CGPET 2011

Solved Paper

Question 1

If in expression $X = 3Y^2$, the dimensions of X and Z are same as the dimensions of capacity and magnetic field respectively, the dimensional formula of Y is

Options:

A. $[M^{-3}L^{-2}T^{-4}A^{-1}]$

B. $[ML^{-2}T^{4}]$

C. $[M^{-3}L^{-2}T^{4}A^{4}]$

D. $[M^{-3}L^{-2}T^{8}A^{4}]$

Answer: D

Solution:

Solution:

Given, $X = 3YZ^2$

$\therefore$ Dimensions of $Y = \frac{[X]}{[3Z^2]}$

$= \frac{[M^{-1}L^{-2}T^{4}A^{2}]}{[MT^{-2}A^{-1}]^2}$

$= \frac{[M^{-1}L^{-2}T^{4}A^{2}]}{[M^{2}T^{-4}A^{-2}]}$

$= [M^{-3}L^{-2}T^{8}A^{4}]$

Question 2

If L, C, R are respectively the inductance, capacitance and resistance, the quantities of dimensions same as of frequency are

Options:

A. $\frac{1}{\sqrt{LC}}, \frac{R}{L}$ and $\frac{1}{RC}$

B. $\sqrt{LC}, \frac{L}{R}$ and RC

C. $\sqrt{\frac{L}{C}}$, LR and $\frac{C}{R}$

D. $\sqrt{\frac{C}{L}}$, $\frac{1}{LR}$ and $\frac{R}{C}$

Answer: A

Solution:

Solution:

Dimension of $\frac{1}{\sqrt{LC}}$

$= \frac{1}{[ML^2T^{-2}A^{-2}]^{1/2}[M^{-1}L^{-2}T^4A^2]^{1/2}}$

$= \frac{1}{[T]} = [T^{-1}]$

Dimension of $\frac{R}{L} = \frac{[ML^2T^{-3}A^{-2}]}{[ML^2T^{-2}A^{-2}]} = [T^{-1}]$

Dimension of $\frac{1}{RC}$

$= \frac{1}{[ML^2T^{-3}A^{-2}][M^{-1}L^{-2}T^4A^2]} = [T^{-1}]$

Dimension of frequency $= [T^{-1}]$

Hence $\frac{1}{\sqrt{LC}}$, $\frac{R}{L}$ and $\frac{1}{RC}$ have the same dimension as frequency.

Question 3

The SI unit of thermal capacity is

Options:

A. Joule

B. Joule/kilogramme

C. Joule/kelvin

D. Joule/kelvin kilogramme

Answer: C

Solution:

Solution:

The SI unit of thermal capacity is joule/kelvin.

Question 4

An object travels in a straight line one-third of the total distance with velocity v_1, second one-third distance with velocity v_2 and the rest one-third distance with velocity v_3. The average velocity of object will be

Options:

A. $\frac{v_1 + v_2 + v_3}{3}$

B. $\frac{v_1v_2 + v_2v_3 + v_3v_1}{3v_1v_2v_3}$

C. $\sqrt{v_1v_2v_3}$

D. $\frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

Answer: D

Solution:

Solution:
An object travels in a straight line one-third of the total distance with velocity v_1. Second one third distance with velocity v_2 and the rest one third distance with velocity v_3. The average velocity
$v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

Question 5

A ball is projected at an angle θ upwards from horizontal. The true statement is

Options:

A. at each point of flight vertical component of momentum remains constant

B. at each point of flight horizontal component of momentum remains constant

C. at the highest point of flight, potential energy is minimum

D. at the highest point of flight, kinetic energy is zero

Answer: B

Solution:

Solution:
A ball is projected at an angle theta upward from horizontal, then at each point of flight horizontal component of momentum remain constant.

Question 6

A packet of mass m is dropped from an aeroplane moving at a height h above the ground with a horizontal velocity u. If g is the acceleration due to gravity, the kinetic energy possessed by the packet on reaching the ground will be

Options:

A. mgh

B. $\frac{1}{2}\mathrm{mu}^2 + \mathrm{mgh}$

C. $\frac{1}{2}mu^2 - mgh$

D. $mgh - \frac{1}{2}mu^2$

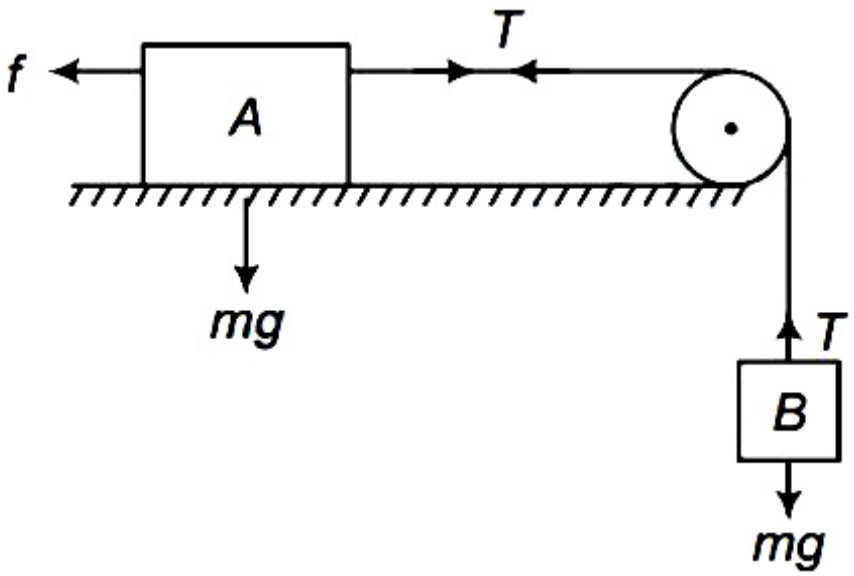
Answer: A

Solution:

Solution:

Question 7

In the diagram below, a block A of mass 10 kg rests on a horizontal table. A massless string attached with it passes over a frictionless pulley attached at the end of table with another block B at its free end. If coefficient of friction between the block A and table surface is 0.2, the minimum mass of block B needed to start motion in block A is



Options:

A. $2\,\text{kg}$

B. $0.2\,\text{kg}$

C. $5\,\text{kg}$

D. $10\,\text{kg}$

Answer: A

Solution:

Solution:
Coefficient of friction $\mu_s = \frac{m_B}{m_A}$
$0.2 = \frac{\text{m}_\text{B}}{10}$
$\text{m}_B = 0.2 \times 10$
$\text{m}_B = 2.0\,\text{kg}$
Therefore the minimum mass of block B needed to start motion in block A is $2.0\,\text{kg}$.

Question 8

The fundamental Newton's law of motion is

Options:

A. $F = \text{ma}$

B. $F = 0$ if $a = 0$

C. $F_{12} = -F_{21}$

D. $F = \frac{\text{d}}{\text{dt}}(\text{m}\,v)$

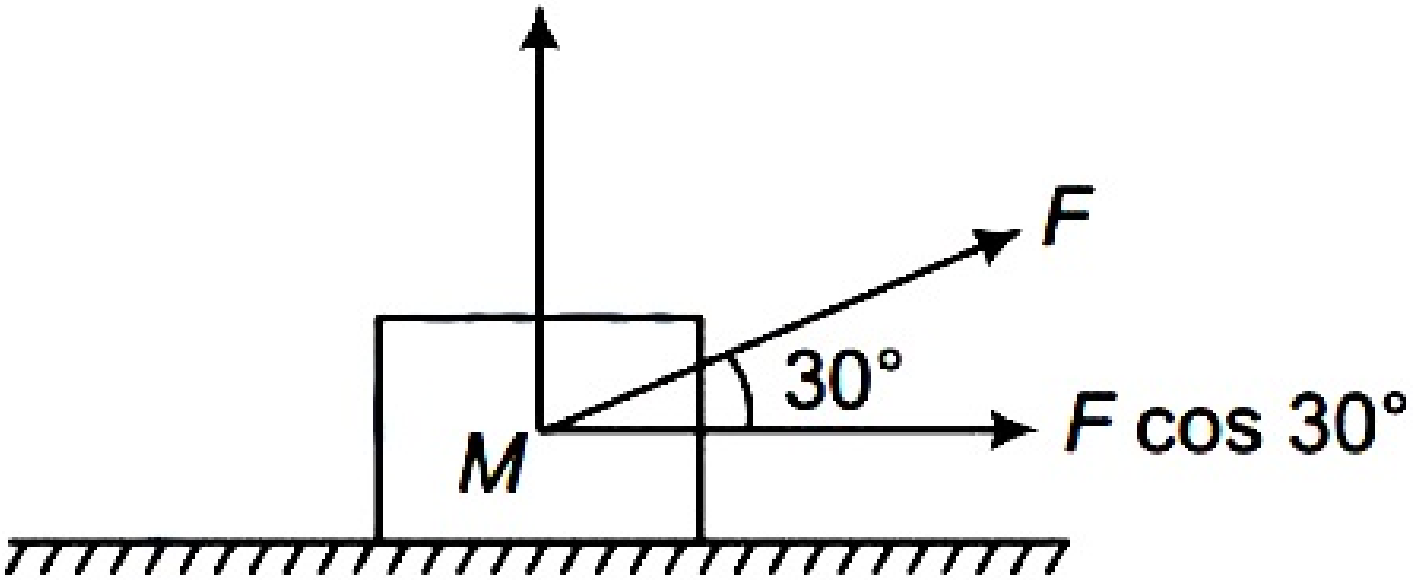
Answer: D

Solution:

Solution:
Newton's second law
$F = \frac{\text{d}}{\text{dt}}(\text{m}\,v)$
This is called fundamental Newton's law of motion because Newton's first and third law can be generated by this.

Question 9

In the diagram below, a body of mass $\text{M} = 5\,\text{kg}$ placed on a horizontal surface is pulled by a force $\text{F} = 40\text{N}$ in a direction making an angle 30° with the horizontal. If $\text{g} = 10\text{ms}^{-2}$ and coefficient of friction between the body and surface is 0.2, the acceleration acquired by the body will be



Options:

