CHAPTER-10 (Gate only)

-:- Columns -:-

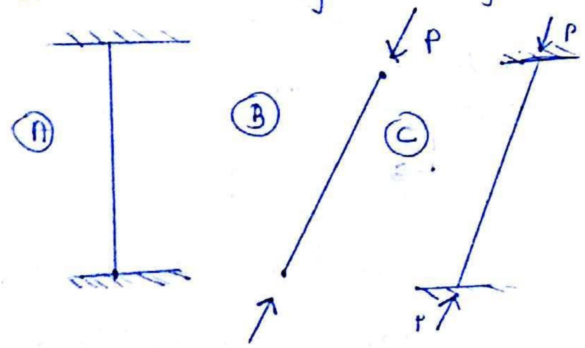
Columns is define as a vertical structral member which is subjected to axial compressive load during it functionality

End condn of Colums may be both ends Hinge, both ends fixed, or fixed and hing or fixed and free.

Type's of Columns :-

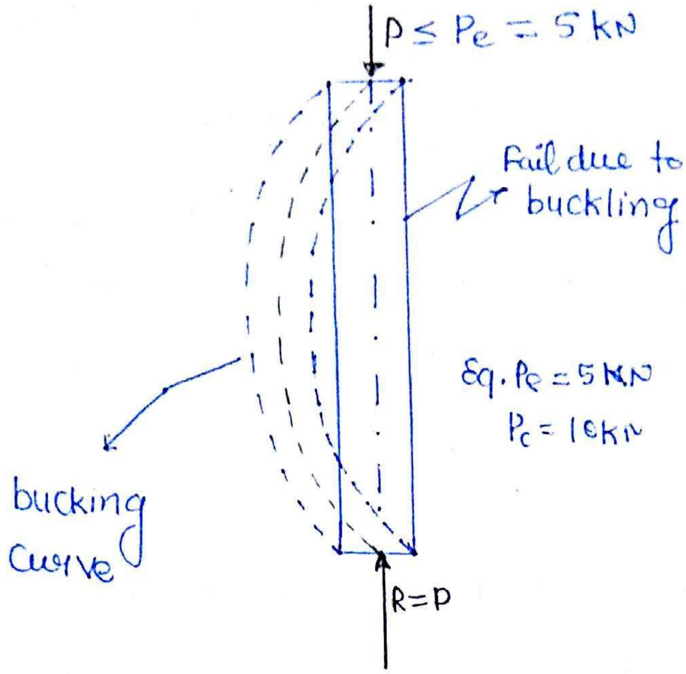
1. long Column's :- fails due to buckling (Strong in Crushing)
2. Medium Column's - fails due to both crushing & buckling
3. short Column's - fails due to crushing (Strong in buckling)

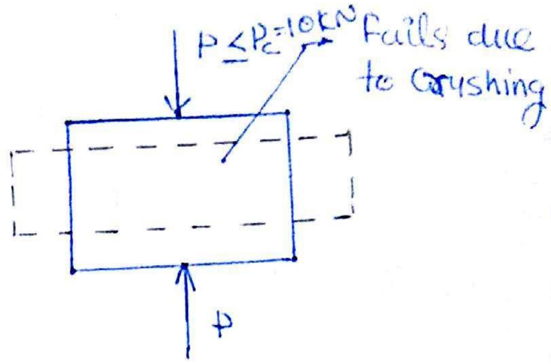
* Column's are always subjected to vertical position



Ⓐ Ⓑ Ⓒ truces
Ⓐ Column's

All columns are trusses but all trusses are not columns.





⇒ long Column

$P \le P_e$; $P \le P_e$

P_e = Buckling load $(P \le P_e)$

P_c = crushing load $(P \gtrless P_e)$

⇒ Short Column

$\sigma_{ind} \le \sigma_{per}$

$P/A \le \frac{S_{yc}}{N}$

$P \le A(\sigma_c)_{per}$

↓

doesn't depend of 'L'

Euler's Buckling load :- (P_e)

$$\boxed{P_e = \frac{\pi^2 E I_{min}}{L_e^2} \quad \text{OR} \quad \frac{n\pi^2 E I_{min}}{L^2}}$$

Based upon assumptions

P_e - Eular's bucking load

L - Actual length of Column

l_e - effective length of Column

$l_e = \alpha L$

where α = length fixity Coeff

$n = \frac{1}{\alpha^2}$ = end fixity Coeff

For buckling in Column Jonshon formula is best bcoz it based on actual experiment

