DAY TWENTY SEVEN

Ray Optics

Learning & Revision for the Day

- Reflection of Light
- Mirror formula
- Refraction of Light
- Lens
- Total Internal Reflection (TIR)
- Deviation by a Prism
- Dispersion by a Prism
- Refraction Through a Prism
- Scattering of Light

Reflection of Light

The phenomena of bouncing back of light on striking a smooth surface is called reflection of light.

- According to the laws of reflection, (i) the incident ray, reflected ray and the normal drawn on the reflecting surface at the point of incidence lie in the same plane and (ii) the angle of incidence $\angle i =$ angle of reflection $\angle r$.
- Laws of reflection are true for reflection from a polished mirror or from an unpolished surface or for diffused reflection.
- Whenever reflection takes place from a denser medium, the reflected rays undergo a phase change of π.

Reflection from a Plane Mirror

- If a ray is incident on a plane mirror at an angle of incidence i, then it suffers a deviation of $(\pi - 2i)$ and for two inclined plane mirrors deviation is $(360° - 2\theta)$.
- While keeping an object fixed, a plane mirror is rotated in its plane by an angle θ, then the reflected ray rotates in the same direction by an angle 2θ.
- Focal length as well as the radius of curvature of a plane mirror is infinity. Power of a plane mirror is zero.
- If two plane mirrors are inclined to each other at an angle θ, the total number of images formed of an object kept between them, is $n = \frac{2\pi}{\theta}$ or $\left(\frac{2\pi}{\theta} - 1\right)$, when it is odd.
- The minimum size of a plane mirror fixed on a wall of a room, so that a person at the centre of the room may see the full image of the wall behind him, should be 1/3rd the size of the wall.

Reflection from a Spherical Mirror

- A spherical mirror is a part of a hollow sphere whose one surface is polished, so that it becomes reflecting. The other surface of the mirror is made opaque.

- Image formed by a concave mirroris is virtual and erect, when the object is placed between the pole and the principal focus of concave mirror. In all other cases, the image formed is real and inverted one.
- Image formed by a convex mirror is virtual, erect and diminished in size irrespective of the position of the object. Moreover, image is formed in between the pole and the principal focus of the mirror.
- The focal length of a spherical mirror is half of its radius of curvature, i.e. $f = \frac{R}{2}$.

Mirror Formula

Let an object be placed at a distance u from the pole of a mirror and its image is formed at a distance v from the pole.

Then, according to mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

- The power of a mirror (in dioptre), is given as

$$P = \frac{1}{f(\text{in metre})}$$

- If a thin object of height h is placed perpendicular to the principal axis of a mirror and the height of its image be h', then the transverse or lateral magnification produced is given by

$$m = \frac{h'}{h} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

Negative sign of magnification means the inverted image and positive sign means an erect image.

- When a small sized object is placed along the principal axis, then its longitudinal (or axial) magnification is given by

$$\text{Axial magnification} = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Refraction of Light

When light passes from one medium, say air, to another medium, say glass, a part is reflected back into the first medium and the rest passes into the second medium. When it passes into the second medium, it either bends towards the normal or away from the normal.

This phenomenon is known as refraction.

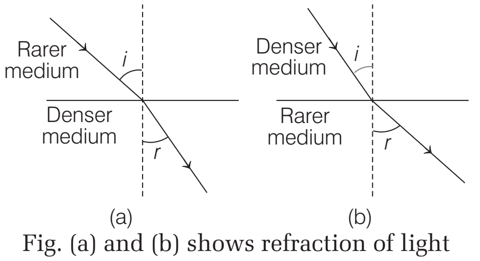


Fig. (a) and (b) shows refraction of light

Refractive Index

For a given pair of media, the ratio of the sine of angle of incidence (i) to the sine of angle of refraction (r) is a constant, which is called the refractive index of second medium, w.r.t. first medium.

Thus, $$\frac{\sin i}{\sin r} = \text{constant} = n_{21} = \frac{n_2}{n_1}$$

This is also called **Snell's law**.

- Refractive index is a unitless, dimensionless and a scalar quantity.
- The refractive index of a medium w.r.t. vacuum (or free space) is known as its absolute refractive index. It is defined as the ratio of the speed of light in vacuum (c) to the speed of light in a given medium (v).

$\therefore$ $$n = \frac{c}{v}$$

Value of absolute refractive index of a medium can be 1 or more than 1, but never less than 1.

- When light travels from one material medium to another, the ratio of the speed of light in the first medium to that in the second medium is known as the relative refractive index of second medium, w.r.t. the first medium. Thus,

$$n_{21} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{n_2}{n_1}$$

- When light undergoing refraction through several media finally enters the first medium itself, then

$$n_{21} \times n_{32} \times n_{13} = 1 \text{ or } n_{32} = \frac{n_{31}}{n_{21}}$$

- When the object is in denser medium and the observer is in rarer medium, then real and apparent depth have the relationship, $\frac{\text{Real depth}}{\text{Apparent depth}} = n_{21}$

i.e. real depth > apparent depth shift, $y = h - h' = \left(1 - \frac{1}{n_{21}}\right)h$

where, h and h' are real and apparent depths.

Refraction from a Spherical Surface

Let an object be placed in a medium of refractive index n_1 at a distance u from the pole of a spherical surface of radius of curvature R and after refraction, its image is formed in a medium of refractive index n_2 at a distance v, then

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

The relation is true for all surfaces, whether the image formed is real or virtual.

Lens

A lens is part of a transparent refracting medium bound by two surfaces, with atleast one of the two surfaces being a curved one. The curved surface may be spherical or cylindrical.

The lens formula is given by $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For a thin object of height h placed perpendicular to the principal axis at a distance u, if the height of image formed is h', then lateral or transverse magnification m is given by

$$m = \frac{h'}{h} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

For a small sized object placed linearly along the principal axis, its **axial** or **longitudinal magnification** is given by

$$\text{Axial magnification} = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f+u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

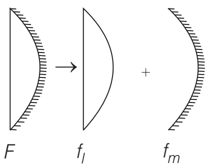
Silvering of Lens

When one surface of a lens is silvered, it behaves as a mirror. The focal length of silvered lens is $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$

where, f_1 is focal length of plane convex lens and f_m is focal length of corresponding mirror.

In case of plano-convex lens

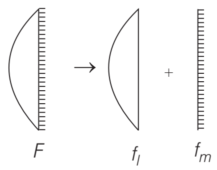
- **When curved surface is silvered**, then focal length of silvered lens is $F = \frac{R}{2\mu}$



where, $R = 2f_m$ or $R = f_l(\mu - 1)$

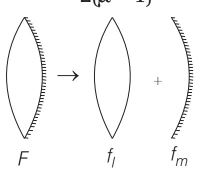
- **When plane surface is silvered**, then

$$F = \frac{R}{2(\mu - 1)} \text{ and } f_l = \frac{R}{(\mu - 1)}, f_m = \infty$$



- **When double convex lens is silvered**, then

$$F = \frac{R}{2(2\mu - 1)} \text{ and } f_l = \frac{R}{2(\mu - 1)} \Rightarrow f_m = \frac{R}{2}.$$



Power of a Lens

The power of a lens is mathematically given by the reciprocal of its focal length, i.e. power $P = \frac{1}{f(\text{m})}$

SI unit of power is dioptre (D). Power of a converging lens is positive and that of a diverging lens is negative.

