26. Laws of Thermodynamics

Short Answer

1. Question

Should the internal energy of a system necessarily increase if heat is added to it?

Answer

No, the internal energy of the system necessarily should not increase if heat is added to it.

Explanation

1. Internal energy is defined as the sum of the kinetic energy and potential energy of molecules of the system. It includes only the energy associated with the random motion of molecules of the system.

2. Random motion of molecules is associated with the temperature of the system. Thus, any change in temperature will change the internal energy of the system.

3. Change in internal energy is given as

$\Delta U=C_V \Delta T$

Where ΔU= change in internal energy

C_V=molar specific heat at constant volume

ΔT= change in temperature.

4. If ΔT =0 then, ΔU will also be zero.

5. In the isothermal process, where the change in temperature is zero, ΔU is also zero.

6. According to First law of thermodynamics,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

7. Since for an isothermal process $\Delta T=0$, therefore $\Delta U=0$. So, from the first law of thermodynamics $\Delta Q=\Delta W$.

8. For an isothermal process, heat supplied to the system is used up entirely in doing work.

2. Question

Should the internal energy of a system necessarily increase if its temperature is increased?

Answer

No, the internal energy of a system will not necessarily increase if its temperature is increased.

Explanation

1. Internal energy is defined as the sum of the kinetic energy and potential energy of molecules of the system. It includes only the energy associated with the random motion of molecules of the system.

2. The important thing about internal energy is that it depends only on the state of the system, not how that state was achieved.

3. Thus, the internal energy of a given mass of gas depends on its state described by specifics values of pressure, volume, and temperature.

4. The internal energy of ideal gas depends on only on temperature.

5. For system other than ideal gas, internal energy depends on pressure, volume and temperature combined.

3. Question

A cylinder containing gas is lifted from the first floor to the second floor. What is the amount of work done on the gas? What is the amount of work done by the gas? Is the internal energy of the gas increased? Is the temperature of the gas increased?

Answer

1. Work done on the gas will be zero.

Explanation

Atmospheric pressure decreases, as we increase the height.

So, when the cylinder is lifted from the first floor to second-floor atmospheric pressure on the cylinder will decrease. Hence, the volume of gas inside the cylinder will increase. In the expansion process, work is always done by the gas. So, in such case, no work will be done on the gas. But gas will do work.

2. Work done by the gas will be $P\Delta V$.

Explanation

When the cylinder is lifted from the first floor to second-floor atmospheric pressure on the cylinder will decrease. Hence the volume of gas inside the cylinder will increase. This means gas expands. In the expansion, process work is done by the gas.

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of gas = ΔV

The pressure at which gas expands =P

Thus, work done by the gas W

$W=P\Delta V$

3. Change in internal energy and temperature will depend on the walls of the container, whether they are insulating or conducting. So, we cannot comment about internal energy and temperature unless we know the nature of the wall.

4. Question

A force F is applied on a block of mass M. the block is displaced through a distance d in the direction of the force. What is the work done by the force on the block? Does the internal energy change because of this work?

Answer

1. Work done by the force on the block is $F \times d$.

Explanation

Given

Force =F

Displacement due to force = d

We know that

Work done= force × displacement × cos θ

θ in our case is zero as displacement is in the direction of the force.

Work done by the force = F ×d ×cos 0

=F ×d (∵ cos0=1)

∴ Work done by the force on the block is F ×d.

2. Internal energy will not change because of work done by the force on the block.

<u>Explanation</u>

Internal energy is defined as the sum of the kinetic energy and potential energy of molecules of the system. It includes only the energy associated with the random motion of molecules of the system.

If the block moves as a whole system with some velocity, the kinetic energy of the box is not to be included in internal energy.

5. Question

The outer surface of a cylinder containing gas is rubbed vigorously by a polishing machine. The cylinder and its gas become warm. Is the energy transferred to the gas heat or work?

Answer

Energy transferred to the gas is heat energy.

<u>Explanation</u>

Since the outer surface of the gas is rubbed and not displaced/shaken, no work will be done on the gas.

We know that,

Work done = force ×displacement

Force =pressure × area

∴work done = pressure × area × displacement

So, work will only be done when there is either displacement or the gas is expanded/compressed.

The cylinder and gas become warm because of the heat generated by friction between the polishing machine and surface of the cylinder. This heat will warm up the gas and outer surface of the cylinder.

6. Question

When we rub our hands, they become warm. Have we supplied heat to the hands?

Answer

Our hands become warm when we rub them due to the friction between our hands. Friction between our hands supplies heat to the hands.

7. Question

A closed bottle contains some liquid. The bottle is shaken vigorously for 5 minutes. It is found that the temperature of the liquid is increased. Is heat transferred to the liquid? Is work done on the liquid? Neglect expansion on heating.

Answer

Work is done when there is displacement in the body.

Work done = force ×displacement

As the bottle is shaken vigorously, that means the bottle is displaced from its position. And this displacement is due to external force. Therefore, work is done on the bottle. Due to this work, the temperature of the liquid increases. No external heat is supplied to the bottle and liquid.

8. Question

The final volume of a system is equal to the initial volume in a certain process. Is the work done by the system necessarily zero? Is it necessarily nonzero?

Answer

Work done by the system is neither necessarily zero nor nonzero when the final volume is equal to the initial volume.

<u>Explanation</u>

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the system W at constant pressure

$W = P\Delta V$

But if $V_2 = V_1$ then, $\Delta V = 0$

Work done by the system $W = 0$

So, in an isobaric process (where pressure remains constant) work done by the system will be zero if initial and final volume are equal.

But we also know that in the cyclic process the system returns to its initial state. But still work done is not zero. In fact, in the cyclic process since the system returns to its initial state, internal energy becomes zero. This is because internal energy is a state variable. It depends on initial and final state only. And if initial and final state becomes equal, then change in internal energy will be zero.

According to First law of thermodynamics,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

In cyclic process $\Delta U=0$.

So, from the first law of thermodynamics $\Delta Q=\Delta W$ i.e. heat supplied to the system is converted entirely into work in a cyclic process.

9. Question

Can work be done by a system without changing its volume?

Answer

Yes, work can be done by a system without changing its volume if the process is cyclic.

<u>Explanation</u>

1. In cyclic process, the system returns to its initial state. Therefore, initial volume becomes equal to final volume and hence $\Delta V=0$.

2. Also, since system returns to initial state, the internal energy of the system remains same.

3. This is because internal energy is a state variable. It depends on initial and final state only. And if initial and final state becomes equal, then change in internal energy will be zero.

4. So, for cyclic process $\Delta V=0$ and $\Delta U=0$.

5. According to First law of thermodynamics,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

6. So, from the first law of thermodynamics $\Delta Q=\Delta W$ i.e. heat supplied to the system is converted entirely into work in a cyclic process.

7. Therefore, work can be done by a system without changing its volume.

10. Question

An ideal gas is pumped into a rigid container having a diathermic wall so that the temperature remains constant. In a certain time interval, the pressure in the container is doubled. Is the internal energy of the contents of the container also doubled in the interval?

Answer

Yes, the internal energy will also be doubled.

Explanation

1. Internal energy for an ideal gas is given as

$U=nC_VT$

Where U= internal energy

N=number of moles

C_V=molar specific heat at constant volume

T= temperature

2. Since the ideal gas is continuously pumped into rigid container number of moles are also increasing.

3. When the pressure becomes doubled, the number of moles also gets doubled.

4. It is given that the temperature remains constant. So internal energy depends only on the number of moles.

5. Internal energy will also be doubled as the number of moles is getting doubled.

11. Question

When a tyre bursts, the air coming out is cooler than the surrounding air. Explain.

Answer

We know that pressure inside the tyre is greater than the atmospheric pressure of surroundings. So, when a tyre bursts there is an adiabatic expansion of the air (adiabatic because no heat is either supplied or released during expansion). In the expansion process, work is done by the gas. Now work done in an adiabatic process is given as

$$W=\frac{\mu R(T_1-T_2)}{\gamma-1} \quad \ldots..(i)$$

Where μ=number of moles

R=gas constant

T_1=initial temperature

T_2=final temperature

γ = ratio of specific heat at constant pressure and volume

Since work is done by the gas in adiabatic expansion, therefore, W>0. From equation (i), for W>0, $T_1>T_2$. This means that air

coming out of tyre will be at a lower temperature than when it was inside the tyre.