I_{min} = min of $[I_{xx}$ & $I_{yy}]$

$P_e \propto I_{min}$

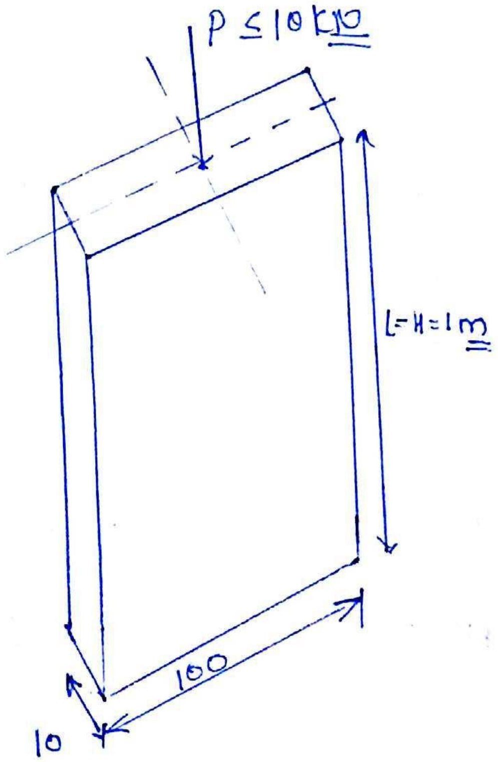
$P_e \propto E$

$P_e \propto \frac{1}{L^2}$

$P_e \propto \left[n \text{ or } \frac{1}{\alpha^2}\right]$

when other parameters Remain Constant.

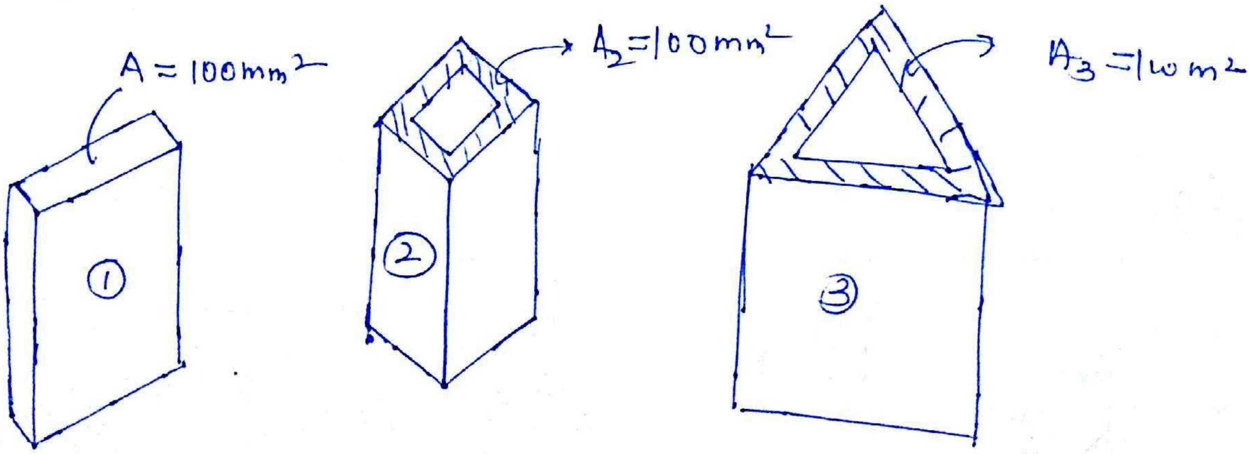
Eg.



$$\frac{(P_e)_{y-y}}{(P_e)_{x-x}} = \frac{\frac{1}{12}(10)(100)^3}{\frac{1}{12}(100)(10)^3} = 100$$

$$(P_e)_{x-x} = \frac{\pi^2 E I_{x-x}}{L_e^2} = 10\,KN$$

$$(P_e)_{y-y} = 1000\,kN$$



$E_1 = E_2 = E_3$

$A_1 = A_2 = A_3$

$L_1 = L_2 = L_3$

$\alpha_1 = \alpha_2 = \alpha_3$

$$\boxed{\frac{(P_e)_{solid}}{(P_e)_{Hollow}} = \frac{1-k^2}{1+k^2} = \frac{3}{5} \quad k = 1/2}$$

