Unit 1

Relations and Functions

Teaching-Learning Points

● Let A and B are two non empty sets then a relation from set A to set B is defined as R = {(a.b) : a $\in$ A and b $\in$ B}. If (a.b) $\in$ R, we say that a is related to b under the relation R and we write as a R b.

● R $R \subseteq A \times B$.

● A relation R in a set A is a subset of A × A.

Types of relations :

(i) empty relation : $R = \phi\, C A \times A$

(ii) Universal relation R = A × A

(iii) Reflexive relation : $(ii, \quad) \in \quad \forall \in A.$

(iv) Symmetric relation : If $(a, b) \in R \Rightarrow (b, a) \in R \quad \forall\, a, b \in A$.

(v) Transitive relation : If $\ddot{\imath}\ddot{w}\ddot{\imath} \quad \in$ and

$\Rightarrow$ $(a, c) \in R \quad \forall\, a, b, c \in A$.

● A relation R is set A is said to be equivalence relation. If R is reflexive, symmetric and transitive.

● Let R is an equivalence relation is set A and R divides A into mutually disjoint subset A called partitions or subdivisions of A subsfying the conditions :

(i) all element of A_i are related to each other, $\forall$ i.

(ii) no element of A_i is related to any element of A_j, it j

(iii) UA_i = A and $A_i \cap$ Aj = φ, $\neq$ j.

● Type of Functions :

(i) one-one (or injective) function : Let F : A $\rightarrow$ B, then for every x_1, x_2 $\in$ A, $f(x_1) = f(x_2)$ $\Rightarrow$ $x_1 = x_2$.

(ii) onto (or surjective function) : Let F : A $\rightarrow$ B, then for every y $\in$ B, there exists an element x $\in$ A such that f(x) = y.

(iii) A function which is not one-one is called many-one function.

● A function which is not onto is called into function.

● A function which is both one-one and onto is called a bijective function.

- Let A be a finite set then an injective function $F : A \rightarrow A$ is subjective and conversely.
- Let $F : A \rightarrow B$ and $g : B \rightarrow C$ be two functions. Then the composition of F and g, denoted as gof is defined as the function $gof : A \rightarrow C$ given by $g\ of\ (x) = g\ [f(x)]$

$$\forall x \in A$$

- Composition of functions need not to be commutative and associative.
- If $F : A \rightarrow B$ and $g : B \rightarrow C$ be one-one (or on to) functions, then $gof : A \rightarrow c$ is also one-one (or on to) but converse is not true.
- A function $F : A \rightarrow B$ is said to be invertible if there exists another function $g : B \rightarrow A$ such that

$gof = I_A$ and $fog = I_B$. The function g is called the inverse of the function F.

- A function $F : A \rightarrow B$ is said to be invertible if and only if F is one-one and onto (i.e. bijective).
- If $F : A \rightarrow B$ and $g : B \rightarrow C$ are invertible functions, then $gof : A \rightarrow C$ is also invertible and $(gof)^{-1} = F^{-1}\ og^{-1}$.

Binary operations :

- A binary operation * on a set A is a function $* A \times A \rightarrow A$ we denoted * (a, b) by a * b.
- A binary operation * on a set A is called commutative if $a * b = b * a \ \forall a, b \in A$.
- A binary operation * on a set A is said to be associative if $a * (b * b) * c \ \forall a, b, c \in A$.
- The element $e \in A$, if it exists, is called identity element for binary operation * if $a * e = a = e * a$ $\forall a \in A$.
- The element $a \in A$ is said to be invertible with respect to the binary operation * if there exile $b \in A$ such that $a * b = e = b * a$. The element b is called morse of a and is denoted as a^{-1}.

Question for Practice

Evaluate the following Integrals

Very Short Answer Type Questions (1 Mark)

Q1. Let R be a relation on A defined as $R = \{(a, b) \in A \times A : a \text{ is a husband of } b\}$ can we say R is symmetric? Explain your answer.

Q2. Let A = {a, b, c} and R is a relation on A given by R = {(a, a), (a, b), (a, c), (b, a), (c, c)}. Is R symmetric? Give reasons.