12. Question

When we heat an object, it expands. Is work done by the object is this process? Is heat given to the object equal to the increase in its internal energy?

Answer

1. We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of object = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the object W

$W=P\Delta V$

During expansion volume increases. Hence work done will be positive and equals $P\Delta V$.

2. According to First law of thermodynamics,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Since work done by the gas is not zero, heat supplied to the object will not be equal to internal energy only.

13. Question

When we stir a liquid vigorously, it becomes warm. Is it a reversible process?

Answer

No, it is not a reversible process.

Explanation

1. When we stir a liquid vigorously, it becomes warm because we do work on the liquid and it increases its temperature.

2. To make it a reversible process, would require it to bring the temperature to its initial value by stirring the liquid in opposite direction. Only then we can call it a reversible process.

3. But we know that this is not possible. The only way we can decrease the temperature is by extracting heat from the liquid and not by stirring it in opposite direction.

14. Question

What should be the condition for the efficiency of a Carnot engine to be equal to 1?

Answer

We know that efficiency η of a Carnot engine is

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \quad \ldots..(i)$$

Where W=work done by the engine

Q_1=heat absorbed by the engine

Q_2=heat released by the engine

As seen from the formula (i), efficiency will be 1 when:

i) $W=Q_1$

ii) $Q_2=0$

15. Question

When an object cools down, heat is withdrawn from it. Does the entropy of the object decrease in this process? If yes, is it a violation of the second law of thermodynamics state in terms of increase in entropy?

Answer

1. The entropy of the system is the measure of molecular disorder or randomness, of a system.

2. When heat is withdrawn from the system, temperature decreases. This means that there is less randomness in the system. So, entropy will decrease.

3. But the heat withdrawn from the system is supplied to surroundings. Therefore, the entropy of the surroundings will increase.

4. The second law of thermodynamics states that net entropy of the universe always increases.

5. So, if the entropy of the system decreases, the entropy of the surroundings increases. Therefore, there is always a net increase in entropy.

6. Hence second law of thermodynamic is not violated in this case.

Objective I

1. Question

The first law of thermodynamics is a statement of

A. conservation of heat

B. conservation of work

C. conservation of momentum

D. conservation of energy

Answer

According to First law of thermodynamics,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

It is a general law of conservation of energy applicable to any systemin which the energy transfer from or to surroundings is considered.

2. Question

If heat is supplied to an ideal gas in an isothermal process,

A. the internal energy of the gas will increase

B. the gas will do positive work

C. the gas will do negative work

D. the said process is not possible.

Answer

According to First law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

For an isothermal process $\Delta T=0$, therefore $\Delta U=0$.

Then,

$\Delta Q = \Delta W$

When heat is supplied to the system gas expands i.e. the volume of the gas increases.

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of object = $\Delta V = V_2 - V_1$

Pressure =P

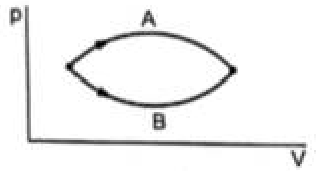
Thus, work done by the object W

$W = P\Delta V$

During expansion volume increases. Hence, work done will be positive and equals $P\Delta V$.

3. Question

Figure shows two process A and B on a system. Let ΔQ_1 and ΔQ_2 be the heat given to the system in processes A and B respectively. Then



A. $\Delta Q_1 > \Delta Q_2$

B. $\Delta Q_1 = \Delta Q_2$

C. $\Delta Q_1 < \Delta Q_2$

D. $\Delta Q_1 \leq \Delta Q_2$

Answer

Initial and final points of both processes A and B are same. Therefore, internal energy in both the processes will be the same because internal energy is a state variable, independent of the path taken.

The area under the P-V curve gives the work done on the system. From the graph, it can be seen the area under the curve for process A is more than the area under the curve for process B., therefore, work done on the system in process A ΔW_1 is more than work done on the system in process B ΔW_2

$\Delta W_1 > \Delta W_2$ .....(i)

According to First law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

Where ΔQ=heat supplied/extracted to/from the system

ΔU=change in internal energy

ΔW=work done by/on the system

For process A

$\Delta Q_1 = \Delta U + \Delta W_1$ ...(ii)

For process B

$\Delta Q_2 = \Delta U + \Delta W_2$ ...(iii)

From equation (i) ,(ii) and (iii) it is clear that $\Delta Q_1 > \Delta Q_2$.

4. Question

Refer to figure. Let ΔU_1 and ΔU_2 be the changes in internal energy of the system in the process A and B. Then

A. $\Delta U_1 > \Delta U_2$

B. $\Delta U_1 = \Delta U_2$

C. $\Delta U_1 < \Delta U_2$

D. $\Delta U_1 \neq \Delta U_2$

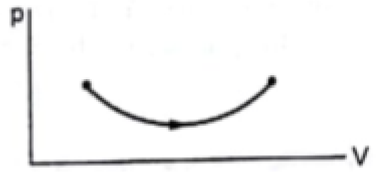
Answer

Initial and final points of both processes A and B are same. Therefore, change in internal energy in both the processes will be the same because internal energy is a state variable, independent of the path taken.

Therefore, $\Delta U_1 = \Delta U_2 = 0$.

5. Question

Consider the process on a system shown in the figure. During the process, the work done by the system.



A. continuously increases

B. continuously decreases

C. first increases then decreases

D. first decreases then increases.

Answer

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure × volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the system W at constant pressure

$W = P\Delta V$

From the graph we can see that $V_2 > V_1$ i.e. final volume is greater than the initial volume.

So, work done by the system continuously increases.

6. Question

Consider the following two statements.

A. If heat is added to a system, its temperature must increase.

B. If positive work in done by a system in a thermodynamic process, its volume must increase.

A. Both A and B are correct

B. A is correct but B is wrong

C. B is correct, but A is wrong

D. Both A and B are wrong

Answer

Statement A: For an isothermal process (where the temperature remains constant) if heat is added to a system temperature will not increases. So, statement A is wrong.

Statement B: We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the system W at constant pressure

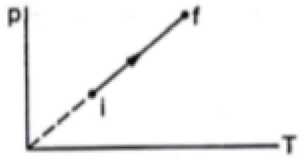
$W = P\Delta V$

$W > 0$ and positive only when $V_2 > V_1$.

Therefore, statement B is correct.

7. Question

An ideal gas goes from the state i to the state f as shown in the figure. The work done by the gas during the process.



A. is positive

B. is negative

C. is zero

D. cannot be obtained from this information.

Answer

Since the graph between P and T is a straight line passing through the origin, therefore $P \propto T$.

P can only be proportional to T when the volume is kept constant. This can be easily proved from the ideal gas equation which is

PV=RT

Since R is already a constant, if volume also becomes constant then $P \propto T$.

Constant volume implies $\Delta V=0$.

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

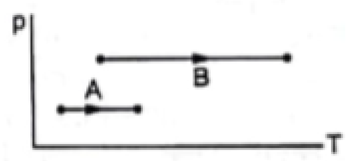
Thus, work done by the system W

$W=P\Delta V$

For $\Delta V=0$, W=0.

8. Question

Consider two processes on a system as shown in the figure.



The volumes in the initial states are the same in the two processes and the volumes in the final states are also the same. Let ΔW_1 and ΔW_2 be the work done by the system in the processes A and B respectively.

A. $\Delta W_1 > \Delta W_2$

B. $\Delta W_1 = \Delta W_2$

C. $\Delta W_1 < \Delta W_2$

D. Nothing can be said about the relation between ΔW_1 and ΔW_2.

Answer

Given $\Delta V_1 = \Delta V_2$

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the system W

$\Delta W = P\Delta V$

For process A $\Delta W_1 = P_1 \Delta V_1$

For process B $\Delta W_2 = P_2 \Delta V_2$

Since $\Delta V_1 = \Delta V_2$ we can write,

$$\frac{\Delta W_1}{P_1} = \frac{\Delta W_2}{P_2}$$

$$\frac{\Delta W_1}{\Delta W_2} = \frac{P_1}{P_2}$$

$P_1 < P_2$ (from graph)

Therefore $\Delta W_1 < \Delta W_2$.

