = \frac{-12}{10}, z = \frac{-4}{9}$ (iv) $x = \frac{-3}{5}, y = \frac{2}{9}, z = \frac{10}{7}$

4. Write the reciprocal of each of the following:

(i) -2 (ii) $\frac{-5}{8}$ (iii) $\frac{7}{-9}$ (iv) $\frac{-3}{4}$

(v) $\frac{2}{7}\times\left(\frac{-3}{15}\right)$ (vi) $\left(\frac{-3}{8}\right)\times\left(\frac{-12}{9}\right)$ (vii) $(-8)\times\frac{5}{6}$ (viii) $3\times\left(\frac{-7}{9}\right)$

5. Verify that x × (y + z) = x × y + x × z when

(i) $x = \frac{3}{5}, y = \frac{25}{24}, z = 10$ (ii) $x = \frac{-5}{4}, y = \frac{8}{5}, z = \frac{16}{15}$

(iii) $x = \frac{-2}{7}, y = \frac{14}{10}, z = \frac{3}{5}$

6. Verify that x × (y – z) = x × y – x × z when

(i) $x = \frac{-2}{3}, y = \frac{3}{4}, z = \frac{6}{7}$ (ii) $x = \frac{-1}{2}, y = \frac{5}{6}, z = \frac{-3}{10}$

(iii) $x = \frac{3}{4}, y = \frac{8}{9}, z = -10$

7. Name the property of multiplication of rational numbers represented by the following statement:

(i) $\frac{-2}{5}\times\frac{3}{4} = \frac{3}{4}\times\left(\frac{-2}{5}\right)$ ....

(ii) $\frac{-3}{8}\times 1 = \frac{-3}{8} = 1\times\frac{-3}{8}$ ....

(iii) $\frac{5}{8}\times\left(\frac{3}{4}+\frac{2}{3}\right) = \frac{5}{8}\times\frac{3}{4}+\frac{5}{8}\times\frac{2}{3}$ ....

(iv) $\left(\frac{-2}{7}\times\frac{5}{4}\right)\times\frac{7}{10} \quad = \quad \frac{-2}{7}\times\left(\frac{5}{4}\times\frac{7}{10}\right)$ ....

(v) $\left(\frac{-7}{9}\right)\times\frac{3}{4}-\left(\frac{-7}{9}\right)\times\frac{5}{10} = \left(\frac{-7}{9}\right)\times\left(\frac{3}{4}-\frac{5}{10}\right)$ ....

8. Multiple Choice Questions :

(i) For rational numbers x, y and z, which of the following is not true :

(a) $x \times y = y \times x$ (b) $x \times (y - z) = x \times y - x \times z$

(c) $x - y = y - x$ (d) $x \times (y \times z) = (x \times y) \times z$

(ii) If $\frac{-5}{8}\times\frac{4}{7}=\frac{4}{7}\times\left(\frac{-5}{8}\right)$ then which of the following property it holds

(a) Closure (b) Commutative (c) Associative (d) Identity

(iii) The multiplicative identity of rational number 'a' is

(a) 1 (b) 0 (c) $\frac{1}{a}$ (d) –a

(iv) The statement $\frac{-5}{8}\times\left(\frac{3}{4}-\frac{2}{3}\right)=\left(\frac{-5}{8}\right)\times\frac{3}{4}-\left(\frac{-5}{8}\right)\times\frac{2}{3}$ holds under the property.

(a) Associative of multiplication

(b) Associative of subtraction

(c) Distribution of multiplication over addition

(d) Distribution of multiplication over subtraction

(v) Which of the following number does not have multiplicative inverse?

(a) 0 (b) –1 (c) 1 (d) $\frac{-2}{-3}$

(vi) Which of the following number is multiplicative inverse of itself ?

(a) 0 (b) –1 (c) 1 (d) Both b and c

1.5 Division of Rational Numbers:

In earlier classes, we have learnt the division of two fractions. We know the division of fractions is the inverse of multiplication. The same rule is applicable for the rational numbers.

i.e. If $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq$ 0) are two rational numbers such that $\frac{c}{d} \neq 0$, then

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times(\text{Reciprocal of }\frac{c}{d})=\frac{a}{b}\times\frac{d}{c}$$

Here, $\frac{a}{b}$ is called the dividend, $\frac{c}{d}$ is called the divisor and $\frac{a}{b}\times\frac{d}{c}$ is called the quotient.