Lens Maker's Formula

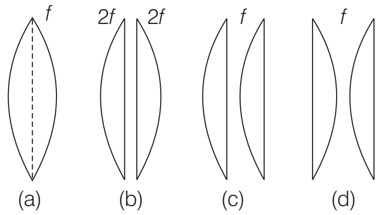
For a lens having surfaces with radii of curvature R_1 and R_2 respectively, its focal length is given by

$$P = \frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

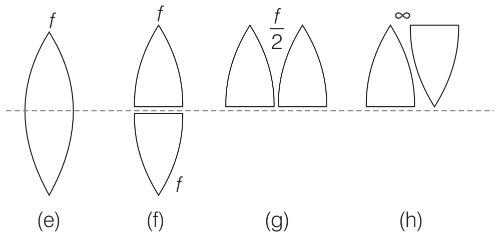
where, $n_{21} = \frac{n_2}{n_1}$ = refractive index of the lens material w.r.t. the surroundings.

Cutting and Combination of a Lens

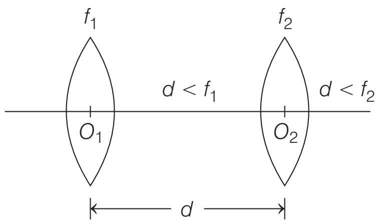
If a symmetrical convex lens of focal length f is cut into two parts along its optic axis, then focal length of each part (a plano-convex lens) is $2f$. However, if the two parts are joined as shown in the figure, the focal length of the combination is again f.



If a symmetrical convex lens of focal length f is cut into two parts along the principal axis, then the focal length of each part remains unchanged, as f [Fig. f]. If these two parts are joined with the curved ends on one side, the focal length of the combination is $\frac{f}{2}$ [Fig. g]. But on joining the two parts in opposite sense, the net focal length becomes ∞ (or net power = 0) (Fig. h).



- The equivalent focal length of co-axial combination of two lenses is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$



- If a number of lenses are in contact, then $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots$
- If two thin lenses of focal lengths f_1 and f_2 are in contact, then their equivalent focal length $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$

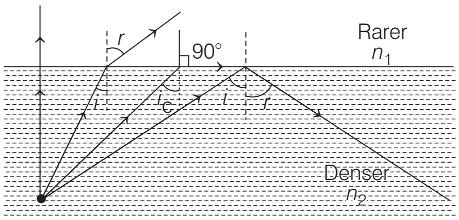
In terms of power, $P_{eq} = P_1 + P_2$

Important Results

- Effective diameter of light transmitting area is called aperture. Intensity of image $\propto$ (Aperture)^2
- Relation between object and image speed, $v_i = \left(\dfrac{f}{f+u}\right)^2 v_o$
- Newton's formula states, $f^2 = x_1 x_2$ where, x_1 and x_2 = distance of object; and image from first and second principlal foci. This formula is also called thin lens formula.
- If lens immersed in a liquid, then $\dfrac{f_l}{f_a} = \dfrac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)} = \dfrac{({}_a\mu_g - 1)}{\left(\dfrac{{}_a\mu_g}{{}_a\mu_l} - 1\right)}$

Total Internal Reflection (TIR)

When a ray of light goes from a denser to a rarer medium, it bends away from the normal. For a certain angle of incidence i_C, the angle of refraction in rarer medium becomes 90°. The angle i_C is called the **critical angle**.



TIR in a medium

$$\sin i_C = \frac{n_1}{n_2} = \frac{1}{n_{21}}$$

For the angle of incidence greater than the critical angle ($i > i_C$) in the denser medium, the light ray is totally internally reflected back into the denser medium itself.

Conditions for Total Internal Reflection

- The light ray should travel from the denser medium towards the rarer medium.
- The angle of incidence should be the greater than the critical angle.

Common Examples of Total Internal Reflection

- **Looming** An optical illusion in cold countries.
- **Mirage** An optical illusion in deserts.
- **Brilliance of diamond** Due to repeated internal reflections diamond sparkles.
- **Optical fibre** Each fibre consists of core and cladding. The refractive index of core material is higher than that of cladding. Light entering at small angle on one end undergoes repeated total internal reflections along the fibre and finally comes out.

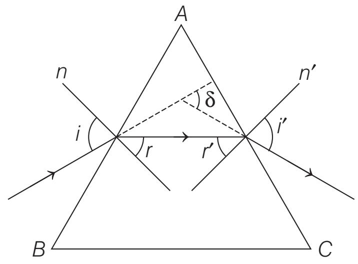
Deviation by a Prism

A prism is a homogeneous, transparent medium bounded by two plane surfaces inclined at an angle A with each other. These surfaces are called as refracting surfaces and the angle between them is called angle of prism A.

Deviation produced by a prism is,

$$\delta = i + i' - A$$

$\Rightarrow \quad r + r' = A$



For grazing incidence
$i = 90°$

and grazing emergence
$i' = 90°$

For minimum deviation

(i) $i = i'$ and $r = r'$ (ii) $\mu = \dfrac{\sin\left(\dfrac{\delta_m + A}{2}\right)}{\sin\dfrac{A}{2}}$

In case of minimum deviation, ray is passing through prism symmetrically.

For maximum deviation (δ_{max}), $i = 90°$ or $i' = 90°$

For thin prism, $\delta = (\mu - 1)A$

Dispersion by a Prism

Dispersion of light is the phenomenon of splitting of white light into its constituent colours on passing light through a prism. This is because different colours have different wavelength and hence different refractive indices.

Angular dispersion $= \delta_v - \delta_r = (n_v - n_r)A$
where, n_v and n_r represent refractive index for violet and red lights.

Dispersive power, $\omega = \dfrac{n_v - n_r}{n - 1}$, where $n = \dfrac{n_v + n_r}{2}$ is the mean refractive index.

By combining two prisms with angle A and A' and refractive index n and n' respectively, we can create conditions of

- Dispersion without deviation when, $A' = -\dfrac{(n-1)A}{(n'-1)}$
- Deviation without dispersion when,
$$A' = -\left[\frac{n_v - n_r}{n'_v - n'_r}\right]A$$

Refraction Through a Prism

A ray of light suffers two refractions at the two surfaces on passing through a prism. Angle of deviation through a prism $\delta = i + e - A$. where, i is the angle of incidence, e is the angle of emergence and A is the angle of prism.

Scattering of Light

Molecules of a medium after absorbing incoming light radiations, emit them in all directions. This phenomenon is called scattering. According to Rayleigh, intensity of scattered light $\propto \dfrac{1}{\lambda^4}$.

