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Vectors

Teaching Learning Point

- A quantity that has magnitude as well as direction is called is called a vector.

- A directed line segment represents a vector and is denoted by $\overrightarrow{AB}$ or $\vec{a}$.

- Position vector of a point P(x, y, z) w.r.t. origin 0(0,0,0) is denoted by $\overrightarrow{OP}$, where $\overrightarrow{OP}$ = $x\hat{i}+y\hat{j}+z\hat{k}$ and $\left|\overrightarrow{OP}\right|$ = $\sqrt{x^2+y^2+z^2}$

- The angles α, β, γ made $\vec{r}$ with positive direction of X, Y and Z-axies respectivily are called direction angles and cosines of these angles are called direction cosines of $\vec{r}$ usually denoted as l = cos α, m = cosβ and n = cos γ where $l_2 + m_2 + n_2 = 1$.

- The numbers a, b, c, propotional to l, m and n are called direction ratios i.e. $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$.

- If two vector $\vec{a}$ and $\vec{b}$ are represented in magnitude and direction by the sides of a triangle taken in order, then their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by the third side of triangle taken in opposite order. This is called triangle law of addition of vectors.

- If $\vec{a}$ is any vector and λ is a scalar, then λ $\vec{a}$ is a vector, collinear with $\vec{a}$ and $\left|\lambda\vec{a}\right|$ = $\left|\lambda\right|\left|\vec{a}\right|$.

- Any vector $\vec{a}$ can be written is $\vec{a}$ = $\left|\vec{a}\right|\hat{a}$ where $\hat{a}$ is a unit vector in the direction of $\hat{a}$.

- If A(x_1, y_1, z_1) and B(x_2, y_2, z_2) be any two points in space, then $\overrightarrow{AB}$ = $(x_2 - x_1)$ $\hat{i}$ + $(y_2 - y_1)$ $\hat{j}+(z_2-z_1)\hat{k}$.

- If $\vec{a}$ and $\vec{b}$ be the position vectors of points A and B, and C is any point dividing AB internally in m : n, then position vector $\vec{c}$ of C is given as $\vec{c}$ = $\frac{m\vec{b}+n\vec{a}}{m+n}$. If C divides $\overrightarrow{AB}$ in m : n externally, then $\vec{C} = \dfrac{m\vec{b} - n\vec{a}}{m-n}$.

● Scalar Product (Dot Product) of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}.\vec{b}$ and is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$ ($0 \le \theta \le \pi$)

● Dot product of two vectors is commutative i.e., $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

● For unlike parallel vectors $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{b}$ = $|\vec{a}||\vec{b}|$, so $\vec{a}.\vec{a}$ = $|\vec{a}|^2$

● For unlike parallel vectors $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{b}$ = $-|\vec{a}||\vec{b}|$

● If $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} \perp \vec{b}$.

● $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0, \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

● If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$.

● Projection of a vector $\vec{a}$ on $\vec{b}$ = $\left|\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}\right|$

● Projection of a vector $\vec{a}$ along $\vec{b}$ = $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}\right)\hat{b}$.

● Cross product (Vector product) of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a} \times \vec{b}$ and is defined as $\vec{a} \times \vec{b}$ = $|\vec{a}||\vec{b}|$ sinθ $\hat{\eta}$, where θ is the angle between $\vec{a}$ and $\vec{b}$ ($0 \le \theta \le \prod$) and $\hat{\eta}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\hat{\eta}$ form a right handed system.

● Vector product of two vectors $\vec{a}$ and $\vec{b}$ is not commutative i.e., $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$, but $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

● If $\vec{a} \times \vec{b} = \vec{\text{O}} \Leftrightarrow \vec{a} = \vec{O}$ or $\vec{b} = \vec{O},$ or $\vec{a} \square \vec{b}.$

● $\vec{a}.\vec{a} = \vec{\text{O}}, \text{So } \hat{i} \times \hat{i} = \vec{\text{O}} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}.$

● If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

● If θ is angle between two non zero vector $\vec{a}$ and $\vec{b}$, then

$$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

● $\left|\vec{a} \times \vec{b}\right|$ is the area of that parallelogram where adjacent sides are vectors $\vec{a}$ and $\vec{b}$.

