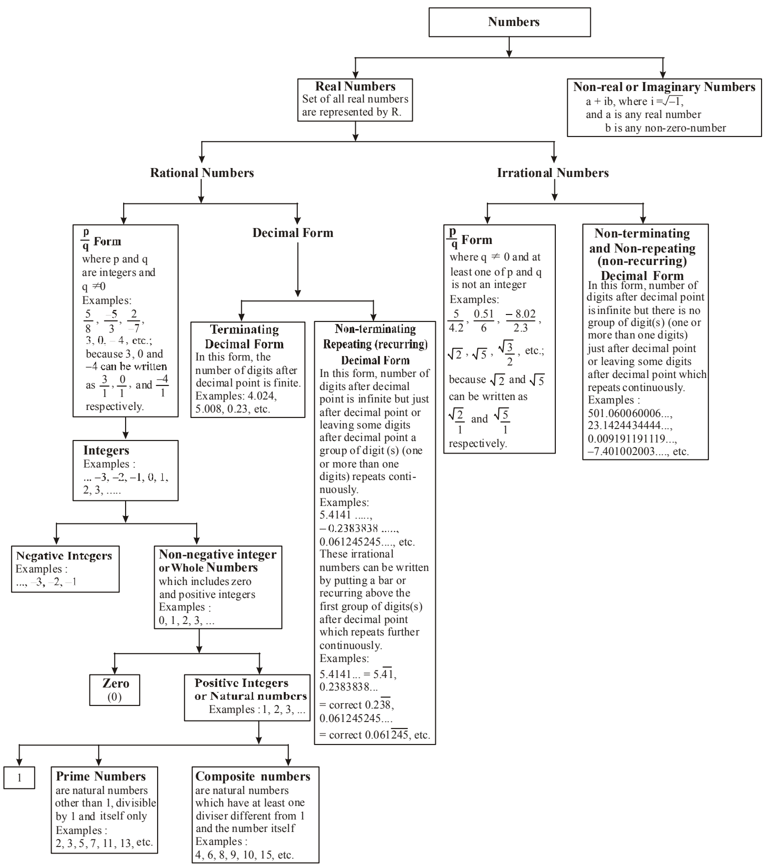
Chapter 1

Number System & Simplification

INTRODUCTION



REMEMBER

- ★ The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits.
- ★ 1 is neither prime nor composite.
- ★ 1 is an odd integer.
- ★ 0 is neither positive nor negative.
- ★ 0 is an even integer.
- ★ 2 is prime & even both.
- ★ All prime numbers (except 2) are odd.

Natural Numbers :

These are the numbers (1, 2, 3, etc.) that are used for counting.
It is denoted by N.
There are infinite natural numbers and the smallest natural number is one (1).

Even numbers :

Natural numbers which are divisible by 2 are even numbers.
It is denoted by E.
E = 2, 4, 6, 8,...
Smallest even number is 2. There is no largest even number.

Odd numbers :

Natural numbers which are not divisible by 2 are odd numbers.
It is denoted by O.
O = 1, 3, 5, 7, ...
Smallest odd number is 1.
There is no largest odd number.

Based on divisibility, there could be two types of natural numbers : Prime and Composite.

Prime Numbers :

Natural numbers which have exactly two factors, i.e., 1 and the number itself are called prime numbers.
The lowest prime number is 2.
2 is also the only even prime number.

Composite Numbers :

It is a natural number that has atleast one divisor different from unity and itself.

Every composite number can be factorised into its prime factors.

For Example : $24 = 2 \times 2 \times 2 \times 3$. Hence, 24 is a composite number.

The smallest composite number is 4.

Twin-prime Numbers:

Pairs of such prime numbers whose difference is 2.

Example: 3 and 5, 11 and 13, 17 and 19.

How to check whether a given number is prime or not ?

Steps : (i) Find approximate square root of the given number.

(ii) Divide the given number by every prime number less than the approximate square root.

(iii) If the given number is exactly divisible by atleast one of the prime numbers, the number is a composite number otherwise a prime number.

Example : Is 401 a prime number?

Sol. Approximate square root of 401 is 20.