There are some phenomenon based on scattering

- Sky looks blue due to scattering.
- At the time of sunrise and sunset, sun looks reddish.
- Danger signals are made of red colour.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 To get three images of a single object, one should have two plane mirrors at an angle of

(a) 60° (b) 90° (c) 120° (d) 30°

2 A source of light lies on the angle bisector of two plane mirrors inclined at angle θ. The values of θ, so that the light reflected from one mirror does not reach the other mirror will be

(a) $\theta \geq 120°$ (b) $\theta \geq 90°$
(c) $\theta \leq 120°$ (d) $\theta < 30°$

3 A beam of light composed of red and green rays is incident obliquely at a point on the face of a rectangular glass slab. When coming out of the opposite parallel face, the red and green rays emerge from

(a) two points propagating in two different non-parallel directions
(b) two points propagating in two different parallel directions
(c) one point propagating in two different directions
(d) one point propagating in the same direction

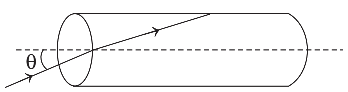
4 The optical path of a monochromatic light is same if it goes through 4.0 cm of glass or 4.5 cm of water. If the refractive index of glass is 1.53, the refractive index of the water is

(a) 1.30 (b) 1.36
(c) 1.42 (d) 1.46

5 On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally as it travels, the light beam → **JEE Main 2015**

(a) becomes narrower
(b) goes horizontally without any deflection
(c) bends downwards
(d) bends upwards

6 A transparent solid cylinder rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

7 A ray of light is incident on the surface of separation of a medium at an angle 45° and is refracted in the medium at an angle 30°. What will be the speed of light in the medium?

(a) 1.96×10^8 ms^{-1} (b) 2.12×10^8 ms^{-1}
(c) 3.18×10^8 ms^{-1} (d) 3.33×10^8 ms^{-1}

8 A beaker contains water upto a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is → **AIEEE 2011**

(a) $\left(1-\frac{1}{\mu_1}\right)h_2 + \left(1-\frac{1}{\mu_2}\right)h_1$ (b) $\left(1+\frac{1}{\mu_1}\right)h_1 + \left(1+\frac{1}{\mu_2}\right)h_2$
(c) $\left(1-\frac{1}{\mu_1}\right)h_1 + \left(1-\frac{1}{\mu_2}\right)h_2$ (d) $\left(1+\frac{1}{\mu_1}\right)h_2 - \left(1+\frac{1}{\mu_2}\right)h_1$

9 A printed page is pressed by a glass of water. The refractive index of the glass and water is 1.5 and 1.33, respectively. If the thickness of the bottom of glass is 1 cm and depth of water is 5 cm, how much the page will appear to be shifted if viewed from the top? → **JEE Main (Online) 2013**

(a) 1.033 cm (b) 3.581 cm
(c) 1.3533 cm (d) 1.90 cm

10 A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the water surface, the radius of this circle in cm, is

(a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$ (c) $36\sqrt{5}$ (d) $4\sqrt{5}$

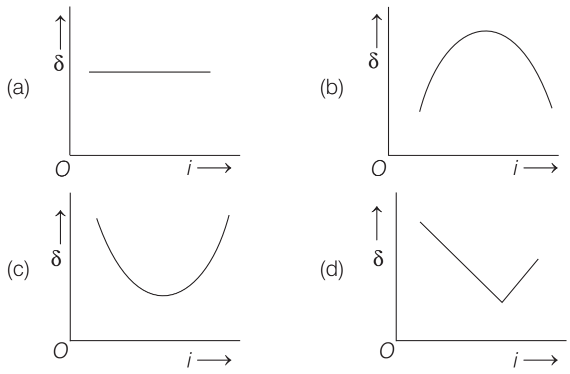
11 A green light is incident from the water to the air-water interface at the critical angle (θ). Select correct statement. → **JEE Main 2014**

(a) The spectrum of visible light whose frequency is more than that of green light will come out to the medium
(b) The entire spectrum of visible light will come out of the water at various angles to the normal
(c) The entire spectrum of visible light come out of the water at an angle of 90° to the normal
(d) The spectrum of visible light whose frequency is less than that of green light will come out to the medium

12 Light is incident from a medium into air at two possible angles of incidence (a) 20° and (b) 40°. In the medium, light travels 3.0 cm in 0.2 ns. The ray will → **JEE Main (Online) 2013**

(a) suffer total internal reflection in both cases (a) and (b)
(b) suffer total internal reflection in case (b) only
(c) have partial reflection and partial transmission in case (b)
(d) have 100% transmission in case (a)

13 The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by



14 The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be the angles of minimum deviation for the red and blue light respectively in a prism of this glass. Then,

(a) $D_1 < D_2$
(b) $D_1 = D_2$
(c) D_1 can be less than or greater than D_2 depending upon the angle of prism
(d) $D_1 > D_2$

15 Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

(a) 30° for both the colours
(b) greater for the violet colour
(c) greater for the red colour
(d) equal but not 30° for both the colours

16 The maximum magnification that can be obtained with a convex lens of focal length 2.5 cm is (The least distance of distinct vision is 25 cm)

(a) 10 (b) 0.1 (c) 62.5 (d) 11

17 A wire mesh consisting of very small squares is viewed at a distance of 8 cm through a magnifying lens of focal length 10 cm, kept close to the eye. The magnification produced by the lens is

(a) 5 (b) 8 (c) 10 (d) 20

18 When the distance between the object and the screen is more than $4F$, we can obtain image of an object on the screen for the two positions of a lens. It is called displacement method. In one case, the image is magnified and in the other case it is diminished. Then, the ratio of the size of image to the diminished image is

(a) $\frac{(D+d)^2}{(D-d)^2}$ (b) $\frac{D}{d}$ (c) $\frac{D^2}{d^2}$ (d) $\frac{D+d}{D-d}$

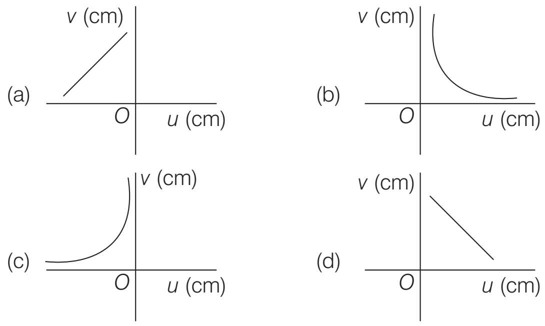
19 A biconvex lens of focal length f forms a circular image of radius r of sun in the focal plane. Then, which option is correct?

(a) $\pi r^2 \propto f$
(b) $\pi r^2 \propto f^2$
(c) If lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$
(d) If f is doubled, intensity will increase

20 In an optics experiments, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be **→ AIEEE 2009**

(a) $(2f, 2f)$ (b) $\left(\frac{f}{2}, \frac{f}{2}\right)$
(c) (f, f) (d) $(4f, 4f)$

21 A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like



22 An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image

(a) moves away from the lens with a uniform speed 5 m/s
(b) moves away from the lens with a uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration

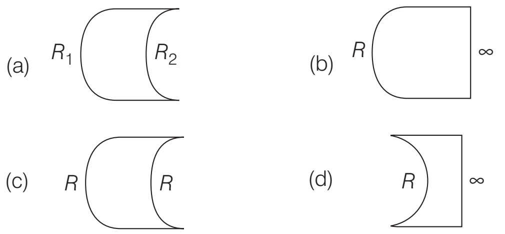
23 In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then,

(a) $x < f$ (b) $f < x < 2f$
(c) $x = 2f$ (d) $x > 2f$

24 The image of an illuminated square is obtained on a screen with the help of a converging lens. The distance of the square from the lens is 40 cm. The area of the image is 9 times that of the square. The focal length of the lens is → **JEE Main (Online) 2013**

(a) 36 cm (b) 27 cm
(c) 60 cm (d) 30 cm

25 Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams.



