Motion in a Plane

Quick Revision

1. **Scalar Quantity** is the physical quantity which has only magnitude but no direction. It is specified completely by a single number, alongwith the proper unit.
   e.g. Temperature, mass, length, time, work, etc.
2. **Vector Quantity** is the physical quantity which has both magnitude and direction and obeys the triangle/ parallelogram laws of vector addition and subtraction.

   e.g. Displacement, acceleration, velocity, momentum, force, etc.
3. **Representation of Vector** A vector is represented by a **bold face** type or by **an arrow** placed over a letter,

   i.e. **F**, **a**, **b** or $\vec{F}, \vec{a}, \vec{b}$.

   The length of the line gives the magnitude and the arrowhead gives the direction.
4. **Types of Vectors** Vectors are classified into two types **polar** and **axial** vectors.
   - **Polar Vectors** Vectors which have a starting point or a point of application are called polar vectors. e.g. Force, displacement, etc.
   - **Axial Vectors** Vectors which represent the rotational effect and act along the axis of rotation are called axial vectors.
     e.g. Angular velocity, angular momentum, torque, etc.
5. **Modulus of a Vector** The magnitude of a vector is called modulus of vector. For a vector **A**, it is represented by $|\mathbf{A}|$ or A.
6. **Unit Vector** It is a vector having unit magnitude. A unit vector of **A** is written as $\hat{\mathbf{A}}$. It is expressed as

   $$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}$$

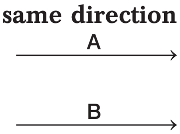
   or $$\mathbf{A} = A\hat{\mathbf{A}}$$

   In cartesian coordinates, $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the unit vectors along X-axis, Y-axis and Z-axis.

   It has no unit or dimensions.
7. **Equal Vectors** Two vectors are said to be equal, if they have equal magnitude and same direction.
8. **Resultant Vector** It is the combination of two or more vectors and it produces the same effect as two or more vectors collectively produce.

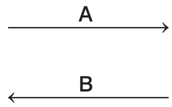
   Two cases for resultant vectors are as follows

   ***Case* I** **When two vectors are acting in the same direction**

   Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

***Case* II When two vectors are acting in mutually opposite directions**



Resultant vector, $\mathbf{R} = \mathbf{A} - \mathbf{B}$

(i) If $\mathbf{B} > \mathbf{A}$, then direction of $\mathbf{R}$ is along $\mathbf{B}$.

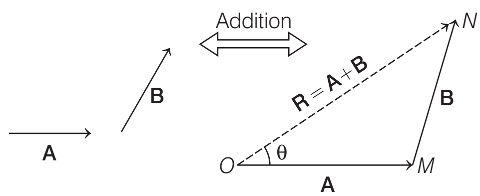
(ii) If $\mathbf{A} > \mathbf{B}$, then direction of $\mathbf{R}$ is along $\mathbf{A}$.

9. **Addition of Two Vectors (Graphical Method)**

Two vectors can be added, if both of them are of same nature. Graphical method of addition of vectors helps us in visualising the vectors and the resultant vector.

This method contains following laws

- **Triangle Law of Vector Addition** This law states that, if two vectors can be represented both in magnitude and direction by two sides of a triangle taken in the same order, then their resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.

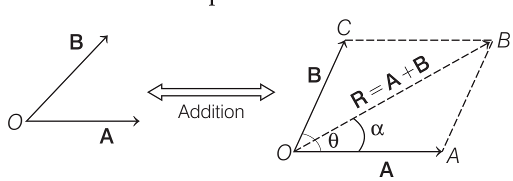


According to triangle law of vector addition,

$$\mathbf{ON} = \mathbf{OM} + \mathbf{MN}$$

Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

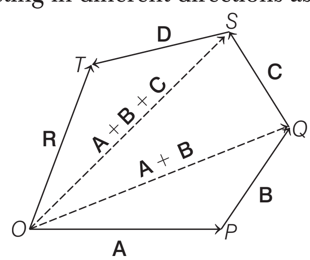
- **Parallelogram Law of Addition of Two Vectors** This law states that, if two vectors are acting on a particle at the same time be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.



The resultant vector formed in this method is also same as that formed in triangle law of addition. i.e. Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

- **Polygon Law of Addition of Vectors** This law states that, when the number of vectors are represented in both magnitude and direction by the sides of an open polygon taken in an order, then their resultant is represented in both magnitude and direction by the closing side of the polygon taken in opposite order.

Consider a number of vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ be acting in different directions as shown



According to this law,

$$\mathbf{OT} = \mathbf{OP} + \mathbf{PQ} + \mathbf{QS} + \mathbf{ST}$$

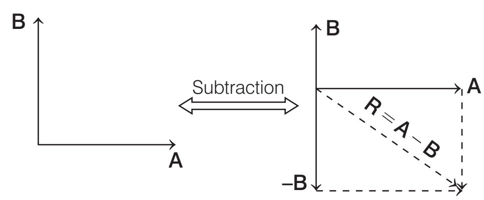
$\therefore$ Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$

10. **Properties of Addition of Vectors**

- It follows commutative law, i.e. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
- It follows associative law, $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$
- It follows distributive law, $\lambda(\mathbf{A} + \mathbf{B}) = \lambda\mathbf{A} + \lambda\mathbf{B}$
- $\mathbf{A} + \mathbf{0} = \mathbf{A}$

11. **Subtraction of Two Vectors (Graphical Method)** If a vector $\mathbf{B}$ is to be subtracted from vector $\mathbf{A}$, then we have to invert the vector $\mathbf{B}$ and then add it with vector $\mathbf{A}$, according to laws of addition of two vectors.

Hence, the subtraction of vector $\mathbf{B}$ from a vector $\mathbf{A}$ is expressed as $\mathbf{R} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$



12. **Properties of Subtraction of Vectors**

- Subtraction of vectors does not follow commutative law

$$\mathbf{A} - \mathbf{B} \neq \mathbf{B} - \mathbf{A}$$

- It does not follow associative law

$$\mathbf{A} - (\mathbf{B} - \mathbf{C}) \neq (\mathbf{A} - \mathbf{B}) - \mathbf{C}$$

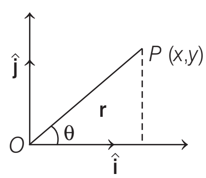
- It follows distributive law

$$\lambda(\mathbf{A} - \mathbf{B}) = \lambda\mathbf{A} - \lambda\mathbf{B}$$

13. **Resolution of Vectors in Plane (In Two-Dimensions)** The process of splitting a single vector into two or more vectors in different directions which collectively produce the same effect as produced by the single vector alone is known as resolution of a vector.

The various vectors into which the single vector is splitted are known as **component vectors**.

Any vector $\mathbf{r}$ can be expressed as a linear combination of two unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ at right angle, i.e. $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.



$\therefore$ Magnitude of resultant vector $= |\mathbf{r}| = \sqrt{x^2 + y^2}$

If θ is the inclination of $\mathbf{r}$ with X-axis, then angle, $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$.

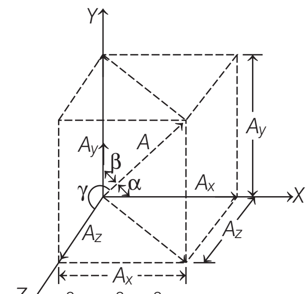
14. **Resolution of a Space Vector (In Three-Dimensions)** We can resolve a general vector $\mathbf{A}$ into three components along X, Y and Z-axes in three dimensions (i.e. space). While resolving we have,

$$A_x = A\cos\alpha,$$
$$A_y = A\cos\beta,\ A_z = A\cos\gamma$$

$\therefore$ Resultant vector,

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

Magnitude of vector $\mathbf{A}$ is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$



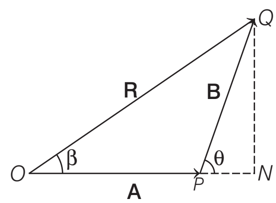
Remember that, $\cos\alpha = \dfrac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = l$

$$\cos\beta = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$$

$$\cos\gamma = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$$

Here, l, m and n are known as **direction cosines** of $\mathbf{A}$.

15. **Addition of Vectors (Analytical Method)** According to triangle law of vector addition, the resultant $(\mathbf{R})$ is given by OQ but in opposite order.



Resultant, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

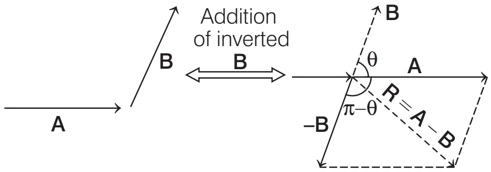
and direction of resultant $\mathbf{R}$,

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Regarding the Magnitude of R

- When $\theta = 0°$, then $R = A + B$ (maximum)
- When $\theta = 90°$, then $R = \sqrt{A^2 + B^2}$
- When $\theta = 180°$, then $R = A - B$ (minimum)

16. **Subtraction of Vectors (Analytical Method)** There are two vectors $\mathbf{A}$ and $\mathbf{B}$ at an angle θ. If we have to subtract $\mathbf{B}$ from $\mathbf{A}$, then first invert the vector $\mathbf{B}$ and then add with $\mathbf{A}$ as shown in figure.



The resultant vector is $\mathbf{R} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$

The magnitude of resultant in this case is

$$R = |\mathbf{R}| = \sqrt{A^2 + B^2 + 2AB\cos(\pi - \theta)}$$

or $$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Regarding the magnitude of R

- When $\theta = 0°$, then $R = A - B$ (minimum)
- When $\theta = 90°$, then $R = \sqrt{A^2 + B^2}$
- When $\theta = 180°$, then $R = A + B$ (maximum)

17. **Dot Product or Scalar Product** It is defined as the product of the magnitudes of vectors **A** and **B** and the cosine angle between them. It is represented by

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

***Case* I** When the two vectors are parallel, then $\theta = 0°$. We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 0° = AB$$

***Case* II** When the two vectors are mutually perpendicular, then, $\theta = 90°$. We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 90° = 0$$

***Case* III** When the two vectors are anti-parallel, then $\theta = 180°$. We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 180° = -AB$$

18. **Properties of Dot Product**

- $\mathbf{a} \cdot \mathbf{a} = (\mathbf{a})^2$
- $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
- $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
- $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$
  $= a_1b_1 + a_2b_2 + a_3b_3$

  where, $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$,

  and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$

  Here, $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$

  $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$

19. **Vector Product or Cross Product**

It is defined as the product of the magnitudes of vectors **A** and **B** and the sine angle between them.

It is represented as, $\mathbf{A} \times \mathbf{B} = AB\sin\theta\ \hat{\mathbf{n}}$

where, $\hat{\mathbf{n}}$ is a unit vector in the direction of $\mathbf{A} \times \mathbf{B}$.

Cross Product of Two Vectors in Terms of Their Components

If $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$,

then $$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2)\hat{\mathbf{i}} - (a_1b_3 - a_3b_1)\hat{\mathbf{j}} + (a_1b_2 - a_2b_1)\hat{\mathbf{k}}$$

where, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$

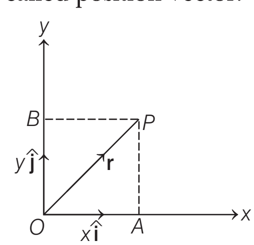
and $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}},\ \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}},\ \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$,

$\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}},\ \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}},\ \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$

20. **Properties of Cross Product**

- $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
- $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
- $(\mathbf{a} \times \mathbf{b}) + (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \times \mathbf{c}) + (\mathbf{a} \times \mathbf{d}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{b} \times \mathbf{d})$
- $m\mathbf{a} \times \mathbf{b} = \mathbf{a} \times m\mathbf{b}$
- $(\mathbf{b} + \mathbf{c})\ \mathbf{a} = \mathbf{b} \times \mathbf{a} + \mathbf{c} \times \mathbf{a}$
- $\mathbf{a} \times \mathbf{a} = 0$
- $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}$
- $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - |\mathbf{a} \cdot \mathbf{b}|^2$
- $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\ \mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\ \mathbf{c}$

21. **Position Vector** A vector that extends from a reference point to the point at which particle is located is called position vector.