Note : Division by 0 is not defined.

Let's discuss some examples.

Example 1.17 **Divide :**

(i) $\frac{3}{10}$ by $\frac{4}{25}$ (ii) $\left(\frac{-8}{9}\right)$ by $\left(\frac{-4}{3}\right)$ (iii) $\left(\frac{-8}{13}\right)$ by $\left(\frac{-5}{26}\right)$

(iv) $\left(\frac{-5}{8}\right)$ by $\left(\frac{15}{16}\right)$ (v) $\frac{7}{15}$ by $\frac{21}{20}$

Solution : (i) $\frac{3}{10} \div \frac{4}{25} = \frac{3}{10} \times (\text{Reciprocal of } \frac{4}{25})$

$$= \frac{3}{10} \times \frac{25}{4} = \frac{3 \times 25^{5}}{10_{2} \times 4} = \frac{15}{8}$$

(ii) $\left(\frac{-8}{9}\right) \div \left(\frac{-4}{3}\right) = \left(\frac{-8}{9}\right) \times \left(\text{Reciprocal of } \frac{-4}{3}\right)$

$$= \left(\frac{-8}{9}\right) \times \left(\frac{3}{-4}\right) = \frac{\left(-8^{2}\right) \times 3^{1}}{9_{3} \times \left(-4_{1}\right)} = \frac{2}{3}$$

(iii) $\left(\frac{-8}{13}\right) \div \left(\frac{-5}{26}\right) = \left(\frac{-8}{13}\right) \times \left(\text{Reciprocal of } \frac{-5}{26}\right)$

$$= \left(\frac{-8}{13}\right) \times \left(\frac{26}{-5}\right) = \frac{(-8) \times 26^{2}}{13_{1} \times (-5)} = \frac{-16}{-5} = \frac{16}{5}$$

(iv) $\left(\frac{-5}{8}\right) \div \frac{15}{16} = \left(\frac{-5}{8}\right) \times \left(\text{Reciprocal of } \frac{15}{16}\right)$

$$= \left(\frac{-5}{8}\right) \times \frac{16}{15} = \frac{\left(-5^{1}\right) \times 16^{2}}{8 \times 15_{3}} = \frac{-2}{3}$$

(v) $\frac{7}{15} \div \frac{21}{20} = \frac{7}{15} \times \left(\text{Reciprocal of } \frac{21}{20}\right)$

$$= \frac{7}{15} \times \frac{20}{21} = \frac{7^{1} \times 20^{4}}{15_{3} \times 21_{3}} = \frac{4}{9}$$

Example 1.18 **The product of two numbers is $\frac{-14}{27}$. If one of the number is $\frac{-7}{9}$ then find the other.**

Solution : Let the other number be x then

$$\left(\frac{-7}{9}\right) \times x = \frac{-14}{27}$$

$$x = \left(\frac{-14}{27}\right) \div \left(\frac{-7}{9}\right)$$

$$= \frac{-\cancel{14}^{2}}{\cancel{27}_{3}} \times \frac{\cancel{9}^{1}}{-\cancel{7}_{1}}$$

$$= \frac{2}{3}$$

Example 1.19 **By what number should we multiply $\frac{3}{-7}$, so that the product may be $\frac{-18}{49}$.**

Solution : Let the required number be x then

$$x \times \left(\frac{3}{-7}\right) = \left(\frac{-18}{49}\right)$$

$$x = \left(\frac{-18}{49}\right) \div \left(\frac{3}{-7}\right) = \left(\frac{-18}{49}\right) \times \left(\frac{-7}{3}\right)$$

$$= \frac{\left(-\cancel{18}^{6}\right) \times \left(-\cancel{7}^{1}\right)}{\cancel{49}_{7} \times \cancel{3}_{1}} = \frac{6}{7}$$

1.5.1 Properties of Division of Rational Numbers

- **Closure Property :** The division of two rational numbers (divisor $\neq 0$) is always a rational number.

i.e. for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ (b, d $\neq 0$)

Such that $\frac{c}{d} \neq 0$ then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number.

e.g. (i) $\left(\frac{-2}{5}\right) \div \frac{4}{9} = \frac{-2}{5} \times \frac{9}{4} = \frac{(-2)\times 9}{5\times 4} = \frac{-9}{10}$ is a rational number.

