Factorisation

Exercise 12.1

Q. 1. B. Find the common factors of the given terms in each.

8x, 24

Answer : ∴ Given terms are 8x and 24

Prime factors of given terms are:-

$8x = 2 \times 2 \times 2 \times x$

$24 = 2 \times 2 \times 2 \times 3$

As the x is an undefined value,

Common factors will be

$2 \times 2 \times 2 = 8$

Q. 1. B. Find the common factors of the given terms in each.

3a, 21ab

Answer : ∴ Given terms are 3a and 21ab

Prime factors of given terms are:-

$3a = 3 \times a$

$21ab = a \times b \times 7 \times 3$

Common factors will be

$\Rightarrow 3 \times a = 3a$

Q. 1. C. Find the common factors of the given terms in each.

$7xy, 35x^2y^3$

Answer : ∴ Given terms are $7xy$ and $35x^2y^3$

Prime factors of given terms are:-

$7xy = 7 \times x \times y$

$35x^2y^3 = x \times x \times y \times y \times y \times 7 \times 5$

Common factors will be

$\Rightarrow 7 \times x \times y = 7xy$

Q. 1. D. Find the common factors of the given terms in each.

$4m^2$, $6m^2$, $8m^3$

Answer : ∴ Given terms are $4m^2$,$6m^2$ and $8m^3$

Prime factors of given terms are:-

$4m^2 = 2 \times 2 \times m \times m$

$6m^2 = 3 \times 2 \times m \times m$

$8m^3 = 2 \times 2 \times 2 \times m \times m \times m$

Common factors will be

$\Rightarrow 2 \times m \times m = 2m^2$

Q. 1. E. Find the common factors of the given terms in each.

15p, 20qr, 25rp

Answer : ∴ Given terms are 15p,20qr and 25rp

Prime factors of given terms are:-

$15p = 3 \times 5 \times p$

$20qr = 2 \times 2 \times 5 \times q \times r$

$25rp = 5 \times 5 \times r \times p$

⇒ Common factors will be 5

Q. 1. F. Find the common factors of the given terms in each.

$4x^2$, $6xy$, $8y^2x$

Answer : ∴ Given terms are $4x^2$,$6xy$ and $8y^2x$

Prime factors of given terms are:-

$4x^2 = 2 \times 2 \times x \times x$

$6xy = 3 \times 2 \times x \times y$

$8y^2x = 2 \times 2 \times 2 \times y \times y \times x$

Common factors will be

$\Rightarrow 2 \times x = 2x$

Q. 1. G. Find the common factors of the given terms in each.

$12x^2y$, $18xy^2$

Answer : Given terms are $12x^2y$and $18xy^2$

Prime factors of given terms are:-

$12yx^2 = 3 \times 2 \times 2 \times y \times x \times x$

$18xy^2 = 3 \times 2 \times 3 \times x \times y \times y$

Common factors will be

$\Rightarrow 2 \times 3 \times x \times y = 6xy$

Q. 2. A. Factorise the following expressions

$5x^2 – 25xy$

Answer : In the given expression

Check the common factors for all terms;

$\Rightarrow [5 \times x \times x - 5 \times 5 \times x \times y]$

$\Rightarrow 5 \times x[x-5 \times y]$

$\Rightarrow 5x[x-5y]$

$\therefore 5x^2 - 25xy = 5x[x-5y]$

Q. 2. B. Factorise the following expressions

$9a^2$ – 6ax

Answer : In the given expression

Check the common factors for all terms;

⇒ [5 × a × a- 2 × 3 × x × a]

⇒ a[5 × a-2 × 3 × x]

⇒ a[5a-6x]

∴ $9a^2$ - 6ax = a[5a-6x]

Q. 2. C. Factorise the following expressions

$7p^2$ + 49pq

Answer : In the given expression

Check the common factors for all terms;

⇒ [7 × p × p + 7 × 7 × p × q]

⇒ 7 × p[p + 7 × q]

⇒ 7p[p + 7q]

∴ $7p^2$ + 49pq = 7p[p + 7q]

Q. 2. D. Factorise the following expressions

$36a^2b$ – 60 a^2bc

Answer : In the given expression

Check the common factors for all terms;

⇒ [2 × 2 × 3 × 3 × a × a × b - 2 × 2 × 3 × 5 × a × a × b × c]

⇒ 2 × 2 × 3 × a × a × b[3 × b-5 × c]

⇒ $12a^2b$[3b-5c]

∴ $36a^2b - 60\,a^2bc = 12a^2b[3b-5c]$

Q. 2. E. Factorise the following expressions

$3a^2bc + 6ab^2c + 9abc^2$

Answer : In the given expression

Check the common factors for all terms;

⇒ $[3 \times a \times a \times b \times c + 2 \times 3 \times a \times b \times b \times c + 3 \times 3 \times a \times b \times c \times c]$

⇒ $3 \times a \times b \times c[a + 2 \times b + 3 \times c]$

⇒ $3abc[a + 2b + 3c]$

∴ $3a^2bc + 6ab^2c + 9abc^2 = 3abc[a + 2b + 3c]$

Q. 2. F. Factorise the following expressions

$4p^2 + 5pq – 6pq^2$

Answer : In the given expression

Check the common factors for all terms;

⇒ $[2 \times 2 \times p \times p + 5 \times p \times q - 2 \times 3 \times p \times q \times q]$

⇒ $p[2 \times 2 \times p + 5 \times q - 2 \times 3 \times q \times q]$

⇒ $p[4p + 5q-6q^2]$

∴ $4p^2 + 5pq – 6pq^2 = p[4p + 5q-6q^2]$

Q. 2. G. Factorise the following expressions

$ut + at^2$

Answer : In the given expression

Check the common factors for all terms;

⇒ $[u \times t + a \times t \times t]$

⇒ $t[u + a \times t]$

⇒ t[u + at]

∴ $ut + at^2 = t[u + at]$

Q. 3. A. Factorise the following:

3ax – 6xy + 8by – 4ab

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

3ax-6xy = 3x[a-2y] -------eq 1

Regrouping the last 2 terms we have,

8by-4ab = -4b[a-2y] -------eq 2

∵ We have to make common parts in both eq 1 and 2

Combining eq 1 and 2

3ax – 6xy + 8by – 4ab = 3x[a-2y] + [-4b[a-2y]]

= 3x[a-2y] - 4b[a-2y]

= [3x-4] [a-2y]

Hence the factors of 3ax – 6xy + 8by – 4ab are [3x-4] and [a-2y]

Q. 3. B. Factorise the following:

$x^3 + 2x^2 + 5x + 10$

Answer : In the given expression

Check whether there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

$x^3 + 2x^2 = x^2[x + 2]$ -------eq 1

Regrouping the last 2 terms we have,

$5x + 10 = 5[x + 2]$ -------eq 2

Combining eq 1 and 2

$x^3 + 2x^2 + 5x + 10 = x^2[x + 2] + 5[x + 2]$

$= [x^2 + 5][x + 2]$

Hence the factors of $x^3 + 2x^2 + 5x + 10$ are $[x^2 + 5]$ and $[x + 2]$

Q. 3. C. Factorise the following:

$m^2 – mn + 4m – 4n$

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

$m^2 - mn= m[m - n]$ -------eq 1

Regrouping the last 2 terms we have,

$4m – 4n = 4[m – n]$ -------eq 2

Combining eq 1 and 2

$m^2 – mn + 4m – 4n = 4[m – n] + m[m - n]$

$= [4 + m][m-n]$

Hence the factors of $m^2 – mn + 4m – 4n$ are $[m – n]$ and $[4 + m]$

Q. 3. D. Factorise the following:

$a^3 – a^2b^2 – ab + b^3$

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

$a^3 – a^2b^2 = a^2[a-b^2]$ -------eq 1

Regrouping the last 2 terms we have,

$– ab + b^3 = -b[a-b^2]$ -------eq 2

∵ We have to make common parts in both eq 1 and 2

Combining eq 1 and 2

$a^3 – a^2b^2 – ab + b^3 = a^2[a-b^2] -b[a-b^2]$

$= [a^2 – b][a – b^2]$

Hence the factors of $a^3 – a^2b^2 – ab + b^3$ are $[a^2 – b]$ and $[a – b^2]$

Q. 3. E. Factorise the following:

$p^2q – pr^2 – pq + r^2$

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

$p^2q – pr^2 = p[pq-r^2]$ -------eq 1

Regrouping the last 2 terms we have,

$– pq + r^2 = -1[pq-r^2]$ -------eq 2

∵ We have to make common parts in both eq 1 and 2

Combining eq 1 and 2

$p^2q – pr^2 – pq + r^2 = p[pq-r^2] -1[pq-r^2]$

$= [p - 1][pq - r^2]$

Hence the factors of $p^2q - pr^2 - pq + r^2$ are $[p - 1]$ and $[pq - r^2]$

Exercise 12.2

Q. 1. A. Factories the following expression-

$a^2 + 10a + 25$

Answer : In the given expression

1st and last terms are perfect square

$\Rightarrow a^2 = a \times a$

$\Rightarrow 25 = 5 \times 5$

And the middle expression is in form of 2ab

$10a = 2 \times 5 \times a$

$\therefore a \times a + 2 \times 5 \times a + 5 \times 5$

Gives $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow$ In $a^2 + 10a + 25$