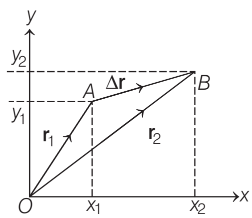


For a particle at point P, its position vector,

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

In three-dimensions, the position vector is represented as $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

22. **Displacement Vector** This vector represent the straight line joining the initial and final positions of a particle.
It does not depends on the actual path undertaken by the particle between the two positions.



Displacement vector, **AB**

$$\Delta \mathbf{r} = (x_2 - x_1)\,\hat{\mathbf{i}} + (y_2 - y_1)\,\hat{\mathbf{j}}$$

Similarly, in three-dimensions, the displacement vector can be represented as

$$\Delta \mathbf{r} = (x_2 - x_1)\,\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\,\hat{\mathbf{k}}$$

23. **Velocity** Rate of change of displacement of a body w.r.t. time is called velocity. It is of two types as given below

- **Average Velocity** It is defined as the ratio of the net displacement and the corresponding time interval.

  Thus, average velocity $= \dfrac{\text{net displacement}}{\text{time taken}}$

  Average velocity,

  $$\mathbf{v}_{\text{av}} = \frac{\Delta \mathbf{r}}{\Delta t} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta y}{\Delta t}\hat{\mathbf{j}}$$

  Velocity can be expressed in the component form as

  $$\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

  where, v_x and v_y are the components of velocity along x-direction and y-direction, respectively.

  The magnitude of $\mathbf{v}$ is given by

  $$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

  and the direction of $\mathbf{v}$ is given by angle θ

  $$= \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

- **Instantaneous Velocity** The velocity at an instant of time (t) is known as instantaneous velocity.

  The average velocity will become instantaneous, if Δt approaches to zero. The instantaneous velocity is expressed as

  $$\mathbf{v}_i = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$$

24. **Acceleration** The rate of change of velocity of a body w.r.t. time is called acceleration. It is of two types as given below

- **Average Acceleration** It is defined as the ratio of change in velocity $(\Delta \mathbf{v})$ and the corresponding time interval (Δt). It can be expressed as

  $$\mathbf{a}_{\text{av}} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1}$$

- **Instantaneous Acceleration** It is defined as the limiting value of the average acceleration as the time interval approaches to zero.

  It can be expressed as, $\mathbf{a}_i = \lim\limits_{\Delta t \to 0} \dfrac{\Delta \mathbf{v}}{\Delta t} = \dfrac{d\,\mathbf{v}}{dt}$

  Instantaneous acceleration, $\mathbf{a}_i = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$

  In terms of x and y, a_x and a_y can be expressed as

  $$a_x = \frac{dv_x}{dt}$$

  and $$a_y = \frac{dv_y}{dt}$$

  The magnitude of instantaneous acceleration is given by

  $$a_i = \sqrt{a_x^2 + a_y^2}$$

  Direction of acceleration, $\theta = \tan^{-1}\left(\dfrac{a_y}{a_x}\right)$

25. **Motion in a Plane with Uniform Velocity** Consider an object moving with uniform velocity $\mathbf{v}$ in xy-plane. Let $\mathbf{r}(0)$ and $\mathbf{r}(t)$ be its position vectors at $t = 0$ and $t = t$, respectively.

Then , $$\mathbf{v} = \frac{\mathbf{r}(t) - \mathbf{r}(0)}{t - 0}$$

$\Rightarrow \qquad \mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}t$

26. **Motion in a Plane with Constant Acceleration** For a body moving with uniform acceleration, we have

$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{v}_0}{t - 0} \Rightarrow \mathbf{v} = \mathbf{v}_0 + \mathbf{a}\,t$$

In terms of rectangular components, we can express it as

$$v_x = v_{0x} + a_x t$$

and $\quad v_y = v_{0y} + a_y t$

Also, $\mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}_0 + \frac{1}{2}\mathbf{a}t$

27. **Relative Velocity in Two-Dimensions** The relative velocity of an object A w.r.t. object B, when both are in motion, is the rate of change of position of object A w.r.t. object B.

Suppose two objects A and B are moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ w.r.t. ground or the earth. Then, relative velocity of object A w.r.t. object B,

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

Relative velocity of object B w.r.t. object A,

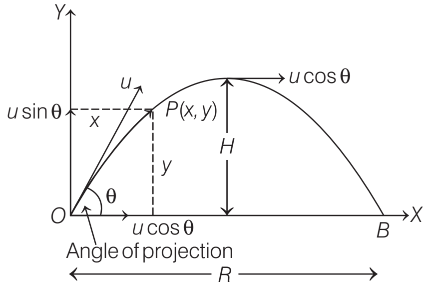
$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

Clearly, $\quad \mathbf{v}_{AB} = -\mathbf{v}_{BA}$

and $\quad |\mathbf{v}_{AB}| = |\mathbf{v}_{BA}|$

28. **Projectile Motion** It is a form of motion in which an object or a particle is thrown with some initial velocity near the earth's surface and it moves along a curved path under the action of gravity alone. The path followed by a projectile is called its **trajectory**. e.g.,
- A tennis ball or a baseball in a flight.
- A bullet fired from a rifle.

29. **Equation of Path of a Projectile** Suppose at any time t_1, the object reaches at point $P(x, y)$.



- Position of the object at time t along horizontal direction is given by

$$x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

- Position of the object at any time t along the vertical direction i.e. OY is

$$y = x \tan\theta - \left(\frac{1}{2}\frac{g}{u^2\cos^2\theta}\right)x^2$$

This equation represents a parabola and is known as **equation** of **trajectory of a projectile.**

30. **Time of Flight** It is defined as the total time for which projectile is in flight, i.e. time during the motion of projectile from O to B. It is denoted by T.

Time of flight, $T = \dfrac{2u\sin\theta}{g}$

Time of flight consist of two parts such as
- Time taken by an object to go from point O to H. It is also known as **time of ascent** (t).
- Time taken by an object to go from point H to B. It is also known as **time of descent** (t).

31. **Maximum Height of a Projectile** It is defined as the maximum vertical height attained by an object above the point of projection during its flight. It is denoted by H.

Maximum height,

$$H = \frac{u^2\sin^2\theta}{2g}$$

32. **Horizontal Range of a Projectile** The horizontal range of the projectile is defined as the horizontal distance covered by the projectile during its time of flight. It is denoted by R and is given as

$$R = u\cos\theta \times T$$

or $\quad R = \dfrac{u^2\sin 2\theta}{g}$

The horizontal range will be maximum, if $\theta = 45°$.

$\therefore$ Maximum horizontal range,

$$R_m = \frac{u^2}{g}$$

33. **Uniform Circular Motion** When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion.

e.g.,

- Motion of the tip of the second hand of a clock.
- Motion of a point on the rim of a wheel rotating uniformly.

34. **Terms Related to Circular Motion**

- **Angular Displacement** It is defined as the angle traced out by the radius vector at the centre of the circular path in the given time. It is denoted by $\Delta\theta$ and expressed in radians. It is a dimensionless quantity.
- **Angular Velocity** It is defined as the time rate of change of its angular displacement. It is denoted by ω and is measured in radians per second. Its dimensional formula is $[M^0 L^0 T^{-1}]$. It is a vector quantity.

  It is expressed as $\omega = \dfrac{\Delta\theta}{\Delta t}$.
- **Angular Acceleration** It is defined as the time rate of change of angular velocity of a particle. It is measured in radian per second square and has dimensions $[M^0 L^0 T^{-2}]$.
- **Time Period** It is defined as the time taken by a particle to complete one revolution along its circular path. It is denoted by T and is measured in second.
- **Frequency** It is defined as the number of revolutions completed per unit time. It is denoted by f and is measured in Hz.
- **Relation between Time Period and Frequency**

  Time period, $T = \dfrac{1}{f}$
- **Relation between Angular Velocity, Frequency and Time Period**

  Angular velocity, $\omega = \dfrac{\theta}{t} = \dfrac{2\pi}{T} = 2\pi f$
- **Relation between Linear Velocity** (v) **and Angular velocity** (ω)

  Linear velocity, $v = r\dfrac{\Delta\theta}{\Delta t} = r\omega$

35. **Centripetal Acceleration** The acceleration associated with a uniform circular motion and whose direction is towards the centre of the path is called centripetal acceleration.

Centripetal acceleration, $a = \dfrac{v^2}{r}$

Objective Questions

Multiple Choice Questions

1. In order to describe motion in two or three dimensions, we use

(a) positive sign (b) vectors
(c) negative sign (d) Both (b) and (c)

2. If length and breadth of a rectangle are 1 m and 0.5 m respectively, then its perimeter will be a

(a) free vector (b) scalar quantity
(c) localised vector (d) Neither (a) nor (b)

3. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are *(NCERT Exemplar)*

(a) impulse, pressure and area
(b) impulse and area
(c) area and gravitational potential
(d) impulse and pressure

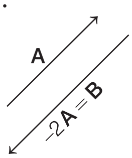
4. Suppose an object is at point P at time t moves to P' and then comes back to P. Then, displacement is a

(a) unit vector (b) null vector
(c) scalar (d) None of these

5. The relation between the vectors $\mathbf{A}$ and $-\lambda\mathbf{A}$ is that,

(a) both have same magnitude
(b) both have same direction
(c) they have opposite directions
(d) None of the above

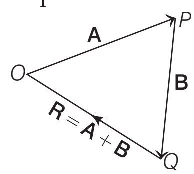
6. Choose the correct option regarding the given figure.

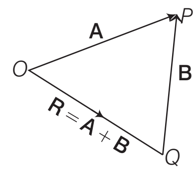


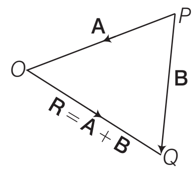
(a) $\mathrm{B} = \mathrm{A}$ (b) $\mathrm{B} = -\mathrm{A}$
(c) $|\mathrm{B}| = |\mathrm{A}|$ (d) $|\mathrm{B}| \neq |\mathrm{A}|$

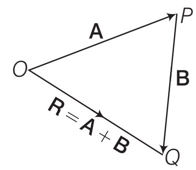
7. A and B are two inclined vectors. R is their sum.

Choose the correct figure for the given description.