● $\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$ is the area of that parallelorgam whose diagonals are $\vec{a}$ and $\vec{b}$.

● If $\vec{a}, \vec{b}$ and $\vec{c}$ forms a triangle, then area of the triangle = $\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$ = $\frac{1}{2}\left|\vec{b} \times \vec{c}\right|$ = $\frac{1}{2}\left|\vec{c} \times \vec{a}\right|.$

● If $\vec{a}, \vec{b}$ and $\vec{c}$ are position vectors of vertices of a triangle, then area of triangle = $\frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$

Scalar triple product of vectors

The scalar triple product of three vectors $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

Geometrical Interpretation : $(\vec{a} \times \vec{b}).\vec{c}$ represents the volume of parallelepiped whose coterminous edges are represented by $\vec{a}\ \vec{b}$ and $\vec{c}$.

Scalar triple product of three vectors remains unchanged as long as their cyclic order remains unchanged.

$(\vec{a} \times \vec{b}).\vec{c}$ = $(\vec{b} \times \vec{c}).\vec{a}$ = $(\vec{c} \times \vec{a}).\vec{b}$

or

$[\vec{a}\ \vec{b}\ \vec{c}]$ = $[\vec{b}\ \vec{c}\ \vec{a}]$ = $[\vec{c}\ \vec{a}\ \vec{b}]$

Scalar triple product changes its sign but not magnitude, when the cyclic order of vectors is changed.

$[\vec{a}\ \vec{b}\ \vec{c}]$ = – $[\vec{a}\ \vec{c}\ \vec{b}]$

Scalar triple product vanishes if any two of its vector are equal.

Scalar triple product vanishes if any two if its vector are parallel or collinear.

Necessary and sufficient condition for three non zero non collinear vectors $\vec{a}\ \vec{b}$ and $\vec{c}$ to be coplanar is that $[\vec{a}\ \vec{b}\ \vec{c}]$ = 0

Scalar triple Product in terms of component

Let = $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

then $[\vec{a}\ \vec{b}\ \vec{c}]$ = $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Question for Practice

Very Short Answer Type Questions (1 Mark)

1. Write direction cosines of vector $2\hat{i} + \hat{j} + 2\hat{k}$

2. For what value of 'x' the vectors

$2\hat{i} - 3\hat{j} + 4\hat{k}$ and $x\hat{i} - 6\hat{j} + 8\hat{k}$ are collinear

3. Write the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$

4. Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having

$\vec{a}.\vec{b} = \sqrt{6}$

5. If $\vec{a}.\vec{a} = 0$ and $\vec{a}.\vec{b} = 0$ = 0 then what can be concluded about vector $\vec{b}$

6. What is the cosine of the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with z axis

7. Write a vector of Magnitude 3 units in the direction of vector $\hat{i} + \hat{j} - \hat{k}$

8. If $\vec{a}$ = $2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b}$ = $6\hat{i} + \lambda\hat{j} + 9\hat{k}$ and $\vec{a} \,\square\, \vec{b}$ find the value of λ

9. For what value of λ $\vec{a}$ = $2\hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{b}$ = $\hat{i} + \hat{j} + \hat{k}$ are prependicular to each other

10. If $\vec{a}$ is a unit vector such that $\vec{a} \times i = \hat{j}$ find $\vec{a}.i$

11. $|\vec{a}|$ = 2 $|\vec{b}|$ = 7, $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ write the angle between $\vec{a}$ and $\vec{b}$