9. Question

A gas is contained in a metallic cylinder fitted with a piston. The piston is suddenly moved in to compress the gas and is maintained at this position. As time passes the pressure of the gas in the cylinder.

A. increases

B. decreases

C. remains constant

D. increases or decreases depending on the nature of the gas.

Answer

1. When the piston is moved to compress the gas, the volume of the gas reduces and pressure and temperature increases.

2. After that piston is maintained at this position. So, volume becomes constant.

3. But during this sudden compression heat is generated. And since the cylinder is metallic this heat can escape out of the cylinder as metals are good conductors of heat.

4. When heat escapes out of the cylinder, the temperature of the gas will decrease which will, in turn, decrease the pressure of the gas.

Objective II

1. Question

The pressure p and volume V of an ideal gas both increase in a process.

A. Such a process is not possible.

B. The work done by the system is positive.

C. The temperature of the system must increase.

D. Heat supplied to the gas is equal to the change in internal energy.

Answer

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the system W

$\Delta W = P\Delta V$

It is given that volume in increasing i.e. $\Delta V>0$. So, $\Delta W>0$ and positive.

From ideal gas equation, we know that

PV=nRT

Where P=pressure

V=volume

n=number of moles

R=gas constant

T=temperature

So, if both pressure and volume are increasing, then

the temperature must also increase as n and R are constant.

2. Question

In a process on a system, the initial pressure and volume are equal to the final pressure and volume.

A. The initial temperature must be equal to the final temperature.

B. The initial internal energy must be equal to the final internal energy.

C. The net heat given to the system in the process must be zero.

D. The net work done by the system in the process must be zero

Answer

From ideal gas equation, we know that

PV=nRT

Where P=pressure

V=volume

n=number of moles

R=gas constant

T=temperature

If $P_iV_i=P_fV_f$

Then $nRT_i=nRT_f$

$\therefore T_i=T_f$

Since the initial state of the system (P_i, V_i, T_i) is equal to final state of the system (P_f, V_f, T_f), initial and final internal energy will also be equal as internal energy is a state variable.

3. Question

A system can be taken from the initial state p_1, V_1 to the final state p_2, V_2 by two different methods. Let ΔQ and ΔW represent the heat given to the system and the work done by the system. Which of the following must be the same in both the methods?

A. ΔQ

B. ΔW

C. ΔQ + ΔW

D. ΔQ – ΔW

Answer

According to First law of thermodynamics,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied/extracted to/from the system

ΔU=change in internal energy

ΔW=work done by/on the system

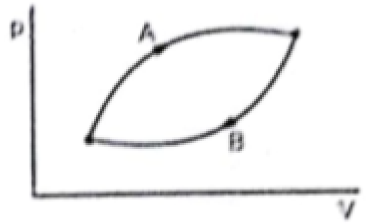
$\therefore \Delta U=\Delta Q-\Delta W$

It is given that initial state (p_1, V_1) and final state (p_2, V_2) is same for both the method.

So, change in internal energy will be the same for both the methods as internal energy is a state variable and is independent of the path taken to achieve a state.

4. Question

Refer to figure. Let ΔU_1 and ΔU_2 be the change in internal energy in processes A and B respectively, ΔQ be the net heat given to the system in process A + B and ΔW be the net work done by the system in the process A + B.



A. $\Delta U_1 + \Delta U_2 = 0$

B. $\Delta U_1 - \Delta U_2 = 0$

C. $\Delta Q - \Delta W = 0$

D. $\Delta Q + \Delta W = 0$

Answer

As seen from the figure initial state of A is the final state of B and vice-versa.

So, the change in internal energy of process A and B will be equal in magnitude but with a negative sign.

So $\Delta U_1 = -\Delta U_2$

$\therefore \Delta U_1 + \Delta U_2 = 0$.

5. Question

The internal energy of an ideal gas decreases by the same amount as the work done by the system.

A. The process must be adiabatic

B. The process must be isothermal

C. The process must be isobaric

D. The temperature must decrease

Answer

1. The internal energy of an ideal gas is proportional to the temperature. Since internal energy is decreasing, therefore, the temperature must decrease.

2. According to the question, change in internal energy is equal to change in work done.

3. According to First law of thermodynamics,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied/extracted to/from the system

ΔU=change in internal energy

ΔW=work done by/on the system

4. We know for adiabatic process $\Delta Q=0$. So, from first law $\Delta U=\Delta W$.

Exercises

1. Question

A thermally insulated, closed copper vessel contains water at 15°C. When the vessel is shaken vigorously for 15 minutes, the temperature rises to 17°C. The mass of the vessel is 100 g and that of the water is 200 g. The specific heat capacities of copper and water are 420 J kg^{-1} K^{-1} and 4200 J kg^{-1} K^{-1} respectively.

Neglect any thermal expansion.

(a) How much heat is transferred to the liquid-vessel system?

(b) How much work has been done on this system?

(c) How much is the increase in internal energy of the system?

Answer

Given

Initial temperature of water T_1=15°C =288K

Final temperature of water T_2=17°C =290K

Specific heat capacity of copper c_c=420 J kg^{-1} K^{-1}

Specific heat capacity of water c_w=4200 J kg^{-1} K^{-1}

Mass of copper vessel m_c=100g = 100×10^{-3}kg

Mass of water m_w=200g = 200×10^{-3}kg

a) It is given that copper vessel is thermally insulated. Therefore, no heat from the surroundings can be transferred to the liquid-vessel system.

b) Work done on this system will be

$\Delta W=m_w c_w \Delta T + m_c c_c \Delta T$

$\Delta T=T_2-T_1=290-288=2K$

So,

$\Delta W = 200\times10^{-3}\times4200\times2 + 100\times10^{-3}\times420\times2$

$\Delta W=1680 + 84=1764J$

∴ work done on the system is 1764J.

c) From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

From part (a) we have concluded that $\Delta Q=0$

So, first law becomes

$\Delta U=-\Delta W$

Work done on the system $\Delta W=1764$ J

So, work done by the system $\Delta W=-1764J$

$\Delta U = -(-1764) = 1764J$

∴ Increase in internal energy of the system = 1764J.

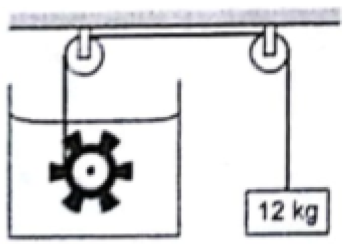
2. Question

Figure shows a paddle wheel coupled to a mass of 12 kg through fixed frictionless pulleys. The paddle is immersed in a liquid of heat capacity 4200 JK^{-1} kept in an adiabatic container. Consider a time interval in which the 12 kg block falls slowly through 70 cm.

(a) How much heat is given to the liquid?

(b) How much work is done on the liquid?

(c) Calculate the rise in the temperature of the liquid neglecting the heat capacity of the container and the paddle.



Answer

Given

Mass attaches to the pulley m = 12kg

Heat capacity of liquid s =4200 JK^{-1}

Height through which mass fall= 70cm=0.7m

a) Paddle immersed in liquid is kept in an adiabatic container. So, no heat can be either supplied or extracted to the liquid. Therefore, heat given to liquid is zero.

b) As no heat is supplied to liquid and pulley is frictionless, work done on the liquid will be equal to the potential energy of the mass attached to the pulley.

Work done on the liquid = potential energy of mass

Potential energy of mass= mgh

Where g=acceleration due to gravity=$10ms^{-2}$

Potential energy of mass= 12×10×0.7=84J

∴ work done on liquid =84J.

c) The mechanical work calculated in the second part will be converted into heat. This heat will be supplied to liquid and due to which temperature of the liquid will rise.

We know that,

Heat capacity $s = \frac{\Delta Q}{\Delta T}$

Where ΔQ = heat supplied

ΔT=rise in temperature

$$\Delta T = \frac{\Delta Q}{s}$$

Since work done is equal to heat supplied, therefore

$$\Delta T = \frac{84}{4200} = \frac{1}{50} = 0.02K$$

∴ the rise in temperature of the liquid will be 0.02K.