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19

401 is not divisible by 2, 3, 5, 7, 11, 13, 17 or 19.

$\therefore$ 401 is a prime number.

(**Hint :** Next prime number after 19 and 23, which is greater than 20, so we need not check further.)

Co-prime Numbers : Co-prime numbers are those numbers which are prime to each other i.e., they don't have any common factor other than 1. Since these numbers do not have any common factor, their HCF is 1 and their LCM is equal to product of the numbers.

Note : Co-prime numbers can be prime or composite numbers. Any two prime numbers are always co-prime numbers.

Example 1 : 3 and 5 : Both numbers are prime numbers.

Example 2 : 8 and 15 : Both numbers are composite numbers but they are prime to each other i.e., they don't have any common factor.

Face value and Place value :

Face Value is absolute value of a digit in a number.

Place Value (or Local Value) is value of a digit in relation to its position in the number.

Example : Face value and Place value of 9 in 14921 is 9 and 900 respectively.

Whole Numbers :

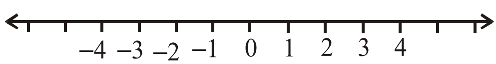
The natural numbers along with zero (0), form the system of whole numbers.
It is denoted by W.
There is no largest whole number and
The smallest whole number is 0.

Integers :

The number system consisting of natural numbers, their negative and zero is called integers.
It is denoted by Z or I.
The smallest and the largest integers cannot be determined.

The Number Line :

The number line is a straight line between negative infinity on the left to positive infinity on the right.



Rational numbers : Any number that can be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

- It is denoted by Q.
- Every integer is a rational number.
- Zero (0) is also a rational number. The smallest and largest rational numbers cannot be determined. Every fraction (and decimal fraction) is a rational number.

$$Q = \frac{p \text{ (Numerator)}}{q \text{ (Denominator)}}$$

REMEMBER

★ If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational number and its value lies between the given two rational numbers x and y.

★ An infinite number of rational numbers can be determined between any two rational numbers.

Irrational numbers : The numbers which are not rational or which cannot be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called irrational number.

It is denoted by Q' or Q^c.

$\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers.

NOTE :

(i) Every positive irrational number has a negative irrational number corresponding to it.

(ii) $\sqrt{2}+\sqrt{3} \neq \sqrt{5}$

$\sqrt{5}-\sqrt{3} \neq \sqrt{2}$

$\sqrt{3}\times\sqrt{2} = \sqrt{3\times 2} = \sqrt{6}$

$\sqrt{6} \div \sqrt{2} = \sqrt{\frac{6}{2}} = \sqrt{3}$

(iii) Some times, product of two irrational numbers is a rational number.

For example : $\sqrt{2}\times\sqrt{2} = \sqrt{2\times 2} = 2$

$(2+\sqrt{3})\times(2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$

(iv) π is an irrational number. π : approximately equal to $\frac{22}{7}$ or 3.14.

Real Numbers :

All numbers that can be represented on the number line are called real numbers.

It is denoted by R.

R^+ : denotes the set of all positive real numbers and

R^- : denotes the set of negative real numbers.

Both rational and irrational numbers can be represented in number line.

Every real number is either rational or irrational.

FRACTIONS

A fraction is a quantity which expresses a part of the whole.

$$\boxed{\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}}$$

TYPES OF FRACTIONS :

1. **Proper fraction :** If numerator is less than its denominator, then it is a proper fraction.

   For example : $\frac{2}{5}, \frac{6}{18}$

2. **Improper fraction :** If numerator is greater than or equal to its denominator, then it is a improper fraction.

   For example : $\frac{5}{2}, \frac{18}{7}, \frac{13}{13}$

NOTE : *If in a fraction, its numerator and denominator are of equal value then fraction is equal to unity i.e. 1.*

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3. **Mixed fraction :** It consists of an integer and a proper fraction.

   For example : $1\frac{1}{2},\ 3\frac{2}{3},\ 7\frac{5}{9}$

NOTE : *Mixed fraction can always be changed into improper fraction and vice versa.*

For example : $7\frac{5}{9} = \frac{7 \times 9 + 5}{9} = \frac{63 + 5}{9} = \frac{68}{9}$

and $\frac{19}{2} = \frac{9 \times 2 + 1}{2} = 9 + \frac{1}{2} = 9\frac{1}{2}$

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4. **Equivalent fractions or Equal fractions :** Fractions with same value.