(ii) $\left(\frac{-3}{10}\right) \div \left(\frac{-5}{9}\right) = \left(\frac{-3}{10}\right) \times \left(\frac{9}{-5}\right) = \frac{(-3)\times(-9)}{10\times 5} = \frac{27}{50}$ is a rational number.

- **Commutative property :** The division of rational numbers is not commutative.

  i.e. For any two non-zero rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ then $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

- **Associative property :** The division of rational numbers ia not associative.

  i.e. For any three non-zero rational numbers.

  $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ then

  $$\left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f} \neq \frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right)$$

1.5.2 Properties of Rational Numbers

Properties/Operations	Addition	Subtraction	Multiplication	Division
Closure	✓	✓	✓	✓
Commutative	✓	×	✓	×
Associative	✓	×	✓	×
Identity	✓	×	✓	×
Inverse	✓	×	✓	×

Example 1.20 **Verify $x \div y \neq y \div x$ when**

(i) $x = \frac{-2}{5}, y = \frac{3}{4}$ (ii) $x = \frac{-3}{13}, y = \frac{-7}{9}$

Solution : (i) LHS : $x \div y = \left(\frac{-2}{5}\right) \div \frac{3}{4} = \left(\frac{-2}{5}\right) \times \frac{4}{3} = \frac{(-2)\times 4}{5\times 3} = \frac{-8}{15}$

RHS : $y \div x = \frac{3}{4} \div \left(\frac{-2}{5}\right) = \frac{3}{4} \times \left(\frac{5}{-2}\right) = \frac{3\times 5}{4\times(-2)} = \frac{-15}{8}$

$\therefore$ LHS $\neq$ RHS

Thus, $x \div y \neq y \div x$

(ii) LHS : $x \div y = \left(\frac{-3}{13}\right) \div \left(\frac{-7}{9}\right) = \left(\frac{-3}{13}\right) \times \left(\frac{9}{-7}\right)$

$$= \frac{(-3)\times 9}{13\times(-7)} - \frac{27}{91}$$

RHS : $y \div x = \left(\frac{-7}{9}\right) \div \left(\frac{-3}{13}\right) = \left(\frac{-7}{9}\right) \times \left(\frac{13}{-3}\right)$

$$= \frac{(-7)\times(-13)}{9\times 3} = \frac{91}{27}$$

$\therefore$ LHS $\neq$ RHS

Thus, $x \div y \neq y \div x$

Example 1.21 **Verify $x + (y + z) \neq (x + y) + z$, when $x = \frac{-2}{3}$, $y = \frac{-5}{6}$, $z = 3$**

Solution : LHS : $x \div (y \div z) = \left(\frac{-2}{3}\right) \div \left(\left(\frac{-5}{6}\right) \div 3\right) = \left(\frac{-2}{3}\right) \div \left(\left(\frac{-5}{6}\right) \div \frac{3}{1}\right)$

$$= \left(\frac{-2}{3}\right) \div \left(\frac{-5}{6} \times \frac{1}{3}\right) = \left(\frac{-2}{3}\right) \div \left(\frac{-5}{18}\right)$$

$$= \left(\frac{-2}{3}\right) \times \left(\frac{18}{-5}\right) = \frac{(-2)\times \overset{6}{\cancel{18}}}{\underset{1}{\cancel{3}}\times(-5)} = \frac{12}{5}$$

RHS : $(x \div y) \div z = \left[\left[\frac{-2}{3}\right] \div \left[\frac{-5}{6}\right]\right] \div 3$

$$= \left(\left(\frac{-2}{3}\right) \times \left(\frac{-6}{5}\right)\right) \div 3 = \left(\frac{(-2)\times\left(-\overset{2}{\cancel{6}}\right)}{\underset{1}{\cancel{3}}\times 5}\right) \div 3$$

$$= \frac{4}{5} \div \frac{3}{1} = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

So, $x \div (y \div z) \neq (x \div y) \div z$

Exercise 1.4

1. **Divide:-**

(i) $\frac{2}{5}$ by $\frac{3}{4}$ (ii) $\left(\frac{-3}{8}\right)$ by $\left(\frac{-2}{3}\right)$ (iii) $\left(\frac{-5}{6}\right)$ by $\frac{3}{4}$

(iv) $\left(\frac{-5}{8}\right)$ by (-3) (v) $\left(\frac{-3}{4}\right)$ by (-6) (vi) $\left(\frac{-2}{3}\right)$ by $\left(\frac{-7}{12}\right)$