$a = a$ and $b = 5$;

$\therefore a^2 + 10a + 25 = (a + 5)^2$

Hence the factors of $a^2 + 10a + 25$ are $(a + 5)$ and $(a + 5)$

Q. 1. B. Factories the following expression-

$l^2 - 16l + 64$

Answer : In the given expression

1st and last terms are perfect square

$\Rightarrow l^2 = l \times l$

$\Rightarrow 64 = 8 \times 8$

And the middle expression is in form of 2ab

$16l = 2 \times 8 \times l$

$\therefore l \times l + 2 \times 8 \times l + 8 \times 8$

Gives $(a-b)^2 = a^2 - 2ab + b^2$

$\Rightarrow$ In $l^2 + 16l + 64$

$a = l$ and $b = 8$;

$\therefore l^2 + 16l + 64 = (l + 8)^2$

Hence the factors of $l^2 + 16l + 64$ are $(l + 8)$ and $(l + 8)$

Q. 1. C. Factories the following expression-

$36x^2 + 96xy + 64y^2$

Answer : In the given expression

1st and last terms are perfect square

$\Rightarrow 36x^2 = 6x \times 6x$

$\Rightarrow 64y^2 = 8y \times 8y$

And the middle expression is in form of 2ab

$96xy = 2 \times 6x \times 8y$

$\therefore 6x \times 6x + 2 \times 8y \times 6x + 8y \times 8y$

Gives $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow$ In $36x^2 + 96xy + 64y^2$

$a = 6x$ and $b = 8y$;

$\therefore 36x^2 + 96xy + 64y^2 = (6x + 8y)^2$

Hence the factors of $36x^2 + 96xy + 64y^2$ are $(6x + 8y)$ and $(6x + 8y)$

Q. 1. D. Factories the following expression-

$25x^2 + 9y^2 – 30xy$

Answer : In the given expression

1st and last terms are perfect square

$\Rightarrow 25x^2 = 5x \times 5x$

$\Rightarrow 9y^2 = 3y \times 3y$

And the middle expression is in form of 2ab

$30xy = 2 \times 5x \times 3y$

$\therefore 5x \times 5x + 2 \times 3y \times 5x + 3y \times 3y$

Gives $(a-b)^2 = a^2-2ab + b^2$

$\Rightarrow$ In $25x^2 – 30xy + 9y^2$

$a = 5x$ and $b = 3y$;

$\therefore 25x^2 - 30xy + 9y^2 = (5x-3y)^2$

Hence the factors of $25x^2 - 30xy + 9y^2$ are $(5x-3y)$ and $(5x-3y)$

Q. 1. E. Factories the following expression-

$25m^2 – 40mn + 16n^2$

Answer : In the given expression

1st and last terms are perfect square

$\Rightarrow 25m^2 = 5m \times 5m$

$\Rightarrow 16n^2 = 4n \times 4n$

And the middle expression is in form of 2ab

$40mn = 2 \times 5m \times 4n$

$\therefore 5m \times 5m - 2 \times 4n \times 5m + 4n \times 4n$

Gives $(a-b)^2 = a^2-2ab + b^2$

$\Rightarrow$ In $25m^2 – 40mn + 16n^2$

$a = 5m$ and $b = 4n$;

$\therefore 25m^2 – 40mn + 16n^2 = (5m-4n)^2$

Hence the factors of $25m^2 – 40mn + 16n^2$ are $(5m-4n)$ and $(5m-4n)$

Q. 1. F. Factories the following expression-

$81x^2 – 198 xy + 121y^2$

Answer : In the given expression

1st and last terms are perfect square

$\Rightarrow 81x^2 = 9x \times 9x$

$\Rightarrow 121y^2 = 11y \times 11y$

And the middle expression is in form of 2ab

$198xy = 2 \times 9x \times 11y$

$\therefore 9x \times 9x - 2 \times 11y \times 9x + 11y \times 11y$

Gives $(a-b)^2 = a^2-2ab + b^2$

$\Rightarrow$ In $81x^2 – 198xy + 121y^2$

$a = 9x$ and $b = 11y$;

$\therefore 81x^2 – 198xy + 121y^2 = (9x-11y)^2$

Hence the factors of $81x^2 – 198xy + 121y^2$ are $(9x-11y)$ and $(9x-11y)$

Q. 1. G. Factories the following expression-

$(x + y)^2 – 4xy$
(Hint: first expand $(x + y)^2$)

Answer : If $(a + b)^2 = a^2 + 2ab + b^2$

Then $(x + y)^2 – 4xy$

$= x^2 + 2xy + y^2 - 4xy$

$= x^2 + y^2 - 2xy$

In given expression

1st and last terms are perfect square

$\Rightarrow x^2 = x \times x$

$\Rightarrow y^2 = y \times y$

And the middle expression is in form of 2ab

$2xy = 2 \times x \times y$

$\therefore x \times x - 2 \times y \times x + y \times y$

Gives $(a-b)^2 = a^2 - 2ab + b^2$

$\Rightarrow$ In $x^2 - 2xy + y^2$

$a = x$ and $b = y$;

$\therefore x^2 - 2xy + y^2 = (x-y)^2$

Hence the factors of $(x + y)^2 - 4xy$ are $(x-y)$and $(x-y)$

Q. 1. H. Factories the following expression-

$l^4 + 4l^2m^2 + 4m^4$

Answer : In given expression

1st and last terms are perfect square

$\Rightarrow l^4 = l^2 \times l^2$

$\Rightarrow m^4 = m^2 \times m^2$

And the middle expression is in form of 2ab

$4l^2m^2 = 2 \times l^2 \times m^2$

$\therefore l^2 \times l^2 + 2 \times m^2 \times l^2 + m^2 \times m^2$

Gives $(a + b)^2 = a^2 + 2ab + b^2$

⇒ In $l^4 + 4l^2m^2 + m^4$

$a = l^2$ and $b = m^2$;

∴ $l^4 – 4l^2m^2 + m^4 = (l^2-m^2)^2$

Hence the factors of $l^4 + 4l^2m^2 + 4m^4$ are (l^2-m^2) and (l^2-m^2)

Q. 2. A. Factories the following

x^2 – 36

Answer : In given expression

Both terms are perfect square

⇒ $x^2 = x \times x$

⇒ $36 = 6 \times 6$

∴ x^2-6^2

Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = x and b = 6;

$x^2 – 36 = (x + 6)(x-6)$

Hence the factors of $x^2 – 36$ are $(x + 6)$ and $(x-6)$

Q. 2. B. Factories the following

$49x^2 – 25y^2$

Answer : In given expression

Both terms are perfect square

⇒ $49x^2 = 7x \times 7x$

⇒ $25y^2 = 5y \times 5y$

∴ $49x^2-25y^2$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = 7x and b = 5y;

$49x^2 - 25y^2 = (7x + 5y)(7x-5y)$

Hence the factors of $49x^2 - 25y^2$ are $(7x + 5y)$ and $(7x-5y)$

Q. 2. C. Factories the following

$m^2 - 121$

Answer : In given expression

Both terms are perfect square

$\Rightarrow m^2 = m \times m$

$\Rightarrow 121 = 11 \times 11$

$\therefore m^2-121$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = m and b = 11;

$m^2 - 121 = (m + 11)(m-11)$

Hence the factors of $m^2 - 121$ are $(m + 11)$ and $(m-11)$

Q. 2. D. Factories the following

$81 - 64x^2$

Answer : In given expression

Both terms are perfect square

$\Rightarrow 64x^2 = 8x \times 8x$

$\Rightarrow 81 = 9 \times 9$

$\therefore 81-64x^2$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = 9 and b = 8x;

$81-64x^2 = (9-8x)(9 + 8x)$

Hence the factors of $81-64x^2$ are$(9-8x)$ and $(9 + 8x)$

Q. 2. E. Factories the following

$x^2y^2 – 64$

Answer : In given expression

Both terms are perfect square

$\Rightarrow y^2x^2 = xy \times xy$

$\Rightarrow 64 = 8 \times 8$

$\therefore x^2y^2 – 64$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = xy and b = 8;