(a)


(b)


(c)


(d)


8. Find the correct option about vector subtraction.

(a) $\mathrm{A} - \mathrm{B} = \mathrm{A} + \mathrm{B}$ (b) $\mathrm{A} + \mathrm{B} = \mathrm{B} - \mathrm{A}$
(c) $\mathrm{A} - \mathrm{B} = \mathrm{A} + (-\mathrm{B})$ (d) None of these

9. $\mathbf{A}$ is a vector with magnitude A, then the unit vector $\hat{\mathbf{a}}$ in the direction of vector $\mathbf{A}$ is

(a) $A\mathrm{A}$ (b) $\mathrm{A} \cdot \mathrm{A}$ (c) $\mathrm{A} \times \mathrm{A}$ (d) $\dfrac{\mathrm{A}}{A}$

10. Unit vector in the direction of the resultant of vectors $\mathbf{A} = -3\hat{i} - 2\hat{j} - 3\hat{k}$ and $\mathbf{B} = 2\hat{i} + 4\hat{j} + 6\hat{k}$ is

(a) $\dfrac{-3\hat{i} + 2\hat{j} - 3\hat{k}}{\sqrt{14}}$ (b) $-\hat{i} + 2\hat{j} + 3\hat{k}$

(c) $\dfrac{-\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$ (d) $-2\hat{i} - 4\hat{j} + 8\hat{k}$

11. If $\mathbf{A} = \mathbf{B} + \mathbf{C}$ have scalar magnitudes of 5, 4, 3 units respectively, then the angle between $\mathbf{A}$ and $\mathbf{C}$ is

(a) $\cos^{-1}(3/5)$ (b) $\cos^{-1}(4/5)$
(c) $\pi/2$ (d) $\sin^{-1}(4/5)$

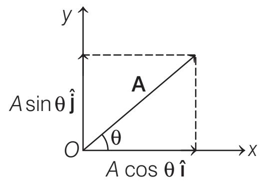
12. For two vectors **A** and **B**, $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$ is always true, when

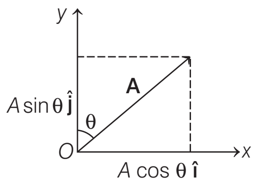
(a) $|\mathbf{A}| = |\mathbf{B}| \neq 0$
(b) $|\mathbf{A}| = |\mathbf{B}| \neq 0$ and A and B are parallel or anti-parallel
(c) either $|\mathbf{A}|$ or $|\mathbf{B}|$ is zero
(d) None of the above

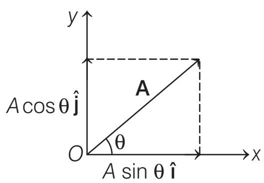
13. Two equal vectors have a resultant equal to either of the two. The angle between them is

(a) 90° (b) 60°
(c) 120° (d) 0°

14. Consider a vector **A** that lies in *xy*-plane. If A_x and A_y are the magnitudes of its *x* and *y* -components respectively, then the correct representation of **A** can be given by

(a) 

(b) 

(c) 

(d) None of the above

15. The component of a vector **r** along *X*-axis will have maximum value if *(NCERT Exemplar)*

(a) **r** is along positive *Y*-axis
(b) **r** is along positive *X*-axis
(c) **r** makes an angle of 45° with the *X*-axis
(d) **r** is along negative *Y*-axis

16. Magnitude of a vector **Q** is 5 and magnitude of its *y*-component is 4. So, the magnitude of the *x*-component of this vector is

(a) 8 (b) 3
(c) 6 (d) 9

17. Three vectors are given as $\mathbf{P} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}, \mathbf{Q} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$ and $\mathbf{R} = (3/4)\hat{\mathbf{i}} - \hat{\mathbf{j}}$, then which of the following is correct?

(a) *P*, *Q* and *R* are equal vectors
(b) *P* and *Q* are parallel but *R* is not parallel
(c) *P*, *Q* and *R* are parallel
(d) None of the above

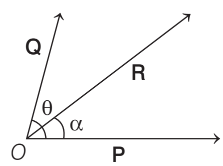
18. A vector is inclined at an angle 60° to the horizontal. If its rectangular component in the horizontal direction is 50 N, then its magnitude in the vertical direction is

(a) 25 N (b) 75 N
(c) 87 N (d) 100 N

19. The angle θ between the vector $\mathbf{p} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and unit vector along *X*-axis is

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$
(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{1}{2}\right)$

20. Two vectors **P** and **Q** are inclined at an angle θ and **R** is their resultant as shown in the figure.



Keeping the magnitude and the angle of the vectors same, if the direction of **P** and **Q** is interchanged, then their is a change in which of the following with regard to $\mathbf{R}$?

(a) Magnitude
(b) Direction
(c) Both magnitude and direction
(d) None of the above

21. It is found that $|\mathbf{A}+\mathbf{B}|=|\mathbf{A}|$. This necessarily implies

(a) $|\mathrm{B}|=0$
(b) A, B are parallel
(c) A, B are perpendicular
(d) $\mathrm{A}; \mathrm{B} \leq 0$

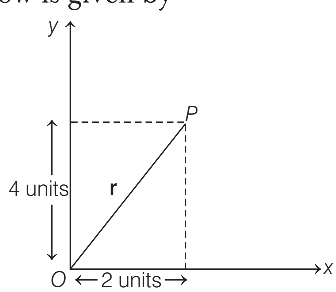
22. The sides of a parallelogram are represented by vectors $\mathbf{p}=5\hat{\mathbf{i}}-4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\mathbf{q}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$. Then, the area of the parallelogram is

(a) $\sqrt{684}$ sq. units (b) $\sqrt{72}$ sq. units
(c) 171 sq. units (d) 72 sq. units

23. If $\mathbf{a} \cdot \mathbf{b}=|\mathbf{a} \times \mathbf{b}|$, then the angle θ between $\mathbf{a}$ and $\mathbf{b}$ will be

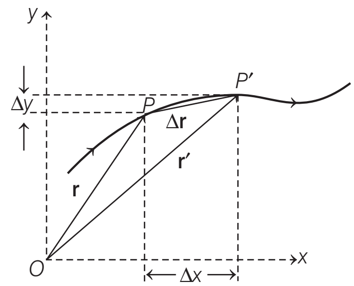
(a) 60° (b) 45° (c) 75° (d) 90°

24. Position vector $\mathbf{r}$ of a particle P located in a plane with reference to the origin of an xy-plane as shown in the figure below is given by



(a) $2\hat{\mathrm{i}}+4\hat{\mathrm{j}}$ (b) $4\hat{\mathrm{i}}+2\hat{\mathrm{j}}$
(c) $6\hat{\mathrm{k}}$ (d) $\hat{\mathrm{i}}+\hat{\mathrm{j}}+4\hat{\mathrm{k}}$

25. Suppose a particle moves along a curve shown by the thick line and the positions of particle are represented by P at t and P' at t'.



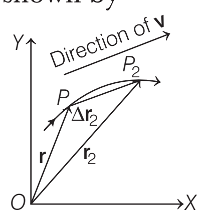
where, coordinates of P is (4, 3) and P' is (7, 6). Net displacement of the particle will be

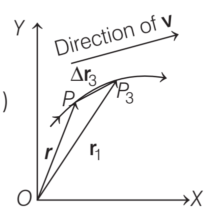
(a) zero (b) $7\hat{\mathrm{i}}+9\hat{\mathrm{j}}$
(c) $10\hat{\mathrm{i}}+18\hat{\mathrm{j}}$ (d) $3\hat{\mathrm{i}}+3\hat{\mathrm{j}}$

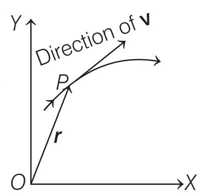
26. A particle moves in xy-plane from positions (2 m, 4 m) to (6 m, 8 m) is 2 s. Magnitude and direction of average velocity is

(a) $\sqrt{2}\ \mathrm{ms}^{-1}$ and 45° (b) $2\sqrt{2}\ \mathrm{ms}^{-1}$ and 45°
(c) $4\sqrt{2}\ \mathrm{ms}^{-1}$ and 30° (d) $3\sqrt{2}\ \mathrm{ms}^{-1}$ and 60°

27. The direction of instantaneous velocity is shown by

(a) 

(b) 

(c) 

(d) None of these

28. The position of a particle is given by $\mathbf{r}=3t\,\hat{\mathbf{i}}+2t^2\,\hat{\mathbf{j}}+5\hat{\mathbf{k}}$, then the direction of $\mathbf{v}(t)$ at $t=1$ s in

(a) 45° with X-axis
(b) 63° with Y-axis
(c) 30° with Y-axis
(d) 53° with X-axis

29. In three dimensional system, the position coordinates of a particle (in motion) are given below

$$x = a\cos\omega t$$
$$y = a\sin\omega t$$
$$z = a\omega t$$

The velocity of particle will be

(a) $\sqrt{2}\,a\omega$ (b) $2\,a\omega$
(c) $a\omega$ (d) $\sqrt{3}\,a\omega$

30. The coordinates of a moving particle at any time t are given by, $x = 2t^3$ and $y = 3t^3$. Acceleration of the particle is given by

(a) $468\,t$ (b) $t\sqrt{468}$ (c) $234\,t^2$ (d) $t\sqrt{234}$

31. A particle starts from origin at $t = 0$ with a velocity $5.0\,\hat{\mathbf{i}}\ \text{ms}^{-1}$ and moves in xy-plane under action of force which produces a constant acceleration of $(3.0\,\hat{\mathbf{i}} + 2.0\,\hat{\mathbf{j}})\ \text{ms}^{-2}$. What is the y-coordinate of the particle at the instant, if x-coordinate is 84 m?

(a) 36 m (b) 24 m
(c) 39 m (d) 18 m

32. A car driver is moving towards a fired rocket with a velocity of $8\hat{\mathbf{i}}\ \text{ms}^{-1}$. He observed the rocket to be moving with a speed of $10\ \text{ms}^{-1}$. A stationary observer will see the rocket to be moving with a speed of

(a) $5\ \text{ms}^{-1}$ (b) $6\ \text{ms}^{-1}$
(c) $7\ \text{ms}^{-1}$ (d) $8\ \text{ms}^{-1}$

33. A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at $10\ \text{kmh}^{-1}$.

He finds that raindrops are hitting his head vertically. The actual speed of raindrops is

(a) $20\ \text{kmh}^{-1}$ (b) $10\sqrt{3}\ \text{kmh}^{-1}$
(c) $20\sqrt{3}\ \text{kmh}^{-1}$ (d) $10\ \text{kmh}^{-1}$

34. A girl can swim with speed $5\ \text{kmh}^{-1}$ in still water. She crosses a river 2 km wide, where the river flows steadily at $2\ \text{kmh}^{-1}$ and she makes strokes normal to the river current. Find how far down the river she go when she reaches the other bank.

(a) 1 km (b) 2 km
(c) 800 m (d) 750 m

35. When a ball is thrown obliquely from the ground level, then the x-component of the velocity

(a) decreases with time
(b) increases with time
(c) remains constant
(d) zero

36. The motion of an object that is in flight after being projected is a result of two simultaneously occurring components of motion, which are the components in

(a) horizontal direction with constant acceleration
(b) vertical direction with constant acceleration
(c) horizontal direction without acceleration
(d) Both (b) and (c)

37. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are

(a) parallel to each other
(b) antiparallel to each other
(c) inclined to each other at an angle of 45°
(d) perpendicular to each other

38. A projectile is given an initial velocity of $(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{ms}^{-1}$, where $\hat{\mathbf{i}}$ is along the ground and $\hat{\mathbf{j}}$ is along vertical. If g is $10\ \text{ms}^{-2}$, then the equation of its trajectory is

(a) $y = x - 5x^2$
(b) $y = 2x - 5x^2$
(c) $4y = 2x - 5x^2$
(d) $4y = 2x - 25x^2$

39. The equations of motion of a projectile are given by $x = 18t$ and $2y = 54t - 9.8t^2$. The angle θ of projection is

(a) $\tan^{-1}(3)$ (b) $\tan^{-1}(1.5)$

(c) $\sin^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{3}\right)$

40. A football player throws a ball with a velocity of 50 ms^{-1} at an angle $30°$ from the horizontal. The ball remains in the air for (Take, $g = 10 \text{ ms}^{-2}$)

(a) 2.5 s (b) 1.25 s

(c) 5 s (d) 0.625 s

41. The ceiling of a hall is 30 m high. A ball is thrown with 60 ms^{-1} at an angle θ, so that it could reach the ceiling of the hall and come back to the ground. The angle of projection θ that the ball was projected is given by

(a) $\sin\theta = \frac{1}{\sqrt{8}}$ (b) $\sin\theta = \frac{1}{\sqrt{6}}$

(c) $\sin\theta = \frac{1}{\sqrt{3}}$ (d) None of these

42. Two projectiles A and B are projected with same speed at angles $30°$ and $60°$ to be horizontal then, which amongst the following relation between their range, maximum height and time of flight is wrong?