A. $5.75 ms^{-2}$

B. $8.0 ms^{-2}$

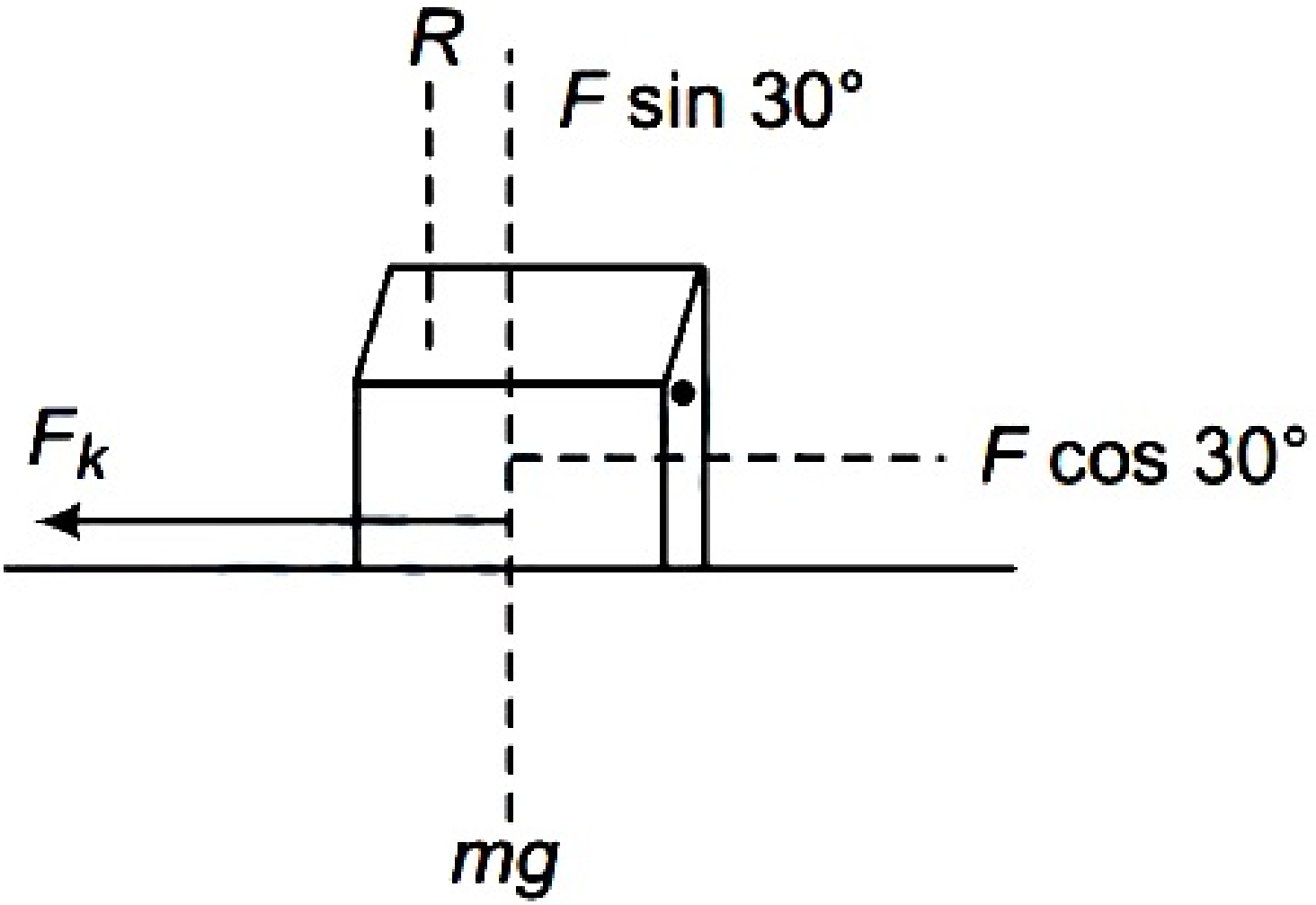
C. $3.17 ms^{-2}$

D. $10.0 ms^{-2}$

Answer: A

Solution:

Solution:



Kinetic friction k $= \mu_k R$
$= 0.2(mg - F sin\ 30^\circ)$
$= 0.2(5 \times 10 - 40 \times \frac{1}{2})$
$= 6N$
$ma = F \cos 30^\circ - \mu_k R$
$a = \frac{F \cos 30 - 6}{m}$
$a = \frac{40 \times \frac{\sqrt{3}}{2} - 6}{5}$
$a = \frac{20\sqrt{3} - 6}{5} = 5.73 ms^{-2}$

Question 10

A particle of mass m is moving in a circular path of radius r with angular momentum L. The centripetal force acting on the particle is

Options:

A. L / mr^2

B. L^2 / mr^2

C. L^2 / mr^3

D. $L^2 / m^2 r^2$

Answer: C

Solution:

Solution:
Angular momentum L = mvr

Centripetal force F = $\frac{mv^2}{r}$

F = $\frac{m^2v^2r^2}{mr}$

F = $\frac{L^2}{mr^3}$

Question 11

A particle of mass m is projected with velocity v at an angle 45° with the horizontal. The magnitude of angular momentum of the particle about the point of projection, at highest point of flight is

Options:

A. $mv^2/2g$

B. $mv^3/2\sqrt{g}$

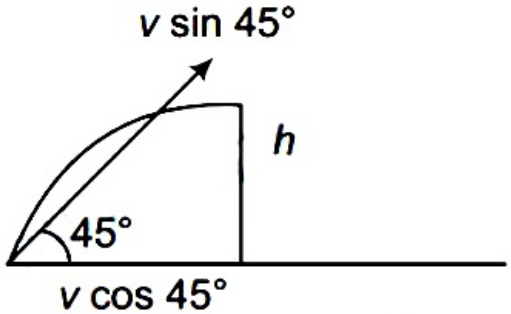
C. $mv^3/4\sqrt{2}g$

D. Zero

Answer: C

Solution:

Solution:



Maximum height H = $\frac{v^2 sin^2 45°}{2g}$

= $\frac{v^2}{4g}$

Momentum of particle at the highest point

p = $mv\cos\theta = mv\cos 45°$

= $\frac{mv}{\sqrt{2}}$

Angular momentum = pH

= $\frac{mv}{\sqrt{2}} \times \frac{v^2}{4g}$

= $\frac{mv^3}{4\sqrt{2}g}$

Question 12

On removing the concentric circular part of radius r from a disc of radius R, an annular disc of mass M is obtained. The moment of inertia of this annular disc about an axis normal to its plane and passing through its centre of gravity is

Options:

A. $\frac{1}{2}M(R^2+r^2)$

B. $\frac{1}{2}M(R^2-r^2)$

C. $\frac{1}{2}M(R^4+r^4)$

D. $\frac{1}{2}M(R^4-r^4)$

Answer: A

Solution:

Solution:
The moment of inertia of annular disc about an axis normal to its plane and passing through its centre of gravity is $\frac{1}{2}m(R^2+r^2)$.