$x^2y^2 – 64 = (xy-8)(xy + 8)$

Hence the factors of $x^2y^2 – 64$ are (xy-8) and (xy + 8)

Q. 2. F. Factories the following

$6x^2 – 54$

Answer : In given expression

Take out the common factor,

$[2 \times 3 \times x \times x-2 \times 3 \times 3 \times 3]$

$\Rightarrow 2 \times 3[x \times x-3 \times 3]$

$\Rightarrow 6[x^2-9]$

Both terms are perfect square

$\Rightarrow x^2 = x \times x$

$\Rightarrow 9 = 3 \times 3$

$\therefore x^2 – 9$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = x and b = 3;

$x^2 – 9 = (x-3)(x + 3)$

Hence the factors of $6x^2 – 54$ are 6,(x-3) and (x + 3)

Q. 2. G. Factories the following

$x^2 – 81$

Answer : In given expression

Both terms are perfect square

⇒ $x^2 = x \times x$

⇒ $81 = 9 \times 9$

∴ $x^2 – 81$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = x and b = 9;

$x^2 – 81 = (x-9)(x + 9)$

Hence the factors of $x^2 – 81$ are (x-9) and (x-9)

Q. 2. H. Factories the following

$2x – 32x^5$

Answer : In given expression

Take out the common factor,

$[2 \times x - 2 \times 2 \times 2 \times 2 \times 2 \times x \times x \times x \times x \times x]$

⇒ $2 \times x[1 - 2 \times 2 \times 2 \times 2 \times x \times x \times x \times x]$

⇒ $2x [1-16x^4] = 2x [1-(2x)^4]$

⇒ In the term $1-(2x)^4$

$= 1-(4x^2)^2$

Both terms are perfect square

⇒ $(4x^2)^2 = 4x^2 \times 4x^2$

⇒ $1 = 1 \times 1$

∴ $1-(4x^2)^2$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where $a = 1$ and $b = 4x^2$;

$1-16x^4 = (1-4x^2)(1 + 4x^2)$

→ $1-4x^2 = 1-(2x)^2$

∴ $1-4x^2$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where $a = 1$ and $b = 2x$;

$1-4x^2 = (1-2x)(1 + 2x)$

∴ $1-16x^2 = (1-2x)(1 + 2x)\ (1 + 4x^2)$

Hence the factors of $2x – 32x^5$ are $2x$,$(1-2x)$,$(1 + 2x)$ and $(1 + 4x^2)$

Q. 2. I. Factories the following

$81x^4 – 121x^2$

Answer : In given expression

Take out the common factor,

$[3 \times 3 \times 3 \times 3 \times x \times x \times x \times x - 11 \times 11 \times x \times x]$

$\Rightarrow x \times x[3 \times 3 \times 3 \times 3 \times x \times x - 11 \times 11]$

$\Rightarrow x^2[81x^2 – 121]$

In expression $81x^2 - 121$

Both terms are perfect square

$\Rightarrow 81x^2 = 9x \times 9x$

$\Rightarrow 121 = 11 \times 11$

∴ $81x^2 – 121$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where $a = 9x$ and $b = 11$;

$81x^2 – 121 = (9x-11)(9x + 11)$

Hence the factors of $81x^4 - 121x^2$ are x^2,(9x-11) and (9x + 11)

Q. 2. J. Factories the following

$(p^2 - 2pq + q^2) - r^2$

Answer : In the given expression $p^2 - 2pq + q^2$

1st and last terms are perfect square

$\Rightarrow p^2 = p \times p$

$\Rightarrow q^2 = q \times q$

And the middle expression is in form of 2ab

$2pq = 2 \times p \times q$

$\therefore p \times p - 2 \times p \times q + q \times q$

Gives $(a-b)^2 = a^2-2ab + b^2$

$\Rightarrow$ In $p^2 - 2pq + q^2$

a = p and b = q;

$\therefore p^2 - 2pq + q^2 = (p-q)^2$

Now the given expression is $(p-q)^2 - r^2$

Both terms are perfect square

$\Rightarrow (p-q)^2 = (p-q) \times (p-q)$

$\Rightarrow r^2 = r \times r$

$\therefore (p-q)^2 - r^2$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = (p-q) and b = r;

$(p-q)^2 - r^2 = (p-q-r)\ (p-q + r)$

Hence the factors of $(p^2 - 2pq + q^2) - r^2$ are (p-q-r) and (p-q + r)

Q. 2. K. Factories the following

$(x + y)^2 – (x – y)^2$

Answer : In the given expression

We know that

$(a + b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2-2ab + b^2$

Hence

If a = x and b = y

$(x + y)^2 – (x – y)^2 = x^2 + y^2 + 2xy – [x^2 + y^2-2xy]$

$= x^2 + y^2 + 2xy -x^2-y^2 + 2xy$

$= 4xy$

Q. 3. A. Factories the expressions-

$lx^2 + mx$

Answer : In the given expression

Take out the common in all the terms,

$\Rightarrow lx^2 + mx$

$\Rightarrow x[lx + m]$

Q. 3. B. Factories the expressions-

$7y^2 + 35z^2$

Answer : In the given expression

Take out the common in all the terms,

$\Rightarrow 7y^2 + 35z^2$

$\Rightarrow 7[y^2 + 5z^2]$

Q. 3. C. Factories the expressions-

$3x^4 + 6x^3y + 9x^2z$

Answer : In the given expression

Take out the common in all the terms,

$\Rightarrow 3x^4 + 6x^3y + 9x^2z$

$\Rightarrow 3x^2[x^2 + 2xy + 3z]$

Q. 3. D. Factories the expressions-

$x^2 – ax – bx + ab$

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

x^2 - ax= x[x - a] -------eq 1

Regrouping the last 2 terms we have,

-bx + ab = -b[x – a] -------eq 2

Combining eq 1 and 2

x^2 – ax – bx + ab = x[x - a] - b[x – a]

= [x - b][x - a]

Hence the factors of [x^2 – ax – bx + ab] are [x - b] and [x - a]

Q. 3. E. Factories the expressions-

3ax – 6ay – 8by + 4bx

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1^{st} 2 terms we have,

3ax – 6ay= 3a[x - 2y] -------eq 1

Regrouping the last 2 terms we have,

-8by + 4bx = 4b[x – 2y] -------eq 2

Combining eq 1 and 2

3ax – 6ay – 8by + 4bx = 3a[x - 2y] + 4b[x – 2y]

= [x – 2y][3a + 4b]

Hence the factors of [3ax – 6ay – 8by + 4bx] are [x – 2y] and [3a + 4b]

Q. 3. F. Factories the expressions-

mn + m + n + 1

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1^{st} 2 terms we have,

mn + m = m[n + 1] -------eq 1

Regrouping the last 2 terms we have,

n + 1 = 1[n + 1] -------eq 2

Combining eq 1 and 2

mn + m + n + 1 = m[n + 1] + 1[n + 1]

= [m + 1][n + 1]

Hence the factors of [mn + m + n + 1] are [m + 1] and [n + 1]

Q. 3. G. Factories the expressions-

$6ab – b^2 + 12ac – 2bc$

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

$6ab – b^2 = b[6a - b]$ -------eq 1

Regrouping the last 2 terms we have,

$12ac – 2bc = 2c[6a - b]$ -------eq 2

Combining eq 1 and 2

$6ab – b^2 + 12ac – 2bc = b[6a - b] + 2c[6a - b]$

$= [6a - b][b + 2c]$

Hence the factors of $[6ab – b^2 + 12ac – 2bc]$ are $[6a - b]$ and $[b + 2c]$

Q. 3. H. Factories the expressions-

$p^2q – pr^2 – pq + r^2$

Answer : In the given expression

Check weather there is any common factors for all terms;

None;

Regrouping the 1st 2 terms we have,

$p^2q – pr^2 = p[pq – r^2]$ -------eq 1

Regrouping the last 2 terms we have,

$– pq + r^2 = -1[pq – r^2]$ -------eq 2

$\because$ we have to make common parts in both eq 1 and 2

Combining eq 1 and 2

$p^2q - pr^2 - pq + r^2 = p[pq - r^2] -1[pq - r^2]$

$= [pq - r^2][p - 1]$

Hence the factors of $[p^2q - pr^2 - pq + r^2]$ are $[pq - r^2]$ and $[p - 1]$

Q. 3. I. Factories the expressions-

$x (y + z) - 5 (y + z)$

Answer : In the given expression

Take out the common in all the terms,

$\Rightarrow x (y + z) - 5 (y + z)$

$\Rightarrow (y + z)(x - 5)$

Hence the factors of $x (y + z) - 5 (y + z)$ are $(y + z)$ and $(x - 5)$

Q. 4. A. Factories the following

$x^4 - y^4$

Answer : In expression $x^4 - y^4$

Both terms are perfect square

$\Rightarrow x^4 = x^2 \times x^2$

$\Rightarrow y^4 = y^2 \times y^2$

$\therefore x^4 - y^4$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where $a = x^2$ and $b = y^2$;