3. Question

A 100 kg block is started with a speed of 2.0 ms^{-1} on a long, rough belt kept fixed in a horizontal position. The coefficient of kinetic friction between the block and the belt is 0.20.

(a) Calculate the change in the internal energy of the block-belt system as the block comes to a stop on the belt.

(b) Consider the situation com a frame of reference moving at 2.0 m s^{-1} along the initial velocity of the block. As seen from this frame, the block is gently put on a moving belt and in due time the block starts moving with the belt at 2.0 ms^{-1}. Calculate the increase in the kinetic energy of the block as it stops slipping past the belt.

(c) Find the work done in this frame by the external force holding the belt.

Answer

Given

Mass of block m=100kg

Initial velocity u=2 m/s

Coefficient of kinetic friction μ = 0.20

a) Since the block comes to stop final velocity will be zero.

So, final velocity v=0 m/s

When the block is moving over belt there is kinetic friction between the lower surface of the block and upper surface of the belt. And we know that heat is produced due to friction between two surfaces. Now because of this heat, the internal energy of block will change.

So,

Kinetic energy lost in heat due to friction = change in the internal energy

Kinetic energy lost = initial kinetic energy- final kinetic energy

$$K.E\ lost = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 100 \times 2 \times 2 - 0 = 200J$$

∴change in internal energy is 200J.

b) Given

The velocity of the frame of reference u_o= 2m/s

So, in this frame of reference initial and final velocity of the block will change.

New initial velocity u'=u-u_o =2-2 = 0m/s

New final velocity v' = v-u_o =0-2 =-2m/s

So, increase in kinetic energy = (final – initial) kinetic energy

$$=\frac{1}{2}mv'^2-\frac{1}{2}mu'^2$$

$$=\frac{1}{2}\times 100\times(-2)^2-0=200J$$

Increase in kinetic energy in com frame of reference is 200J.

c) Total work done in com frame of reference will be work done due to friction plus work done to give final velocity.

We know that force of friction f=μN

Where μ= coefficient of friction =0.02

N=normal reaction =mg

f = 0.02×100×10 = 200N

from newton's second law of motion

force= mass ×acceleration

so,

200=100×acceleration

$$acceleration\ 'a'=\frac{200}{100}=2ms^{-2}$$

Using the third equation of motion,

$v'^2-u'^2=2as$

where s=displacement as seen in com frame of reference

$(-2)^2-0=2\times2\times s$

s=1m

work done due to friction W_f =force ×displacement

$W_f=200\times1=200J$

Now to calculate work done to give final velocity, we will work-energy theorem.

According to work-energy theorem,

Work done = change in kinetic energy

So,

$$W'=\frac{1}{2}mv'^2-\frac{1}{2}mu'^2=\frac{1}{2}\times 100\times(-2)^2-0=200J$$

Total work done $W=W' + W_f$ =200+200=400J.

∴ work done in com frame of reference= 400J.

4. Question

Calculate the change in internal energy of a gas kept in a rigid container when 100 J of heat is supplied to it.

Answer

Given

Heat supplied ΔQ = 100J

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the gas

$\Delta W=P\Delta V$

Since the gas is kept in a rigid container, therefore ΔV=0 in this case.

So, ΔW=0

Thus, first law will become

$\Delta Q=\Delta U = 100J$

∴ change in internal energy will be 100J.

5. Question

The pressure of gas changes linearly with volume from 10 kPa, 200 cc to 50 kPa, 50 cc.

(a) Calculate the work done by the gas.

(b) If no heat is supplied or extracted from the gas, what is the change in the internal energy of the gas?

Answer

Given

Initial pressure P_1=10kPa=10×10^3Pa

Final pressure P_2=50kPa=50×10^3Pa

Initial volume V_1=200cc=$200\times10^{-6}m^3$

Final volume V_2=50cc= $50\times10^{-6}m^3$

a) We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure ×volume

Let change in the volume of system = $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the gas

$\Delta W = P\Delta V$

Here we have given two values of pressure. So, we will take the average value of pressure

Average pressure P

$$P = \frac{P_1 + P_2}{2}$$

$$P = \frac{10 \times 10^3 + 50 \times 10^3}{2} = 30 \times 10^3 Pa$$

Therefore, $\Delta W = 30\times10^3\times(50-200)\times10^{-6}$

$\Delta W = -4.5J$

∴ Work done by the gas is -4.5J.

b) Given that no heat is supplied or extracted from the gas.

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Since $\Delta Q=0$

Therefore $\Delta U = -\Delta W = -(-4.5)\ J=4.5J$

∴ the change in internal energy of the gas is 4.5J.

6. Question

An ideal gas is taken from an initial state i to a final state f in such a way that the ratio of the pressure to the absolute temperature remains constant. What will be the work done by gas?

Answer

Given

Initial pressure P_i

Final pressure P_f

Initial temperature T_i

Final temperature T_f

It is given that

$$\frac{P_i}{T_i}=\frac{P_f}{T_f}\ \ldots\ldots(i)$$

From ideal gas equation, we know

PV=nRT

Where V=volume of gas

R=gas constant

n= number of moles

applying ideal gas equation for both processes, we get

$$\therefore \frac{P_i}{T_i} = \frac{nR}{V_i} \dots\dots (ii)$$

And

$$\frac{P_f}{T_f} = \frac{nR}{V_f} \dots\dots (iii)$$

From equation (i), (ii) and (iii), we get

$$\frac{nR}{V_i} = \frac{nR}{V_f}$$

$\therefore V_i = V_f$

We know that,

Work done = force ×displacement

$$pressure = \frac{force}{area}$$

Volume = area ×displacement

Therefore,

Work done=pressure × volume

Let change in the volume of system = $\Delta V = V_f - V_i$

Pressure =P

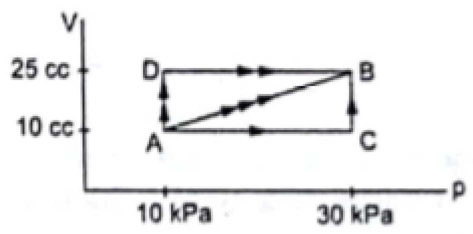
Thus, work done by the gas

$\Delta W = P\Delta V = P\ (V_f - V_i) = 0$

$\therefore$ work done by the gas is zero.

7. Question

Figure shows three paths through which a gas can be taken from the state A to the state B. Calculate the work done by the gas in each of the three paths.



Answer

We know that work done by the gas is given as

$\Delta W = P\Delta V$

From graph we can write

$V_A = V_C$ = 10cc=$10\times10^{-6}m^3$

$V_D=V_B$ = 25cc= $25\times10^{-6}m^3$

$P_B=P_C$=30kPa= 30×10^3Pa

$P_A=P_D$=10kPa=10×10^3Pa

Work done in path ADB $W_{ADB}=W_{AD}+W_{DB}$

$W_{ADB} = P_A(V_D-V_A) + 0$ ($\because W_{DB}=0$ because $V_D=V_B$)

$=10\times10^3\times(25-10)\times10^{-6}$

=0.15J

Work done in path AB $W_{AB}= P_{avg}(V_B-V_A)$

$$P_{avg} = \frac{P_B + P_A}{2} = \frac{10\times10^3 + 30\times10^3}{2} = 20\times10^3 Pa$$

$W_{AB}=20\times10^3\times(25-10)\times10^{-6}$

=0.30J

Work done in path ACB $W_{ACB}= W_{AC}+W_{BC}$

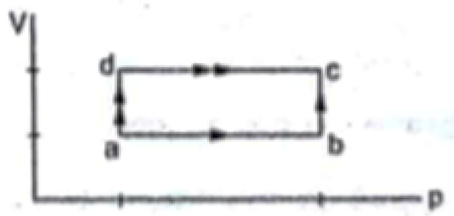
$W_{ACB} = 0+P_B(V_B-V_C)$ ($\because W_{AC}=0$ because $V_A=V_C$)

$=30\times10^3\times(25-10)\times10^{-6}$

=0.45J

8. Question

When a system is taken through the process abc shown in figure 80J of heat is absorbed by the system and 30 J of work is done by it. If the system does 10 J of work during the process adc, how much heat flows into it during the process?