26 A double convex lens made of glass (refractive index $n = 1.5$) has the radii of curvature of both the surfaces as 20 cm. Incident light rays parallel to the axis of the lens will converge at a distance L such that

(a) $L = 20$ cm (b) $L = 10$ cm
(c) $L = 40$ cm (d) $L = \frac{20}{3}$ cm

27 When monochromatic red light is used instead of blue light in a convex lens, its focal length will → **AIEEE 2011**

(a) not depend on colour of light
(b) increase
(c) decrease
(d) remain same

28 A concave lens and a convex lens have the same focal length of 20 cm and both are kept in contact. The combination is used to view an object 5 cm long kept at a distance of 20 cm from the lens combination. As compared to the object, the image will be

(a) magnified and inverted
(b) diminished and erect
(c) of the same size and erect
(d) of the same size and inverted

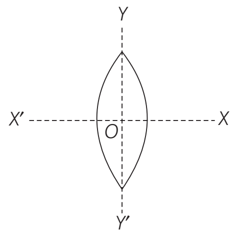
29 To make an achromatic combination, a convex lens of focal length 42 cm having dispersive power of 0.14 is placed in contact with a concave lens of dispersive power 0.21. The focal length of the concave lens should be

(a) 63 cm (b) 21 cm
(c) 42 cm (d) 14 cm

30 A convex lens is in contact with a concave lens. The magnitude of the ratio of their focal length is $\frac{3}{2}$. Their equivalent focal length is 30 cm. What are their individual focal lengths?

(a) $-75, 50$ (b) $-10, 15$ (c) $75, 50$ (d) $-15, 10$

31 An equiconvex lens is cut into two halves along (i) XOX' and (ii) along YOY' as shown in figure. Let f, f' and f'' be the focal lengths of the complete lens, of each half in case (i), and of each half in case (ii) respectively. Choose the correct statement from the following.



(a) $f' = 2f, f'' = f$ (b) $f' = f, f'' = f$
(c) $f' = 2f, f'' = 2f$ (d) $f' = f, f'' = 2f$

Direction (Q. Nos. 32-36) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

32 **Statement I** The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small as compared to their radii of curvature.

Statement II Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

33 **Statement I** Endoscopy involves use of optical fibres to study internal organs.

Statement II Optical fibres are based on the phenomenon of total internal reflection.

34 **Statement I** The refractive index of diamond is $\sqrt{6}$ and that of liquid is $\sqrt{3}$. If the light travels from diamond to the liquid, it will be totally reflected when the angle of incidence is 30°.

Statement II $n = \frac{1}{\sin C}$, where n is the refractive index of diamond with respect to liquid.

35 **Statement I** A double convex lens $(n = 1.5)$ has a focal length 10 cm. When the lens is immersed in water $(n = 4/3)$, its focal length becomes 40 cm.

Statement II $\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

36 A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

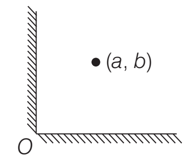
Statement I When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement II The centre of the interference pattern is dark. → **AIEEE 2011**

DAY PRACTICE SESSION 2

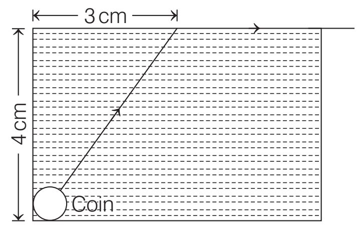
PROGRESSIVE QUESTIONS EXERCISE

1 Two plane mirrors are inclined at 90°. An object is placed between them whose coordinates are (a, b). The position vectors of all the images formed is



(a) $-a\hat{\mathbf{i}} - b\hat{\mathbf{j}}, a\hat{\mathbf{i}} + b\hat{\mathbf{j}}, -a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$
(b) $-a\hat{\mathbf{i}} + b\hat{\mathbf{j}}, -a\hat{\mathbf{j}} - b\hat{\mathbf{j}}, a\hat{\mathbf{i}} - b\hat{\mathbf{j}}$
(c) $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}, -a\hat{\mathbf{i}} - b\hat{\mathbf{j}}, a\hat{\mathbf{i}} - b\hat{\mathbf{j}}$
(d) None of the above

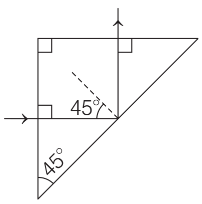
2 A small coin is resting on the bottom of a beaker filled with a liquid. A ray of light from the coin travels upto the surface of the liquid and moves along its surface (see figure).



How fast is the light travelling in the liquid?
(a) 1.8×10^8 ms^{-1} (b) 2.4×10^8 ms^{-1}
(c) 3.0×10^8 ms^{-1} (d) 1.2×10^8 ms^{-1}

3 The reflective surface is given by $y = 2 \sin x$ and it is facing positive axis. What is the least value of coordinate of the point where a ray parallel to positive x-axis becomes parallel to positive y-axis after reflection?
(a) $\left(\frac{\pi}{3}, \sqrt{3}\right)$ (b) $\left(\frac{\pi}{2}, \sqrt{2}\right)$ (c) $\left(\frac{\pi}{3}, \sqrt{2}\right)$ (d) $\left(\frac{\pi}{4}, \sqrt{3}\right)$

4 A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that for the refractive index n as



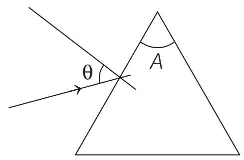
(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$ (c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

5 The speed at which the image of the luminous point object is moving, if the luminous point object is moving at speed v_0 towards a spherical mirror, along its axis is (Given, R = radius of curvature, u = object distance)
(a) $v_i = -v_0$ (b) $v_i = -v_0\left[\frac{R}{2u - R}\right]^2$
(c) $v_i = -v_0\left(\frac{2u - R}{R}\right)$ (d) $v_i = -v_0\left(\frac{R}{2u - R}\right)$

6 Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of lens
(a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

7 A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object?
(a) 20 cm (b) 30 cm (c) 60 cm (d) 80 cm

8 Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided

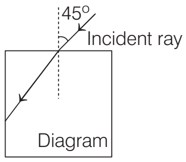


→ **JEE Main 2015**

(a) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right]$
(b) $\theta < \sin^{-1}\left[\mu \sin\left(A + \sin^{-1}\frac{1}{\mu}\right)\right]$
(c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\frac{1}{\mu}\right)\right]$
(d) $\theta < \cos^{-1}\left[\mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right]$

9 A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquid having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has focal lengths f_1 and f_2, respectively. The correct relation between focal length is → **JEE Main 2014**
(a) $f_2 > f$ and f_1 becomes one
(b) f_1 and f_2 both becomes one
(c) $f_1 = f_2 < f$
(d) $f_1 > f$ and f_2 becomes one

10 A light ray falls on a square glass slab as showin in the diagram. The index of refraction of the glass, if total internal reflection is to occur at the vertical face, is equal to **→ JEE Main 2013**



(a) $\frac{(\sqrt{2}+1)}{2}$ (b) $\sqrt{\frac{5}{2}}$ (c) $\frac{3}{2}$ (d) $\sqrt{\frac{3}{2}}$

11 A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is **→ AIEEE 2012**

(a) 58.59° (b) 58.77°
(c) 58.65° (d) 59°

12 A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is **→ AIEEE 2011**

(a) $\frac{1}{15}$ m/s (b) 10 m/s (c) 15 m/s (d) $\frac{1}{10}$ m/s

13 A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is **→ JEE Main 2017 (Offline)**

(a) virtual and at a distance of 40 cm from convergent lens
(b) real and at a distance of 40 cm from the divergent lens
(c) real and at a distance of 6 cm from the convergent lens
(d) real and at a distance of 40 cm from convergent lens