Q3. Let R = {(a, b), (c, d), (e, f)}, write R^{-1}.

Q4. Let L be the set of are straight lines in a given plane and $R = \{(x, y) : x \perp y \ \forall x, y \in L\}$. Can we say that R is transitive? Give reasons.

Q5. The relation R in a set $A = \{x : x \in z \text{ and } 0 \leq x \leq 12\}$ is given by $R = \{(a, b) : |a - b| \text{ is a multiple if } 4\}$ is an equivalence relation. Find the equivalence class related to {3}.

Q6. Let R_1 be the relation on R defined as R = {(a, b) : a $\leq$ b^2}. Can we say that R is reflexive? Give reasons.

Q7. Let R {(a, b) : a, b $\in$ Z (Integers) and |a – b| $\leq$ 5}. Can we say that R is transitive? Give reason.

Q8. If A = {2, 3, 4, 5}, then write the relation R on A, where R = {(a, b) : a + b = 6}.

Q9. If A {1, 2}, and B = {a, b, c}, then what is the number of relations on A × B?

Q10. State reason for the relation R in the set {1, 2, 3} given by R {(1, 2), (2, 1)} not to be transitive.

Q11. If f is invertible function, find the inverse of f(x) = $\frac{3x-2}{5}$.

Q12. If f(x) = x + 7 and g(x) = x – 7, x $\in$ R, find fog (x).

Q13. Write the inverse of the function f(x) = 5x + 7, x $\in$ R.

Q14. Show that f : R $\rightarrow$ R defined as f(x) = x^2 + 1 is not one-one.

Q15. Show that the function f : N $\rightarrow$ N defined by f(x) = 3x is not an onto function.

Q16. Let * be a binary operation on Z defiand by a * b = 2a + b – 3, find 3 * 4.

Q17. Let * be a binary operation on N defined by a * b = a^2 + b and O be a binary operation on N defined by aob = 3a – b find (2 * 1) 02.

Q18. let * be a binary operation on R defined by a * b = a – b. Show * is not commutative on R.

Q19. Let * be a binary operation on N given by a * b = l.c.m (a, b), a, b $\in$ N find (2 * 3) * 6.

Q20. Can we say that division is a binary operation on R? Give reasons.

Q21. Show that * : R × R $\rightarrow$ R given by a * b = a + 2b is not associative.

Q22. Explain that addition operation on N does not have any identity.

Q23. What is inverse of the element 2 for addition operation on R?

Q24. Let * be the binary operation on N given by l.c.m (a : b) find the identify element for * on N.

Q25. Let * be the binary operation on N defined by a * b = HCF (a, b). Deos there exist identify element for * on N?

Short Answer Type Questions (4 Marks)

Q26. Show that f : N $\rightarrow$ N givne by

f(x) = { x + 1 if x is odd

x – 1 is x is even, is bijective

Q27. Let * be a binary operation on the set A = {0, 1, 2, 3, 4, 5} as

a * b = { a + b if a + b < 6

a + b – 6 if a + b $\geq$ 6,

Show that O is the identify element for this operation and each element a of the set is invertible with 6 – a being the inverse of a.

Q28. Let N be the set of all natural numbers and R be a relation on N × N, defined by (a, b) R (c, d) $\Rightarrow$ ad = bc $\forall$ (a, b), (c, d) $\in$ N × N. Show that R is an equivalence relation.

Q29. Let f : R $\rightarrow$ R be defined by f(x) = 3x + 2. Show that f is invertible. Find f : R $\rightarrow$ R.

Q30. Let * be a binary operation on N × N defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative as well as associative. Find the identity element for * on N × N if any.

Q31. Let T is a set of all triangles in a plane and R be a relation as R : T $\rightarrow$ T = {($\Delta 1$, $\Delta 2$) : $\Delta 1 \cong \Delta 2$ $\forall$ $\Delta 1$, $\Delta 2 \in$T}. Show that R is an equivalence relation.