$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$,

$\therefore x^2 - y^2$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where a = x and b = y;

$x^2 - y^2 = (x- y)(x+ y)$,

$\Rightarrow x^4 – y^4 = (x– y)(x+ y), (x^2 + y^2)$

Hence the factors of $x^4 – y^4$ are $(x– y)$,$(x+ y)$ and $(x^2 + y^2)$

Q. 4. B. Factories the following

$a^4 – (b + c)^4$

Answer : In expression $a^4 – (b + c)^4$

Both terms are perfect square

$\Rightarrow a^4 = a^2 \times a^2$

$\Rightarrow (b + c)^4 = (b + c)^2 \times (b + c)^2$

$\therefore a^4 – (b + c)^4$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where $a = a^2$ and $b = (b + c)^2$;

$a^4 – (b + c)^4 = (a^2 – (b + c)^2)(a^2 + (b + c)^2),$

$\therefore a^2 – (b + c)^2$ Seems to be in identity $a^2-b^2 = (a + b)(a-b)$

Where $a = a$ and $b = (b + c)$;

$a^2 – (b + c)^2 = (a– (b + c))(a+ (b + c)),$

$\Rightarrow a^4 – (b + c)^4 = (a– (b + c))(a+ (b + c)), (a^2 + (b + c)^2)$

$\Rightarrow a^4 – (b + c)^4 = (a–b–c)(a + b + c), (a^2 + b^2 + c^2 + 2bc)$

Hence the factors of $a^4 – (b + c)^4$ are $(a–b–c)$,$(a + b + c)$,$(a^2 + b^2 + c^2 + 2bc)$

Q. 4. C. Factories the following

$l^2 – (m – n)^2$

Answer : In the given expression $l^2 – (m – n)^2$

Both terms are perfect square

$\Rightarrow l^2 = l \times l$

$\Rightarrow (m – n)^2 = (m – n) \times (m – n)$

∴ $l^2 - (m - n)^2$ Seems to be in identity $a^2 - b^2 = (a + b)(a - b)$

Where a = a and b = (m - n);

∴ $l^2 - (m - n)^2 = (l + m - n)(l - m + n)$

Hence the factors of $l^2 - (m - n)^2$ are $(l + m - n)(l - m + n)$

Q. 4. D. Factories the following

$$49x^2 - \frac{16}{25}$$

Answer :

In the given expression $49x^2 - \frac{16}{25}$

Both terms are perfect square

⇒ $49x^2 = 7x \times 7x$

⇒ $(\frac{4}{5})^2 = \frac{4}{5} \times \frac{4}{5}$

$49x^2 - \frac{16}{25}$ Seems to be in identity $a^2 - b^2 = (a + b)(a - b)$

Where a = 7x and b = $\frac{4}{5}$;

∴ $(7x)^2 - (\frac{4}{5})^2 = (7x - \frac{4}{5})(7x + \frac{4}{5})$

Hence the factors of $49x^2 - \frac{16}{25}$ are $(7x - \frac{4}{5})$ and $(7x + \frac{4}{5})$

Q. 4. E. Factories the following

$x^4 – 2x^2y^2 + y^4$

Answer : In the given expression

1st and last terms are perfect square

$\Rightarrow x^4 = x^2 \times x^2$

$\Rightarrow y^4 = y^2 \times y^2$

And the middle expression is in form of 2ab

$2x^2y^2 = 2 \times x^2 \times y^2$

$\therefore x^2 \times x^2 - 2 \times x^2 \times y^2 + y^2 \times y^2$

Gives $(a-b)^2 = a^2-2ab + b^2$

$\Rightarrow x^4 – 2x^2y^2 + y^4$

$a = x^2$ and $b = y^2$;

$\therefore x^4 – 2x^2y^2 + y^4 = (x^2 – y^2) (x^2 + y^2)$

Hence the factors of $x^4 – 2x^2y^2 + y^4$ are $(x^2 – y^2)$ and $(x^2 + y^2)$

Q. 4. F. Factories the following

$4 (a + b)^2 – 9 (a – b)^2$

Answer : In the given expression

We know that

$(a + b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2-2ab + b^2$

Hence

$4[a^2 + 2ab + b^2] – 9[a^2-2ab + b^2]$

$4a^2 + 8ab + 4b^2 - 9a^2 + 18ab - 9b^2$

$26ab – 5a^2 - 5b^2$

$25ab + ab – 5a^2 – 5b^2$

$[25ab – 5a^2] + [ab – 5b^2]$

$5a[5b – a] – b[5b – a]$

$[5a – b][5b – a]$

Hence the factors $4 (a + b)^2 – 9 (a – b)^2$ are $[5a – b]$ and $[5b – a]$

Q. 5. A. Factories the following expressions

$a^2 + 10a + 24$

Answer : The given expression looks as

$x^2 + (a + b)x + ab$

Where $a + b = 10$; and $ab = 24$;

Factors of 24 their sum

1×24 $1 + 24 = 25$

12×2 $2 + 12 = 14$

6×4 $6 + 4 = 10$

∴ The factors having sum 10 are 6 and 4

$a^2 + 10a + 24 = a^2 + (6 + 4)a + 24$

$= a^2 + 6a + 4a + 24$

$= a(a + 6) + 4(a + 6)$

$= (a + 6)(a + 4)$

Hence the factors of $a^2 + 10a + 24$ are $(a + 6)$ and $(a + 4)$

Q. 5. B. Factories the following expressions

$x^2 + 9x + 18$

Answer : The given expression looks as

$x^2 + (a + b)x + ab$

Where $a + b = 9$; and $ab = 18$;

Factors of 18 their sum

1×18 $1 + 18 = 19$

9×2 $2 + 9 = 11$

6×3 $6 + 3 = 9$

∴ The factors having sum 9 are 6 and 3

$x^2 + 9x + 18 = x^2 + (6 + 3)x + 18$

$= x^2 + 6x + 3x + 18$

$= x(x + 6) + 3(x + 6)$

$= (x + 6)(x + 3)$

Hence the factors of $x^2 + 9x + 18$ are $(x + 6)$ and $(x + 3)$

Q. 5. C. Factories the following expressions

$p^2 – 10p + 21$

Answer : The given expression looks as

$x^2 + (a + b)x + ab$

where $a + b = -10$; and $ab = 21$;

factors of 21 their sum

-1×-21 $-1-18 = -19$

-7×-3 $-7-3 = -10$

∴ The factors having sum -10 are -7 and -3

$p^2 + 9p + 18 = p^2 + (-7-3)p + 21$

$= p^2 - 7p - 3p + 21$

$= p(p-7) - 3(p-7)$

$= (p-7)(p-3)$

Hence the factors of $p^2 + 9p + 18$ are $(p-7)$ and $(p-3)$

Q. 5. D. Factories the following expressions

$x^2 – 4x – 32$

Answer : The given expression looks as

$x^2 + (a + b)x + ab$

where $a + b = -4$; and $ab = -32$;

factors of -32 their sum

1×-32 $1-32 = -31$

-16×2 $2 -16 = - 14$

-8×4 $4 -8 = -4$

∴ the factors having sum -4 are -8 and 4

$x^2 – 4x – 32 = x^2 + (4 -8)x - 32$

$= x^2 + 4x - 8x - 32$

$= x(x + 4) -8(x + 4)$

$= (x + 4)(x-8)$

Hence the factors of $x^2 – 4x – 32$ are $(x + 4)$ and $(x-8)$

Q. 6. The lengths of the sides of a triangle are integrals, and its area is also integer. One side is 21 and the perimeter is 48. Find the shortest side.

Answer : $A = \sqrt{s(s-a)(s-b)(s-c)}$

If the area is an integer

Then $[s(s-a)(s-b)(s-c)]$ should be proper square

$\text{If } s = \frac{\text{parameter}}{2} \text{ Then } s = \frac{48}{2} = 24$

Hence :

A = √ 24(24-a)(24-b)(24-c)

If side of triangle are

a = 21 and b + c = 27

let c be smallest side

then b = 27-c

∴ √ 24(24-21)(24-27 + c)(24-c)

⇒ √ 24 × 3 × (c-3)(24-c)

⇒ √ 2 × 2 × 2 × 3 × 3 × (c-3)(24-c)

⇒ 2 × 3√2(c-3)(24-c)

⇒ 6√2(c-3)(24-c)

∴ The value of [2(c-3)(24-c)] must be a perfect square for area to be a integer

For getting square 2(c-3) should be equal to (24-c)

2(c-3) = (24-c)

2c-6 = 24-c

2c + c = 24 + 6

3c = 10

c = 10; b = 27-c = 27-10 = 17

Hence the size of smallest size is 10.