Answer

Given

Heat absorbed in process abc ΔQ_1=80J

Work done by the system in process abc ΔW_1=30J

Work done by the system in process adc ΔW_2=10J

Let heat absorbed into the system during process adc =ΔQ_2

Now initial point a and final point c is the same for both the processes is the same. So, change in internal energy will be the same for both the process, as internal energy is a state function independent of the path taken.

Therefore,

$\Delta U_1=\Delta U_2=\Delta U$ ……(i)

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Using first law of thermodynamics for process abc

$\Delta Q_1=\Delta U_1+\Delta W_1$

$\Delta U_1=\Delta Q_1-\Delta W_1=80-30=50J$

Using first law of thermodynamics for process adc

$\Delta Q_2=\Delta U_2+\Delta W_2$

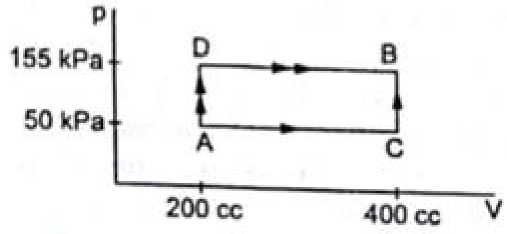
$=\Delta U_1+\Delta W_2$ (from (i))

= 50+10=60J

∴ heat absorbed into the system during process adc= 60J.

9. Question

50 cal of heat should be supplied to take a system from the state A to the state B through the path ACB as shown in the figure. Find the quantity of heat to be supplied to take it from A to B via ADB.



Answer

From graph we can write

$V_A = V_D$ = 200cc=200×10^{-6}m^3

$V_B = V_C$ = 400cc= 400×10^{-6}m^3

$P_B = P_D$=155kPa= 155×10^{3}Pa

$P_A = P_C$=50kPa=50×10^{3}Pa

Given

Heat absorbed in process ABC ΔQ_1=50cal= 50×4.2 J=210J

Let heat absorbed into the system during process ADC =ΔQ_2

We know that work done by the gas is given as

$\Delta W = P\Delta V$

Work done in path ACB $W_{ACB} = \Delta W_1 = W_{AC} + W_{BC}$

$\Delta W_1 = P_A(V_C - V_A) + 0$ (W_{BC}=0 because $V_B = V_C$)

$=50\times10^3\times(400-200)\times10^{-6}$

=10J

Work done in path ADB $W_{ADB} = \Delta W2 = W_{AD} + W_{DB}$

$\Delta W_2 = P_B (V_D - V_B) + 0$ (∵ W_{AD}=0 because $V_A = V_D$)

$=155\times10^3\times(400-200)\times10^{-6}$

=31 J

Now initial point A and final point C is the same for both the processes is the same. So, change in internal energy will be the same for both the process, as internal energy is a state function independent of the path taken.

Therefore,

$\Delta U_1 = \Delta U_2 = \Delta U$ ……(i)

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Using first law of thermodynamics for process ABC

$\Delta Q_1=\Delta U_1+\Delta W_1$

$\Delta U_1=\Delta Q_1-\Delta W_1=210-10=200J$

Using first law of thermodynamics for process adc

$\Delta Q_2=\Delta U_2+\Delta W_2$

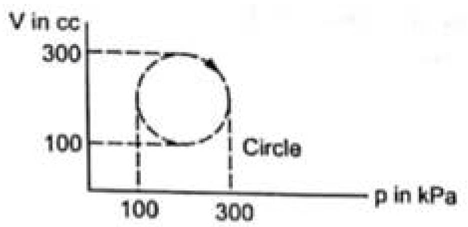
$=\Delta U_1+\Delta W_2$ (from (i))

$=200+31=231J.$

∴ heat supplied to the system during process ADC=231J.

10. Question

Calculate the heat absorbed by a system in going through the cyclic process shown in the figure.



Answer

We know that in the cyclic process the system returns to its initial state. So, change internal energy in the cyclic process will be zero as internal energy is a state function.

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Since $\Delta U=0$, first law becomes

$\Delta Q = \Delta W$

In a PV graph work done is equal to the area under the curve.

ΔW= area of a circle

$\Delta Q = \Delta W$=area of circle

Diameter of the circle =300-100=200

$$radius = \frac{diameter}{2} = \frac{200}{2} = 100$$

Area of the circle $= \pi \times (\text{radius})^2$

$= \pi \times 100 \times 100 \times 10^{-6} \times 10^3$

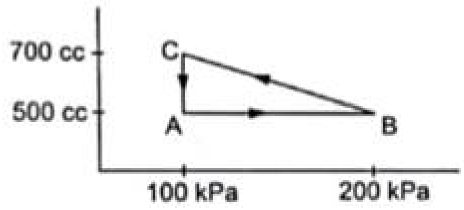
($10^{-6} \times 10^3$ is because volume and pressure are given in cc and kPa respectively)

Area of circle =3.14×10=31.4

∴ Heat absorbed by a system = 31.4J.

11. Question

A gas is taken through a cyclic process ABCA as shown in the figure. If 2.4 cal of heat is given in the process, what is the value of J?



Answer

'J' is mechanical equivalent of heat a conversion factor between two different units of energy: calorie to joule

From the graph we can write

$V_A = V_B = 500cc = 500 \times 10^{-6} m^3$

$V_C = 700cc = 700 \times 10^{-6} m^3$

$P_A = P_C = 100kPa = 100 \times 10^3$ Pa

$P_B = 200kPa = 200 \times 10^3 Pa$

We know that work done by the gas is given as

$\Delta W = P \Delta V$

Work done in path AB=0 as $V_A=V_B$.

Work done in path CA $\Delta W_1=P_A(V_A-V_C)$

$=100\times10^3\times(500-700)\times10^{-6}$

=-20J

Work done in path BC $\Delta W_2= P_{avg}(V_C-V_B)$

$$P_{avg}=\frac{P_B+P_C}{2}=\frac{100\times10^3+200\times10^3}{2}=150\times10^3 Pa$$

$\Delta W_2=150\times10^3\times(700-500)\times10^{-6}$

=30J

Total work done in process ABCA=$\Delta W=\Delta W_1+\Delta W_2$

=30-20=10J

We know that in the cyclic process the system returns to its initial state. So, change internal energy in the cyclic process will be zero as internal energy is a state function.

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Since $\Delta U=0$, first law becomes

$\Delta Q=\Delta W$=10J

But it is given in question that ΔQ=2.4cal

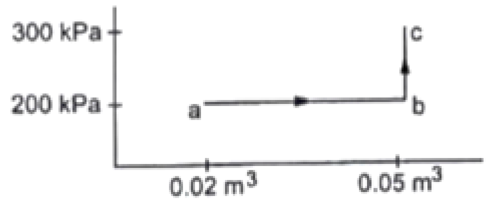
So, 2.4×J=10Joule

$$J=\frac{10}{2.4}=4.17 Joule/cal$$

∴ value of 'J' is 4.17J/cal.

12. Question

A substance is taken through the process abc as shown in the figure. If the internal energy of the substance increases by 5000 J and heat of 2625 cal is given to the system, calculate the value of J.



Answer

'J' is mechanical equivalent of heat a conversion factor between two different units of energy: calorie to the joule.

Given

Heat given to system =2625cal =2625×J J

Change in internal energy= 5000J

From graph

$V_a=0.02m^3$

$V_b=V_c=0.05m^3$

$P_a=P_b=200kPa=200\times10^3Pa$

$P_c=300kPa=300\times10^3Pa$

We know that work done by the gas is given as

$\Delta W=P\Delta V$

Work done in process abc=$\Delta W=W_{ab}+W_{bc}$

$\Delta W=P_a(V_b-V_a)+0$ ($\because$ $W_{bc}=0$ because $V_b=V_c$)

$\Delta W=200\times10^3\times(0.05-0.02)=6000J$

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

$\Delta Q=5000+6000=11000J$

But ΔQ=2625cal =2625×J J

Therefore,

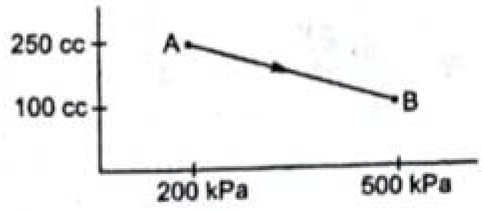
2625×J=11000

$$J = \frac{11000}{2625} = 4.19 joule/cal$$

∴ value of 'J' is 4.19joule/cal.