(a) $R_A = R_B$ (b) $H_B = 3H_A$

(c) $T_B = \sqrt{3}T_A$ (d) None of these

43. A man can throw a stone to a maximum distance of 80 m. The maximum height to which it will rise, is

(a) 30 m (b) 20 m

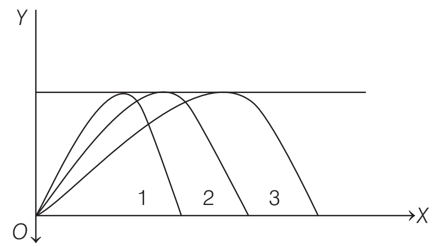
(c) 10 m (d) 40 m

44. Two stones were projected simultaneously in the same vertical plane from same point obliquely, with different speeds and angles with the horizontal. The trajectory of path followed by one, as seen by the other, is

(a) parabola (b) straight line

(c) circle (d) hyperbola

45. Given below figure show three paths of a rock with different initial velocities. The correct increasing order for the respective initial horizontal velocity component (ignoring the effect of air resistance) is



(a) $1 < 2 < 3$ (b) $3 < 2 < 1$

(c) $2 < 1 < 3$ (d) $3 < 1 < 2$

46. What is the centripetal acceleration of a point mass which is moving on a circular path of radius 5m with speed 25 ms^{-1}?

(a) 125 ms^{-2} (b) 90 ms^{-2}

(c) 60 ms^{-2} (d) None of these

47. The displacement of a particle moving on a circular path, when it makes $60°$ at the centre is

(a) $2r$ (b) r

(c) $\sqrt{2}r$ (d) None of these

48. If a car is executing a uniform circular motion, then its centripetal acceleration represents

(a) a scalar quantity (b) constant vector

(c) not a constant vector (d) None of these

49. A car revolves uniformly in a circle of diameter 0.80m and completes 100 rev min^{-1}. Its angular velocity is

(a) 10.467 rads^{-1} (b) 0.6 rads^{-1}

(c) 46.26 rads^{-1} (d) 8 rads^{-1}

50. If 2 balls are projected at angles $45°$ and $60°$ and the maximum heights reached are same, then the ratio of their initial velocities is ……… .

(a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : \sqrt{2}$

(c) 3:2 (d) 2:3

51. Two projectiles having different masses m_1 and m_2 are projected at an angle α and $(90° - \alpha)$ with the same speed from some point. The ratio of their maximum heights is ....

(a) $\cot\alpha : \sin\alpha$ (b) $1 : 1$
(c) $\tan^2\alpha : 1$ (d) $1 : \tan\alpha$

52. A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be ....

(a) $2R$ (b) $R/2$
(c) R (d) $4R$

53. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal accelerations is ....

(a) $m_1r_1 : m_2r_2$ (b) $m_1 : m_2$
(c) $r_1 : r_2$ (d) $1:1$

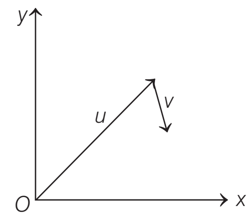
54. Which one of the following statement is correct? *(NCERT Exemplar)*

(a) A scalar quantity is the one that is conserved in a process.
(b) A scalar quantity is the one that can never take negative values.
(c) A scalar quantity is the one that does not vary from one point to another in space.
(d) A scalar quantity has the same value for observers with different orientation of the axes.

55. For two vectors **A** and **B** which lie in a plane. Which of the following statement is correct?

(a) If magnitude of A and B vector is 3 and 4 and they add upto give vector having magnitude of 5, then they must be perpendicular vector.
(b) If they add up to give more than 5, then they must be inclined at obtuse angle.
(c) If they add upto give less than 5, then they must be inclined at acute angle.
(d) None of the above

56. Figure shows the orientation of two vectors **u** and **v** in the *xy*-plane.
If $\mathbf{u} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $\mathbf{v} = p\hat{\mathbf{i}} + q\hat{\mathbf{j}}$



Which of the following statement is correct? *(NCERT Exemplar)*

(a) a and p are positive, while b and q are negative.
(b) a, p and b are positive, while q is negative.
(c) a, q and b are positive, while p is negative.
(d) a, b, p and q are all positive.

57. Match the Column I (example of motion) with Column II (type of motion) and select the correct answer from the codes given below.

	Column I		Column II
A.	Free fall	p.	One-dimensional motion
B.	Projectile motion	q.	Two-dimensional motion
C.	Circular motion	r.	Three-dimensional motion
D.	Motion along a straight road		

Codes

	A	B	C	D
(a)	q	p	r	p
(b)	p	q	r	q
(c)	p	q	q	p
(d)	p	r	q	p

58. Match the Column I (magnitude of vectors **A** and **B**) with Column II (relationship between **A** and **B**) and select the correct answer from the codes given below.

Column I		Column II
A.	$\xrightarrow{\quad \|\mathbf{A}\| = l \quad}$ $\xrightarrow{\quad \|\mathbf{B}\| = 2l \quad}$	p. $\mathbf{A} = -\mathbf{B}$
B.	$\xrightarrow{\quad \|\mathbf{A}\| = l \quad}$ $\xleftarrow{\quad \|\mathbf{B}\| = l \quad}$	q. $\mathbf{A} = \mathbf{B}$
C.	$\xrightarrow{\quad \|\mathbf{A}\| = 2l \quad}$ $\xleftarrow{\quad \mathbf{B} = l \quad}$	r. $2\mathbf{A} = \mathbf{B}$
D.	$\xrightarrow{\quad \|\mathbf{A}\| = l \quad}$ $\xrightarrow{\quad \|\mathbf{B}\| = l \quad}$	s. $\mathbf{A} = -2\,\mathbf{B}$

Codes

	A	B	C	D		A	B	C	D
(a)	q	s	p	q	(b)	r	p	s	q
(c)	r	p	q	s	(d)	q	r	s	p

59. A vector is given by $\mathbf{A} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, where α, β and γ are the angles made by $\mathbf{A}$ with coordinate axes. Then, match the Column I with Column II (respective values) and select the correct option from the codes given below.

Column I	Column II
A. α	p. $\cos^{-1}(1/\sqrt{2})$
B. β	q. $\cos^{-1}(4/5\sqrt{2})$
C. γ	r. $\cos^{-1}(3/5\sqrt{2})$

Codes

	A	B	C		A	B	C
(a)	p	q	r	(b)	q	r	p
(c)	r	q	p	(d)	p	p	q

Assertion-Reasoning MCQs

*For question numbers 60 to 69, two statements are given-one labelled **Assertion** (A) and the other labelled **Reason** (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) are as given below*

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

60. **Assertion** Magnitude of resultant of two vectors may be less than the magnitude of either vector.

Reason Vector addition is commutative.

61. **Assertion** Vector addition of two vectors is always greater than their vector subtraction.

Reason At $\theta = 90^{\circ}$, addition and subtraction of vectors are unequal.

62. **Assertion** In case of projectile motion, the magnitude of rate of change of velocity is variable.

Reason In projectile motion, magnitude of velocity first increases and then decreases during the motion.

63. **Assertion** At highest point of a projectile, dot product of velocity and acceleration is zero.

Reason At highest point, velocity and acceleration are mutually perpendicular.

64. **Assertion** A particle is projected with speed u at an angle θ with the horizontal. At any time during motion, speed of particle is v at angle α with the vertical, then $v \sin \alpha$ is always constant throughout the motion.

Reason In case of projectile motion, magnitude of radial acceleration at topmost point is minimum.

65. **Assertion** For projection angle $\tan^{-1}(4)$, the horizontal and maximum height of a projectile are equal.

Reason The maximum range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.

66. **Assertion** The range of a projectile is maximum at 45°.

Reason At $\theta = 45^{\circ}$, the value of $\sin\theta$ is maximum.

67. **Assertion** Sum of maximum height for angles α and $90°-\alpha$ is independent of the angle of projection.

Reason For angles α and $90°-\alpha$, the horizontal range R is different.

68. **Assertion** The maximum height of projectile is always 25% of the maximum range.

Reason For maximum range, projectile should be projected at 90°.

69. **Assertion** Uniform circular motion is uniformly accelerated motion.

Reason Kinematic equations for uniform acceleration motion can be applied in the case of uniform circular motion.

Case Based MCQs

Direction *Answer the questions from 70-74 on the following case.*

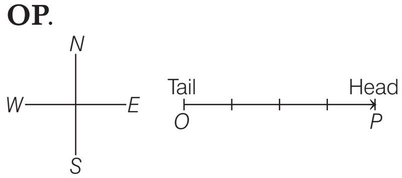
Vectors

Vectors are the physical quantities which have both magnitudes and directions and obey the triangle/parallelogram laws of addition and subtraction. It is specified by giving its magnitude by a number and its direction. e.g. Displacement, acceleration, velocity, momentum, force, etc. A vector is represented by a bold face type and also by an arrow placed over a letter, i.e.

F, **a**, **b** or $\vec{F}, \vec{a}, \vec{b}$.

The length of the line gives the magnitude and the arrowhead gives the direction.

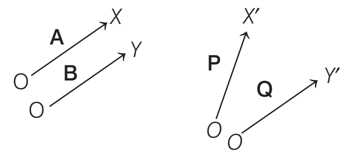
The point P is called head or terminal point and point O is called tail or initial point of the vector **OP**.



70. Amongst the following quantities, which is not a vector quantity?

(a) Force (b) Acceleration
(c) Temperature (d) Velocity

71. Set of vectors **A** and **B**, **P** and **Q** are as shown below



Length of **A** and **B** is equal, similarly length of **P** and **Q** is equal. Then, the vectors which are equal, are

(a) A and P (b) P and Q
(c) A and B (d) B and Q

72. $|\lambda \mathbf{A}| = \lambda |\mathbf{A}|$, if

(a) $\lambda > 0$ (b) $\lambda < 0$
(c) $\lambda = 0$ (d) $\lambda \neq 0$

73. Among the following properties regarding null vector which is incorrect?

(a) $A + 0 = A$ (b) $\lambda 0 = \lambda$
(c) $0 A = 0$ (d) $A - A = \mathbf{0}$

74. The x and y components of a position vector **P** have numerical values 5 and 6, respectively. Direction and magnitude of vector **P** are

(a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$ (b) $\tan^{-1}\left(\frac{5}{6}\right)$ and $\sqrt{61}$
(c) 60° and 8 (d) 30° and 9

Direction *Answer the questions from 75-79 on the following case.*

Relative Velocity

Every motion is relative as it has to be observed with respect to an observer. Relative velocity is a measurement of velocity of an object with respect to other observer. It is defined as the time rate of change of relative position of one object with respect to another.

For example, if rain is falling vertically with a velocity v_r and a man is moving horizontally with v_m, the man can protect himself from the rain if he holds his umbrella in the direction of relative velocity of rain w.r.t. man.