Q. 7. Find the values of 'm' for which x^2 + 3xy + x + my –m has two linear factors in x and y, with integer coefficients.

Answer : For the given 2 degree equation

That must be equal to(ax + by + c)(dx + e)

= ad.x^2 + bd.xy + cd.x + ea.x + be.y + ec

= ad.x^2 + bd.xy + (cd + ea).x + be.y + ec

x^2 + 3xy + x + my–m = ad.x^2 + bd.xy + (cd + ea).x + be.y + ec

Compare the equation

And take out the coefficient of every term

a.d = 1 ----------1

b.d = 3 ----------2

c.d + e.a = 1 ----------3

b.e = m ----------4

e.c = -m ----------5

⇒ from eq 1; a = d = 1 ∵ all coefficient are integers

After putting result in eq 3; c + e = 1 -------6

After putting result in eq 2; b = 3 --------7

⇒ divide eq 4 and 5

$$\frac{be}{ec} = \frac{m}{-m}$$

$$\frac{b}{c} = -1$$

∴ That implies b = -c = -3 ∵ eq 7

Put value of c in eq 6

-3 + e = 1

e = 1 + 3 = 4

Putting value of b and e in eq 4

$m = b \times e$

$m = 3 \times 4 = 12$

Exercise 12.3

Q. 1. A. Carry out the following divisions

$48a^3$ by $6a$

Answer : In the given term

Dividend = $48a^3 = 2 \times 2 \times 2 \times 2 \times 3 \times a \times a \times a$

Divisor = $6a = 2 \times 3 \times a$

$$\frac{48a^3}{6a} = \frac{2 \times 2 \times 2 \times 2 \times 3 \times a \times a \times a}{2 \times 3 \times a}$$

$$\frac{48a^3}{6a} = 2 \times 2 \times 2 \times a \times a$$

$$\frac{48a^3}{6a} = 8a^2$$

Hence dividing $48a^3$ by $6a$ gives $8a^2$

Q. 1. B. Carry out the following divisions

$14x^3$ by $42x^2$

Answer : In the given term

Dividend = $14x^3 = 2 \times 7 \times x \times x \times x$

Divisor = $42x^2 = 2 \times 3 \times 7 \times x \times x$

$$\frac{14x^3}{42x^2} = \frac{2 \times 7 \times x \times x \times x}{2 \times 3 \times 7 \times x \times x}$$

$$\frac{14x^3}{42x^2} = \frac{(x)}{3}$$

Hence dividing $14x^3$ by $42x^2$ gives $\frac{x}{3}$

Q. 1. C. Carry out the following divisions

$72a^3b^4c^5$ by $8ab^2c^3$

Answer : In the given term

Dividend = $72a^3b^4c^5 = 2 \times 2 \times 2 \times 3 \times 3 \times a \times a \times a \times b \times b \times b \times b \times c \times c \times c \times c \times c$

Divisor = $8ab^2c^3 = 2 \times 2 \times 2 \times a \times b \times b \times c \times c \times c$

$$\frac{72a^3b^4c^5}{8ab^2c^3} = \frac{2 \times 2 \times 2 \times 3 \times 3 \times a \times a \times a \times b \times b \times b \times b \times c \times c \times c \times c \times c}{2 \times 2 \times 2 \times a \times b \times b \times c \times c \times c}$$

$$\frac{72a^3b^4c^5}{8ab^2c^3} = 3 \times 3 \times a \times a \times b \times b \times c \times c$$

$$\frac{72a^3b^4c^5}{8ab^2c^3} = 9a^2b^2c^2$$

Hence dividing $72a^3b^4c^5$ by $8ab^2c^3$ gives $9a^2b^2c^2$

Q. 1. D. Carry out the following divisions

$11xy^2z^3$ by $55xyz$

Answer : In the given term

Dividend = $11xy^2z^3 = 11 \times x \times y \times y \times z \times z \times z$

Divisor = 55xyz = 5 × 11 × x × y × z

$$\frac{72xy^2z^3}{55xyz} = \frac{11 \times x \times y \times y \times z \times z \times z}{5 \times 11 \times x \times y \times z}$$

$$\frac{72xy^2z^3}{55xyz} = \frac{y \times z}{5}$$

$$\frac{72xy^2z^3}{55xyz} = \frac{yz}{5}$$

Hence dividing $11xy^2z^3$ by 55xyz gives $\frac{yz}{5}$

Q. 1. E. Carry out the following divisions

$-54l^4m^3n^2$ by $9l^2m^2n^2$

Answer : In the given term

Dividend = $-54l^4m^3n^2$ = (-1) × 2 × 3 × 3 × 3 × l × l × l × l × m × m × m × n × n

Divisor = $9l^2m^2n^2$ = 3 × 3 × l × l × m × m × n × n

$$\frac{-54l^4m^3n^2}{9l^2m^2n^2} = \frac{(-1) \times 2 \times 3 \times 3 \times 3 \times l \times l \times l \times l \times m \times m \times m \times n \times n}{3 \times 3 \times l \times l \times m \times m \times n \times n}$$

$$\frac{-54l^4m^3n^2}{9l^2m^2n^2} = (-1) \times 3 \times 2 \times l \times l \times m$$

$$\frac{-54l^4m^3n^2}{9l^2m^2n^2} = -6l^2m$$

Hence dividing $-54l^4m^3n^2$ by $9l^2m^2n^2$ gives $-6l^2m$

Q. 2. A. Divide the given polynomial by the given monomial

$(3x^2 - 2x) \div x$

Answer : In the given term

Dividend = $(3x^2 – 2x)$

Take out the common part in binomial term

= $(3 \times x \times x – 2 \times x)$

= $x(3x-2)$

Divisor = x

$$\frac{3x^2-2x}{x} = \frac{x(3x-2)}{x}$$

$$\frac{3x^2-2x}{x} = 3x-2$$

Hence dividing $(3x^2 – 2x)$ by x gives out $3x-2$

Q. 2. B. Divide the given polynomial by the given monomial

$(5a^3b – 7ab^3) \div ab$

Answer : In the given term

Dividend = $(5a^3b – 7ab^3)$

Take out the common part in binomial term

= $(5 \times a \times a \times a \times b – 7 \times a \times b \times b \times b)$

= $ab(5a^2 – 7b^2)$

Divisor = ab

$$\frac{(5a^3b - 7ab^3)}{ab} = \frac{ab(5a^2 - 7b^2)}{ab}$$

$$\frac{(5a^3b - 7ab^3)}{ab} = (5a^2 - 7b^2)$$

Hence dividing $(5a^3b – 7ab^3)$ by ab gives out $(5a^2 – 7b^2)$

Q. 2. C. Divide the given polynomial by the given monomial

$(25x^5 – 15x^4) \div 5x^3$

Answer : In the given term

Dividend = $(25x^5 – 15x^4)$

Take out the common part in binomial term

= $(5 \times 5 \times x \times x \times x \times x \times x – 3 \times 5 \times x \times x \times x \times x)$

= $(5x – 3)5x^4$

Divisor = $5x^3$

$$\frac{(25x^5 - 15x^4)}{5x^3} = \frac{(5x-3)5x^4}{5x^3}$$

$$\frac{(25x^5 - 15x^4)}{5x^3} = (5x-3)x$$

= $5x^2 – 3x$

Hence dividing $(25x^5 – 15x^4)$ by $5x^3$ gives out $5x^2 – 3x$

Q. 2. D. Divide the given polynomial by the given monomial

$4(l^5 – 6l^4 + 8l^3) \div 2l^2$

Answer : In the given term

Dividend = $(4l^5 – 6l^4 + 8l^3)$

Take out the common part in binomial term

= $(2 \times 2 \times l \times l \times l \times l \times l – 3 \times 2 \times l \times l \times l \times l + 2 \times 2 \times 2 \times l \times l \times l)$