A, E, L Same

$I_3 > I_2 > I_1 \Rightarrow (P_e)_3 > (P_e)_2 > (P_e)_1$

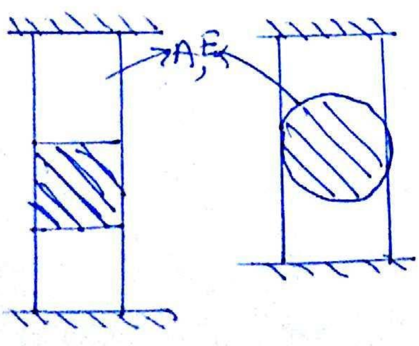
End Cond^{ns} ($\longrightarrow$) Value of (α) $\downarrow$	B.H.	B.F.	F & H	F & F
α	1	$1/2$	$\frac{1}{\sqrt{2}}$	2
$n = \frac{1}{\alpha^2}$	1	4	2	$\frac{1}{4} = 0.25$

$$(P_e)_{B.F.} > (P_e)_{F\&H} > (P_e)_{B.H} > (P_e)_{F\&F}$$

$$(P_e)_{B.F.} = 2\,(P_e)_{F\&H} = 4\,(P_e)_{BH} = 16\,(P_e)_{F\&F}$$

for given E, I_{min} & L

eg



$A = 100\,m^2$

$l_1 = 1\,m$

$l_2 = 0.8\,m$

$I_1 > I_2 \Rightarrow (P_e)_1 \not> (P_e)_2$

$L_1 > L_2 \Rightarrow (P_e)_1 < (P_e)_2$

$$\frac{(P_e)_1}{(P_e)_2} = \left(\frac{I_1}{I_2}\right) \times \left(\frac{L_2}{L_1}\right)^2$$

Imp.

Slenderness Ratio (S)

It is used to compare buckling load of diff. Column made of same area & same x-s/c Area

$$S = \frac{L_e}{k} = \frac{\text{effective length of Column}}{\text{Radius of gyration}}$$

where $k = \sqrt{\frac{I_{min}}{A}}$

$$\sigma_e = \text{Buckling stress} = \frac{P_e}{A}$$

$$\sigma_e = \frac{P_e}{A} = \frac{\pi^2 E I_{min}}{L_e^2 A} = \frac{\pi^2 E k^2}{L_e^2}$$

$$\sigma_e = \frac{P_e}{A} = \frac{\pi^2 E}{(L_e/k)^2}$$

$$\boxed{\sigma_e = \frac{P_e}{A} = \frac{\pi^2 E}{S^2}}$$

* For a given 'E' & 'A' (σ_e or P_e) $\propto \frac{1}{S^2}$

$S\downarrow \rightarrow P_e\uparrow \Rightarrow$ buckling tendency ($\downarrow$)

$S\uparrow \rightarrow P_e\downarrow \Rightarrow$ buckling tendency ($\uparrow$)

$(S)_{L.C.} > (S)_{M.C.} > (S)_{S.C.}$

$[\because (P_e)_{S.C.} > (P_e)_{M.C} > (P_e)_{L.C.}]$

For steel Coloumns

Short Column S.C. $\Rightarrow$ $S' \leq 30$

Medium Column M.C. $\Rightarrow$ $30 < S' \leq 100$

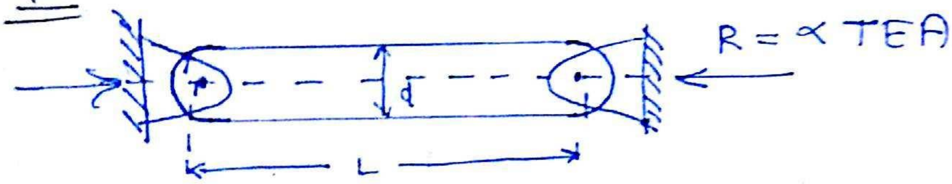
Long Column L.C. $\Rightarrow$ $S > 100$

$\sigma_c \leq P.L. \Rightarrow L.C$ (Propotional limit)

$\sigma_e > S_{yc} \Rightarrow S.C.$ (Yield strength of Coulmn) (S_{Y_c})

$P.L. < \sigma_c \leq S_{YC} \Rightarrow M.C$

<u>Q.6</u>



$$P_e = \frac{n\pi^2 E I_{min}}{L^2} = \frac{(1)\pi^2 E \, \pi d^4}{L^2 \times 64}$$

$$P_e = \frac{\pi^3 E d^4}{64L^2} = \text{Const.}$$

$T\uparrow \Rightarrow R\uparrow$ at a particular temp $R_e > P_e$ Bucking will occur

$$\Rightarrow R > \frac{\pi^3 E d^4}{64L^2} \Rightarrow \alpha T E \frac{\pi d^2}{4} > \frac{\pi^3 E d^4}{64L^2} \Rightarrow T > \frac{\pi^2 d^2}{16\alpha L^2} > 49.35°C$$