Sample Question Paper - 25
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours | ***Maximum Marks : 40***

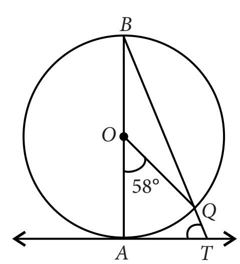
General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

2. In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$.



3. For the following data, find the modal class.

Class interval	Frequency
Less than 20	15
Less than 40	37
Less than 60	56
Less than 80	87
Less than 100	115

4. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21^{st} term?

OR

Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

5. A pole 14 m high casts a shadow $14\sqrt{3}$ m long on the ground. Find the sun's elevation.

6. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. $\left[\text{Take } \pi = \frac{22}{7}\right]$

OR

Two cubes each of volume 125 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

SECTION - B

7. Find the median of the following frequency distribution:

Weekly wages (in ₹)	60-69	70-79	80-89	90-99	100-109	110-119
No. of workers	5	15	20	30	20	8

8. Draw a line segment of length 8.4 cm and divide it internally in the ratio of 5 : 9. Measure the two parts.

OR

Construct a tangent to a circle of radius 5 cm from a point on the concentric circle of radius 7 cm.

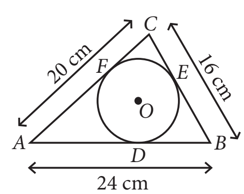
9. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Take $\sqrt{3} = 1.73$)

10. The mean weight of 150 students in a class is 60 kg. The mean weight of boys is 70 kg while that of girls is 55 kg. Find the number of boys and girls in the class.

SECTION - C

11. The sums of n, $2n$, $3n$ terms of an A.P. are S_1, S_2, S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

12. A circle is inscribed in a ΔABC having sides 16 cm, 20 cm and 24 cm as shown in figure. Find AD, BE and CF.



OR

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Case Study - 1

13. Quadratic equations started around 3000 B.C. with the Babylonians. They were one of the world's first civilisation, and came up with some great ideas like agriculture, irrigation and writing. There were many reasons why Babylonians needed to solve quadratic equations. For example to know what amount of crop you can grow on the square field.

Based on the above information, represent the following questions in the form of quadratic equation.

(i) The sum of squares of two consecutive integers is 650.

(ii) A natural number whose square diminished by 84 is thrice of 8 more of given number.

Case Study - 2

14. Alok and his family went for a vacation to Jaipur. There they had a stay in tent for a night. Alok found that the tent in which they stayed is in the form of a cone surmounted on a cylinder. The total height of the tent is 42 m, diameter of the base is 42 m and height of the cylinder is 22 m.

Based on the above information, answer the following questions.

(i) How much canvas is needed to make the tent?

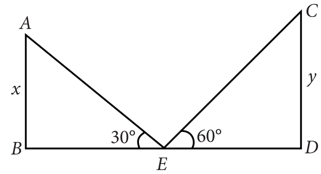
(ii) Find the volume of the tent.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Let AB and CD be two towers of height x and y respectively.



$\because$ E is the midpoint of BD. $\therefore$ $BE = ED$

Now, in ΔABE, $\tan 30° = \frac{AB}{BE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{BE} \Rightarrow x = \frac{BE}{\sqrt{3}}$

And in ΔEDC, $\tan 60° = \frac{CD}{ED}$

$\Rightarrow \sqrt{3} = \frac{y}{ED} \Rightarrow y = \sqrt{3}BE \quad (\because BE = ED)$

$\therefore \frac{x}{y} = \frac{BE}{\sqrt{3}} \times \frac{1}{\sqrt{3}BE} = \frac{1}{3}$

Thus, $x : y = 1 : 3$.

2. Since, angle subtended by an arc at the centre is double the angle subtended by the same arc at the remaining part of the circle.