   For example : $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}\left(= \frac{2}{3}\right).$

NOTE : *Value of fraction is not changed by multiplying or dividing both the numerator or denominator by the same number.*

For example :

(i) $\frac{2}{5} = \frac{2\times5}{5\times5} = \frac{10}{25}$ So, $\frac{2}{5} = \frac{10}{25}$

(ii) $\frac{36}{16} = \frac{36\div4}{16\div4} = \frac{9}{4}$ So, $\frac{36}{16} = \frac{9}{4}$

5. **Like fractions:** Fractions with same denominators.

   For example : $\frac{2}{7}, \frac{3}{7}, \frac{9}{7}, \frac{11}{7}$

6. **Unlike fractions :** Fractions with different denominators.

   For example : $\frac{2}{5}, \frac{4}{7}, \frac{9}{8}, \frac{9}{2}$

NOTE : *Unlike fractions can be converted into like fractions.*

For example : $\frac{3}{5}$ and $\frac{4}{7}$

$\frac{3}{5}\times\frac{7}{7} = \frac{21}{35}$ and $\frac{4}{7}\times\frac{5}{5} = \frac{20}{35}$

7. **Simple fraction :** Numerator and denominator are integers.

   For example : $\frac{3}{7}$ and $\frac{2}{5}$.

8. **Complex fraction :** Numerator or denominator or both are fractional numbers.

   For example : $\frac{2}{\frac{5}{7}}, \frac{2\frac{1}{3}}{5\frac{2}{3}}, \frac{2+\frac{1+\frac{2}{7}}{3}}{2}$

9. Decimal fraction : Denominator with the powers of 10.

For example : $\frac{2}{10} = (0.2), \frac{9}{100} = (0.09)$

10. Comparison of Fractions

Comparison of two faction can be easily understand by the following example:

To compare two fraction $\frac{3}{5}$ and $\frac{7}{9}$, multiply each fraction by the LCM (45) of their denominators 5 and 9.

$$\frac{3}{5} \times 45 = 3 \times 9 = 27$$

$$\frac{7}{9} \times 45 = 7 \times 5 = 35$$

Since $27 < 35$

$$\therefore \quad \frac{3}{5} < \frac{7}{9}$$

SHORT CUT METHOD

$$\frac{3}{5} \times \frac{7}{9}$$

$$27 < 35$$

[Write the each product on their numerator side]

$$\therefore \quad \frac{3}{5} < \frac{7}{9}$$

ADDITION OF MIXED FRACTIONS

You can easily understand the addition of mixed fractions by the following example:

$$1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{9} = \frac{8}{5} + \frac{17}{9} + \frac{14}{5}$$

$$= \frac{72 + 85 + 126}{45} = \frac{283}{45} = 6\frac{23}{45}$$

SHORT CUT METHOD

$$1\frac{3}{5} + 1\frac{8}{9} + 2\frac{4}{5} = (1 + 1 + 2) + \left(\frac{3}{5} + \frac{8}{9} + \frac{4}{5}\right)$$

$$= 4 + \frac{27 + 40 + 36}{45}$$

$$= 4 + \frac{103}{45} = 4 + 2\frac{13}{45} = 6\frac{13}{45}$$

Rounding off (Approximation) of Decimals :

There are some decimals in which numbers are found upto large number of decimal places.

For example : 3.4578, 21.358940789.

But many times we require decimal numbers upto a certain number of decimal places. Therefore,

If the digit of the decimal place is five or more than five, then the digit in the preceding decimal place is increased by one and if the digit in the last place is less than five, then the digit in the precedence place remains unchanged.

CONVERSION OF RATIONAL NUMBER OF THE FORM NON-TERMINATING RECURRING DECIMAL INTO THE RATIONAL NUMBER OF THE FORM $\frac{p}{q}$

First write the non-terminating repeating decimal number in recurring form i.e., write

64.20132132132..... as $64.20\overline{132}$

Then using formula given below we find the required $\frac{p}{q}$ form of the given number.