= $(2l^2 – 3l + 4)2l^3$

Divisor = $2l^2$

$$\frac{(4l^5 - 6l^4 + 8l^3)}{2l^2} = \frac{(2l^2 - 3l + 4)2l^3}{2l^2}$$

$$\frac{(25x^5 - 15x^4)}{5x^3} = (2l^2 - 3l + 4)l$$

$= (2l^3 – 2l^2 + 4l)$

Hence dividing $4(l^5 – 6l^4 + 8l^3)$ by $2l^2$ gives out $(2l^3 – 2l^2 + 4l)$

Q. 2. E. Divide the given polynomial by the given monomial

$15 (a^3b^2c^2 – a^2b^3c^2 + a^2b^2c^3) \div 3abc$

Answer : In the given term

Dividend = $15 (a^3b^2c^2 – a^2b^3c^2 + a^2b^2c^3)$

Take out the common part in binomial term

$= 3 \times 5(a \times a \times a \times b \times b \times c \times c – a \times a \times b \times b \times b \times c \times c + a \times a \times b \times b \times c \times c \times c)$

$= 15 a^2b^2c^2(a – b + c)$

Divisor = 3abc

$$\frac{15 (a^3b^2c^2 - a^2b^3c^2 + a^2b^2c^3)}{3abc} = \frac{15 a^2b^2c^2(a - b + c)}{3abc}$$

$$\frac{15 (a^3b^2c^2 - a^2b^3c^2 + a^2b^2c^3)}{3abc} = 5abc[a-b + c]$$

$= [5a^2bc – 5ab^2c + 5abc^2]$

Hence dividing $15 (a^3b^2c^2 – a^2b^3c^2 + a^2b^2c^3)$ by 3abc gives out $[5a^2bc – 5ab^2c + 5abc^2]$

Q. 2. F. Divide the given polynomial by the given monomial

$(3p^3 – 9p^2q - 6pq^2) \div (–3p)$

Answer : In the given term

Dividend = $(3p^3 - 9p^2q - 6pq^2)$

Take out the common part in binomial term

= (3 × p × p × p– 3 × 3 × p × p × q - 2 × 3 × p × q × q)

= $3 \times p(p^2 - 3pq - 2q^2)$

Divisor = (–3p)

$$\frac{(3p^3 - 9p^2q - 6pq^2)}{-3p} = \frac{3p(p^2 - 3pq - 2q^2)}{-3p}$$

$$\frac{(3p^3 - 9p^2q - 6pq^2)}{-3p} = (-1)\ (p^2 - 3pq - 2q^2)$$

= $(2q^2 + 3pq - p^2)$

Hence dividing $(3p^3 - 9p^2q - 6pq^2)$ by (–3p) gives out $(2q^2 + 3pq - p^2)$

Q. 2. G. Divide the given polynomial by the given monomial

$$\left(\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^2\right) \div \frac{1}{2}abc$$

Answer : In the given term

$$\text{Dividend} = \left(\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^2\right)$$

Take out the common part in binomial term

$= (\frac{2}{3} \times a \times a \times b \times b \times c \times c + \frac{2}{3} \times 2 \times a \times b \times b \times c \times c)$

$= \frac{2}{3} \times a \times b \times b \times c \times c\ (a + 2)$

$= \frac{2}{3}ab\ �^{2}c^{2}(a + 2)$

Divisor $= \frac{1}{2}abc$

$$\frac{(\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^2)}{\frac{1}{2}abc} = \frac{\frac{2}{3}ab^2c^2(a+2)}{\frac{1}{2}abc}$$

$$\frac{(3p^3 - 9p^2q - 6pq^2)}{-3p} = \frac{2}{3} \times \frac{2}{1} \times bc(a + 2)$$

$= \frac{4}{3}bc(a + 2)$

Hence dividing $(\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^2)$ by $\frac{1}{2}abc$ gives out $\frac{4}{3}bc(a + 2)$

Q. 3. A. Workout the following divisions:

(49x – 63) ÷ 7

Answer : In the given term

Dividend = (49x – 63)

Take out the common part in binomial term

= (7 × 7 × x - 7 × 9)

$= 7(7 \times x - 9)$

$= 7(7x - 9)$

Divisor = 7

$$\frac{(49x - 63)}{7} = \frac{7(7x - 9)}{7}$$

$$\frac{(49x - 63)}{7} = (7x - 9)$$

Hence dividing (49x – 63) by 7 gives out (7x - 9)

Q. 3. B. Workout the following divisions:

12x (8x – 20) ÷ 4(2x – 5)

Answer : In the given term

Dividend = 12x (8x – 20)

Take out the common part in binomial term

$= 2 \times 2 \times 3 \times x(2 \times 2 \times 2 \times x - 2 \times 2 \times 5)$

$= 2 \times 2 \times 2 \times 2 \times 3 \times x(2 \times x - 5)$

$= 48x (2x - 5)$

Divisor = 4(2x – 5)

$$\frac{12x\,(8x - 20)}{4(2x - 5)} = \frac{48x\,(2x - 5)}{4(2x - 5)}$$

$$\frac{12x\,(8x - 20)}{4(2x - 5)} = 12x$$

Hence divides 12x (8x – 20) by 4(2x – 5) gives out 12x

Q. 3. C. Workout the following divisions:

$11a^3b^3(7c - 35) \div 3a^2b^2(c - 5)$

Answer : In the given term

Dividend = $11a^3b^3(7c – 35) \div 3a^2b^2(c – 5)$

Take out the common part in binomial term

= 11 × a × a × a × b × b × b (7 × c - 5 × 7)

= 11 × a × a × a × b × b × b × 7 (c - 5)

= $77a^3b^3(c - 5)$

Divisor = $3a^2b^2(c – 5)$

$$\frac{11a^3b^3(7c-35)}{3a^2b^2(c-5)} = \frac{77a^3b^3(c-5)}{3a^2b^2(c-5)}$$

$$\frac{11a^3b^3(7c-35)}{3a^2b^2(c-5)} = \frac{77}{3}ab$$

Hence dividing $11a^3b^3(7c – 35)$ by $3a^2b^2(c – 5)$ gives out $\frac{77}{3}$ ab

Q. 3. D. Workout the following divisions:

54lmn (l + m) (m + n) (n + l) ÷ 81mn (l + m) (n + l)

Answer : In the given term

Dividend = 54lmn (l + m) (m + n) (n + l)

Divisor = 81mn (l + m) (n + l)

$$\frac{54lmn\,(l+m)\,(m+n)\,(n+l)}{81mn\,(l+m)\,(n+l)} = \frac{2l \times (m+n) \times 27mn\,(l+m)\,(n+l)}{3 \times 27mn\,(l+m)\,(n+l)}$$

$$\frac{54lmn\,(l+m)\,(m+n)\,(n+l)}{81mn\,(l+m)\,(n+l)} = \frac{2}{3}\,l(m+n)$$

Hence dividing 54lmn (l + m) (m + n) (n + l) by 81mn (l + m)(n + l) gives out $\frac{2}{3}$ l(m + n)

Q. 3. E. Workout the following divisions:

36 (x + 4) (x^2 + 7x + 10) ÷ 9 (x + 4)

Answer : In the given term

Dividend = 36 (x + 4) (x^2 + 7x + 10)

Divisor = 9 (x + 4)

$$\frac{36\,(x+4)\,(x^2+7x+10)}{9\,(x+4)} = \frac{4 \times \left(x^2+7x+10\right) \times 9 \times (x+4)}{9\,(x+4)}$$

$$\frac{(3p^3 - 9p^2q - 6pq^2)}{-3p} = 4(x^2 + 7x + 10)$$

= ($4x^2$ + 27x + 40)

Hence dividing 36 (x + 4) (x^2 + 7x + 10) by 9 (x + 4) gives out

($4x^2$ + 27x + 40)

Q. 3. F. Workout the following divisions:

a (a + 1) (a + 2) (a + 3) ÷ a (a + 3)

Answer : In the given term

Dividend = a (a + 1) (a + 2) (a + 3)

Divisor = a (a + 3)

$$\frac{a\,(a+1)\,(a+2)\,(a+3)}{a\,(a+3)} = \frac{(a+1) \times (a+2) \times (a \times (a+3))}{a\,(a+3)}$$

$$\frac{a\,(a+1)\,(a+2)\,(a+3)}{a\,(a+3)} = (a+1)\ (a+2)$$

Hence dividing a (a + 1) (a + 2) (a + 3) by a (a + 3) gives out

(a + 1) (a + 2)

Q. 4. A. Factorize the expressions and divide them as directed:

$(x^2 + 7x + 12) \div (x + 3)$

Answer : In the given term

Dividend = $(x^2 + 7x + 12)$

The given expression looks as

$x^2 + (a + b)x + ab$

Where a + b = 7; and ab = 12;