12. If $\vec{a}, \vec{b}$ represent Diagonal of rhombus, then write the value of $\vec{a}.\vec{b}$

13. Write unit vector in the direction of $\vec{a} + \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

14. Write $|\vec{a} - \vec{b}|$, If two vector $\vec{a}$ and $\vec{b}$ are such that

$|\vec{a}|$ = 2, $|\vec{b}|$ = 3, $\vec{a}.\vec{b}$ = 4

15. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} \times \vec{b}$ is also a unit vector. Find the angle between $\vec{a}$ and $\vec{b}$

16. What is the value of $|\vec{x}|$ If for unit vector $\vec{a}$

$(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$

17. Write the value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.((\hat{k} \times \hat{i}) + \hat{k}.(\hat{j} \times \hat{i})$

18. Find the value of λ so that the vectors

$\vec{a}$ = $2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b}$ = $\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{j} + \lambda\hat{k}$ are coplanar

19. Find the value $[\hat{i}\ \hat{j}\ \hat{k}]$

20. Find the volume of parallelepiped whose coterminous edges are represented by the vectors

$\vec{a}$ = $\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b}$ = $3\hat{i} + 7\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} + 5\hat{j} + 3\hat{k}$

Short Answer Type Questions (4 Marks)

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$ then prove that $\vec{a}$ = ±2 $(\vec{b} \times \vec{c})$

2. Find a vector of magnitude 5 units, prependicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

3. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}, \vec{b} = \vec{a}.\vec{c}$

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a} \neq 0$ then prove that $\vec{b} = \vec{c}$

4. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$

5. Using vectors find the area of triangle with vertices A(1, 1, 2) B(2, 3, 5) and (1, 5, 5)

6. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of ΔABC respectively, find an expression for the area of ΔABC and Hence deduce the condition for the points A, B, C to be collinear

7. Show that the area of || gm having Diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ is $5\sqrt{3}$ sq unit.

8. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of magnitude 3, 4, 5 units respectively. If each of these is ⊥ to the sum of other two vectors, then find $\left|\vec{a} + \vec{b} + \vec{c}\right|$

9. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$

$|\vec{a}|$ = 3 $|\vec{b}|$ = 5 $|\vec{c}|$ = 7 find the angle between $\vec{a}$ and $\vec{b}$

10. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then show that

(i) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} - \vec{b}\right|$

(ii) $\cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} + \vec{b}\right|$

11. If a unit vector $\vec{a}$ makes $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with x axis and y axis respectively and an obtuse angle θ with z axis, then find θ and the components of $\vec{a}$ along axis.

12. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $2\vec{a} - 4\vec{b}$ and $10\vec{a} + 8\vec{b}$ are perpendicular to each other. Find the angle between $\vec{a}$ and $\vec{b}$

13. Find a vector $\vec{a}$ such that

$\vec{a}.(\hat{i} + \hat{j}) = 2 \quad \vec{a}.(\hat{i} - \hat{j}) = 3 \quad \vec{a}.\hat{k} = 0$

14. If $\vec{a}$ makes equal angles with the coordinate axes and has magnitude 3, then find the angle between $\vec{a}$ and each of three coordinate axes.

15. A girl walks 4 km west wards, then she walks 3 km is a direction 30° east of north and stops. Determine the girls displacement from her initial point of departure

16. Prove the following

(i) $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}\,\vec{b}\,\vec{c}]$

(ii) $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$

17. Show that the points with position vectors

$6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{i} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$ are coplanar.

18. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}, \vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $\vec{a} - \vec{d}$ is || to $\vec{b} - \vec{c}$

19. Let $\vec{a} = \hat{i} - \hat{j}$ $\vec{b} = 3\hat{i} - \hat{k}$ $\vec{c} = 7\hat{i} - \hat{k}$ find a vector $\vec{d}$ which is ⊥ to both $\vec{a}$ and $\vec{b}$

and $\vec{c}.\vec{d} = 1$

20. If $|\vec{a}| = |\vec{b}| = |\vec{a}+\vec{b}| = 1$ then find $|\vec{a}-\vec{b}|$

21. Prove $(\vec{a}\times\vec{b})^2$ = $\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

22. If $|\vec{a}| = 2 \quad |\vec{b}| = 5 \quad |\vec{a}\times\vec{b}| = 8$ find $\vec{a}.\vec{b}$

23. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w} = \vec{0}$

If $|\vec{u}| = 3 |\vec{v}| = 4 |\vec{w}| = 5$ find $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$

24. In a Regular hexagon ABCDEF if $\overrightarrow{AB} = \vec{a}\ \overrightarrow{BC} = \vec{b}$ then express $\overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{EF}, \overrightarrow{FA}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AE},$ and $\overrightarrow{CE}$ is terms of $\vec{a}$ and $\vec{b}$

25. Given $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ express $\vec{b} = \vec{b_1} + \vec{b_2}$ where $\vec{b},$ is parallel to $\vec{a}$ and $\vec{b_2}$ is $\perp$ $\vec{a}$

Answers

1. $\left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle$ 2. x = –4

3. 0 4. $\frac{\pi}{4}$

5. $\vec{b} = \vec{a}$ 6. cosθ = $\frac{1}{2}$

7. $\sqrt{3}(\hat{i} + \hat{j} - \hat{k})$ 8. –3

9. λ = –1 10. 0

11. $\frac{\pi}{6}$ 12. 0

13. $\frac{1}{3}(2\hat{i}+2\hat{j}+\hat{k})$ 14. $\sqrt{5}$

15. $\frac{\pi}{2}$ 16. 4

17. 1 18. λ = –1

19. 1

20. 39 cubic unit

Answer of four marks question

1. Given $\vec{a}.\vec{b}=0$ and $\vec{a}.\vec{c}=0$ ⇒ is ⊥ to both $\vec{b}$ and $\vec{c}$

Also $\vec{a}$ is a unit vector ∴ = ± $\frac{\vec{b}\times\vec{c}}{\left|\vec{b}\times\vec{c}\right|}$

But $\left|\vec{b}\times\vec{c}\right|=\left|\vec{b}\right|\left|\vec{c}\right|\sin\frac{\pi}{6}=1\times1\times\frac{1}{2}=\frac{1}{2}$

2. $\pm\frac{5}{\sqrt{6}}(-\hat{i}+2\hat{j}-\hat{k})$

3. $\vec{a}.\vec{b}=\vec{a}.\vec{c}\Rightarrow\vec{a}.\vec{b}-\vec{a}.\vec{c}=0\Rightarrow\vec{a}.(\vec{b}-\vec{c})=0$ $\vec{a}=\vec{0}$ or $\vec{b}-\vec{c}=\vec{0}$ or $\vec{a}\perp(\vec{b}-\vec{c})$ (1) but $\vec{a}\neq\vec{0}$ (given) $\vec{a}\times\vec{b}$ = $\vec{a}\times\vec{c}$ ⇒ $\vec{a}\times\vec{b}-\vec{a}\times\vec{c}=\vec{0}$ ⇒ $\vec{a}\times(\vec{b}-\vec{c})=\vec{0}$ $\vec{a}=\vec{0}$ or $\vec{b}-\vec{c}=\vec{0}$ or $\vec{a}\,\square\,\vec{b}-\vec{c}$ (2) but $\vec{a}\neq\vec{0}$ from (1) and (2) we get $\vec{b}=\vec{c}$ ($\vec{a}\perp\vec{b}-\vec{c}$ and $\vec{a}\,\square\,\vec{b}-\vec{c}$ can not held simultaneously)

4. $\vec{c}=\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$ 5. $\frac{1}{2}\sqrt{61}$

6. Area of ΔABC = $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$= \dfrac{1}{2}\left|(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right|$