75. Two bodies are held separated by 9.8 m vertically one above the other. They are released simultaneously to fall freely under gravity. After 2 s, the relative distance between them is

(a) 4.9 m (b) 19.6 m (c) 9.8 m (d) 39.2 m

76. If two objects P and Q move along parallel straight lines in opposite direction with velocities $\mathbf{v}_P$ and $\mathbf{v}_Q$ respectively, then relative velocity of P w.r.t. Q,

(a) $v_{PQ} = v_P = v_Q$ (b) $v_P - v_Q$
(c) $v_P + v_Q$ (d) $v_Q - v_P$

77. A train is moving towards East and a car is along North, both with same speed. The observed direction of car to the passenger in the train is

(a) East-North direction
(b) West-North direction
(c) South-East direction
(d) None of the above

78. Buses A and B are moving in the same direction with velocities $20\hat{\mathbf{i}}$ ms^{-1} and $15\hat{\mathbf{i}}$ ms^{-1}, respectively. Then, relative velocity of A w.r.t. B is

(a) $35\,\hat{i}$ ms^{-1} (b) $5\,\hat{i}$ ms^{-1}
(c) $5\,\hat{j}$ ms^{-1} (d) $35\,\hat{j}$ ms^{-1}

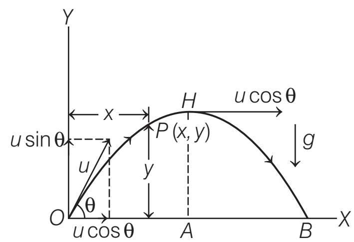
79. A girl riding a bicycle with a speed of 5 ms^{-1} towards east direction sees raindrops falling vertically downwards. On increasing the speed to 15 ms^{-1}, rain appears to fall making an angle of 45° of the vertical. Find the magnitude of velocity of rain.

(a) 5 ms^{-1} (b) $5\sqrt{5}$ ms^{-1}
(c) 25 ms^{-1} (d) 10 ms^{-1}

Direction *Answer the questions from 80-84 on the following case.*

Projectile Motion

Projectile motion is a form of motion in which an object or particle is thrown with some initial velocity near the earth's surface and it moves along a curved path under the action of gravity alone. The path followed by a projectile is called its trajectory, which is shown below. When a projectile is projected obliquely, then its trajectory is as shown in the figure below



Here velocity u is resolved into two components, we get (a) $u\cos\theta$ along OX and (b) $u\sin\theta$ along OY.

80. The example of such type of motion is

(a) motion of car on a banked road
(b) motion of boat in sea
(c) a javelin thrown by an athlete
(d) motion of ball thrown vertically upward

81. The acceleration of the object in horizontal direction is

(a) constant
(b) decreasing
(c) increasing
(d) zero

82. The vertical component of velocity at point H is

(a) maximum
(b) zero
(c) double to that at O
(d) equal to horizontal component

83. A cricket ball is thrown at a speed of 28 m/s in a direction 30° with the horizontal.

The time taken by the ball to return to the same level will be

(a) 2.0 s (b) 3.0 s
(c) 4.0 s (d) 2.9 s

84. In above case, the distance from the thrower to the point where the ball returns to the same level will be

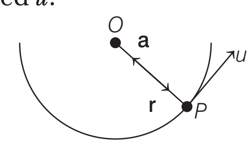
(a) 39 m (b) 69 m
(c) 68 m (d) 72 m

Direction *Answer the questions from 85-89 on the following case.*

Uniform Circular Motion

When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion. The word uniform refers to the speed which is uniform (constant) throughout the motion. Although the speed does not vary, the particle is accelerating because the velocity changes its direction at every point on the circular track.

The figure shows a particle P which moves along a circular track of radius r with a uniform speed u.



85. A circular motion

(a) is one-dimensional motion
(b) is two-dimensional motion
(c) it is represented by combination of two variable vectors
(d) Both (b) and (c)

86. For a particle performing uniform circular motion, choose the incorrect statement from the following.

(a) Magnitude of particle velocity (speed) remains constant.
(b) Particle velocity remains directed perpendicular to radius vector.
(c) Direction of acceleration keeps changing as particle moves.
(d) Angular momentum is constant in magnitude but direction keeps changing.

87. Two cars A and B move along a concentric circular path of radius r_A and r_B with velocities v_A and v_B maintaining constant distance, then $\frac{v_A}{v_B}$ is equal to

(a) $\frac{r_B}{r_A}$ (b) $\frac{r_A}{r_B}$
(c) $\frac{r_A^2}{r_B^2}$ (d) $\frac{r_B^2}{r_A^2}$

88. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular lap. The average velocity and average speed for each circular lap, respectively is

(a) 0, 0
(b) 0, 10 ms^{-1}
(c) 10 ms^{-1}, 10 ms^{-1}
(d) 10 ms^{-1}, 0

89. A particle is revolving at 1200 rpm in a circle of radius 30 cm. Then, its acceleration is

(a) 1600 ms^{-2} (b) 4740 ms^{-2}
(c) 2370 ms^{-2} (d) 5055 ms^{-2}

ANSWERS

Multiple Choice Questions

1. (b)	2. (b)	3. (b)	4. (b)	5. (c)	6. (d)	7. (d)	8. (c)	9. (d)	10. (c)
11. (a)	12. (c)	13. (c)	14. (a)	15. (b)	16. (b)	17. (c)	18. (c)	19. (a)	20. (b)
21. (a)	22. (a)	23. (b)	24. (a)	25. (d)	26. (b)	27. (c)	28. (d)	29. (a)	30. (b)
31. (a)	32. (b)	33. (a)	34. (c)	35. (c)	36. (d)	37. (d)	38. (b)	39. (b)	40. (c)
41. (b)	42. (d)	43. (b)	44. (b)	45. (a)	46. (a)	47. (b)	48. (c)	49. (a)	50. (b)
51. (c)	52. (d)	53. (c)	54. (d)	55. (a)	56. (b)	57. (c)	58. (b)	59. (b)	

Assertion-Reasoning MCQs

60. (b)	61. (d)	62. (d)	63. (a)	64. (c)	65. (b)	66. (c)	67. (c)	68. (c)	69. (d)

Case Based MCQs

70. (c)	71. (c)	72. (a)	73. (b)	74. (a)	75. (c)	76. (c)	77. (b)	78. (b)	79. (b)
80. (c)	81. (d)	82. (b)	83. (d)	84. (b)	85. (d)	86. (c)	87. (b)	88. (b)	89. (b)

SOLUTIONS

1. In order to describe two dimensional or three dimensional motions, we use vectors. However, direction of the motion of an object along a straight line is shown by positive and negative signs.

2. The perimeter of the rectangle would be the sum of the lengths of the four sides, i.e. 1.0 m + 0.5 m + 1.0 m + 0.5 m = 3.0 m.

Since, length of each side is a scalar, thus the perimeter is also a scalar.

3. We know that, impulse $J = F \cdot \Delta t = \Delta p$, where F is force, Δt is time duration and Δp is change in momentum. As Δp is a vector quantity, hence impulse is also a vector quantity. Sometimes area can also be treated as vector.

4. In the given case, the initial and final positions coincides, so the displacement will be zero. Thus, it is a null vector.

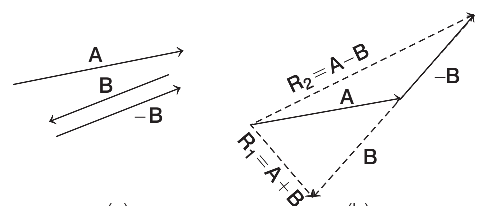
5. Multiplying a vector $\mathbf{A}$ by a negative number λ gives a vector $\lambda\mathbf{A}$, whose direction is opposite to the direction of $\mathbf{A}$ and its magnitude is $-\lambda$ times $|\mathbf{A}|$.

6. $|\mathbf{B}| = -2|\mathbf{A}|$. So, when $\mathbf{A}$ is multiplied by -2, then its direction gets reversed and magnitude would be 2 times $|\mathbf{A}|$.

Thus, $|\mathbf{B}| \neq |\mathbf{A}|$.

7. Vectors obey the triangle law of addition, according to which, if vector $\mathbf{B}$ is placed with its tail at the head of vector $\mathbf{A}$. Then, when we join the tail of $\mathbf{A}$ to the head of $\mathbf{B}$. The line OQ represents a vector $\mathbf{R}$, i.e. the sum of the vectors $\mathbf{A}$ and $\mathbf{B}$. Thus, figure given in option (d) is correct.

8. To subtract $\mathbf{B}$ from $\mathbf{A}$, we can add $-\mathbf{B}$ and $\mathbf{A}$. So, $\mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B} = \mathbf{R}_2$. This is as shown below



(a) (b)

Hence, option (c) is correct about vector subtraction.

9. Unit vector of $\mathbf{A}$ is $\hat{\mathbf{a}} = \dfrac{\mathbf{A}}{|\mathbf{A}|} = \dfrac{\mathbf{A}}{A}$

10. Resultant vector of $\mathbf{A}$ and $\mathbf{B}$ is

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (-3\hat{i} - 2\hat{j} - 3\hat{k}) + (2\hat{i} + 4\hat{j} + 6\hat{k})$$

$$= -\hat{i} + 2\hat{j} + 3\hat{k}$$

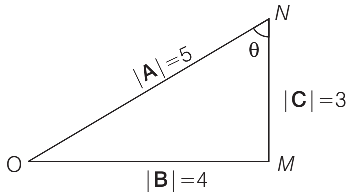
$$|\mathbf{R}| = \sqrt{(-1)^2 + (2)^2 + (3)^2}$$

$$= \sqrt{1 + 4 + 9} = \sqrt{14}$$

Unit vector in the direction of **R** is

$$\hat{R} = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{-\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$$

11. Here, triangle OMN with its adjacent sides as vectors **A, B** and **C** are shown below



As, $\cos\theta = \dfrac{MN}{ON}$

$\Rightarrow \theta = \cos^{-1}\dfrac{|\mathbf{C}|}{|\mathbf{A}|} = \cos^{-1}\left(\dfrac{3}{5}\right)$

12. Given, $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$

$\Rightarrow \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta}$

$= \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos\theta}$

$\Rightarrow |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta$

$= |\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos\theta$

$\Rightarrow 4|\mathbf{A}||\mathbf{B}|\cos\theta = 0$

$\Rightarrow |\mathbf{A}||\mathbf{B}|\cos\theta = 0$

$\Rightarrow |\mathbf{A}| = 0$

or $|\mathbf{B}| = 0$

or $\cos\theta = 0$

$\Rightarrow \theta = 90°$

Thus, $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$ is always true, when either $|\mathbf{A}|$ or $|\mathbf{B}|$ is zero or **A** and **B** are perpendicular to each other.

13. Let two vectors are **A** and **B**, inclined at an angle θ.

Resultant of the two vectors **A** and **B**,

$$|\mathbf{R}| = \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta} \quad \ldots\text{(i)}$$

Let, $|\mathbf{A}| = |\mathbf{B}| = a$

According to the question, $|\mathbf{R}| = a$

From Eq. (i),

$$a = \sqrt{a^2 + a^2 + 2aa\cos\theta}$$

$\Rightarrow a^2 = a^2 + a^2 + 2a^2\cos\theta$

$\Rightarrow 2a^2\cos\theta = -a^2$

$\Rightarrow \cos\theta = -1/2$

$\Rightarrow -\cos\theta = 1/2$

$\Rightarrow \cos(180° - \theta) = \cos 60°$

$\Rightarrow \theta = 120°$

14. Vector along X-axis (x-component)

$= A_x\hat{\mathbf{i}} = |\mathbf{A}|\cos\theta\hat{\mathbf{i}}$

$= A\cos\theta\hat{\mathbf{i}}$

Vector along Y-axis (y-component)

$= A_y\hat{\mathbf{j}} = |\mathbf{A}|\sin\theta\hat{\mathbf{j}}$

$= A\sin\theta\hat{\mathbf{j}}$

15. Let **r** makes an angle θ with positive x-axis, so the component of **r** along X-axis

$r_x = |\mathbf{r}|\cos\theta$

$(r_x)_{\text{maximum}} = |\mathbf{r}|(\cos\theta)_{\text{maximum}}$

$= |\mathbf{r}|\cos 0° = |\mathbf{r}|$

($\because$ $\cos\theta$ is maximum of $\theta = 0°$)

At $\theta = 0°$, **r** will be along positive X-axis.