Factors of 12 their sum

1 × 12 1 + 12 = 13

6 × 2 2 + 6 = 8

4 × 3 4 + 3 = 7

∴ the factors having sum 7 are 4 and 3

$x^2 + 7x + 12 = x^2 + (4 + 3)x + 12$

$= x^2 + 4x + 3x + 12$

$= x(x + 4) + 3(x + 4)$

$= (x + 4)(x + 3)$

Divisor = $(x + 3)$

$$\frac{(x^2 + 7x + 12)}{(x + 3)} = \frac{(x + 3)(x + 4)}{(x + 3)}$$

$$\frac{(x^2 + 7x + 12)}{(x + 3)} = (x + 4)$$

Hence dividing $(x^2 + 7x + 12)$ by $(x + 3)$ gives out $(x + 4)$

Q. 4. B. Factorize the expressions and divide them as directed:

$(x^2 – 8x + 12) \div (x – 6)$

Answer : In the given term

Dividend = $(x^2 - 8x + 12)$

The given expression looks as

$x^2 + (a + b)x + ab$

where a + b = -8; and ab = 12;

factors of 12 their sum

-1×-12 $-1-12 = -13$

-6×-2 $-2-6 = -8$

-4×-3 $-4-3 = -7$

∴ the factors having sum 7 are 4 and 3

$x^2 - 8x + 12 = x^2 + (-6-2)x + 12$

$= x^2 - 6x - 2x + 12$

$= x(x - 6) -2(x - 6)$

$= (x - 6)(x - 2)$

Divisor = $(x - 6)$

$$\frac{(x^2 - 8x + 12)}{(x-6)} = \frac{(x-6)(x-2)}{(x-6)}$$

$$\frac{(x^2 - 8x + 12)}{(x-6)} = (x - 2)$$

Hence dividing $(x^2 – 8x + 12)$ by $(x – 6)$ gives out $(x – 2)$

Q. 4. C. Factorize the expressions and divide them as directed:

$(p^2 + 5p + 4) \div (p + 1)$

Answer : In the given term

Dividend = $(p^2 + 5p + 4)$

The given expression looks as

$x^2 + (a + b)x + ab$

where a + b = 5; and ab = 4;

factors of 4 their sum

1×4 $1 + 4 = 5$

2×2 $2 + 2 = 4$

∴ the factors having sum 5 are 4 and 1

$(p^2 + 5p + 4) = p^2 + (4 + 1)p + 4$

$= p^2 + 4p + p + 4$

$= p(p + 4) + 1(p + 4)$

$= (p + 1)(p + 4)$

Divisor = $(p + 1)$

$$\frac{(p^2 + 5p + 4)}{(p + 1)} = \frac{(p + 1)(p + 4)}{(p + 1)}$$

$$\frac{(p^2 + 5p + 4)}{(p + 1)} = (p + 4)$$

Hence dividing $(p^2 + 5p + 4)$ by $(p + 1)$ gives out $(p + 4)$

Q. 4. D. Factorize the expressions and divide them as directed:

$15ab\ (a^2 – 7a + 10) \div 3b\ (a – 2)$

Answer : In the given term

Dividend = $15ab\ (a^2 – 7a + 10)$

The given expression $(a^2 – 7a + 10)$ looks as

$x^2 + (a + b)\ x + ab$

where $a + b = -7$; and $ab = 10$;

factors of 10 their sum

-1×-10 $-1-10 = -11$

-2×-5 $-2-5 = -7$

∴ the factors having sum -7 are -2 and -5

$(a^2 – 7a + 10) = a^2 + (-2-5)a + 10$

$= a^2 – 5a – 2a + 10$

$= a(a – 5) – 2(a – 5)$

$= (a – 5)(a – 2)$

Divisor = $3b\ (a – 2)$

$$\frac{15ab\ (a^2-7a+10)}{3b(a-2)} = \frac{15ab\ (a-5)(a-2)}{3b(a-2)}$$

$$\frac{15ab\ (a^2-7a+10)}{3b(a-2)} = 5a(a-5)$$

Hence dividing $15ab\ (a^2 – 7a + 10)$ by $3b\ (a – 2)$ gives out $5a(a – 5)$

Q. 4. E. Factorize the expressions and divide them as directed:

$15lm\,(2p^2 – 2q^2) \div 3l\,(p + q)$

Answer : In the given term

Dividend = $15lm\,(2p^2 – 2q^2)$

In given expression $(2p^2 – 2q^2)$

Take out the common factor in binomial term

$\Rightarrow (2 \times p \times p – 2 \times q \times q)$

$\rightarrow 2(p^2 – q^2)$

Both terms are perfect square

$\Rightarrow p^2 = p \times p$

$\Rightarrow q^2 = q \times q$

$\therefore (p^2 – q^2)$ Seems to be in identity $a^2 - b^2 = (a + b)(a - b)$

Where $a = p$ and $b = q$;

$p^2 – q^2 = (p + q)(p – q)$

Hence the factors of $p^2 – q^2$ are $(p + q)$ and $(p – q)$

Divisor = $3l\,(p + q)$

$$\frac{15lm\,(2p^2-2q^2)}{3l\,(p+q)} = \frac{15lm \times 2(p+q)(p-q)}{3l\,(p+q)}$$

$$\frac{15lm\,(2p^2-2q^2)}{3l\,(p+q)} = \frac{10m(p-q) \times 3l \times (p+q)}{3l\,(p+q)}$$

$$\frac{15lm\,(2p^2-2q^2)}{3l\,(p+q)} = 10m(p-q)$$

Hence dividing 15lm $(2p^2–2q^2)$ by 3l (p + q) gives out 10m(p – q)

Q. 4. F. Factorize the expressions and divide them as directed:

$26z^3(32z^2–18) \div 13z^2(4z – 3)$

Answer : In the given term

Dividend = $26z^3(32z^2–18)$

Take out the common factor in binomial term

⇒ 2 × 13 × z × z × z (2 × 2 × 2 × 2 × 2 × z × z – 2 × 3 × 3)

⇒ 2 × 2 × 13 × z × z × z (2 × 2 × 2 × 2 × z × z – 3 × 3)

⇒ $52z^3(16z^2 – 9)$

In given expression $(16z^2 – 9)$

Both terms are perfect square

⇒ $16z^2$ = 4z × 4z

⇒ 9 = 3 × 3

∴ $(16z^2 – 9)$ Seems to be in identity a^2-b^2 = (a + b)(a-b)

Where a = 4z and b = 3;

$(16z^2 – 9)$ = (4z + 3)(4z – 3)