14 In an experiment for determination of refractive index of glass of a prism by i - δ, plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index? **→ JEE Main 2016 (Offline)**

(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8

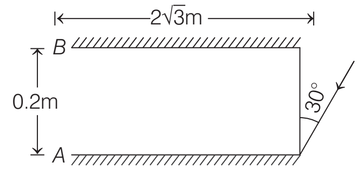
15 An object 2.4 m infront of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? **→ AIEEE 2012**

(a) 7.2 m (b) 2.4 m (c) 3.2 m (d) 5.6 m

16 Let the zx-plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\,\mathbf{i} + 8\sqrt{3}\,\mathbf{j} - 10\,\mathbf{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is **→ AIEEE 2011**

(a) 45° (b) 60° (c) 75° (d) 30°

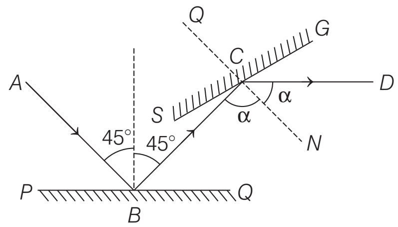
17 Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure.



The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is

(a) 28 (b) 30 (c) 32 (d) 34

18 A light ray is incident on a horizontal plane mirror at an angle of 45°. At what angle should a second plane mirror be placed in order that the reflected ray finally be reflected horizontally from the second mirror as shown in figure, is



(a) $\theta = 45°$ (b) $\theta = 60°$ (c) $\theta = 22.5°$ (d) $\theta = 15.3°$

ANSWERS

SESSION 1	**1** (b)	**2** (a)	**3** (b)	**4** (b)	**5** (d)	**6** (d)	**7** (b)	**8** (c)	**9** (d)	**10** (b)
	11 (d)	**12** (b)	**13** (c)	**14** (a)	**15** (a)	**16** (d)	**17** (a)	**18** (a)	**19** (b)	**20** (a)
	21 (c)	**22** (c)	**23** (b)	**24** (a)	**25** (c)	**26** (a)	**27** (b)	**28** (c)	**29** (a)	**30** (d)
	31 (d)	**32** (c)	**33** (a)	**34** (d)	**35** (a)	**36** (b)				
SESSION 2	**1** (b)	**2** (a)	**3** (a)	**4** (b)	**5** (b)	**6** (c)	**7** (a)	**8** (a)	**9** (d)	**10** (d)
	11 (c)	**12** (a)	**13** (d)	**14** (a)	**15** (d)	**16** (a)	**17** (b)	**18** (c)		

Hints and Explanations

SESSION 1

1 Number of images

$$n = \frac{360°}{\theta} - 1$$

where, θ is angle between mirrors.

$$\therefore \quad 3 = \frac{360°}{\theta} - 1 \quad \text{or} \quad \theta = 90°$$

2 $n = \dfrac{360}{\theta}$

Number of images, $N = n$, which is odd $= n - 1$

For the given condition, no successive reflection takes place. So, the number of images will be $N \le 2$

$$n - 1 \le 2$$

$$n \le 3$$

$$\frac{360}{\theta} \le 3$$

$$120 \le \theta \quad \Rightarrow \quad \theta \ge 120°$$

3 In any medium other than air or vacuum, the velocities of different colours are different. Therefore, both red and green colours are refracted at different angles of refractions.

Hence, after emerging from glass slab through opposite parallel face, they appear at two different points and move in two different parallel directions.

4 As optical paths are equal, hence

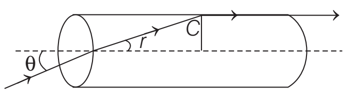
$$n_g \cdot x_g = n_w \cdot x_w$$

$$\Rightarrow \quad n_w = n_g \cdot \frac{x_g}{x_w}$$

$$= 1.53 \times \frac{4.0}{4.5} = 1.36$$

5 A horizontal beam is travelling from a denser to a rarer medium, so it bends upwards (away from normal).

6 $\sin C = \dfrac{\sqrt{3}}{2}$ ...(i)



$$\sin r = \sin (90° - C) = \cos C = \frac{1}{2}$$

$$\frac{\sin \theta}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\sin \theta = \frac{2}{\sqrt{3}} \times \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

7 $n = \dfrac{\sin i}{\sin r} = \dfrac{c}{v}$

Hence, $v = \dfrac{c \sin r}{\sin i}$

$$= \frac{3 \times 10^8 \times \sin 30°}{\sin 45°}$$

$$= \frac{3 \times 10^8}{\sqrt{2}}$$

$$= 2.12 \times 10^8 \text{ ms}^{-1}$$

8 We know, apparent shift,

$$\Delta h = \left(1 - \frac{1}{\mu}\right) h$$

$\therefore$ Apparent shift produced by water,

$$\Delta h_1 = \left(1 - \frac{1}{\mu_1}\right) h_1$$

and apparent shift produced by kerosene,

$$\Delta h_2 = \left(1 - \frac{1}{\mu_2}\right) h_2$$

$$\therefore \quad \Delta h = \Delta h_1 + \Delta h_2$$

$$= \left(1 - \frac{1}{\mu_1}\right) h_1 + \left(1 - \frac{1}{\mu_2}\right) h_2$$

9 Given, $\mu = 1.5, t_1 = 5\,\text{cm}$,

$\mu_2 = 1.33$ and $t_2 = 1\,\text{cm}$

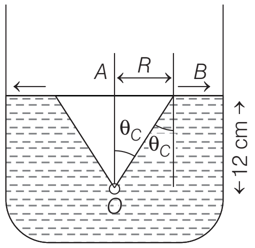
Change in path $= \Delta t_1 + \Delta t_2$

$$= \left(1 - \frac{1}{\mu_1}\right) \times t_1 + \left(1 - \frac{1}{\mu_2}\right) \times t_2$$

$$= \left(1 - \frac{1}{1.5}\right) \times 5 + \left(1 - \frac{1}{1.33}\right) \times 1$$

$$\approx 1.90 \text{ cm}$$

10 The situation is shown in figure.



$$\sin \theta_C = \frac{1}{\mu}$$

$$\tan \theta_C = \frac{AB}{OA}$$

$$\therefore \quad AB = OA \tan \theta_C$$

$$\text{or } AB = \frac{OA}{\sqrt{n^2 - 1}} = \frac{12}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{36}{\sqrt{7}}$$

11 $\mu_r < \mu_g < \mu_v$

$\Rightarrow \sin(\theta_r) > \sin(\theta_g) > \sin \theta_v$

$\theta_r > \theta_g > \theta_v$ [$\theta_r, \theta_g, \theta_v$ are critical angles of corresponding colours]. Thus, all colours from red to green will emerge out of water.

12 Speed of light in medium

$$= \frac{3 \times 10^{-2}}{0.2 \times 10^{-9}}$$

$$= \frac{3}{2} \times 10^8 \text{ ms}^{-1} = 1.5 \times 10^8 \text{ ms}^{-1}$$

As, $\dfrac{\mu_2}{\mu_1} = \dfrac{v_1}{v_2}$

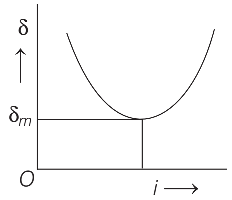
$$\frac{\mu}{1} = \frac{3 \times 10^8}{1.5 \times 10^8} \Rightarrow \mu = 2$$

We have, $\sin C = \dfrac{1}{2}$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

When the value of incidence angle is greater than critical angle than total internal reflection take place.
In the second case we get total interval reflection.