16. Given, $|\mathbf{Q}| = 5$

$Q_y = 4$

$Q_x = ?$

As, $|\mathbf{Q}| = \sqrt{Q_x^2 + Q_y^2}$

$\Rightarrow |\mathbf{Q}|^2 = Q_x^2 + Q_y^2$

Substituting the given values, we get

$(5)^2 = Q_x^2 + 4^2$

$\Rightarrow Q_x = \sqrt{9} = 3$

17. Given, $\mathbf{P} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

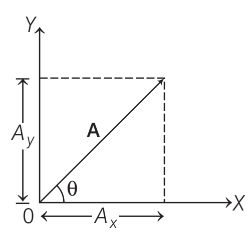
and $\mathbf{Q} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} = 2(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = 2\mathbf{P}$

Also, $\mathbf{R} = \dfrac{3}{4}\hat{\mathbf{i}} - \hat{\mathbf{j}} = \dfrac{1}{4}(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \dfrac{\mathbf{P}}{4}$

So, **P**, **Q** and **R** are parallel with unequal magnitude.

Thus, they are not equals vectors.

18. Given, vector can be shown below as



where, $\theta = 60°$

Then, $\tan\theta = \dfrac{A_y}{A_x}$

or $A_y = A_x \tan\theta$

$\Rightarrow A_y = 50 \tan 60° = 50 \times \sqrt{3}$ $(\because \sqrt{3} = 1.732)$

$= 86.6 \simeq 87$ N

19. Given, $\mathbf{p} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and unit vector along X-axis, $\mathbf{x} = \hat{\mathbf{i}}$.

So, the angle θ between them can be determine by

$$\cos\theta = \frac{\mathbf{p}\cdot\mathbf{x}}{|\mathbf{p}||\mathbf{x}|} = \frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}})}{\sqrt{1^2+1^2+1^2}\cdot\sqrt{1^2}} = \frac{1}{\sqrt{3}}$$

$\therefore \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

20. Since, the magnitude and angle between the vectors is unchanged, so the magnitude of the resultant **R** will be same. However, the direction of **R** will get changed.

21. Given that, $|\mathbf{A} + \mathbf{B}| = |\mathbf{A}|$ or $|\mathbf{A} + \mathbf{B}|^2 = |\mathbf{A}|^2$

$\Rightarrow |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta = |\mathbf{A}|^2$

where, θ is angle between **A** and **B**.

$\Rightarrow |\mathbf{B}|(|\mathbf{B}| + 2|\mathbf{A}|\cos\theta) = 0$

$\Rightarrow |\mathbf{B}| = 0$ or $|\mathbf{B}| + 2|\mathbf{A}|\cos\theta = 0$

$\Rightarrow \cos\theta = -\dfrac{|\mathbf{B}|}{2|\mathbf{A}|}$ ...(i)

If **A** and **B** are anti-parallel, then $\theta = 180°$.

Hence, from Eq. (i),

$\cos 180° = -1 = -\dfrac{|\mathbf{B}|}{2|\mathbf{A}|} \Rightarrow |\mathbf{B}| = 2|\mathbf{A}|$

Hence, the given condition can only be implied of either $|\mathbf{B}| = 0$ or **A** and **B** are anti-parallel provided $|\mathbf{B}| = 2|\mathbf{A}|$.

22. Area of a parallelogram $= |\mathbf{a} \times \mathbf{b}|$

where, **a** and **b** are sides of parallelogram.

Given, $\mathbf{a} = \mathbf{p} = 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\mathbf{b} = \mathbf{q} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$$\therefore \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 5 & -4 & 3 \\ 3 & 2 & -1 \end{vmatrix}$$

$\Rightarrow \mathbf{a} \times \mathbf{b} = \hat{\mathbf{i}}(4-6) - \hat{\mathbf{j}}(-5-9) + \hat{\mathbf{k}}(10+12)$

$\Rightarrow \mathbf{a} \times \mathbf{b} = -2\hat{\mathbf{i}} + 14\hat{\mathbf{j}} + 22\hat{\mathbf{k}}$

Thus, $|\mathbf{a} \times \mathbf{b}| = \sqrt{(2)^2 + (14)^2 + (22)^2}$

$= \sqrt{684}$ sq. units

23. Given, $\mathbf{a}\cdot\mathbf{b} = |\mathbf{a} \times \mathbf{b}|$

$\Rightarrow ab\cos\theta = ab\sin\theta$

$(\because \mathbf{a}\cdot\mathbf{b} = ab\cos\theta$ and $\mathbf{a} \times \mathbf{b} = ab\sin\theta)$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{ab}{ab} = 1$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45°$

$\therefore \theta = 45°$

24. Position vector **r** of an object in xy-plane at point P with its components along X and Y-axes as x and y, respectively is given as $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.

Given, $x = 2$ units and $y = 4$ units.

So, position vector at P will be $\mathbf{r} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$.

25. Position vector of the particle at P,

$\mathbf{r} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

Position vector of the particle at P',

$\mathbf{r}' = 7\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$\therefore$ Displacement of the particle is $\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}$

$\Rightarrow \Delta\mathbf{r} = (7\hat{\mathbf{i}} + 6\hat{\mathbf{j}}) - (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$

$= (7-4)\hat{\mathbf{i}} + (6-3)\hat{\mathbf{j}} = 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

26. Displacement,

$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$\therefore \mathbf{v}_{av} = \dfrac{\Delta\mathbf{r}}{\Delta t} = \dfrac{4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}}{2} = 2(\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-1}$

$\Rightarrow$ Magnitude of velocity,

$|\mathbf{v}_{av}| = 2\sqrt{1^2 + 1^2} = 2\sqrt{2}\ \text{ms}^{-1}$

Direction,

$\theta = \tan^{-1}\left(\dfrac{\Delta v_y}{\Delta v_x}\right) = \tan^{-1}\left(\dfrac{2}{2}\right) = \tan^{-1} 1 = 45°$

27. The direction of instantaneous velocity at any point on the path of an object is tangential to the path at that point and is in the direction of motion. Also, direction of average velocity is same as that of $\Delta\mathbf{r}$.

So, amongst the given figures we can say that, options (a) and (b) are depicting the direction of averge velocity but option (c) is correctly depicting the direction of instantaneous velocity.

28. Given, $\mathbf{r} = 3t\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$$\therefore \mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(3t\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = 3\hat{\mathbf{i}} + 4t\hat{\mathbf{j}}$$

At $t = 1$ s, $\quad \mathbf{v} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Thus, its direction is $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$\cong 53°$ with X-axis

29. Given that the position coordinates of a particle

$$\left.\begin{aligned} x &= a\cos\omega t \\ y &= a\sin\omega t \\ z &= a\omega t \end{aligned}\right\} \quad \ldots\text{(i)}$$

So, the position vector of the particle is

$$\hat{\mathrm{r}} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$$

$$\Rightarrow \quad \hat{\mathrm{r}} = a\cos\omega t\,\hat{\mathrm{i}} + a\sin\omega t\,\hat{\mathrm{j}} + a\,\omega t\,\hat{\mathrm{k}}$$

$$\hat{\mathrm{r}} = a[\cos\omega t\,\hat{\mathrm{i}} + \sin\omega t\,\hat{\mathrm{j}} + \omega t\,\hat{\mathrm{k}}]$$

Therefore, the velocity of the particle is

$$\because \quad \hat{\mathrm{v}} = \frac{d\mathrm{r}}{dt} = \frac{d[a][\cos\omega t\,\hat{\mathrm{i}} + \sin\omega t\hat{\mathrm{j}} + \omega t\,\hat{\mathrm{k}}]}{dt}$$

$$\Rightarrow \quad \hat{\mathrm{v}} = -a\omega\sin\omega t\,\hat{\mathrm{i}} + a\omega\cos\omega t\,\hat{\mathrm{j}} + a\omega\hat{\mathrm{k}})$$

The magnitude of velocity is

$$|\mathrm{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\text{or} \quad |\mathrm{v}| = \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + (a\omega)^2}$$

$$= \omega a\sqrt{(-\sin\omega t)^2 + (\cos\omega t)^2 + (1)^2}$$

$$= \sqrt{2}\,\omega a$$

30. Given, $x = 2t^3$

$$\therefore \quad v_x = \frac{dx}{dt} = 6t^2$$

$$\Rightarrow \quad a_x = \frac{dv_x}{dt} = 12t$$

Also, $\quad y = 3t^3$

$$\Rightarrow \quad v_y = \frac{dy}{dt} = 9t^2$$

$$\Rightarrow \quad a_y = \frac{dv_y}{dt} = 18t$$

$\therefore$ Acceleration, $a = \sqrt{a_x^2 + a_y^2} = t\sqrt{468}$

31. Given, initial velocity of the particle at $t = 0$ s,

$\mathbf{v}_0 = 5.0\,\hat{\mathbf{i}}\ \text{ms}^{-1}$, acceleration,

$\mathbf{a} = (3.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\ \text{ms}^{-2}$

The position of the particle is given by

$$\mathbf{r}(t) = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$= 5.0\,\hat{\mathbf{i}}\,t + (1/2)(3.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})t^2$$

$$= (5.0t + 1.5t^2)\hat{\mathbf{i}} + 1.0t^2\hat{\mathbf{j}} \quad \ldots\text{(i)}$$

As, $\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} \quad \ldots\text{(ii)}$

Comparing Eqs. (i) and (ii), we get

$x(t) = 5.0t + 1.5t^2$ and $\quad y(t) = +1.0t^2$

Given, $\qquad x(t) = 84\text{m}$

$\Rightarrow \qquad 5.0t + 1.50\,t^2 = 84$

or $\quad 1.50\,t^2 + 5.0t - 84 = 0$

Solving the above quadratic equation, the value of t is given as

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(1.50)(-84)}}{2(1.50)}$$

$$= \frac{-5 \pm \sqrt{25 + 504}}{3} = \frac{-5 \pm \sqrt{529}}{3} = \frac{-5 \pm 23}{3}$$

$= 6$ or -9.33

(Neglecting the negative values as t can never be negative)

$\Rightarrow\ t = 6$ s

At $t = 6$ s, $\quad y = 1.0(6)^2 = 36$ m

32. The velocity of car driver $= 8\,\hat{\mathbf{i}}\ \text{ms}^{-1}$

Velocity of rocket $= v_y\hat{\mathbf{j}}\ \text{ms}^{-1}$

Relative velocity of rocket w.r.t. car $= 8\hat{\mathbf{i}} - v_y\,\hat{\mathbf{j}}$

Since, the speed of the rocket observed by the car driver is 10 ms^{-1}.

$\therefore\ (v_y)^2 + (8)^2 = (10)^2$

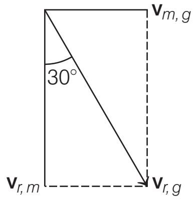
$v_y^2 = 100 - 64 = 36$

$\Rightarrow \qquad v_y = 6\ \text{ms}^{-1}$

Velocity of rocket, $v_y\,\hat{\mathbf{j}} = (6\hat{\mathbf{j}})\ \text{ms}^{-1}$

$\therefore$ Relative speed of rocket w.r.t. a stationary observer $= 6 - 0 = 6\ \text{ms}^{-1}$

33. When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground.



Here, $\mathbf{v}_{r,g}$ = velocity of the rain with respect to the ground,

$\mathbf{v}_{m,g}$ = velocity of the man with respect to the ground $= 10\ \text{kmh}^{-1}$

and $\mathbf{v}_{r,m}$ = velocity of the rain with respect to the man.