Hence the factors of $(16z^2 - 9)$are $(4z + 3)$ and $(4z - 3)$

Divisor = $13z^2(4z - 3)$

Exercise 12.4

Q. 1. A. Find the errors and correct the following mathematical sentences

$3(x - 9) = 3x - 9$

Answer : If LHS is

$3(x - 9)$

Then RHS would be

$\Rightarrow 3(x - 9)$

$= 3 \times x - 3 \times 9$

$= 3x - 27$

The error is 27 instead of 9

Hence $3(x - 9) = 3x - 27$

Q. 1. B. Find the errors and correct the following mathematical sentences

$x(3x + 2) = 3x^2 + 2$

Answer : If LHS is

$x(3x + 2)$

Then RHS would be

$\Rightarrow x(3x + 2)$

$= 3 \times x \times x - 2 \times x$

$= 3x^2 - 2x$

The error is 2x instead of 2

Hence $x(3x + 2) = 3x^2 + 2x$

Q. 1. C. Find the errors and correct the following mathematical sentences

$2x + 3x = 5x^2$

Answer : If LHS is

$2x + 3x$

Then RHS would be

$\Rightarrow 2x + 3x$

$= x(2 + 3)$

$= 5x$

The error is $5x$ instead of $5x^2$

Hence $2x + 3x = 5x$

Q. 1. D. Find the errors and correct the following mathematical sentences

$2x + x + 3x = 5x$

Answer : If LHS is

$2x + x + 3x = 5x$

Then RHS would be

$\Rightarrow 2x + x + 3x$

$= x(2 + 1 + 3)$

$= 6x$

The error is $6x$ instead of $5x$

Hence $2x + x + 3x = 6x$

Q. 1. E. Find the errors and correct the following mathematical sentences

$4p + 3p + 2p + p – 9p = 0$

Answer : If LHS is

$4p + 3p + 2p + p – 9p$

Then RHS would be

$\Rightarrow 4p + 3p + 2p + p – 9p$

$= p(4 + 3 + 2 + 1–9)$

$= p(10–9)$

$= p$

The error is p instead of 0

Hence $4p + 3p + 2p + p–9p = p$

Q. 1. F. Find the errors and correct the following mathematical sentences

$3x + 2y = 6xy$

Answer : If RHS is

$6xy$

Then LHS would be

$\Rightarrow 6xy$

$= 2 \times 3 \times x \times y$

$= 3 \times x \times 2 \times y$

$= 3x \times 2y$

The error is sign of multiplication instead of sign of addition

Hence $3x \times 2y = 6xy$

Q. 1. G. Find the errors and correct the following mathematical sentences

$(3x)^2 + 4x + 7 = 3x^2 + 4x + 7$

Answer : If LHS is

$(3x)^2 + 4x + 7$

Then RHS would be

⇒ $(3x)^2 + 4x + 7$

$= 3^2 \times x^2 + 4x + 7$

$= 9x^2 + 4x + 7$

The error is $9x^2$ instead of $3x^2$

Hence $(3x)^2 + 4x + 7 = 9x^2 + 4x + 7$

Q. 1. H. Find the errors and correct the following mathematical sentences

$(2x)^2 + 5x = 4x + 5x = 9x$

Answer : If LHS is

$(2x)^2 + 5x$

Then RHS would be

⇒ $(2x)^2 + 5x$

$= 2^2 \times x^2 + 5x$

$= 4x^2 + 5x$

The error is $4x^2$ instead of $4x$

Hence $(2x)^2 + 5x = 4x^2 + 5x$

Q. 1. I. Find the errors and correct the following mathematical sentences

$(2a + 3)^2 = 2a^2 + 6a + 9$

Answer : If LHS is

$(2a + 3)^2$

Then RHS would be

⇒ $(2a + 3)^2$

$= (2a)^2 + 3^2 + 2 \times 2a \times 3$

$= 4a^2 + 9 + 12a$

$= 4a^2 + 12a + 9$

The error is $4a^2$ instead of $2a^2$ and 12a instead of 6a

Hence $= (2a + 3)^2 = 4a^2 + 9 + 12a$

Q. 1. J. Find the errors and correct the following mathematical sentences

Substitute x = – 3 in
(a) $x^2 + 7x + 12 = (–3)^2 + 7 (–3) + 12 = 9 + 4 + 12 = 25$

Answer : If LHS is

$x^2 + 7x + 12$

Then RHS would be

$\Rightarrow x^2 + 7x + 12$

Putting x = (-3)

$= (–3)^2 + 7 (–3) + 12$

= 9 + (-21) + 12

= 21-21

= 0

The error is (-21) instead of 4 and end result 0 instead of 25

Hence putting x = (-3) in $x^2 + 7x + 12$ results to 0

Q. 1. K. Find the errors and correct the following mathematical sentences

Substitute x = – 3 in
(b) $x^2– 5x + 6 = (–3)^2 –5 (–3) + 6 = 9 – 15 + 6 = 0$

Answer : If LHS is

$x^2– 5x + 6$

Then RHS would be

$\Rightarrow x^2 - 5x + 6$

Putting x = (-3)

$= (-3)^2 - 5(-3) + 6$

= 9 + 15 + 6

= 30

The error is + 15 instead of (-15) and end results to 30 instead of 0

Hence putting x = (-3) in $x^2 - 5x + 6$ results to 30

Q. 1. L. Find the errors and correct the following mathematical sentences

Substitute x = – 3 in
(c) $x^2 + 5x = (-3)^2 + 5(-3) + 6 = -9 - 15 = -24$

Answer : If LHS is

$x^2 + 5x$

Then RHS would be

$\Rightarrow x^2 + 5x$

Putting x = (-3)

$= (-3)^2 + 5(-3)$

= 9 + (-15)

= -6

The error is (+ 9) instead of (-9) and end results to (-6) instead of (-24)

Hence putting x = (-3) in $x^2 + 5x$ results to (-6)

Q. 1. M. Find the errors and correct the following mathematical sentences

$(x - 4)^2 = x^2 - 16$

Answer : If LHS is

$(x - 4)^2$

Then RHS would be

$\Rightarrow (x - 4)^2$

$= (x)^2 + 4^2 - 2 \times x \times 4$

$= x^2 + 16 - 8x$

The error is $x^2 + 16 - 8x$ instead of $x^2 - 16$

Hence $(x - 4)^2 = x^2 + 13 - 8x$

Q. 1. N. Find the errors and correct the following mathematical sentences

$(x + 7)^2 = x^2 + 49$

Answer : If LHS is

$(x + 7)^2$

Then RHS would be

$\Rightarrow (x + 7)^2$

$= (x)^2 + 7^2 + 2 \times x \times 7$

$= x^2 + 49 + 14$

The error is $x^2 + 14x + 49$ instead of $x^2 + 49$

Hence $(x + 7)^2 = x^2 + 14x + 49$

Q. 1. O. Find the errors and correct the following mathematical sentences

$(3a + 4b)(a - b) = 3a^2 - 4a^2$

Answer : For getting in the equation

$(a^2 - b^2) = (a + b)(a-b)$

RHS would be

$3a^2 - 4b^2$

Then LHS would be

$\Rightarrow 3a^2 – 4b^2$

$= (3a – 4b)(3a + 4b)$

The error is (a – b) instead of (3a – 4b)

$3a^2 – 4b^2$ instead of $3a^2 – 4a^2$

Hence $3a^2 – 4b^2 = (3a – 4b)(3a + 4b)$

Q. 1. P. Find the errors and correct the following mathematical sentences

$(x + 4) (x + 2) = x^2 + 8$

Answer : If LHS is

$(x + 4) (x + 2)$

Then RHS would be

$\Rightarrow (x + 4) (x + 2)$

$= x^2 + 4 \times x + 2 \times x + 2 \times 4$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$

The error is $x^2 + 6x + 8$ instead of $x^2 + 8$

Hence $(x + 4) (x + 2) = x^2 + 6x + 8$

Q. 1. Q. Find the errors and correct the following mathematical sentences

$(x – 4) (x – 2) = x^2 – 8$

Answer : If LHS is

$(x – 4) (x – 2)$

Then RHS would be

$\Rightarrow (x – 4) (x – 2)$

$= x^2 – 4 \times x – 2 \times x + (-2) \times (-4)$

$= x^2 - 4x - 2x + 8$

$= x^2 - 6x + 8$

The error is $x^2 - 6x + 8$ instead of $x^2 - 8$

Hence $(x - 4)(x - 2) = x^2 - 6x + 8$

Q. 1. R. Find the errors and correct the following mathematical sentences

$5x^3 \div 5x^3 = 0$

Answer : If LHS is

$5x^3 \div 5x^3$

Then RHS would be

$\Rightarrow 5x^3 \div 5x^3$

$= \frac{5x^3}{5x^3}$

$= 1$

The error is1 instead of 0

Hence $5x^3 \div 5x^3 = 1$

Q. 1. S. Find the errors and correct the following mathematical sentences

$2x^3 + 1 \div 2x^3 = 1$

Answer : If LHS is

$(2x^3 + 1) \div 2x^3$

Then RHS would be

$\Rightarrow (2x^3 + 1) \div 2x^3$

$= \frac{2x^3 + 1}{2x^3}$

$= \frac{2x^3}{2x^3} + \frac{1}{2x^3}$

$= 1 + \frac{1}{2x^3}$

The error is $1 + \frac{1}{2x^3}$ instead of 1

Hence $(2x^3 + 1) \div 2x^3 = 1 + \frac{1}{2x^3}$

Q. 1. T. Find the errors and correct the following mathematical sentences

$3x + 2 \div 3x = \frac{2}{3x}$

Answer : If LHS is

$(3x + 2) \div 3x$

Then RHS would be

$\Rightarrow (3x + 2) \div 3x$

$= \frac{3x + 2}{3x}$

$= \frac{3x}{3x} + \frac{2}{3x}$

$= 1 + \frac{2}{3x}$

The error is $1 + \frac{2}{3x}$ instead of $\frac{2}{3x}$

Hence $(3x + 2) \div 3x = 1 + \frac{2}{3x}$

Q. 1. U. Find the errors and correct the following mathematical sentences

3x + 5 ÷ 3 = 5

Answer : If LHS is

For the complete and perfect division

There must be 3x instead of x

(3x + 5) ÷ 3x

Then RHS would be

⇒ (3x + 5) ÷ 3x

$= \frac{3x + 5}{3x}$

$= \frac{3x}{3x} + \frac{5}{3x}$

$= 1 + \frac{5}{3x}$

The error is $1 + \frac{5}{3x}$ instead of 5 and 3x instead of x

Hence $= (3x + 5) \div 3x = 1 + \frac{5}{3x}$

Q. 1. V. Find the errors and correct the following mathematical sentences

$\frac{4x+3}{3}$ = **x + 1**

Answer : If LHS is

$$\frac{4x + 3}{3}$$

Then RHS would be

$\Rightarrow \frac{4x+3}{3}$

$= \frac{4}{3}x + \frac{3}{3}$

$= \frac{4}{3}x + 1$

The error is $\frac{4}{3}x + 1$ instead of $x + 1$

Hence $\frac{4x+3}{3} = \frac{4}{3}x + 1$