13 Angle of deviation depends upon the angle of incidence. If we determine experimentally the angles of deviation corresponding to different angles of incidence, then the plot between i and δ that we will get is shown below



14 $D = (n - 1) A$

For blue light n is greater than that for red light, so $D_2 > D_1$.

15 At minimum deviation $(\delta = \delta_m)$

$$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30°$$

(For both colours)

16 Maximum magnification is obtained when image is formed at near point of eye. In that case,

$$m = 1 + \frac{D}{f} = 1 + \frac{25}{2.5} = 11$$

17 $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ ...(i)

Given, $f = 10\,\text{cm}$

(as lens is converging)

$u = -8\,\text{cm}$

(as object is placed on left side of the lens)

On substituting these values in Eq. (i), we get

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-8}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{8}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{8-10}{80}$$

$$\therefore \quad v = \frac{80}{-2} = -40\,\text{cm}$$

Hence, magnification produced by the lens

$$m = \frac{v}{u} = \frac{-40}{-8} = 5$$

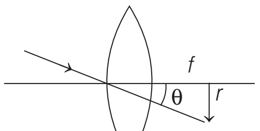
18 In displacement method, the ratio of the diminished image to the object is

$$m_2 = \frac{I_2}{O} = \frac{D-d}{D+d}$$

and the ratio of image to the object is $\frac{D+d}{D-d}$

Hence, $\frac{m_1}{m_2} = \frac{I_1}{O} \times \frac{O}{I_2} = \frac{(D+d)^2}{(D-d)^2}$

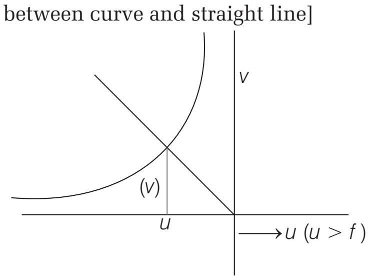
19



$r = f \tan\theta$ or $r \propto f$

$\therefore \quad \pi r^2 \propto f^2$

20 It is possible when object kept at centre of curvature because then only position of object and image would be same. i.e. $u = v$ [which is the point of intersection between curve and straight line]



$u = v$

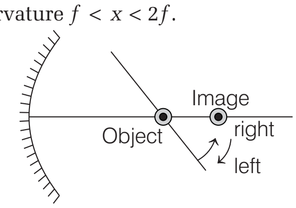
$u = 2f, v = 2f$

21 $\therefore \; \frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \text{constant},$

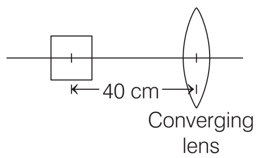
which is a rectangular hyperbola.

22 When object would approach the lens then image would move away from the lens with non-uniform acceleration. This can be seen from the fact that when object is far away, image would be formed at focus P. When object approaches the lens then image would move away such that when object approaches to focus, then image would approach to infinity.

23 Since, object and image move in opposite directions, the positioning should be as shown in the figure. Object lies between focus and centre of curvature $f < x < 2f$.



24 As magnification $= 9 = \frac{v}{u}$



$\therefore \quad v = 9u = 9 \times 40 = 360$

Now, $\quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{-40} = \frac{1}{f}$$

$$\frac{1}{360} + \frac{1}{40} = \frac{1}{f}$$

$$\Rightarrow \quad \left[\frac{1}{9} + 1\right]\frac{1}{40} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{10}{9} \times \frac{1}{40} = \frac{1}{f}$$

$$\Rightarrow \quad f = 36\,\text{cm}$$

25 $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For no dispersion, $\frac{1}{f} = 0$

or $R_1 = R_2 = R$.

So, (c) is correct option.

26 Here, $n = 1.5$, as per sign convention followed

$R_1 = +20\,\text{cm}$ and $R_2 = -20\,\text{cm}$

$$\therefore \quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5-1)\left[\frac{1}{(+20)} - \frac{1}{(-20)}\right]$$

$$= 0.5 \times \frac{2}{20} = \frac{1}{20}$$

$\Rightarrow \quad f = 20\,\text{cm}$

Incident rays travelling parallel to the axis of lens will converge at its second principal focus.

Hence, $L = +20\,\text{cm}$

27 $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Also, by Cauchy's formula,

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots$$

$$\Rightarrow \quad \mu \propto \frac{1}{\lambda}$$

As $\lambda_{\text{blue}} < \lambda_{\text{red}} \Rightarrow \mu_{\text{blue}} > \mu_{\text{red}}$

Hence, $\quad f_{\text{red}} > f_{\text{blue}}$

28 When a concave lens is joined in contact with a convex lens of same focal length, the combination behaves as a glass plate of infinite focal length or zero power.

Hence, the image of an object will be of the same size and erect.

29 For an achromatic combination, the condition is

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

Here, $\omega_1 = 0.14$, $f_1 = 42\,\text{cm}$

thus, we get $f_2 = -63\,\text{cm}$

30 Let focal length of convex lens is $+f$, then focal length of concave lens would be $-\frac{3}{2}f$.

From the given condition,

$$\frac{1}{30} = \frac{1}{f} - \frac{2}{3f} = \frac{1}{3f}$$

$$\therefore \quad f = 10\,\text{cm}$$

Therefore, focal length of convex lens $= +10\,\text{cm}$ and that of concave lens $= -15\,\text{cm}$.

31 (i) Cutting along YOY'

Focal length get doubled.

$$f'' = 2f$$

(ii) Cutting along XOX'

Focal length unchanged.

32 Laws of reflection are valid for plane surfaces, irregular surface and curved surface.

33 Optical fibre consists of a very long and thin fibre of quartz glass. When a light ray is incident at one end of the fibre making a small angle of incidence, it suffers refraction from air to quartz and strikes the fibre-layer interface at an angle of incidence greater than the critical angle.

It therefore, suffers total internal reflection and strikes its opposite interface. At this interface also, the angle of incidence is greater than the critical angle. So, it again suffers total internal reflection. Thus, optical fibre is based on total internal reflection. Endoscopy is a process for viewing internal organs of human body. This process use a device endoscope which is based on total internal reflection.

34 Refraction index of diamond w.r.t. liquid

$$_{l}n_{d} = \frac{1}{\sin C}$$

$$\therefore \quad \frac{\sqrt{6}}{\sqrt{3}} = \frac{1}{\sin C}$$

or $\quad \sin C = \frac{1}{\sqrt{2}} = \sin 45°$

$$\therefore \quad C = 45°$$

For total internal reflection angle should be greater than critical angle.

But here angle of incidence is lower than critical angle, so total internal reflection does not occur in light.