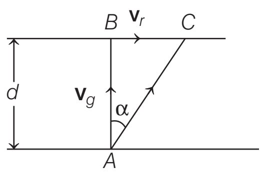
$$\therefore \quad v_{r,g} = \frac{10}{\sin 30^\circ} = 20\ \text{kmh}^{-1}$$

34. Given, speed of girl, $v_g = 5$ km h^{-1}

Speed of river, $v_r = 2$ km h^{-1}

Width of river, $d = 2$ km

The given condition is as shown in the figure below



Since, the girl dive the river normal to the flow of the river, time taken by the girl to cross the river, so

$$t = \frac{d}{v_g} = \frac{2\ \text{km}}{5\ \text{kmh}^{-1}} = \frac{2}{5}\ \text{h}$$

In this time, the girl will go down the river by the distance AC due to river current.

$\therefore$ Distance travelled along the river

$$= v_r \times t = 2 \times \frac{2}{5}$$

$$= \frac{4}{5}\ \text{km} = \frac{4000}{5}\ \text{m} = 800\ \text{m}$$

35. After the object has been projected, the x-component of the velocity remains constant throughout the motion and only the y-component changes, like an object in free-fall in vertical direction.

36. An object that is in flight after being thrown or projected is called a projectile. The motion of projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.

37. Velocity is in horizontal direction and acceleration is vertical downwards. Therefore, the direction of velocity and acceleration of the projectile are perpendicular to each other.

38. Given, initial velocity,

$$\mathbf{u} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{ms}^{-1}$$

Magnitude of velocity,

$$u = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\ \text{ms}^{-1}$$

Equation of trajectory of projectile,

$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2 \sec^2\theta}{2u^2}$$

$$= x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

$$[\because \sec^2\theta = 1 + \tan^2\theta]$$

Substituting the given values, we get

$$\therefore \quad y = x \times 2 - \frac{10(x)^2}{2(\sqrt{5})^2}[1 + (2)^2]$$

$$\left[\because \tan\theta = \frac{u_y}{u_x} = \frac{2}{1} = 2\right]$$

$$= 2x - \frac{10(x^2)}{2 \times 5}(1 + 4) = 2x - 5x^2$$

39. Given, equations of motion are

$$x = 18t,\ 2y = 54t - 9.8t^2$$

General equations of projectile are

$$x = u\cos\theta \cdot t \text{ and } y = u\sin\theta \cdot t - \frac{1}{2}gt^2$$

where, θ is the angle of projection.

Comparing it with given equation, we have

$$u\cos\theta = 18 \text{ and } u\sin\theta = \frac{54}{2}$$

$$\Rightarrow \frac{u \sin \theta}{u \cos \theta} = \frac{54/2}{18}$$

$$\therefore \quad \tan \theta = \frac{54}{2 \times 18} = 1.5 \Rightarrow \quad \theta = \tan^{-1}(1.5)$$

40. Time of flight,

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 50 \times \sin 30°}{10} = 5 \text{ s}$$

41. Given, $u = 60 \text{ ms}^{-1}$

Maximum height H that the ball will achieve

= height of ceiling of the hall

= 30 m

As, maximum height, $H = \frac{u^2 \sin^2 \theta}{2g}$

$$\Rightarrow \quad 30 = \frac{(60)^2 \sin^2 \theta}{2g}$$

$$\Rightarrow \quad \sin^2 \theta = \frac{30 \times 2g}{60 \times 60} = \frac{10}{60} \quad [\because g = 10 \text{ ms}^{-2}]$$

$$\Rightarrow \quad \sin \theta = \frac{1}{\sqrt{6}}$$

42. $T \propto \sin \theta$, $\frac{T_A}{T_B} = \frac{\sin 30°}{\sin 60°} = \frac{1}{\sqrt{3}}$ or $T_B = \sqrt{3}\, T_A$

$H \propto \sin^2 \theta$, $\frac{H_A}{H_B} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{1}{3}$

or $\quad H_B = 3\, H_A$

As, $\quad R_\theta = R_{90° - \theta}$

$\therefore \quad R_A = R_B$

43. Given, maximum horizontal range,

$$R_{\max} = 80 \text{ m}$$

As, range of a projectile, $R = \frac{u^2 \sin 2\theta}{g}$

and it is maximum $\theta = 45°$

$$\therefore \quad \frac{u^2}{g} = 80 \text{ m}$$

Maximum height, $h = \frac{u^2 \sin^2 \theta}{2g}$

$$= \frac{80}{2} (\sin^2 45°)$$

$$= 40 \times \frac{1}{2} = 20 \text{ m}$$

44. Velocities of the stones at some instant t can be given as

$$\mathbf{v}_1 = u_1 \cos \theta_1 \hat{\mathbf{i}} + (u_1 \sin \theta_1 - gt)\, \hat{\mathbf{j}}$$

and $\mathbf{v}_2 = u_2 \cos \theta_2 \hat{\mathbf{i}} + (u_2 \sin \theta_2 - gt) \hat{\mathbf{j}}$

Relative velocity,

$$\mathbf{v}_1 - \mathbf{v}_2 = (u_1 \cos \theta_1 - u_2 \cos \theta_2) \hat{\mathbf{i}} + (u_1 \sin \theta_1 - u_2 \cos \theta_2)\, \hat{\mathbf{j}}$$

= constant

Since, their relative velocity is constant.

So, the trajectory of path followed by one as seen by other will be straight line, making a constant angle with horizontal.

45. Range of a projectile,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g}$$

$$= \frac{2(u \sin \theta)\,(u \cos \theta)}{g} = \frac{2u_x u_y}{g}$$

$\Rightarrow R \propto$ horizontal initial velocity component (u_x)

$\therefore$ From the given plot, we can see that for path 3, range is maximum. This implies that the rock has the maximum horizontal velocity component in this path. Thus, the correct order will be $1 < 2 < 3$.

46. Given, speed, $v = 25 \text{ ms}^{-1}$

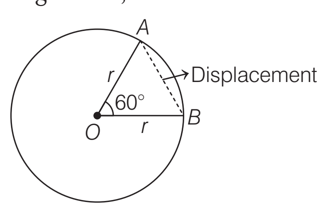
and radius, $r = 5$ m

Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{25 \times 25}{5}$

$$= 125 \text{ ms}^{-2}$$

47. In the figure, AB is the required displacement of the particle.

In triangle OAB, $OA = OB$ and $\angle AOB = 60°$



Therefore, ΔAOB is an equilateral triangle, so

$$OA = OB = r = AB$$

48. For a uniform circular motion,

centripetal acceleration, $a_c = \frac{v^2}{R}$

Since, v and R are constants, the magnitude of the centripetal acceleration of the car is also constant.

However, the direction changes pointing towards the centre. Therefore, a centripetal acceleration is not a constant vector.

49. Radius, $r = \frac{\text{diameter}}{2} = \frac{0.80 \text{ m}}{2} = 0.40 \text{ m}$

Frequency, $\nu = 100 \text{ rev min}^{-1} = \frac{100}{60} \text{ rev s}^{-1}$

Time period, $T = \frac{1}{\nu} = \frac{60}{100} = 0.60$

$\therefore$ Angular velocity, $\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{0.60}$

$= 10.467 \text{ rad s}^{-1}$

50. Given, $H_1 = H_2$

$$\Rightarrow \frac{u_1^2 \sin^2 45°}{2g} = \frac{u_2^2 \sin 60°}{2g}$$

$$\therefore \frac{u_1}{u_2} = \frac{\sin 60°}{\sin 45°} = \frac{\sqrt{3}/2}{(1/\sqrt{2})} = \sqrt{3} : \sqrt{2}$$

51. Maximum height, $H = \frac{u^2 \sin^2 \alpha}{2g}$

For same speed of projection,

$$H \propto \sin^2 \alpha$$

$$\therefore \frac{H_1}{H_2} = \frac{\sin^2 \alpha}{\sin^2(90° - \alpha)} = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \tan^2 \alpha$$

So, $H_1 : H_2 = \tan^2 \alpha : 1$

52. $R = \frac{u^2 \sin 2\theta}{g}$

$\therefore R \propto u^2$

If initial velocity be doubled at same angle of projection, then range will become four times.

53. As, centripetal acceleration is given as

$$a_c = \frac{v^2}{r}$$

For the first body of mass m_1, $a_{c_1} = \frac{v_1^2}{r_1}$

For the second body of mass m_2, $a_{c_2} = \frac{v_2^2}{r_2}$

Also, time taken by both the cars to complete one revolution is same.

Hence, $T_1 = T_2$

$$\Rightarrow \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{r_1}{r_2} \quad \ldots\text{(i)}$$

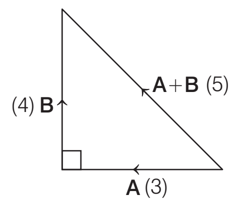
i.e. $a_{c_1} : a_{c_2} = \frac{v_1^2}{r_1} \times \frac{r_2}{v_2^2} = \frac{r_1^2}{r_2^2} \times \frac{r_2}{r_1} = \frac{r_1}{r_2} = r_1 : r_2$

[from Eq. (i)]

54. A scalar quantity is independent of direction hence has the same value for observers with different orientations of the axes.

Hence, the statement given in option (d) is correct, rest are incorrect.

55. Since, $(5)^2 = (3)^2 + (4)^2$, which is in accordance to Pythagoras theorem. So, the vectors can be shown in the figure



$\therefore$ **A** and **B** are perpendicular.

However, if the length of **A** + **B** vector is more than or less than 5, then they should be inclined at acute and obtuse angle, respectively.

Thus, the statement given in option (a) is correct, rest are incorrect.

56. Clearly from the given figure,

u is in the first quadrant, hence both components a and b will be positive.

For $\mathbf{v} = p\hat{\mathbf{i}} + q\hat{\mathbf{j}}$, as it is in positive x-direction and located downward hence x-component p will be positive and y-component q will be negative.

57. The correct sequence is

Hence, A $\to$ p, B $\to$ q, C $\to$ q and D $\to$ p.

58. A. As, $|\mathbf{B}| = 2|\mathbf{A}|$ and they both are in the same direction, so $2\mathbf{A} = \mathbf{B}$.

B. As, $|\mathbf{A}| = |\mathbf{B}|$ but both are in opposite directions, so $\mathbf{A} = -\mathbf{B}$.

C. As, $|\mathbf{A}| = 2|\mathbf{B}|$ but both are in opposite directions, so $\mathbf{A} = -2\mathbf{B}$.

D. As, $|\mathbf{A}| = |\mathbf{B}|$ and both are in same direction, so $\mathbf{A} = \mathbf{B}$.

Hence, A $\to$ r, B $\to$ p, C $\to$ s and D $\to$ q.

59. Magnitude of $\mathbf{A} = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$= \sqrt{(4)^2 + (3)^2 + (5)^2}$$

$$= \sqrt{16 + 9 + 25} = 5\sqrt{2}$$

Angles made by **A** with coordinate axes,

$$\cos\alpha = \frac{A_x}{|\mathbf{A}|} = \frac{4}{5\sqrt{2}}$$

or $$\alpha = \cos^{-1}\left(\frac{4}{5\sqrt{2}}\right)$$

$$\cos\beta = \frac{A_y}{|\mathbf{A}|} = \frac{3}{5\sqrt{2}}$$

or $$\beta = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$$

$$\cos\gamma = \frac{A_z}{|\mathbf{A}|} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

or $$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

Hence, A $\rightarrow$ q, B $\rightarrow$ r and C $\rightarrow$ p.