(vii) $\left(\frac{-16}{21}\right)$ by $\left(\frac{-4}{9}\right)$ (viii) $\frac{10}{9}$ by $\left(\frac{-25}{12}\right)$

2. **Verify $x \div y \neq y \div x$, when**

(i) $x = \frac{5}{7}$ $y = \frac{-3}{4}$ (ii) $x = \frac{-7}{10}, y = \frac{-5}{12}$

(iii) $x = \frac{-3}{4}, y = \frac{-9}{16}$

3. **Verify $x \div (y \div z) \neq (x \div y) \div z$, when**

(i) $x = \frac{-3}{15}, y = \frac{-2}{3}, z = 2$ (ii) $x = \frac{-1}{4}, y = \frac{-3}{2}, z = \frac{-5}{6}$

4. **The product of two rational numbers is $\frac{-8}{9}$. If one of the number is $\frac{-2}{5}$, find the other.**

5. **The product of two rational numbers is –10. If one of the number is 15, find the other.**

6. **By what number should $\frac{-3}{4}$ be multiplied so that the product is $\frac{15}{16}$?**

Looking Outcome

After completion of this chapter, the students are now able to:

- *Know about number system.*
- *Apply different operations addition,subtraction, multiplication and division on rational numbers.*
- *Know about the properties of rational numbers under different operation.*

Answer

Exercise 1.1

1. (i) $\frac{-1}{12}$ (ii) $\frac{-4}{33}$ (iii) $\frac{3}{8}$ (iv) $\frac{-1}{24}$ (v) $\frac{-23}{30}$

(vi) $\frac{-1}{2}$ (vii) $\frac{11}{18}$ (viii) $\frac{8}{75}$

4 . (i) $\frac{5}{11}$ (ii) $\frac{-8}{9}$ (iii) $\frac{15}{13}$ (iv) $\frac{-2}{9}$ (v) $\frac{3}{8}$

(vi) $\frac{2}{7}$ (vii) $\frac{-18}{11}$ (viii) 0

5. (i) $\frac{-12}{15}$ (ii) $\frac{1}{56}$ (iii) $\frac{-77}{18}$ (iv) $\frac{1}{2}$ (v) $\frac{47}{48}$

6. (i) b (ii) c (iii) b (iv) a

Exercise 1.2

1. (i) $\frac{2}{5}$ (ii) 1 (iii) $\frac{11}{8}$ (iv) $\frac{31}{42}$ (v) $\frac{1}{6}$

(vi) $\frac{19}{66}$ (vii) $\frac{1}{3}$ (viii) $\frac{-81}{50}$

4. (i) $\frac{17}{24}$ (ii) $\frac{-119}{24}$ (iii) $\frac{-19}{30}$ (iv) $\frac{-11}{30}$ (v) $\frac{33}{72}$

Exercise 1.3

1. (i) $\frac{35}{44}$ (ii) $\frac{-15}{28}$ (iii) $\frac{-10}{99}$ (iv) $\frac{-112}{35}$ (v) $\frac{16}{5}$ (vi) $\frac{4}{5}$

(vii) $\frac{3}{10}$ (viii) $\frac{4}{5}$ (ix) $\frac{-1}{2}$ (x) $\frac{5}{6}$ (xi) $\frac{16}{3}$ (xii) $\frac{-9}{5}$

4. (i) $\frac{-1}{2}$ (ii) $\frac{-8}{5}$ (iii) $\frac{-9}{7}$ (iv) $\frac{-4}{3}$ (v) $\frac{-35}{2}$

(vi) 2 (vii) $\frac{-3}{20}$ (viii) $\frac{-3}{7}$

7. (i) Commutative Property (ii) Existence of Identity (iii) Distribution over addition
(iv) Associative property (v) Distribution over subtraction

8. (i) c (ii) b (iii) a (iv) d (v) a (vi) d

Exercise 1.4

1. (i) $\frac{8}{15}$ (ii) $\frac{9}{16}$ (iii) $\frac{-10}{9}$ (iv) $\frac{5}{24}$ (v) $\frac{1}{8}$ (vi) $\frac{8}{7}$ (vii) $\frac{12}{7}$

(viii) $\frac{-8}{15}$

4. $\frac{20}{9}$ 5. $\frac{-2}{3}$ 6. $\frac{-5}{4}$