13. Question

A gas is taken along the path AB as shown in the figure. If 70 cal of heat is extracted from the gas in the process, calculate the change in the internal energy of the system.



Answer

Given

Heat extracted from the system ΔQ=-70cal=-70×4.2=-294J

The negative sign is because heat is extracted from the system.

From graph

$V_A=250cc=250\times10^{-6}m^3$

$V_B=100cc=100\times10^{-6}m^3$

$P_A=200kPa=200\times10^3Pa$

$P_B=500kPa=500\times10^3Pa$

We know that work done by the gas is given as

ΔW=PΔV

Here since we have two values of pressure we will take average pressure.

$$P_{avg} = \frac{P_A + P_B}{2} = \frac{200 \times 10^3 + 500 \times 10^3}{2} = 350 \times 10^3 Pa$$

$\Delta W=P_{avg}(V_A-V_B)$

$= 350\times10^3\times (250-100) \times10^{-6}$

=-52.5J

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Therefore,

$\Delta U=-294-(-52.5)=-241.5J$

∴ change in internal energy is -241.5J.

14. Question

The internal energy of a gas is given by U = 1.5 pV. It expands from 100 cm^3 to 200 cm^3 against a constant pressure of 1.0×10^5 Pa. Calculate the heat absorbed by the gas in the process.

Answer

Given

Constant pressure $p=1.0\times10^5$ Pa

Change in volume $\Delta V=(200-100)\times10^{-6}m^3=10^{-4}m^3$

Internal energy U= 1.5pV

So, change in internal energy $\Delta U=1.5p\Delta V$

$=1.5\times1.0\times10^5\times10^{-4}$

$=15J$

We know that work done by the gas is given as

$\Delta W=p\Delta V=1.0\times10^5\times10^{-4}=10J$

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Therefore,

$\Delta Q=15+10=25J$

∴ heat absorbed by the system=25J.

15. Question

A gas is enclosed in a cylindrical vessel fitted with a frictionless piston. The gas is slowly heated for some time. During the process, 10 J of heat is supplied, and the piston is found to move out 10 cm. Find the increase in the internal energy of the gas. The area of cross section of the cylinder = 4 cm^2 and the atmospheric pressure = 100 kPa.

Answer

Given

Heat supplied to system $\Delta Q = 10J$

Atmospheric pressure $P= 100kPa =100\times10^3Pa$

Displacement of the piston d=10cm=0.1m

Area of cross section of cylinder $A=4cm^2 =4\times10^{-4}m^2$

Gas will expand when is heat to the system. Therefore, the volume of gas expanded ΔV

ΔV=area ×displacement

=A ×d

$\Delta V=4\times0.1\times10^{-4}=40\times10^{-6}m^3$

We know that work done by the gas is given as

$\Delta W=P\Delta V$

$=100\times10^3\times40\times10^{-6}$

$\Delta W =4J$

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

$\Delta U=\Delta Q-\Delta W = 10-4 =6J$

Thus, the increase in internal energy is 6J.

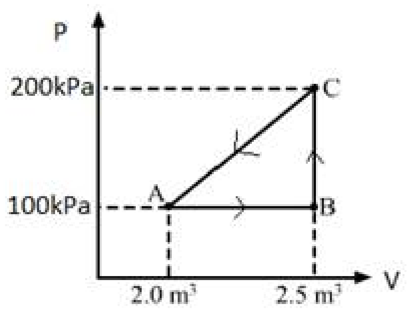
16. Question

A gas is initially at a pressure of 100 kPa and its volume is 2.0 m^3. Its pressure is kept constant and the volume is changed from 2.0 m^3 to 2.5 m^3. Its volume is now kept constant and the pressure is increased from 100 kPa to 200 kPa. The gas is brought back to its initial state, the pressure varying linearly with its volume.

(a) Whether the heat is supplied to or extracted from the gas in the complete cycle?

(b) How much heat was supplied or extracted?

Answer



From graph

$V_a=2m^3$

$V_b=V_c=2.5m^3$

$P_a=P_b=100kPa=100\times10^3Pa$

$P_c=200kPa=200\times10^3Pa$

Work done in process ABCA = area enclosed by the triangle ABC

$\Delta W=0.5\times BC\times AB$

Where BC= height of the triangle

AB= base of triangle

So,

$\Delta W=0.5\times(200-100)\times10^3\times(2.5-2)$

$\Delta W=0.5\times100\times0.5\times10^3=25000J$

a) Process ABCA is a cyclic process. The system is brought back to its initial state. Since internal energy is a state function, change in internal energy will be zero.

So, $\Delta U=0$

From first law of thermodynamics, we know that,

$\Delta Q = \Delta U + \Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

$\therefore \Delta Q = \Delta W = 25000J$

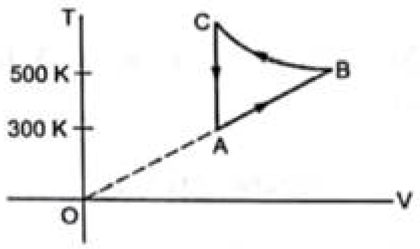
Since work done is positive, work is done by the gas. When work is done by the gas heat is supplied to the system.

b) Amount of heat supplied = work done by the gas in the cyclic process

$\therefore \Delta Q = \Delta W = 25000J$.

17. Question

Consider the cyclic process ABCA, shown in the figure, performed on a sample 2.0 mol of an ideal gas. A total of 1200 J of heat is withdrawn from the sample in the process. Find the work done by the gas during the part BC.



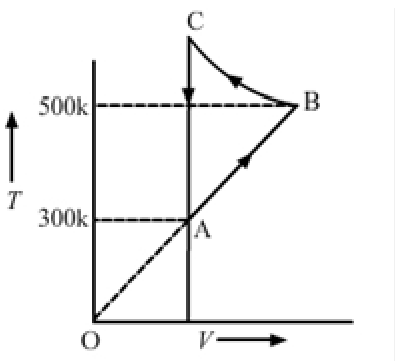
Answer

Given

Heat extracted from the system $\Delta Q = -1200J$

(negative sign is because heat is extracted from the system)

Number of moles of the gas n=2.0



From the graph we can write

T_A=300K

T_B=500K

V_A=V_C

We know that work done by the gas is given as

ΔW=$P\Delta V$

Where ΔV =change in volume

P =pressure

So, work done along line CA will be zero, as V_A=V_C.

Thus, total work done will be

$\Delta W=W_{AB}+W_{BC}$

$W_{AB} = P(V_B-V_A)$

But we know that ideal gas equation is

PV=nRT

Where n= number of moles

R=gas constant

T=temperature

$\therefore P\Delta V=nR\Delta T$

Thus, we can write $W_{AB} = P(V_B-V_A) = nR(T_B-T_A)$

$\therefore \Delta W= nR(T_B-T_A) + W_{BC}$

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Process ABCA is a cyclic process. The system is brought back to its initial state. Since internal energy is a state function, change in internal energy will be zero.

So, ΔU=0.

So, first law becomes

$\Delta Q = \Delta W$

$\Delta Q = nR(T_B - T_A) + W_{BC}$

We know that R=8.31J/K mol

$W_{BC} = \Delta Q - nR(T_B - T_A)$

$= -1200 - 2 \times 8.31 \times (500-300)$

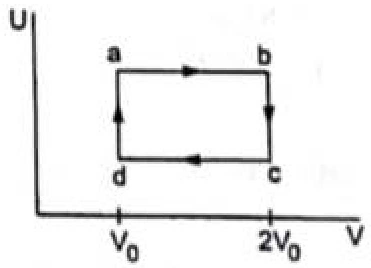
$= -1200 - 3324$

$= -4524J$

Thus, work done along path BC =-4524J

18. Question

Figure shows the variation in the internal energy U with the volume V of 2.0 mol of an ideal gas in a cyclic process abcda. The temperatures of the gas at b and c are 500 K and 300 K respectively. Calculate the heat absorbed by the gas during the process.