Rational number in the form $\frac{p}{q}$

$$= \frac{\left[\begin{array}{l}\text{Complete number neglecting} \\ \text{the decimal and bar over} \\ \text{repeating digit (s)}\end{array}\right] - \left[\begin{array}{l}\text{Non-recurring part of} \\ \text{the number neglecting} \\ \text{the decimal}\end{array}\right]}{m \text{ times 9 followed by } n \text{ times 0}}$$

where m = number of recurring digits in decimal part
and n = number of non-recurring digits in decimals part

Thus, $\frac{p}{q}$ form of $64.20\overline{132} = \frac{6420132 - 6420}{99900}$

$$= \frac{6413712}{99900} = \frac{534476}{8325}$$

In short; $0.\overline{a} = \frac{a}{9}, 0.\overline{ab} = \frac{ab}{99}, 0.\overline{abc} = \frac{abc}{999},$ etc. and

$$0.a\overline{b} = \frac{ab - a}{90},\ 0.\overline{abc} = \frac{abc - a}{990},\ 0.ab\overline{c} = \frac{abc - ab}{900},$$

$$0.a\overline{bcd} = \frac{abcd - ab}{9900},\ ab.c\overline{de} = \frac{abcde - abc}{990}, \text{etc.}$$

PROPERTIES OF OPERATIONS :

The following properties of addition, subtraction and multiplication are valid for real numbers a , b and c.

(a) Commutative property of addition :
$a + b = b + a$

(b) Associative property of addition :
$(a + b) + c = a + (b + c)$

(c) Commutative property of multiplication:
$a \times b = b \times a$

(d) Associative property of multiplication :
$(a \times b) \times c = a \times (b \times c)$

(e) Distributive property of multiplication with respect to addition :
$(a + b) \times c = a \times c + b \times c$

DIVISIBILITY RULES

Divisibility by 2 :

A number is divisible by 2 if its unit's digit is even or 0.

Divisibility by 3 :

A number is divisible by 3 if the sum of its digits are divisible by 3.

Divisibility by 4 :

A number is divisible by 4 if the last 2 digits are divisible by 4, or if the last two digits are 0's.

Divisibility by 5 :

A number is divisible by 5 if its unit's digit is 5 or 0.

Divisibility by 6 :

A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisiblity by 7 :

A number is divisible by 7 if unit's place digit is multiplied by 2 and subtracted from the remaining digits and the number obtained is divisible by 7.

For example,

$$1680\boxed{7} = 1680 - 7 \times 2 = 1666$$

It is difficult to decide whether 1666 is divisible by 7 or not. In such cases, we continue the process again and again till it become easy to decide whether the number is divisible by 7 or not.

$$166\boxed{6} \longrightarrow 166 - 6 \times 2 = 154$$

Again $15\boxed{4} \longrightarrow 15 - 4 \times 2 = 7$, divisible by 7

Hence 16807 is divisible by 7.

Divisibility by 8 :

A number is divisible by 8 if the last 3 digits of the number are divisible by 8, or if the last three digits of a number are zeros.

Divisibility by 9 :

A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility by 10 :

A number is divisible by 10 if its unit's digit is 0.

Divisibility by 11 :

A number is divisible by 11 if the sum of digits at odd and even places are equal or differ by a number divisible by 11.

Divisibility by 12 :

A number is divisible by 12 if the number is divisible by both 4 and 3.

Divisibility by 13 :

A number is divisible by 13 if its unit's place digit is multiplied by 4 and added to the remaining digits and the number obtained is divisible by 13.

For example,

$$219\boxed{7} \longrightarrow 219 + 7 \times 4 = 247$$

Again $24\boxed{7} \longrightarrow 24 + 7 \times 4 = 52$, divisible by 13.

Hence 2197 is divisible by 13.

Divisibility by 14 :

A number is divisible by 14 if the number is divisible by both 2 and 7.

Divisibility by 15 :

A number is divisible by 15 if the number is divisible by both 3 and 5.