35 In water,

$$\frac{1}{f_w} = \left(\frac{n_l - n_m}{n_m}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

$$\frac{1}{f_a} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

$$\frac{f_w}{f_a} = \frac{(n_l - 1)}{\left(\frac{n_l - n_m}{n_m}\right)} = \frac{(1.5 - 1)(4/3)}{(1.5 - 4/3)}$$

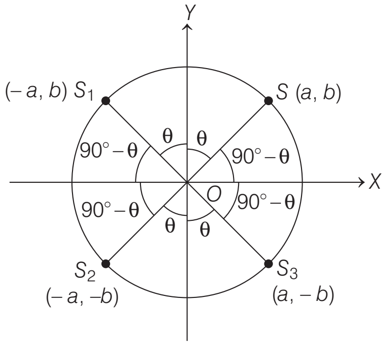
$$= \frac{0.5 \times 4/3}{\frac{3}{2} - \frac{4}{3}} = \frac{(2/3)6}{1}$$

$$\therefore \quad f_w = 4f_a = 4 \times 10 = 40\text{cm}$$

36 Both statements I and II are correct but statement II does not explain statement I.

SESSION 2

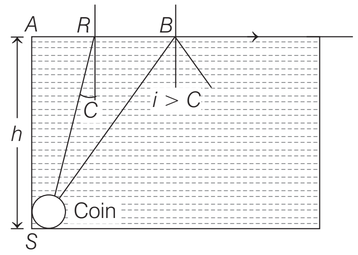
1 The images formed by combination of two plane mirrors are lying on a circle whose radius is equal to OS.



The centre of the circle is lying on meeting point of mirrors (i.e. O). The position of images from diagram is for S_1, $r_1 = -a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$, $r_2 = -a\hat{\mathbf{i}} - b\hat{\mathbf{j}}$, $r_3 = a\hat{\mathbf{i}} - b\hat{\mathbf{j}}$.

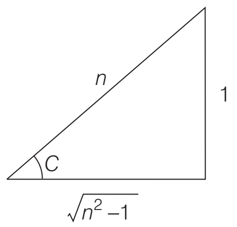
Hence, option (b) is true.

2 As shown in figure, a light ray from the coin will not emerge out of liquid, if $i > C$.



Therefore, minimum radius R corresponds to $i = C$.

In ΔSAB,



$$\frac{R}{h} = \tan C$$

or $\quad R = h \tan C$

or $\quad R = \frac{h}{\sqrt{n^2 - 1}}$

$$\left[\because \sin C = \frac{1}{n} \text{ and } \tan C = \frac{\sin C}{\sqrt{1 - \sin^2 C}}\right]$$

Given, $R = 3\text{cm}$, $h = 4\text{cm}$

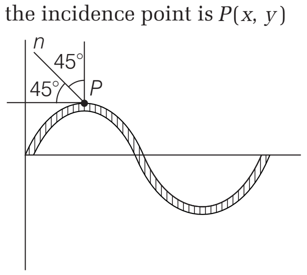
Hence, $\quad \frac{3}{4} = \frac{1}{\sqrt{n^2 - 1}}$

or $\quad n^2 = \frac{25}{9} \quad$ or $\quad n = \frac{5}{3}$

But $\quad n = \frac{c}{v}$

or $\quad v = \frac{c}{n} = \frac{3 \times 10^8}{5/3} = 1.8 \times 10^8 \text{ ms}^{-1}$

3 Let the incidence point is $P(x, y)$



$$m = \tan 45° = 1$$

(from law of reflection)

$$y = 2 \sin x$$

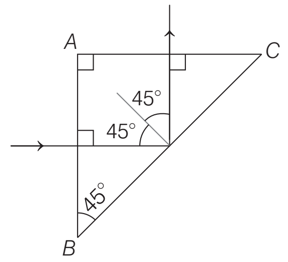
$$\therefore \quad m = \frac{dy}{dx} = 2\cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

The corresponding value of y is $2 \sin \frac{\pi}{3} = \sqrt{3}$

4 For total internal reflection from glass-air interface, critical angle C must be less than angle of incidence.



i.e. $\quad C < i$

or $\quad C < 45° \qquad (\because \angle i = 45°)$

But $\quad n = \frac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\frac{1}{n}\right)$

or $\quad \sin^{-1}\left(\frac{1}{n}\right) < 45°$

or $\quad \frac{1}{n} < \sin 45°$

or $\quad n > \frac{1}{\sin 45°}$

or $\quad n > \frac{1}{(1/\sqrt{2})}$

or $\quad n > \sqrt{2}$

5 $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating both sides

$$-\frac{1}{v^2}\frac{dv}{dt} = \frac{1}{u^2}\frac{du}{dt}$$

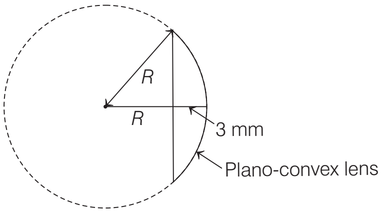
$$\frac{dv}{dt} = v_i = -\left(\frac{v}{u}\right)^2 \frac{du}{dt}$$

$$= -\left(\frac{v}{u}\right)^2 v_0$$

Again, $\quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{2}{R} - \frac{1}{u} = \frac{2u - R}{Ru}$

$$v = \frac{uR}{2u - R}$$

$$v_i = -\left(\frac{v}{u}\right)^2 v_0 = -v_0\left(\frac{R}{2u - R}\right)^2$$

6 

By Pythagoras theorem,

$$R^2 = 3^2 + (R - 3\,\text{mm})^2$$

$$\Rightarrow \quad R^2 = 3^2 + R^2 - 2R(3\,\text{mm}) + (3\,\text{mm})^2$$

$$\Rightarrow \quad R \simeq 15\,\text{cm}$$

Also, $\mu = \frac{c}{v} \Rightarrow \mu = \frac{3\times10^8}{2\times10^8} = \frac{3}{2}$

As, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{f} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{15} - 0\right]$$

$$\Rightarrow \quad f = 30\text{cm}$$

7 A plano-convex lens behaves as a concave mirror if its one surface (curved) is silvered. The rays refracted from plane surface are reflected from curved surface and again refract from plane surface. Therefore, in this lens two refractions and one reflection occur.

Let the focal length of silvered lens is F.

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m} = \frac{2}{f} + \frac{1}{f_m}$$

where, f = focal length of lens before silvering,

f_m = focal length of spherical mirror.

$$\frac{1}{F} = \frac{2}{f} + \frac{2}{R} \quad \text{...(i)}$$

$(\because R = 2f_m)$

Now,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

Here, $R_1 = \infty, R_2 = 30$ cm

$$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{30}\right)$$

or $\frac{1}{f} = \frac{0.5}{30} = -\frac{1}{60}$

or $f = -60$ cm

Hence, from Eq. (i), we get

$$\frac{1}{F} = \frac{2}{60} + \frac{2}{30} = \frac{6}{60}$$

$$F = 10\text{ cm}$$

Again given that,
size of object = size of image

i.e. $O = I$

$$\therefore \quad m = -\frac{v}{u} = \frac{I}{O}$$

$$\Rightarrow \quad \frac{v}{u} = -1$$

or $v = -u$

Thus, from lens formula,

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u}$$

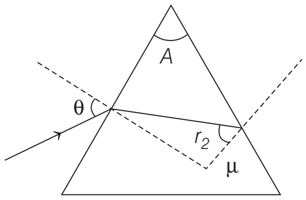
$$\frac{1}{10} = \frac{1}{-u} - \frac{1}{u}$$

$$\frac{1}{10} = -\frac{2}{u}$$

$$\therefore \quad u = -20\text{ cm}$$

Hence, to get a real image, object must be placed at a distance 20 cm on the left side of lens.