Answer

Given

Number of moles n=2

Temperature at b T_b=500K

Temperature at c T_c=300K

From the graph it is clear that

$T_b = T_a$ and $T_d = T_c$

Thus, path ab and cd are isothermal paths.

We know that work done by the gas is given as

$\Delta W = P\Delta V$

Where ΔV =change in volume

P =pressure

Again, from the graph we can see that $\Delta V=0$ for path bc and da.

Therefore, work done along path bc and da are zero.

So, total work done $\Delta W=W_{ab}+W_{cd}$

We know that work done in an isothermal process is given as

$$W = nRT\ ln\frac{V_f}{V_i}$$

Where n=number of moles

R=gas constant =8.31J/Kmol

T=temperature

V_f=final volume

V_i=initial volume

$$W_{ab} = nRT_a\ ln\frac{V_b}{V_a}$$

$$W_{ab} = 2 \times 8.31 \times 500 \times ln\frac{2V_o}{V_o}$$

W_{ab} = 8310×ln2

Similarly,

$$W_{cd} = nRT_c\ ln\frac{V_d}{V_c}$$

$$W_{cd} = 2 \times 8.31 \times 300 \times ln\frac{V_o}{2V_o}$$

$$W_{cd} = -2 \times 8.31 \times 300 \times ln\frac{2V_o}{V_o}$$

W_{cd} = -4986 ×ln2

So, total work done $\Delta W=W_{ab}+W_{cd}$

=8310×ln2 - 4986×ln2

=3324×ln2

ΔW =3324×0.693=2304.02J

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

Process ABCA is a cyclic process. The system is brought back to its initial state. Since internal energy is a state function, change in internal energy will be zero.

So, ΔU=0.

So, first law becomes

ΔQ=ΔW=2304.02J

Thus, heat absorbed by the system is 2304.02J.

19. Question

Find the change in the internal energy of 2 kg of water as it is heated from 0°C to 4°C. The specific heat capacity of water is 4200 J kg^{-1} K^{-1} and its densities at 0°C and 4°C are 999.9 kg m^{-3} and 1000 kg m^{-3} respectively. Atmospheric pressure = 10^5 Pa.

Answer

Given

Mass of water m = 2kg

Change in temperature ΔT =4°C -0°C =4°C

Specific heat capacity of water c =4200 J kg^{-1} K^{-1}

Density of water at 0°C=999.9 kg m^{-3}

Density of water at 4°C=1000 kg m^{-3}

Atmospheric pressure P = 10^5 Pa.

We know that specific heat capacity is given by

$$c = \frac{\Delta Q}{m\Delta T}$$

Where ΔQ = heat supplied to the system

Therefore, ΔQ=cmΔT

=4200×2×4=33600J

We know that work done by the gas is given as

ΔW=PΔV

Where ΔV =change in volume

P =pressure

Also,

$$volume = \frac{mass}{density}$$

Volume at 0^oC V_1

$$V_o = \frac{m}{density\ at\ 0°C} = \frac{2}{999.9} m^3$$

Similarly, volume at 4^oC V_2

$$V_2 = \frac{m}{density\ at\ 4°C} = \frac{2}{1000} m^3$$

∴ $\Delta W = P(V_2 - V_1)$

$$\Delta W = 10^5 \times \left(\frac{2}{1000} - \frac{2}{999.9}\right) = -0.02J$$

From first law of thermodynamics, we know that,

ΔQ=ΔU+ΔW

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

ΔU=ΔQ-ΔW

=33600-(-0.02)

=33599.98J

Thus, change in internal energy is 33599.98J.

20. Question

Calculate the increase in the internal energy of 10g of water when it is heated from 0°C to 100°C and converted into steam at 100 kPa. The density of steam = 0.6 kg m^{-3}. The specific heat capacity of water = 4200 J kg^{-1} $°C^{-1}$ and the latent heat of vaporization of water = 2.25×10^6 J kg^{-1}.

Answer

Given

The density of steam $\rho' = 0.6\ kg\ m^{-3}$

Mass of water m=10g =0.010kg

Specific heat capacity of water c = 4200 J $kg^{-1}\ °C^{-1}$

latent heat of vaporization of water L = $2.25 \times 10^6\ J\ kg^{-1}$.

Pressure P =100kPa =100×10^5Pa

Change in temperature ΔT= (100-0) ^oC =100^oC

Density of water ρ =1000 kg m^{-3}

We know that specific heat capacity is given by

$$c = \frac{\Delta Q}{m\Delta T}$$

Where ΔQ = heat supplied to the system

Therefore, ΔQ= cmΔT

Also, ΔQ =mL

Where m= mass of the substance

L=latent heat

Therefore, ΔQ=mL + cmΔT

$=0.010\times2.25 \times 10^6 + 4200\times0.01\times100$

=22500+4200

=26700J

We know that work done by the gas is given as

ΔW=PΔV

Where ΔV =change in volume

P =pressure

Also,

$$volume = \frac{mass}{density}$$

$$\Delta W = P \times \left(\frac{m}{\rho'} - \frac{m}{\rho}\right)$$

$$\Delta W = 10^5 \times \left(\frac{0.01}{0.06} - \frac{0.01}{1000}\right)$$

$\Delta W = 10^5 \times 0.01699 = 1699$ J

From first law of thermodynamics, we know that,

$\Delta Q = \Delta U + \Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

$\Delta U = \Delta Q - \Delta W$

=26700-1699

=25001J

Thus, change in internal energy is 25001J.

21. Question

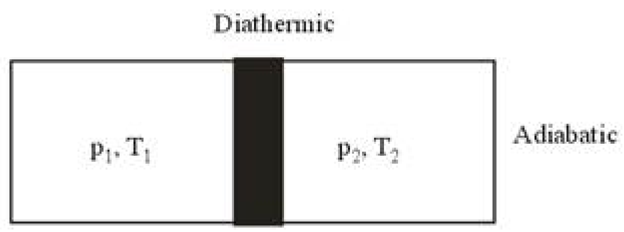
Figure shows a cylindrical tube of volume V with adiabatic walls containing an ideal gas. The internal energy of this ideal gas is given by 1.5 nRT. The tube is divided into two equal parts by a fixed diathermic wall. Initially, the pressure and the temperature are p_1, T_1 on the left and p_2, T_2 on the right. The system is left for sufficient time so that the temperature becomes equal on the two sides.

(a) How much work has been done by the gas on the left part?

(b) Find the final pressures on the two sides.

(c) Find the final equilibrium temperature.

(d) How much heat has flown from the gas on the right to the gas on the left?



Answer

a) According to the question, the diathermic separator between both the part is fixed. So, no change in volume will be observed. And as we know work done on gas is $P\Delta V$. Therefore, no work will be done on the left part during the process as the volume is not changing.

First, we will calculate the final temperature and then final pressure.

(c) Given

Pressure of left chamber $=P_1$

Pressure of right chamber $=P_2$

Temperature of left chamber $=T_1$

Temperature of right chamber $=T_2$

Let the number of moles in the left chamber be n_1

Number of moles in the right chamber be n_2

Diathermic wall has divided the tube in two equal part. So, the volume of the left and the right chamber will be V/2.

Applying ideal gas equation in the left chamber,

$$\frac{P_1V}{2} = n_1RT_1$$

$$n_1 = \frac{P_1V}{2RT_1}$$

Similarly applying ideal gas equation in the right chamber,

$$\frac{P_2V}{2} = n_2RT_2$$

$$n_2 = \frac{P_2V}{2RT_2}$$

Total number of moles $n=n_1+n_2$

$$n = \frac{P_1V}{2RT_1} + \frac{P_2V}{2RT_2}$$

$$n = \frac{V}{2R}\left(\frac{P_1}{T_1} + \frac{P_2}{T_2}\right)$$

$$n = \frac{V}{2R}\left(\frac{T_2P_1 + P_2T_1}{T_1T_2}\right) \quad \ldots..(i)$$

The internal energy of ideal gas is given as

$U=nC_vT$

Where C_v=molar specific heat at constant volume

T=temperature.