Q32. Let * be the binary operation on Q (Rational numbers) defiend by a * b = |a – b|, show that

(i) * is commutative

(ii) * is not associative

(iii) * does not have identity element

Q33. Show that f : R $\rightarrow$ R defined by f(x) = x^3 – 1; is invertible. Find f(x).

Q34. Show that if f : B $\rightarrow$ A is defined by f(x) = $\dfrac{3x+4}{5x-7}$ and g : A $\rightarrow$ B is defined by g(x) = $\dfrac{7x+1}{5x-3}$, then Fog = I_A and gof = I_B, where A = R – $\left\{\dfrac{3}{5}\right\}$ and B = R – $\left\{\dfrac{7}{5}\right\}$.

Q35. Show that the function F : Q – {3} $\rightarrow$ Q, given by F(x) = $\dfrac{2x+3}{x-3}$ is not a bijective function.

Answers

Very Short Answer (1 Mark)

1. No, if is a husband of b, then b being a female can not be husband of anybody.

2. No, because (a, c) $\in$ R but $(c, a) \notin R$.

3. R^{-1} = {(b, a), (d, c), (f, e)}

4. No, If x $\perp$ y & y $\perp$ z $\Rightarrow$ x || z.

5. {3, 7, 11}.

6. No, example $\dfrac{1}{3} \not\leq \left(\dfrac{1}{3}\right)^2$.

7. No, Let a = 5, b = 10, c = 12, then (a, b) $\in$ R, (b, c) $\in$ R but $(a, c) \notin R.$

8. R = $\{(2,4),(3,3),(4,2)\}$ 9. 64

10. $(1,1) \notin R.$ 11. $f^{-1}(x) = \frac{5x+2}{3}$

12. x 13. $\frac{x-7}{5}$

16. 7 17. 13

19. 6

20. No, because Number divided by 0 does not belong to R.

21. Let a = 2, b = 5, c = 8, (a * b) * c = (2 + 2 × 5) * 8 = 12 * 8

= $12 + 2 \times 8 = 28$ and $a*(b*c) = 2*(5*) = 2*(5+2\times 8)$

= $2*21 = 2 + 2 \times 21 = 44$.

22. Because O + number = Number but O does not belong to N.

23. –2 24. 1

25. No

Very Short Answer (4 Mark)

29. f^{-1} = (x) = $\frac{x-2}{3}$ 30. Identity does not exist

33. $f^{-1}(x) = (x+1)^{1/3}$

35. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \Rightarrow$ is one-one.

Let y $\in$ codomain then f(x) = y

$\Rightarrow$ x = $\frac{-3-3y}{2-y} \not\in Q - \{3\}$ for some $y \in Q$

Example 2 $\in$ codomain but

= $\frac{-3-3\times 2}{2-2}$ = Not defined, does not belong to domain

Unit 2

Inverse Trigonometric Functions

Teaching-Learning Points

● The sine function is defined as

sin : R $\rightarrow$ [–1, 1]

Which is not a one-one function over the whole domain and hence its inverse does not exist but if we restrict the domain to $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ then the sine function becomes a one-one and onto function and therefore we com define the inverse of the function sin : $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ $\rightarrow$ [–1, 1] as $\sin^{-1}:[-1,1]\rightarrow\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ In fact there are other intervals also like $\left[\frac{-3\pi}{2},\frac{-\pi}{2}\right],\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ etc which may also be taken as range of the function $\sin^{-1}$. Corresponding to each interval we get branch of $\sin^{-1}$. The branch with range $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ is called principal value branch similiary for other inverse trigonometric functions we have principal value branches.

● List of principal value branches and the domain of inverse trigonometric functions.