8 $1\sin\theta = \mu\sin r_1$



$$\mu\sin\theta_c = 1\sin 90^\circ$$

$$\sin\theta_c = \frac{1}{\mu}$$

$$r_2 < \theta_c$$

$$\sin r_2 < \sin\theta_c$$

$$r_1 + r_2 = A,\ r_1 = A - r_2,\ r_1 = A - \theta_c$$

$$\sin r_1 > \sin(A - \theta_c)$$

$$\Rightarrow \quad \mu\sin r_1 > \mu\sin(A - \theta_c)$$

$$\Rightarrow \quad \sin\theta > \mu\sin(A - \theta_c)$$

$$\theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right]$$

9 $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$= \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

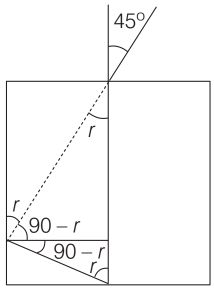
$$= \frac{-1}{10}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(iii)}$$

From Eq. (i), $f = \frac{2}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

From Eq. (ii), $f_1 = \frac{8}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

From Eq. (iii), $f_2 = \frac{-10}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

10 From figure, we get $r = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$



For critical angle, $\sin C = \frac{1}{\mu}$

Now, by Snell's law, we have

$$\frac{\mu}{1} = \frac{\sin i}{\sin r} = \frac{\sin 45^\circ}{\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{3}}},\quad \mu = \sqrt{\frac{3}{2}}$$

11 1 Vernier scale division $= \frac{29}{30}$ main scale division

$$1\text{ VSD} = \frac{29}{30}\times 0.5^\circ = \left(\frac{29}{60}\right)^\circ$$

Thus, least count = 1 MSD – 1VSD

$$= \left(\frac{1}{2}\right)^\circ - \left(\frac{29}{60}\right)^\circ = \left(\frac{1}{60}\right)^\circ$$

So, reading = main scale reading + vernier scale reading

$$= \text{MSR} + n\times\text{LC}$$

$$= 58.5^\circ + 9\times\left(\frac{1}{60}\right)^\circ = 58.65^\circ$$

12 $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{-1}{u^2}\frac{du}{dt} - \frac{1}{v^2}\frac{dv}{dt} = 0$

$$\frac{dv}{dt} = \frac{-v^2}{u^2}\left(\frac{du}{dt}\right)$$

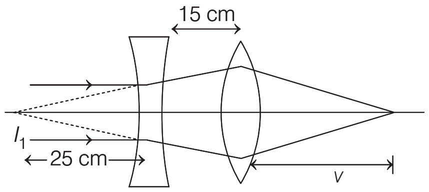
But $\frac{v}{u} = \frac{f}{u - f}$

$$\therefore \quad \frac{dv}{dt} = -\left(\frac{f}{u - f}\right)^2\left(\frac{du}{dt}\right)$$

$$= \left(\frac{0.2}{-2.8 - 0.2}\right)^2 \times 15 = \frac{1}{15}\text{ms}^{-1}$$

13 Focal length of diverging lens is 25 cm.

As the rays are coming parallel, so the image (I_1) will be formed at the focus of diverging lens i.e. at 25 cm towards left of diverging lens.



Now, the image (I_1) will work as object for converging lens.

For converging lens, distance of object u (i.e. distance of I_1)

$$= -(25+15) = -40 \text{ cm}$$

$$f = 20 \text{ cm}$$

$\therefore$ From len's formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{90}$$

$\Rightarrow$ $v = 40$ cm

v is positive so image will be real and will form at right side of converging lens at 40 cm.

14 If μ is refractive index of material of prism, then from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin(A+\delta_m)/2}{\sin A/2} \quad \ldots\text{(i)}$$

where, A is angle of prism and δ_m is minimum deviation through prism.

Given, $i = 35^\circ, \delta = 40^\circ, e = 79^\circ$.

So, angle of deviation by a glass prism,

$$\delta = i + e - A$$

$\Rightarrow$ $40^\circ = 35^\circ + 79^\circ - A$

i.e. Angle of prism $\Rightarrow A = 74^\circ$.

Such that, $r_1 + r_2 = A = 74^\circ$.

Let us put $\mu = 1.5$ in Eq. (i), we get

$$1.5 = \frac{\sin\left(\frac{A+\delta_{\min}}{2}\right)}{\sin A/2}$$

$$\Rightarrow \quad 1.5 = \frac{\sin\left(\frac{74^\circ+\delta_{\min}}{2}\right)}{\sin 37^\circ}$$

$$\Rightarrow \quad 0.9 = \sin\left(37^\circ + \frac{\delta_{\min}}{2}\right)$$

$(\because \sin 37^\circ \approx 0.6)$

$$\sin 64^\circ = \sin\left(37^\circ + \frac{\delta_{\min}}{2}\right)$$

$(\because \sin 64^\circ = 0.9)$

$$37^\circ + \frac{\delta_{\min}}{2} = 64^\circ \Rightarrow \delta_{\min} \approx 54^\circ$$

This angle is greater than the 40° deviation angle already given. For greater μ, deviation will be even higher. Hence, μ of the given prism should be lesser than 1.5. Hence, the closest option will be 1.5.

15 Shift in image position due to glass plate,

$$S = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{1}{1.5}\right) \times 1 \text{ cm}$$

$$= \frac{1}{3} \text{ cm}$$

For focal length of the lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{-240}$$

or $$\frac{1}{f} = \frac{20+1}{240} \Rightarrow f = \frac{240}{21} \text{ cm}$$

Now, to get back image on the film, lens has to form image at $\left(12 - \frac{1}{3}\right)\text{cm} = \frac{35}{3}$ cm such that the glass plate will shift the image on the film.

As, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{3}{35} - \frac{21}{240}$$

$$= \frac{48 \times 3 - 7 \times 21}{1680} = -\frac{1}{560}$$

$\Rightarrow$ $u = -5.6$ m

16 As refractive index for $z > 0$ and $z \le 0$ is different, xy-plane should be boundary between two media.

Angle of incidence,

$$\cos i = \left|\frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}\right| = \frac{1}{2}$$

$\therefore$ $i = 60^\circ$

From Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\sqrt{3}}{\sqrt{2}}$$

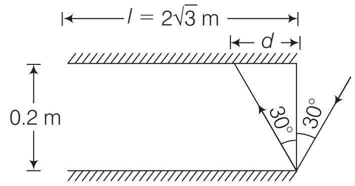
$$\sin r = \frac{\sqrt{2}}{\sqrt{3}} \times \sin 60^\circ$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{\sqrt{2}} = 45^\circ$$

$\Rightarrow$ $r = 45^\circ$

17 $d = 0.2 \tan 30^\circ = \frac{0.2}{\sqrt{3}}$



$$\frac{l}{d} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$$

Therefore, maximum number of reflections are 30.

18 From the figure CD = emergent ray, and CD is parallel to PQ and BC is a line intersecting these parallel lines.

So, $\angle DCB + \angle CBQ = 180^\circ$

$$\angle DCN + \angle NCB + \angle CBQ = 180^\circ$$

$$\alpha + \alpha + 45^\circ = 180^\circ$$

$$\alpha = 67.5^\circ$$

But $\angle NCS = 90^\circ$

So, second mirror is at angle of 22.5° with horizontal.