$\therefore$ $2\angle ABQ = \angle AOQ$

$\Rightarrow \angle ABT = \frac{58°}{2} \Rightarrow \angle ABT = 29°$

Also, $\angle BAT = 90°$ ($\because$ Tangent is perpendicular to the radius through the point of contact)

In ΔABT, $\angle ABT + \angle BAT + \angle ATB = 180°$

$\Rightarrow 29° + 90° + \angle ATQ = 180°$

$\Rightarrow \angle ATQ = 180° - 119° = 61°$

3. The frequency distribution table from the given data can be drawn as :

Class interval	Frequency
0–20	15
20–40	37 – 15 = 22
40–60	56 – 37 = 19
60–80	87 – 56 = 31
80–100	115 – 87 = 28

The modal class is 60–80 as it has the maximum frequency.

4. We have, first term, $a = 3$,

common difference, $d = 15 - 3 = 12$

n^{th} term of an A.P. is given by $a_n = a + (n - 1)\,d$

$\therefore$ $a_{21} = 3 + (20) \times 12 = 3 + 240 = 243$

Let the r^{th} term of the A.P. be 120 more than the 21^{st} term.

$\Rightarrow a + (r - 1)d = 243 + 120 \Rightarrow 3 + (r - 1)\,12 = 363$

$\Rightarrow (r - 1)\,12 = 360 \Rightarrow r - 1 = 30 \Rightarrow r = 31$

OR

Let the four terms are $a - 3d$, $a - d$, $a + d$ and $a + 3d$.

Sum of four terms = 50 [Given]

$\Rightarrow 4a = 50 \Rightarrow a = 25/2$ ...(i)

According to the question, $a + 3d = 4(a - 3d)$

$\Rightarrow a + 3d = 4a - 12d \Rightarrow 15d = 3a$

$\Rightarrow d = a/5 = \frac{25}{2 \times 5}$ [Using (i)]

$\Rightarrow d = 5/2$

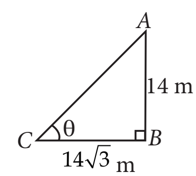
$\therefore$ Numbers are 5, 10, 15 and 20.

5. Let AB be the pole and BC be its shadow.

Then, $AB = 14$ m, $BC = 14\sqrt{3}$ m

Also, let θ be the sun's elevation, *i.e.*, $\angle ACB = \theta$

In right ΔABC,



$\tan\theta = \frac{AB}{BC} = \frac{14}{14\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

But we know that, $\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ $\theta = 30°$

So, required angle of elevation is 30°.

6. Let radius of hemisphere = r cm

Total surface area of hemisphere = 462 cm^2

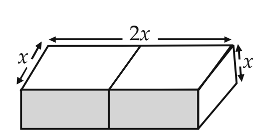
$\Rightarrow 3\pi r^2 = 462 \Rightarrow r^2 = \frac{462 \times 7}{3 \times 22} = 49 \Rightarrow r = 7$

$\therefore$ Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 718.67 \text{ cm}^3$

OR

Let the side of each cube be x.



Given, volume of each cube

$= 125 \text{ cm}^3$

$\therefore$ $x^3 = 125 \text{ cm}^3$

$\Rightarrow x = 5$ cm

Now, length of resulting cuboid $(l) = 2x = 10$ cm

Breadth of resulting cuboid $(b) = x = 5$ cm

Height of resulting cuboid $(h) = x = 5$ cm

$\therefore$ Surface area of the cuboid $= 2\,(lb + bh + hl)$

$= 2\,(10 \times 5 + 5 \times 5 + 5 \times 10) = 2\,[50 + 25 + 50]$

$= 250 \text{ cm}^2$

7. Here, the frequency distribution table is given in inclusive form. So, we first convert it into exclusive form.

$\therefore$ The cumulative frequency distribution table in exclusive form is as follows :

Weekly wages (in ₹)	No. of workers	Cumulative frequency
59.5-69.5	5	5
69.5-79.5	15	20
79.5-89.5	20	40
89.5-99.5	30	70
99.5-109.5	20	90
109.5-119.5	8	98
	$N = \sum f_i = 98$	

We have, $n = 98 \therefore n/2 = 49$

The cumulative frequency just greater than $n/2$ is 70 and the corresponding class is 89.5–99.5. So, 89.5–99.5 is the median class.

$\therefore$ $l = 89.5, h = 10, f = 30$ and $cf = 40$

$$\therefore \text{ Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 89.5 + \left(\frac{49 - 40}{30}\right) \times 10 = 92.5$$

8. Steps of construction :

Step-I : Draw a line segment $AB = 8.4$ cm.