Divisibility by 16 :

A number is divisible by 16 if its last 4 digits is divisible by 16 or if the last four digits are zeros.

Divisibility by 17 :

A number is divisible by 17 if its unit's place digit is multiplied by 5 and subtracted from the remaining digits and the number obtained is divisible by 17.

For example,

$$491\boxed{3} \longrightarrow 491 - 3 \times 5 = 476$$

Again, $47\boxed{6} \longrightarrow 47 - 6 \times 8 = 17,$ divisible by 17.

Hence 4913 is divisible by 17.

Divisibility by 18 :

A number is divisible by 18 if the number is divisible by both 2 and 9.

Divisibility by 19 :

A number is divisible by 19 if its unit's place digit is multiplied by 2 and added to the remaining digits and the number obtained is divisible by 19.

For example,

$4873\boxed{7} \longrightarrow 4873 + 7 \times 2 = 4887$

$488\boxed{7} \longrightarrow 488 + 7 \times 2 = 502$

$50\boxed{2} \longrightarrow 50 + 2 \times 2 = 54$ not divisible by 19.

Hence 48737 is not divisible by 19.

Properties of Divisibility

(i) The product of 3 consecutive natural numbers is divisible by 6.

(ii) The product of 3 consecutive natural numbers, the first of which is even, is divisible by 24.

(iii) Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.

(iv) Any number written in the form $(10^n - 1)$ is divisible by 3 and 9.

(v) Any six-digits, twelve-digits, eighteen-digits or any such number with number of digits equal to multiple of 6, is divisible by each of 7, 11 and 13 if all of its digits are same.

For example 666666, 888888, 333333333333 are all divisible by 7, 11 and 13.

(vi) Any number in the form abcabc (a, b, c are three different digits) is divisible by 1001.

(vii) (a) $(a^n - b^n)$ is divisible both by $(a + b)$ and $(a - b)$, when n is even.
(b) $(a^n - b^n)$ is divisible only by $(a - b)$, when n is odd.

DIVISION ALGORITHM :

Dividend = (Divisor × Quotient) + Remainder

where, Dividend = The number which is being divided

Divisor = The number which performs the division process

Quotient = Greatest possible integer as a result of division

Remainder = Rest part of dividend which cannot be further divided by the divisor.

Complete remainder :

A complete remainder is the remainder obtained by a number by the method of successive division.

Complete remainder = [I divisor × II remainder] + I remainder

$$\text{C.R.} = d_1 r_2 + r_1$$
$$\text{C.R.} = d_1 d_2 r_3 + d_1 r_2 + r_1$$

☞ *Shortcut Approach*

Two different numbers x and y when divided by a certain divisor D leave remainder r_1 and r_2 respectively. When the sum of them sis divided by the same divisor, the remainder is r_3. Then,

divisor $D = r_1 + r_2 - r_3$

See Example : Refer ebook Solved Examples/Ch-1

Method to find the number of different divisors (or factors) (including 1 and itself) of any composite number N :

STEP I : Express N as a product of prime numbers as
$N = x^a \times y^b \times z^c$ ....

STEP II : Number of different divisors (including 1 and itself)
$= (a + 1)(b + 1)(c + 1)$ ....

HIGHEST COMMON FACTOR (HCF) OR GREATEST COMMON DIVISOR (GCD)

The highest (i.e. largest) number that divides two or more given numbers is called the highest common factor (HCF) of those numbers.

Methods to Find The HCF or GCD

There are two methods to find HCF of the given numbers

(i) Prime Factorization Method

When a number is written as the product of prime numbers, then it is called the prime factorization of that number. For example, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$. Here, $2 \times 2 \times 2 \times 3 \times 3$ or $2^3 \times 3^2$ is called prime factorization of 72.

To find the HCF of given numbers by this methods, we perform the prime factorization of all the numbers and then check for the common prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given numbers. The HCF is the product of all such prime factors with their respective least indices.

(ii) Division Method

To find the HCF of two numbers by division method, we divide the larger number by the smaller number. Then we divide the smaller number by the first remainder, then first remainder by the second remainder.. and so on, till the remainder becomes 0. The last divisor is the required HCF.

