Complex Numbers and Quadratic Equations

Multiple Choice Questions

1 Value of $i^9 + i^{10} + i^{11}$ is equal to
(a)$1 - i$ (b)$1 + i$ (c)-1 (d)$-i$

2 Value of $\sqrt{-25} \times \sqrt{-36}$ is
(a)30 (b)$30i$ (c)$-30i$ (d)-30

3 If $z = 3 - 2i$ then $\bar{z}$ is equal to :
(a)$3 + 2i$ (b)$-3 + 2i$ (c)$-3 - 2i$ (d)$3 - 2i$

4 If $x + iy = \frac{1}{\sqrt{3}-2i}$ then y is equal to :
(a)$\frac{\sqrt{3}}{7}$ (b)$\frac{2}{7}$ (c)$-\frac{2}{7}$ (d)$-\frac{\sqrt{3}}{7}$

5 If $z = 3 - 4i$ then $|z|$ is equal to :
(a)7 (b)5 (c)-5 (d)-12

6 Amplitude or argument of $z = 1 + i$ is
(a)60° (b) 30° (c) 90° (d) 45°

7 If $z = \frac{1-i}{1+i}$ then $R_e(z)$ is equal to :
(a)-1 (b)0 (c)1 (d)$\frac{1}{2}$

8 If for $ax^2 + bx + c = 0$, $b^2 - 4ac < 0$ then roots of quadratic equation are :
(a)Real and unequal (b)Real and equal (c)Complex conjugate (d)none of these

9 Roots of $x^2 + 289 = 0$ are :
(a)± 17 (b)$\pm 17i$ (c)0 (d)none of these

10 If $x^2 + x + 1 = 0$ then value of x is :
(a)$\frac{1 \pm \sqrt{3}i}{2}$ (b) $\frac{\sqrt{3}+i}{2}$ (c) $\frac{-1 \pm \sqrt{3}i}{2}$ (d) $\frac{\sqrt{3}-i}{2}$

2,4 & 6 Marks Questions

1. Express the following complex numbers in $a + ib$ form :

(i)$(2 - 5i)^3$ (ii)$\frac{2+3i}{1-i}$ (iii) $i^9 + i^{19} + i^{29} + i^{39}$

(iv)$\frac{1}{4+5i}$ (v) $\frac{2-3i}{3+2i} - \frac{5+7i}{4-i}$

2. Find the multiplicative inverse of the following complex numbers :

(i)$\sqrt{5} - 3i$ (ii)$\frac{2}{4-3i}$ (iii)$\frac{1+i}{5-7i} + \frac{1-i}{7-5i}$ (iv)$\frac{3+1}{\sqrt{2}-\sqrt{3}i}$

3. Find the real values of x and y if z_1 is equal to the conjugate of z_2 in each of the following :

(i)$z_1 = 3x - 7 + 2iy$, $z_2 = -5y - (5 + x)i$

(ii)$z_1 = (x + iy)(2 - 3i)$, $z_2 = 4 - i$

4. If $z_1 = 3 - 4i$ and $z_2 = 7 + 2i$ then find the values of the following :

   (i)$R_e\left(\frac{1}{z_1\overline{z_2}}\right)$ (ii)$I_m\left(\frac{z_1 z_2}{\overline{z_1}}\right)$

5. If z_1, z_2 are any two complex numbers then prove that :

   (i)$R_e(z_1 z_2) = R_e(z_1)R_e(z_2) - I_m(z_1)I_m(z_2)$

   (ii)$I_m(z_1 z_2) = R_e(z_1)I_m(z_2) + R_e(z_2)I_m(z_1)$

6. Write the following complex numbers in the polar form :

   (i)$z = \sqrt{3} - \sqrt{5}i$ (ii) $z = \sqrt{3} - i$ (iii)$\frac{8-3i}{3+i} - \frac{1+7i}{4-3i}$

7. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ then prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

8. If $(x + iy)^3 = u + iv$ then prove that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

9. If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of .

10. Solve the following quadratic equations:

    (i)$x^2 + x + 1 = 0$ (ii)$x^2 - 2x + \frac{3}{2} = 0$ (iii)$ix^2 + 3x - i5 = 0$ (iv)$x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

11. Find the square root of the following complex numbers :

    (i)$z = -5 + 12i$ (ii)$z = -7 - 24i$ (iii)$z = -8 - 6i$ (iv)$-11 - 60i$

12. If α and β are different complex numbers with $|\beta| = 1$,find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$.

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