Functions	Domain	Range (Principal value Branch)
$y = \sin^{-1}x$	$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \pi/2$
$y = \cos^{-1}x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1}x$	$-\infty < x < \infty$	$\frac{-\pi}{2} < y < \frac{\pi}{2}$
$y = \cot^{-1}x$	$-\infty < x < \infty$	$0 < y < \pi$
$y = \sec^{-1}x$	$\begin{cases} -\infty < x \le -1 \\ 1 \le x < \infty \end{cases}$	$\frac{\pi}{2} < y \le \pi$ $0 \le y < \frac{\pi}{2}$
$y = \text{cosec}^{-1}x$	$\begin{cases} -\infty < x \le -1 \\ 1 \le x < \infty \end{cases}$	$\frac{-\pi}{2} \le y < 0$ $0 < y \le \frac{\pi}{2}$

● Properties of inverse trigonometric functions :

1. (i) $\sin^{-1}(\sin x) = x$, $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\sin(\sin^{-1} x) = x$, $x \in [-1, 1]$

(iii) $\cos^{-1}(\cos x) = x$, $x \in [0, \pi]$

(iv) $\cos(\cos^{-1} x) = x,\quad x \in [-1, 1]$

(v) $\tan^{-1}(\tan x) = x,\quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(vi) $\tan(\tan^{-1} x) = x,\quad x \in R.$

2. (i) $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}x,\quad |x| \ge 1$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x,\quad |x| \ge 1$

(iii) $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x,$ x > 0

3. (i) $\sin^{-1}(-x) = -\sin^{-1} x,$ x $\in$ [–1, 1]

(ii) $\tan^{-1}(-x) = -\tan^{-1} x,$ x $\in$ R

(iii) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x,$ |x| $\geq$ 1

(iv) $\cos^{-1}(-x) = \pi - \cos^{-1} x,$ x $\in$ [–1, 1]

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x,$ |x| $\geq$ 1

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x,$ x $\in$ R

4. (i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1, 1]$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in \mathrm{R}.$

(iii) $\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}, |x| \geq 1$

5. (i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}, xy < 1$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}, xy > -1$

6. (i) $2\tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}, |x| < 1$

(ii) $2\tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}, |x| \leq 1$

(iii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}, x \geq 0$

Question for Practice

Evaluate the following Integrals

Very Short Answer Type Questions (1 Mark)

Q1. Write the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Q2. Write the principal value of $\text{cosec}^{-1}(-\sqrt{2})$.

Q3. Write the principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

Q4. Write the principal value of $\tan^{-1}(-\sqrt{3})$.

Q5. Write the principal value of $\sec^{-1}(-\sqrt{2})$.

Q6. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$.

Q7. Show that $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$.

Q8. Show that $\cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$.

Q9. Show that $\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

Q10. Show that $\sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$.

Q11. Show that $\cos^{-1} x = 2\sin^{-1}\sqrt{\frac{1-x}{2}}$.

Q12. Write $\sin^{-1}(3x-4x^3)$ in the simplest form.

Q13. Write $\cos^{-1}(4x^3-3x)$ in the simplest form.

Q14. Evaluate $\operatorname{cosec}^{-1}\left\{\operatorname{cosec}\left(\frac{-\pi}{4}\right)\right\}$.

Q15. Evaluate $\cos\left\{\frac{\pi}{3}-\cos^{-1}\left(\frac{1}{2}\right)\right\}$.

Q16. Show that $\cos^{-1} x = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$.

Q17. Write $\cos^{-1}$ $(2x^2-1)$ in the simplest form.

Q18. Write $\cos^{-1}(1-2x^2)$ in the simplest form.

Q19. Write $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$, $0 \le x < \pi$.

Q20. Show that $\sin^{-1} 2x\sqrt{1-x^2} = 2\sin^{-1} x$.

Q21. Evaluate : $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}$.

Q22. Evaluate : $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

Q23. Find x, if $\tan^{-1} x = \pi/4.$

Q24. Evaluate $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

Q25. Evaluate $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

Q26. Evaluate $\sin^{-1}(\sin 2\pi/3)$.

Q27. Evaluate $\operatorname{cosec}^{-1}\left\{\operatorname{cosec}\frac{3\pi}{4}\right\}$.

Q28. Evaluate $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$.

Q29. Write $\tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}, |x| < a$ in the simplest form.

Q30. Find x, if $\cot^{-1} x + \tan^{-1} 7 = \frac{\pi}{2}$.