60. Resultant of two vectors **A** and **B** is given as

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$\therefore$ We can say that

(i) If θ is an obtuse angle, then magnitude of **R** will be less than magnitude of the either vectors **A** or **B**.

e.g. if $|\mathbf{A}| = 4$, $|\mathbf{B}| = 3$ and $\theta = 120°$, then

$$|\mathbf{R}| = \sqrt{4^2 + 3^2 + 2 \times 4 \times 3\cos(120°)}$$

$$= \sqrt{25 - 12} = \sqrt{13}$$

$$\left(\because \cos 120° = -\frac{1}{2}\right)$$

$$\therefore \quad |\mathbf{R}| < |\mathbf{A}|$$

(ii) If the vectors are in opposite direction and are equal in magnitude, then also the magnitude of **R** will be less than the magnitude of either vectors **A** or **B**.

e.g. If $|\mathbf{A}| = |\mathbf{B}| = a$ (say) and $\theta = 180°$

then, $$|\mathbf{R}| = \sqrt{a^2 + a^2 - 2a^2\cos(180°)}$$

$$= \sqrt{2a^2 - 2a^2} \qquad [\because \cos 180° = -1]$$

$$\therefore \quad |\mathbf{R}| < |\mathbf{A}| \text{ or } |\mathbf{B}|$$

Also, vector addition is commutative in nature.

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

Therefore, both A and R are true but R is not the correct explanation of A.

61. $$|\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$|\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

So, for example, when $90° < \theta < 270°$,

$$|\mathbf{A} + \mathbf{B}| < |\mathbf{A} - \mathbf{B}|$$

Thus, vector addition of two vectors is not always greater than their vector subtraction.

Also, at $\theta = 90°$, $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2}$

Therefore, A is false and R is also false.

62. In projectile motion, rate of change of velocity, $\left|\frac{dv}{dt}\right| = |a| = 9.8 \text{ ms}^{-2}$ = constant

Also, in case of projectile motion, the magnitude of velocity first decreases and then increases during the motion.

Therefore, A is false and R is also false.

63. Velocity is horizontal and acceleration is vertical. i.e. both are perpendicular to each other, hence their dot product is zero.

Therefore, both A and R are true and R is the correct explanation of A.

64. Horizontal component of velocity = $v \sin\alpha$ = constant

$$a_r = \sqrt{g^2 - a_t^2}$$

At highest point $a_t = 0$. Therefore, a_r is maximum.

Therefore, A is true but R is false.

65. $H = R$ or $\dfrac{u^2\sin^2\theta}{2g} = \dfrac{2u^2\sin\theta\cos\theta}{g}$

$$\Rightarrow \qquad \tan\theta = 4$$

Maximum horizontal range (at $\theta = 45°$) is given by $R_{\max} = \dfrac{u^2}{g}$

Therefore, both A and R are true but R is not the correct explanation of A.

66. Horizontal range, $R = \dfrac{u^2\sin 2\theta}{g}$

At $\theta = 45°$, $\sin 2\theta = 1$

$$\therefore R_{\max} = \frac{u^2}{g} = \text{maximum range}$$

$\because \sin\theta = 1$ (maximum), at $\theta = 90°$

Therefore, A is true but R is false.

67. Maximum height, $H_1 = \dfrac{u^2 \sin^2 \alpha}{2g}$

and $H_2 = \dfrac{u^2 \sin^2(90° - \alpha)}{2g} = \dfrac{u^2 \cos^2 \alpha}{2g}$

$$\Rightarrow H_1 + H_2 = \frac{u^2}{2g}(\sin^2 \alpha + \cos^2 \alpha) = \frac{u^2}{2g}$$

Thus, the sum of height for angles α and $90° - \alpha$ is independent of the angle of projection.

As, horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

So, for same value of initial velocity, horizontal range of projectile is same for complementary angles.

Therefore, A is true but R false.

68. To obtain maximum range, angle of projection must be 45°, i.e. $\theta = 45°$.

So, $R_{max} = \dfrac{u^2 \sin 2 \times 45°}{g} = \dfrac{u^2 \sin 90°}{g} = \dfrac{u^2}{g}$ ...(i)

$$\therefore \quad H_{max} = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

[from Eq. (i)]

So, H_{max} is 25% of R_{max}.

Therefore, A is true but R is false.

69. In uniform circular motion the velocity of the object is changing continuously in direction, the object undergoes uniform acceleration which is not a constant vector. However, for a uniformly accelerated motion, the acceleration of the object should be constant. Hence, it is not an example of uniformly accelerated motion.
Kinematic equations for constant acceleration is not applicable for uniform circular motion. Since, in this case the magnitude of acceleration is constant but its direction is changing.

Therefore, A is false and R is also false.

70. Temperature is not a vector quantity because it has magnitude only.

However, force, acceleration and velocity have both a magnitude and a direction. So, these are vectors in nature.

71. Two vectors are said to be equal, if and only if they have the same magnitude and direction.

Among the given vectors **A** and **B** are equal vectors as they have same magnitude (length) and direction.

However, **P** and **Q** are not equal even though they are of same magnitude because their directions are different.

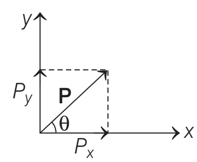
72. $|\lambda \mathbf{A}| = \lambda |\mathbf{A}|$, if $\lambda > 0$, as multiplication of vector **A** with a positive number λ gives a vector whose magnitude is changed by the factor λ but the direction is same as that of **A**.

73. Null vector **0** is a vector, whose magnitude is zero and its direction cannot be specified. So, it means, $|\mathbf{0}| = 0$.

Thus, $\lambda \mathbf{0} = \mathbf{0}$.

Hence, property given in option (b) is incorrect.

74. Let **P** be as shown in the figure, then according to the given information



$$P_x = 5, \; P_y = 6$$

$$\therefore \quad |\mathbf{P}| = \sqrt{P_x^2 + P_y^2} = \sqrt{25 + 36}$$

$$\Rightarrow \quad |\mathbf{P}| = \sqrt{61}$$

and $\tan \theta = \dfrac{P_y}{P_x} = \dfrac{6}{5} \Rightarrow \theta = \tan^{-1}\left(\dfrac{6}{5}\right)$

75. Since, they are following freely, so both the bodies will fall same distance in same time interval.

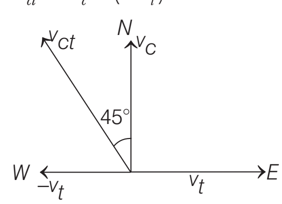
So, the relative separation between them will remain unchanged.

76. Relative velocity of P w.r.t. Q is given by

$$\mathbf{v}_{PQ} = \mathbf{v}_P - (-\mathbf{v}_Q) = \mathbf{v}_P + \mathbf{v}_Q$$

77. Velocity of car w.r.t. train, $\mathbf{v}_{ct} = \mathbf{v}_c - \mathbf{v}_t$

$$\Rightarrow \quad \mathbf{v}_{ct} = \mathbf{v}_c + (-\mathbf{v}_t)$$



Velocity of car w.r.t. train (v_{ct}) is towards West-North.

78. Given, $\mathbf{v}_A = 20\hat{\mathbf{i}}\ \text{ms}^{-1}$

$$\mathbf{v}_B = 15\hat{\mathbf{i}}\ \text{ms}^{-1}$$

Relative velocity of A w.r.t. B,

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B = 20\hat{\mathbf{i}} - 15\hat{\mathbf{i}} = 5\hat{\mathbf{i}}\ \text{ms}^{-1}$$

79. Given, velocity of girl, $v_g = 5\hat{\mathbf{i}}\ \text{ms}^{-1}$

Let velocity of rain, $v_r = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}\ \text{ms}^{-1}$

Relative velocity of rain

$$= v_r - v_g = (v_x - 5)\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

Now, it is vertical, so $\tan\theta = \dfrac{v_x - 5}{v_y} = 0$

$\Rightarrow v_x - 5 = 0 \Rightarrow v_x = 5$ ...(i)

On increasing the speed of the girl, relative velocity becomes $(v_x - 15)\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

$$\tan\theta = \tan 45^\circ = \frac{v_x - 15}{v_y} = 1$$

$\Rightarrow v_x - 15 = v_y \Rightarrow v_y = -10$ [using Eq. (i)]

$\therefore$ Velocity of rain $= (5\hat{\mathbf{i}} - 10\hat{\mathbf{j}})\ \text{ms}^{-1}$

$\Rightarrow$ Magnitude of velocity of rain

$$= \sqrt{(5)^2 + (10)^2}$$

$$= \sqrt{125} = 5\sqrt{5}\ \text{ms}^{-1}$$

80. A javelin thrown by an athlete is an example of projectile motion.

81. The horizontal component of velocity ($u\cos\theta$) is constant throughout the motion, so there will be no acceleration in horizontal direction.

82. As the vertical components of velocity ($u\sin\theta$) decreases continuously with height, from O to H, due to downward force of gravity and becomes zero at H.

83. The time taken by the ball to return to the same level,

$$T = \frac{2v_0\sin\theta}{g} = \frac{2\times 28\times\sin 30^\circ}{9.8} \approx 2.9\ \text{s}$$

84. The distance from the thrower to the point where the ball returns to the same level is

$$R = \frac{v_0^2\sin 2\theta}{g} = \frac{28\times 28\times\sin 60^\circ}{9.8} \approx 69\ \text{m}$$

85. Circular motion is an example of two-dimensional motion with radius vector as

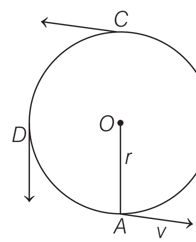
$$\mathbf{r} = a\cos\omega t\,\hat{\mathbf{i}} + a\sin\omega t\,\hat{\mathbf{j}}$$

Both the components $a\cos\omega t\,\hat{\mathbf{i}}$ and $a\sin\omega t\,\hat{\mathbf{j}}$ are perpendicular to each other.

86. If a particle is performing uniform circular motion, then its

(a) speed will be constant throughout the motion.

(b) velocity will be tangential in the direction of motion at a particular point.



(c) acceleration, $a = \dfrac{v^2}{r}$ will always be towards centre of the circular path.

(d) angular momentum (mvr) is constant in magnitude but direction keeps on changing.

87. Angular velocity ω is constant.

$$v = r\omega$$

$$\therefore\ v \propto r \text{ or } \frac{v_A}{v_B} = \frac{r_A}{r_B}$$

88. On a circular path in completing one turn,the distance travelled is $2\pi r$, while displacement is zero.

Hence, average velocity

$$= \frac{\text{displacement}}{\text{time interval}} = \frac{0}{t} = 0$$

Average speed $= \dfrac{\text{Distance}}{\text{Time interval}}$

$$= \frac{2\pi r}{t} = \frac{2\times 3.14\times 100}{62.8} = 10\ \text{ms}^{-1}$$

89. Given, $\nu = 1200\ \text{rpm} = \dfrac{1200}{60}\ \text{rps}$

$$r = 30\ \text{cm} = \frac{30}{100}\ \text{m}$$

Acceleration of the particle = Centripetal acceleration $= \omega^2 r = (2\pi\nu)^2 r$

$$= \left(2\times\frac{22}{7}\times\frac{1200}{60}\right)^2\times\frac{30}{100} \approx 4740\ \text{ms}^{-2}$$