Step-II : Draw any ray AX making an acute angle with AB.

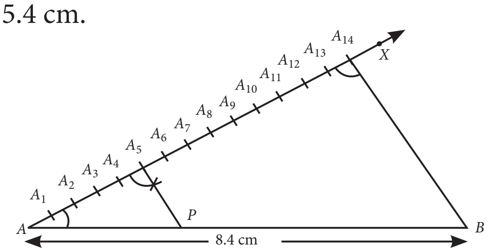
Step-III : On ray AX, mark $5 + 9 = 14$ points $A_1, A_2, A_3, ..., A_{14}$ such that $AA_1 = A_1A_2 = A_2A_3 = ... = A_{13}A_{14}$.

Step-IV : Join $A_{14}B$.

Step -V : From A_5, draw $A_5P \parallel A_{14}B$, meeting AB at P.

Thus, P divides AB in the ratio 5 : 9.

On measuring the two parts, we get $AP = 3$ cm and $PB = 5.4$ cm.

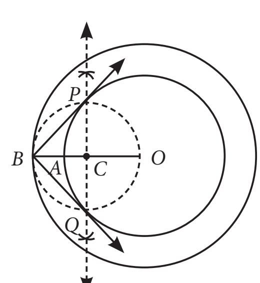


OR

Steps of construction :

Step-I : Draw two concentric circle with centre O and radii 5 cm and 7 cm. Take a point B on the outer circle and then join OB.

Step-II : Draw the perpendicular bisector of OB. Let the bisector intersects OB at C.

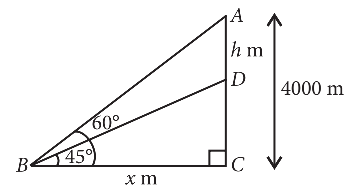


Step-III : With C as centre and OC as radius, draw a circle which intersects the inner circle at P and Q.

Step-IV : Join BP and QB.

Thus, BP and BQ are the required tangents.

9. Let one aeroplane be at A and second be at D such that vertical distance between two planes is h m.



In ΔABC, $\tan 60° = \frac{AC}{BC}$

$\Rightarrow \sqrt{3} = \frac{4000}{x} \Rightarrow x = \frac{4000}{\sqrt{3}}$ ...(i)

In ΔDBC, $\tan 45° = \frac{DC}{BC}$

$\Rightarrow 1 = \frac{4000 - h}{x} \Rightarrow x = 4000 - h$

$\Rightarrow \frac{4000}{\sqrt{3}} = 4000 - h$ [Using (i)]

$\Rightarrow h = 4000 - \frac{4000}{\sqrt{3}} = 4000 - \frac{4000\sqrt{3}}{3}$

$\Rightarrow h = \frac{12000 - 6920}{3} = \frac{5080}{3} = 1693.33$

Hence, vertical distance between the aeroplanes at that instant was 1693.33 m.

10. Total number of students = 150

Mean weight = 60 kg

$\therefore$ Total weight of 150 students = $150 \times 60 = 9000$ kg

Let the total number of boys be x.

$\therefore$ Total number of girls = $150 - x$

Mean weight of boys = 70 kg

$\therefore$ Total weight of boys = $70 \times x = 70x$ kg

Mean weight of girls = 55 kg

$\therefore$ Total weight of girls = $(150 - x)55$ kg

Now, Total weight = Weight of boys + Weight of girls

$\Rightarrow 9000 = 70x + (150 - x)55$

$\Rightarrow 9000 = 70x + 150 \times 55 - 55x$

$\Rightarrow 9000 - 8250 = 70x - 55x$

$\Rightarrow 750 = 15x \Rightarrow x = 50$

$\therefore$ Number of boys = 50 and number of girls = 100

11. Let a be the first term and d be the common difference of the A.P.

$\therefore S_1 = \frac{n}{2}[2a + (n - 1)d]$ ...(i)

$S_2 = \frac{2n}{2}[2a + (2n - 1)d]$ ...(ii) and $S_3 = \frac{3n}{2}[2a + (3n - 1)d]$ ...(iii)