$= \dfrac{1}{2}\left|\vec{b}-\vec{c}-\vec{b}\times\vec{a}-\vec{a}\times\vec{c}+\vec{a}\times\vec{a}\right|$

$= \dfrac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$

$(\vec{a}\times\vec{a}=\vec{0})$ $\quad \vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

Point A, B, C to be collinear if area of ΔABC = 0 $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

7. Hint Area of 11gm $\dfrac{1}{2}\left|\vec{d}_1\times\vec{d}_2\right|$ $\vec{d}_1$ & $\vec{d}_2$ are the vectors along diagonal

8. $5\sqrt{2}$

9. $\vec{a}+\vec{b}+\vec{c}=0$ ⇒ $\vec{a}+\vec{b}=-\vec{c}$

⇒ $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=(-\vec{c}).(-\vec{c})$

$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=|\vec{c}|^2$

$|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=|\vec{c}|^2$

$3_2 + 5_2 + 2 \times 3 \times 5 \cos\theta = 7_2$ $\theta = 60°$

11. $\theta = \dfrac{2\pi}{3}$, $\dfrac{1}{\sqrt{2}}\hat{i}, \dfrac{1}{2}\hat{j}, -\dfrac{1}{2}\hat{k}$

12. angle between $\vec{a}$ and $\vec{b}$ is 120°.

13. $\vec{a}=\dfrac{5}{2}\hat{i}-\dfrac{1}{2}j$

14. Let $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$

cosθ = $\frac{\vec{a}.\hat{i}}{|\vec{a}||\hat{i}|}$ = $\frac{x_1}{3}$

x_1 = 3cosθ

Similarly y_1 = 3cosθ z_1 = 3cosθ

$|\vec{a}|$ = 3 $\sqrt{x_1^2 + y_1^2 + z_1^2}$ = 3 ⇒ $\sqrt{9\cos^2\theta + 9\cos^2\theta + 9\cos^2\theta}$ = 3

$3\sqrt{3}\cos\theta = 3$ ⇒ $\cos\theta = \frac{1}{\sqrt{3}}$ ⇒ $\cos^{-1}\frac{1}{\sqrt{3}}$

15. $\vec{OB}$ = $\vec{OA} + \vec{AB}$ $\vec{OA}$ = $-4\hat{i}$

In ∆AMB

AM = AB cos60° MB = ABsin60°

AM = $\frac{3}{2}$ MB = $\frac{3\sqrt{3}}{2}$

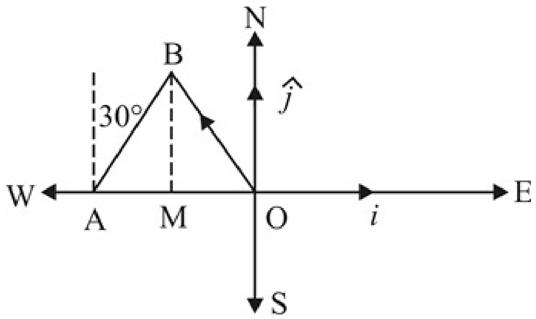
$\vec{AB}$ = $\vec{AM} + \vec{MB}$

$\overrightarrow{AB}$ = $\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Girls displacement from initial point of departure

$$-4\hat{i} + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right) = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

19. $\frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}$



22. 6

23. –25

24. $\overrightarrow{CD} = \vec{b} - \vec{a}$ $\overrightarrow{DE} = -\vec{a}$ $\overrightarrow{EF} = -\vec{b}$ $\overrightarrow{FA} = \vec{a} - \vec{b}$ $\overrightarrow{AC} = \vec{a} + \vec{b}$ $\overrightarrow{AD} = 2\vec{b}$

$\overrightarrow{AE} = 2\vec{b} - \vec{a}$ $\overrightarrow{CE} = \vec{b} - 2\vec{a}$

25. $\vec{b}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$

$\vec{b}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

$A(\vec{a})$

$C(\vec{c})$

$B(\vec{b})$