According to question,

$U=1.5nRT$

$\therefore nC_vT=1.5nRT$

So, $C_v=1.5R$

The internal energy of the left chamber $U_1=n_1C_vT_1$

The internal energy of right chamber $U_2= n_2C_vT_2$

Total internal energy $U= U_1+U_2$

$1.5nRT= n_1C_vT_1+ n_2C_vT_2$

$1.5nRT=C_v(n_1T_1+n_2T_2)$

Substituting the value of C_v

$1.5nRT=1.5R(n_1T_1+n_2T_2)$

$nT=n_1T_1+n_2T_2$

substituting the value of n_1 and n_2 in above equation

$$nT = \frac{P_1V}{2RT_1} \times T_1 + \frac{P_2V}{2RT_2} \times T_2$$

$$nT = \frac{V(P_1 + P_2)}{2R}$$

$$T = \frac{V(P_1 + P_2)}{2Rn}$$

Substituting the value of n in the above equation,

$$T = \frac{V(P_1 + P_2)}{2R} \times \frac{T_1T_2}{T_2P_1 + P_2T_1} \times \frac{2R}{V}$$

$$T = \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$$

Thus, final equilibrium temperature $T=\frac{(T_1T_2)\times(P_1+P_2)}{T_2P_1+P_2T_1}$.

(b) Now we will find final pressures on both sides.

Let final pressure of left chamber P_1'

Final pressure of right chamber P_2'

Applying ideal gas equation in the left chamber before and after equilibrium

$$\frac{P_1 V}{2} = nRT_1 \quad .....(i)$$

$$\frac{P'_1 V}{2} = nRT \quad .....(ii)$$

From equation (i) and (ii),

$$\frac{P_1}{T_1} = \frac{P'_1}{T}$$

$$P'_1 = \frac{P_1}{T_1} T$$

Substituting the value of T,

$$P'_1 = \frac{P_1}{T_1} \times \frac{(T_1 T_2) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1}$$

Applying ideal gas equation in the right chamber before and after equilibrium

$$\frac{P_2 V}{2} = nRT_2 \quad .....(iii)$$

$$\frac{P'_2 V}{2} = nRT \quad .....(iv)$$

From equation (iii) and (iv),

$$\frac{P_2}{T_2} = \frac{P'_2}{T}$$

$$P'_2 = \frac{P_2}{T_2} T$$

Substituting the value of T,

$$P'_2 = \frac{P_2}{T_2} \times \frac{(T_1 T_2) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1}$$

(d) The internal energy of ideal gas is given as

$U = nC_V T$

Where C_V=molar specific heat at constant volume

T=temperature.

As stated in part (a) no work will be done on either chamber of the vessel as the diathermic separator is fixed.

So, $\Delta W = 0$ for the right chamber of the tube.

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

$\Delta Q=\Delta U$

Change in internal energy of the right chamber after equilibrium has reached will be

$\Delta U=n_2C_vT_2 - n_2C_vT$

Substituting the value of n_2, C_v and T in the above equation

$$\Delta U=\frac{P_2V}{2RT_2}\times 1.5\times R\times T_2-\frac{P_2V}{2RT_2}\times 1.5\times R\times\frac{(T_1T_2)\times(P_1+P_2)}{T_2P_1+P_2T_1}$$

$$\Delta U=\frac{P_2V}{2RT_2}\times 1.5\times R\times T_2\left(1-\frac{(T_1)\times(P_1+P_2)}{T_2P_1+P_2T_1}\right)$$

$$\Delta Q=\Delta U=\frac{P_2V}{2}\times 1.5\left(1-\frac{(T_1)\times(P_1+P_2)}{T_2P_1+P_2T_1}\right)$$

$$\Delta Q=\frac{3P_2V}{4}\times\left(\frac{T_2P_1+P_2T_1-(T_1)\times(P_1+P_2)}{T_2P_1+P_2T_1}\right)$$

$$\Delta Q=\frac{3P_2V}{4}\times\left(\frac{T_2P_1-T_1P_1}{T_2P_1+P_2T_1}\right)$$

$$\Delta Q=\frac{3P_1P_2V}{4}\times\left(\frac{T_2-T_1}{T_2P_1+P_2T_1}\right)$$

Thus, heat flown from left to right chamber is $\Delta Q=\frac{3P_1P_2V}{4}\times\left(\frac{T_2-T_1}{T_2P_1+P_2T_1}\right)$.

22. Question

An adiabatic vessel of total volume V is divided into two equal parts by a conducting separator. The separator is fixed in this position. The part on the left contains one mole of an ideal gas (U = 1.5 nRT) and the part on the right contains two moles of the same gas. Initially, the pressure on each side is p. The system is left for sufficient time so that a steady state is reached. Find

(a) the work done by the gas in the left part during the process.

(b) the temperature on the two sides in the beginning,

(c) the final common temperature reached by the gases,

(d) the heat given to the gas in the right part and

(e) the increase in the internal energy of the gas in the left part.

Answer

a) According to the question, conducting separator between both the part is fixed. So, no change in volume will be observed. And as we know work done a gas is $P\Delta V$. Therefore, no work will be done on the left part during the process as the volume is not changing.

b) Given

The vessel is divided into two equal parts.

So, the volume of left part V_1 and volume of right part V_2 will half of the total volume.

$$V_1 = V_2 = \frac{V}{2}$$

Initial pressure on each side is p

Number of moles on left side $n_1 = 1$

Number of moles on right side $n_2 = 2$

Let initial temperature for the left and right part be T_1 and T_2 respectively.

For the left part, applying the ideal gas equation

$$\frac{pV}{2} = 1 \times R \times T_1$$

$$T_1 = \frac{pV}{2R}$$

Similarly, for the right part

$$\frac{pV}{2} = 2 \times R \times T_2$$

$$T_2 = \frac{pV}{4R}$$

c) It is given that the internal energy of the gas is

U=1.5nRT

Where n=number of moles

R=gas constant

T=final equilibrium temperature of both parts

The internal energy of ideal gas is given as

$U=nC_vT$

Where C_v=molar specific heat at constant volume

T=temperature.

Total moles $n=n_1+n_2=1+2=3$

$U=3C_vT$

Let U_1 and U_2 be the internal energy of the left and right part respectively.

So, $U_1=n_1C_vT_1= C_vT_1$

And $U_2=n_2C_vT_2=2C_vT_2$

(gas is same on both the part so Cv will be same)

Now

$U=U_1+U_2$

$3C_vT = C_vT_1+ 2C_vT_2$

$3T=T_1+2T_2$

$$3T = \frac{pV}{2R} + \frac{2pV}{4R}$$

$$T = \frac{pV}{3R}$$

d) As stated in part (a) no work will be done on either part of the vessel as conducting separator is fixed.

So, $\Delta W=0$ for the right part of the vessel.

From first law of thermodynamics, we know that,

$\Delta Q=\Delta U+\Delta W$

Where ΔQ=heat supplied to the system

ΔU=change in internal energy

ΔW=work done by the system

$\therefore \Delta Q=\Delta U$

It is given that the internal energy of the gas is

$U=1.5nRT$

Where n=number of moles

R=gas constant

T=final equilibrium temperature of both parts

$\Delta U=1.5nR\Delta T$

For the right part, change in internal energy after equilibrium has reached will be due to the change in temperature from T_2 to T.

$\Delta U=1.5n_2R(T-T_2)$

$$\Delta U = 1.5 \times 2 \times R \times \left(\frac{pV}{3R} - \frac{pV}{4R}\right)$$

$$\Delta U = 3 \times R \times \frac{pV}{3 \times 4 \times R}$$

$$\Delta U = \frac{pV}{4}$$

Thus, heat given to the right part is $\Delta Q = \frac{pV}{4}$.

e) Since ΔW is zero as the volume is fixed, therefore, the first law of thermodynamics.

But since heat is given to the right part that means heat is extracted from the left part. So, the internal energy of the left part of will decrease.

Therefore, heat given to the right part will be equal to the negative internal energy of the left part.

$\Delta Q=-\Delta U$ (left part)

$\therefore \Delta U = -\frac{pV}{4}$.