Q31. Find x, if $\sin^{-1} x = \frac{\pi}{6} + \cos^{-1} x.$

Q32. Find x, if $4\sin^{-1} x = \pi - \cos^{-1} x.$

Q33. Find x, if $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}.$

Q34. Write $\sin^{-1}\left(\frac{2x}{1+x^2}\right) -1 \le x \le 1$, in the simplest form.

Q35. Write $\sin^{-2}\left(2x\sqrt{1-x^2}\right)$ in the simplest form.

Short Answer Questions Carrying 4 Marks each

Q36. Solve for x : $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}.$

Q37. Solve for x : $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

Q38. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, prove that a + b + c = abc.

Q39. Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x, x > 0$.

Q40. Solve for x : $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1} 7$.

Q41. Solve for x : $\tan^{-1}\left(\frac{2x}{1+x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{-\pi}{2}$.

Q42. Solve for x : $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\frac{23}{36}$.

Q43. Solve for x : $\sin^{-1}\frac{8}{17} = \sin^{-1} x - \sin^{-1}\frac{3}{5}$.

Q44. Solve for x : $\tan^{-1}(2x) + \tan^{-1}(3x) = n\pi + \frac{3\pi}{4}$.

Q45. Solve for x : $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) - \tan^{-1} 3x = 0$.

Q46. Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$.

Q47. Prove that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

Q48. Prove that $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \tan^{-1}\frac{4}{7}$.

Q49. Prove that $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}$.

Q50. Prove that $\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}=\cos^{-1}\left(\frac{253}{325}\right)$.

Q51. Prove that $\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{2}$.

Q52. Prove that $\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{27}{11}\right)$.

Q53. Prove that $\cos^{-1}\left(\frac{63}{65}\right)+2\tan^{-1}\left(\frac{1}{5}\right)=\sin^{-1}\left(\frac{3}{5}\right)$.

Q54. Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.

Q55. Prove that : $2\tan^{-1}\left(\frac{5}{12}\right)-\tan^{-1}\left(\frac{1}{70}\right)+\tan^{-1}\left(\frac{1}{99}\right)=\frac{\pi}{4}$.

Q56. Prove that : $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2}$.

Q57. Prove that $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$.

Q58. Prove that $\cos\left(2\tan^{-1}\frac{1}{7}\right)=\sin\left(4\tan^{-1}\frac{1}{3}\right)$.

Q59. Prove that : $2\tan^{-1}\frac{1}{5}+\operatorname{cosec}^{-1}5\sqrt{2}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

Q60. Prove that : $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$.

Answers

1. $\frac{\pi}{3}$ 2. $\frac{-\pi}{4}$ 3. $\frac{2\pi}{3}$ 4. $\frac{-\pi}{3}$

5. $\frac{3\pi}{4}$ 6. $\frac{\pi}{3}$ 12. $3\sin^{-1}x$ 13. $3\cos^{-1}x$

14. $\frac{-\pi}{4}$ 15. 1 17. $2\cos^{-1}x$ 18. $2\sin^{-1}x$

19. $\frac{x}{2}$ 21. 1 22. π 23. $\frac{\pi}{4}$

24. $\frac{-\pi}{4}$ 25. $\frac{5\pi}{6}$ 26. $\pi/3$ 27. $\pi/4$

28. $\pi/3$ 29. $\sin^{-1}\left(\frac{x}{a}\right)$ 30. 7 31. $\frac{\sqrt{3}}{2}$

32. $\frac{1}{2}$ 33. $\sqrt{3}$ 34. $2\tan^{-1}x$ 35. $2\sin^{-1}x$

36. $-8, \frac{1}{4}$ 37. $\frac{1}{6}$ 39. $\frac{1}{\sqrt{3}}$ 40. x = 2

41. x = 1 42. $\frac{-3}{8}, \frac{4}{3}$ 43. $\frac{77}{85}$ 44. $\frac{-1}{6}, 1$

45. $0, \frac{1}{2}, -\frac{1}{2}$