R.H.S. $= 3(S_2 - S_1)$

$= 3\left[\frac{2n}{2}(2a+2nd-d)-\frac{n}{2}(2a+nd-d)\right]$

$= 3\cdot\frac{n}{2}[2(2a + 2nd - d) - (2a + nd - d)]$

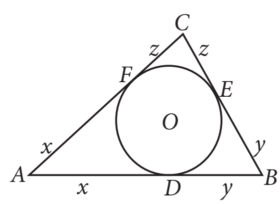
$= \frac{3n}{2}[4a + 4nd - 2d - 2a - nd + d]$

$= \frac{3n}{2}[2a + 3nd - d]$

$= \frac{3n}{2}[2a + (3n - 1)d] = S_3$ [From (iii)]

= L.H.S.

12. Since, tangents drawn from an external point to a circle are equal.



$\therefore$ $AD = AF = x$ (say)

$BD = BE = y$ (say)

$CE = CF = z$ (say)

According to the question,

$AB = x + y = 24$ cm ...(i)

$BC = y + z = 16$ cm ...(ii)

$AC = x + z = 20$ cm ...(iii)

Subtracting (iii) from (i), we get

$y - z = 4$ ...(iv)

Adding (ii) and (iv), we get

$2y = 20 \Rightarrow y = 10$ cm

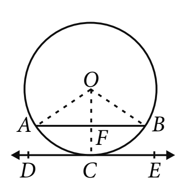
Substituting the value of y in (i) and (ii), we get

$x = 14$ cm, $z = 6$ cm

$\therefore$ $AD = 14$ cm, $BE = 10$ cm and $CF = 6$ cm.

OR

Let us consider a circle with centre O and C is the mid point of arc ACB and DE is a tangent to the circle.



Now, we need to prove that $AB \parallel DE$

Join OA, OB and OC.

Since C is the mid point of arc ACB.

$\therefore$ $\angle AOF = \angle BOF$

[$\because$ OA and OB are equally inclined with OC]

Now, in ΔOAF and ΔOBF,

$OA = OB$ [Radii of the circle]

$\angle AOF = \angle BOF$ [Proved above]

$OF = OF$ [Common]

$\therefore$ $\Delta OAF \cong \Delta OBF$ [By SAS congruence criterion]

$\Rightarrow$ $\angle AFO = \angle BFO$ [By CPCT]

Now, $\angle AFO + \angle BFO = 180°$ [Linear pair]

$\Rightarrow 2\angle AFO = 180° \Rightarrow \angle AFO = 90°$

Also, $\angle OCD = 90°$

[$\because$ Tangent is perpendicular to radius through the point of contact.]

$\therefore$ $\angle AFO = \angle OCD$ [Each 90°]

But these are corresponding angles.

$\therefore$ $AB \parallel DE$

13. (i) Let two consecutive integers be x, $x + 1$.

Given, $x^2 + (x + 1)^2 = 650$

$\Rightarrow 2x^2 + 2x + 1 - 650 = 0$

$\Rightarrow 2x^2 + 2x - 649 = 0$

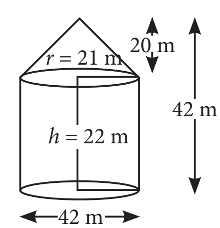
(ii) Let the number be x.

According to question, $x^2 - 84 = 3(x + 8)$

$\Rightarrow x^2 - 84 = 3x + 24 \Rightarrow x^2 - 3x - 108 = 0$

14. (i) Required area of canvas = Curved surface area of cone + Curved surface area of cylinder

$= \pi rl + 2\pi rh = \pi r(l + 2h)$



$= \frac{22}{7} \times 21(29 + 44)$

$\left[\because\ l = \sqrt{r^2 + h_1^2} = \sqrt{(21)^2 + (20)^2} = \sqrt{841} = 29 \text{ m}\right]$

$= 4818 \text{ m}^2$

(ii) Volume of tent = Volume of cone + Volume of cylinder $= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h = \pi r^2\left(\frac{1}{3}h_1 + h\right)$, where h_1 is the height of cone.

$= \frac{22}{7} \times (21)^2\left[\frac{20}{3} + 22\right] = \frac{9702}{7} \times \frac{86}{3} = 39732 \text{ m}^3$