☞ ***Shortcut Approach***

To find the HCF of any number of given numbers, first find the difference between two nearest given numbers. Then find all factors (or divisors) of this difference. Highest factor which divides all the given numbers is the HCF.

See Example : Refer ebook Solved Examples/Ch-1

LEAST COMMON MULTIPLE (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

Methods to Find The LCM

There are two methods to find the LCM.

(i) Prime Factorization Method

After performing the prime factorization of all the given numbers, we find the highest index of all the prime numbers among the given numbers. The LCM is the product of all these prime numbers with their respective highest indices because LCM must be divisible by all of the given numbers.

(ii) Division Method

To find the LCM of 5, 72, 196 and 240, we use the division method in the following way:

Check whether any prime number that divides at least two of all the given numbers. If there is no such prime number, then the product of all these numbers is the required LCM, otherwise find the smallest prime number that divides at least two of the given numbers. Here, we see that smallest prime number that divides at least two given numbers is 2.

Divide those numbers out of the given numbers by 2 which are divisible by 2 and write the quotient below it. The given number(s) that are not divisible by 2 write as it is below it and repeat this step till you do not find at least two numbers that are not divisible by any prime number.

$$\begin{array}{c|cccc} 2 & 5, & 72, & 196, & 240 \\ \hline 2 & 5, & 36, & 98, & 120 \\ \hline 2 & 5, & 18, & 49, & 60 \\ \hline 3 & 5, & 9, & 49, & 30 \\ \hline 5 & 5, & 3, & 49, & 10 \\ \hline & 1, & 3, & 49, & 2 \end{array}$$

After that find the product of all divisors and the quotient left at the end of the division. This product is the required LCM.

Hence, LCM of the given numbers = product of all divisors and the quotient left at the end.

$$= 2 \times 2 \times 2 \times 3 \times 5 \times 3 \times 49 \times 2 = 35280$$

☞ Shortcut Approach

Using idea of co-prime, you can find the LCM by the following shortcut method:

LCM of 9, 10, 15 and 36 can be written directly as $9 \times 10 \times 2$.

The logical thinking that behind it is as follows:

Step 1: If you can see a set of 2 or more co-prime numbers in the set of numbers of which you are finding the LCM, write them down by multiply them.

In the above situation, since we see that 9 and 10 are co-prime to each other, we start off writing the LCM by writing 9×10 as the first step.

Step 2: For each of the other numbers, consider what prime factor(s) of it is/are not present in the LCM (if factorised into primes) taken in step 1. In case you see some prime factors of each of the other given numbers separately are not present in the LCM (if factorised into primes) taken in step 1, such prime factors will be multiplied in the LCM taken in step 1.

Prime factorisation of $9 \times 10 = 3 \times 3 \times 2 \times 5$

Prime factorisation of $15 = 3 \times 5$

Prime factorisation of $36 = 2 \times 2 \times 3 \times 3$

Here we see that both prime factors of 15 are present in the prime factorisation of 9×10 but one prime factor 2 of 36 is not present in the LCM taken in step 1. So to find the LCM of 9, 10, 15 and 36; we multiply the LCM taken in step 1 by 2.

Thus required LCM $= 9 \times 10 \times 2 = 180$

See Example : Refer ebook Solved Examples/Ch-1

RULE FOR FINDING HCF AND LCM OF FRACTIONS

(I) HCF of two or more fractions

$$= \frac{\text{HCF of numerator of all fractions}}{\text{LCM of denominator of all fractions}}$$

(II) LCM of two or more fractions

$$= \frac{\text{LCM of numerator of all fractions}}{\text{HCF of denominator of all fractions}}$$

SIMPLIFICATION

FUNDAMENTAL OPERATIONS :

1. Addition :

(a) Sum of two positive numbers is a positive number.
For example : $(+5) + (+2) = +7$

(b) Sum of two negative numbers is a negative number.
For example : $(-5) + (-3) = -8$

(c) Sum of a positive and a negative number is the difference between their magnitudes and give the sign of the number with greater magnitude.
For example : $(-3) + (+5) = 2$ and $(-7) + (+2) = -5$

2. Subtractions :

Subtraction of two numbers is same as the sum of a positive and a negative number.

For Example :

$(+9)-(+2)=(+9)+(-2)=7$

$(-3)-(-5)=(-3)+5=+2.$

NOTE : *In subtraction of two negative numbers, sign of second number will change and become positive.*

3. Multiplication :

(a) Product of two positive numbers is positive.

(b) Product of two negative numbers is positive.

(c) Product of a positive number and a negative number is negative.

(d) Product of more than two numbers is positive or negative depending upon the presence of negative quantities.

If the number of negative numbers is even then product is positive and if the number of negative numbers is odd then product is negative.

For Example :

$(-3)\times(+2)=-6$

$(-5)\times(-7)=+35$

$(-2)\times(-3)\times(-5)=-30$

$(-2)\times(-3)\times(+5)=+30$

4. Division :

(a) If both the dividend and the divisor are of same sign, then quotient is always positive.

(b) If the dividend and the divisor are of different sign, then quotient is negative,

For Example :

$(-36)\div(+9)=-4$

$(-35)\div(-7)=+5$

Brackets :

Types of brackets are :

(i) Vinculum or bar –

(ii) Parenthesis or small or common brackets : ( )

(iii) Curly or middle brackets : { }

(iv) Square or big brackets : [ ]

The order for removal of brackets is (), {}, []

NOTE : *If there is a minus (–) sign before the bracket then while removing bracket, sign of each term will change.*

'BODMAS' RULE

Now a days it becomes 'VBODMAS' where,

'V' stands for "Vinculum"

'B' stands for "Bracket"

'O' stands for "Of"

'D' stands for "Division"

'M' stands for "Multiplication"

'A' stands for "Addition"

'S' stands for "Subtraction"

Same order of operations must be applied during simplification.

☞ Shortcut Approach

To simplify an expression, add all the positive numbers together and all the negative numbers separately and add or subtract the resulting numbers as the case will.

See Example : Refer ebook Solved Examples/Ch-1

POWERS OR EXPONENTS

When a number is multiplied by itself, it gives the square of the number. i.e., $a \times a = a^2$ (Example $5 \times 5 = 5^2$)

If the same number is multiplied by itself twice we get the cube of the number i.e., $a \times a \times a = a^3$ (Example $4 \times 4 \times 4 = 4^3$)

In the same way $a \times a \times a \times a \times a = a^5$

and $a \times a \times a \times \ldots$ upto n times $= a^n$

There are five basic rules of powers which you should know:

If a and b are any two real numbers and m and n are positive integers, then

(i) $a^m \times a^n = a^{m+n}$ (Example: $5^3 \times 5^4 = 5^{3+4} = 5^7$)

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$, if $m > n$ $\left(\text{Example}: \dfrac{6^5}{6^2} = 6^{5-2} = 6^3\right)$

$\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$, if $m < n$ $\left(\text{Example}: \dfrac{4^3}{4^8} = \dfrac{1}{4^{8-3}} = \dfrac{1}{4^5}\right)$

and $\dfrac{a^m}{a^n} = a^0 = 1$, if $m = n$ $\left(\text{Example}: \dfrac{3^4}{3^4} = 3^{4-4} = 3^0 = 1\right)$

(iii) $(a^m)^n = a^{mn} = (a^n)^m$ (Example: $(6^2)^4 = 6^{2\times 4} = 6^8 = (6^4)^2$

(iv) (a) $(ab)^n = a^n \cdot b^n$ (Example: $(6 \times 4)^3 = 6^3 \times 4^3$)

(b) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, b \neq 0$ $\left(\text{Example}: \left(\dfrac{5}{3}\right)^4 = \dfrac{5^4}{3^4}\right)$

(v) $a^{-n} = \dfrac{1}{a^n}$ $\left(\text{Example}: 5^{-3} = \dfrac{1}{5^3}\right)$

(vi) For any real number a, $a^0 = 1$

ALGEBRIC IDENTITIES

Standard Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow (a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(ii) $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$\Rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(iii) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(v) $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a+b+c=0$ then $a^3 + b^3 + c^3 = 3abc$