Question 13

A body is moved from rest in a straight line by means of a machine supplying constant power. The distance S moved by the body in time t is proportional to

Options:

A. $t^{1/4}$

B. $t^{3/4}$

C. $t^{3/2}$

D. t^2

Answer: C

Solution:

Solution:
Power $= Fv = P =$ constant

or $m\frac{dv}{dt}v = P$

$\Rightarrow \int vdv = \int \frac{P}{m}dt$

$\Rightarrow \frac{v^2}{2} = \frac{P}{m}t + A$

$\Rightarrow$ v = 0 at t = 0, so A = 0

$\therefore v = (\frac{2Pt}{m})^{1/2}$

$\Rightarrow S = (\frac{2P}{m})^{1/2} \cdot \frac{2}{3}t^{3/2} + B$

$\because t = 0, s = 0, so, B = 0$

$S = (\frac{8P}{9m})^{1/2} t^{3/2}$

$S \propto t^{3/2}$

Question 14

A body of mass 5.0 kg is moving with linear momentum 10 $kg - ms^{-1}$. A force of 0.2N is applied on the body for 10 s in the direction of motion of body. The increase in kinetic energy of body will be

Options:

A. 2.8J

B. 3.2J

C. 3.6J

D. 4.4J

Answer: D

Solution:

Solution:

Mass of body m = 5.0 kg

$P = 10\,kg - ms^{-1}$

F = 0.2N

and t = 10 s

$\because P = mu$

$\therefore u = \frac{P}{m} = \frac{10}{5} = 2m/s$

v = u + at

$v = u + \frac{F}{m}t \ (\because a = \frac{F}{m})$

$v = 2 + \frac{0.2}{5} \times 10 = 2.4$

The increase in kinetic energy

$= \frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2} \times 5[(2.4)^2 - (2)^2]$

$= \frac{1}{2} \times 5[5.76 - 4] = 4.4J$

Question 15

Work needed to move a particle of mass m from the surface of a solid sphere of mass M and radius R to its centre will be

Options:

A. $\frac{GMm}{R}$

B. $\frac{2\,GMm}{R}$

C. $\frac{GMm}{3R}$

D. $\frac{GMm}{2R}$

Answer: A

Solution:

Solution:
Work done against the gravitational force by particle of mass m is $\frac{GMm}{R}$ where M is the mass of sphere and R is the radius of sphere.

Question 16

A particle of mass m is placed at each of the three corners of an equilateral triangle of side a. The particles revolve in circular orbits under the influence of mutual gravitational force such that they always lie at the vertices of the triangle. The velocity of each particle is

Options:

A. $\sqrt{\frac{Gm}{a}}$

B. $\sqrt{\frac{2\,Gm}{a}}$

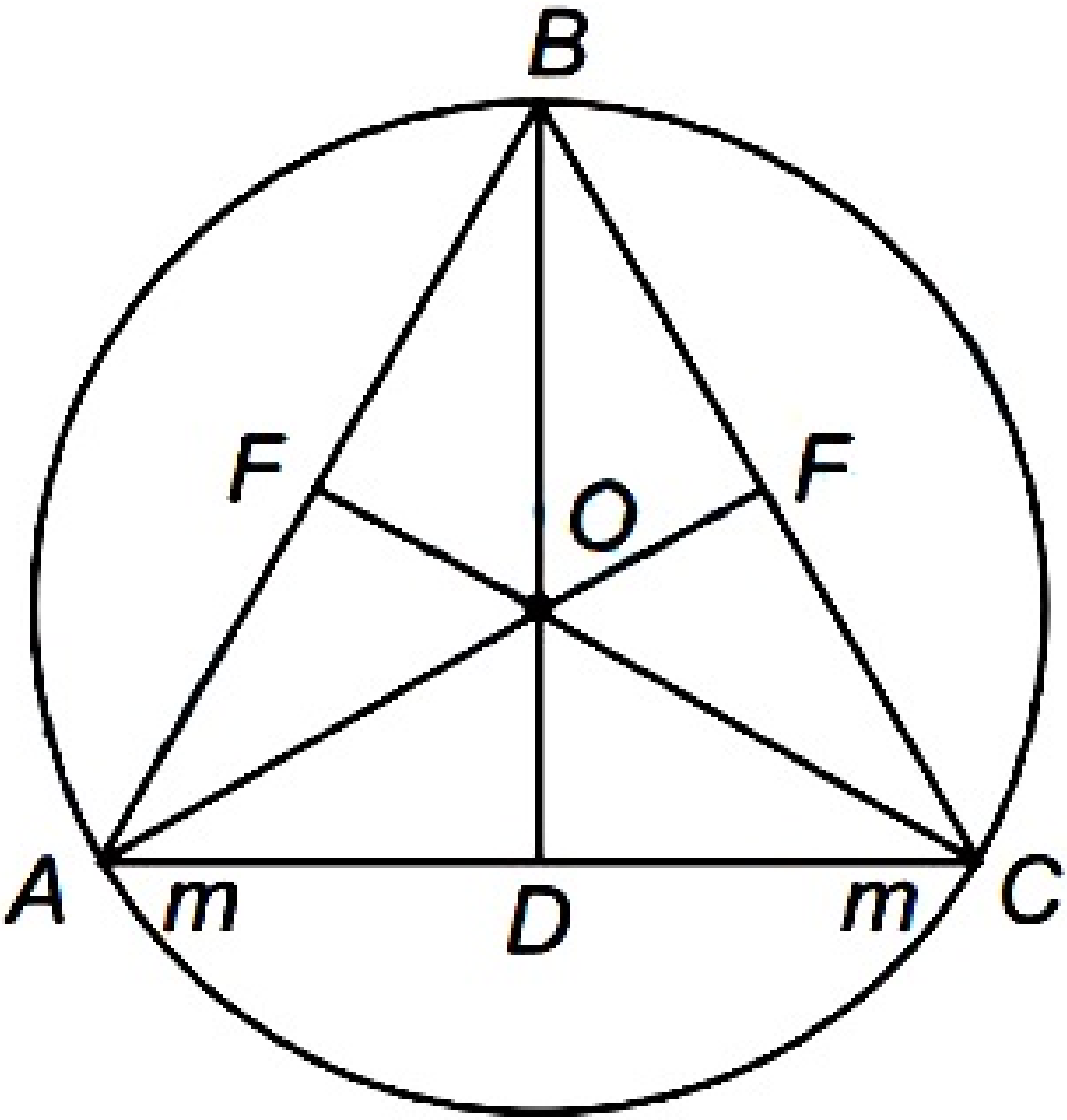
C. $\sqrt{\frac{Gm}{2a}}$

D. $\sqrt{\frac{3}{2}\frac{Gm}{a}}$

Answer: A

Solution:

Solution:



As the gravitational force between any two particles is F = $\frac{Gmm}{a^2}$, the resultant force on each.

Particle due to the other two

$F_R = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ}$

$F_R = \sqrt{3}F = \sqrt{3}\,\frac{G^2}{a^2}$

$\because$ AD = $\frac{a}{2}$

$\therefore$ BD = $\sqrt{AB^2 - AD^2}$

BD = $\sqrt{a^2 - (\frac{a}{2})^2} = \frac{\sqrt{3}a}{2}$

and radius BO = AO = OC = $\frac{\sqrt{3}a}{2} \times \frac{2}{3} = \frac{a}{\sqrt{3}}$

$\frac{mv^2}{r} = \frac{\sqrt{3}Gm^2}{a^2}$

$\frac{mv^2}{\frac{a}{\sqrt{3}}} = \frac{\sqrt{3}G^2}{a^2}$

$\Rightarrow$ v = $\sqrt{\frac{Gm}{a}}$

Question 17

The relationship between Young's modulus Y, bulk modulus K and modulus of rigidity η is

Options:

A. $Y = \frac{9\eta K}{\eta + 3K}$

B. $Y = 9\eta K(K + 3\eta)$

C. $\eta = \frac{9YK}{3K + Y}$

D. $Y = \frac{3\eta K}{9\eta + K}$

Answer: A

Solution:

Solution:
The relationship between Young's modulus Y, bulb modulus K and modulus of rigidity η is
$Y = \frac{9\eta K}{\eta + 3K}$

Question 18

Two soap bubbles of radii 2 mm and 4 mm coalesce to form a double bubble. The radius of its internal common interface will be

Options:

A. 2 mm

B. 4 mm

C. 6 mm

D. 3 mm

Answer: B

Solution:

Solution:
The radius of its internal common interface
$r = \frac{r_1 r_2}{r_1 - r_2}$
$= \frac{2 \times 4}{4 - 2} = \frac{8}{2}$
$= 4\,\text{mm}$

Question 19

The rate of flow of liquid through a capillary tube under a constant pressure difference is Q. On doubling the length of tube and reducing the diameter of tube to half, the rate of flow will become

Options:

A. $\frac{Q}{4}$

B. $\frac{Q}{8}$

C. $\frac{Q}{32}$

D. $16Q$

Answer: B

Solution:

Solution:
Rate of flow of liquid

$\therefore \quad Q = \frac{\pi pr^4}{8\eta 1}$

$\therefore \quad Q' = \frac{\pi p(\frac{r}{2})^4}{8\eta \times 2l}$

$Q' = \frac{\pi pr^4}{8\eta \times 321}$

or

$Q' = \frac{\pi pr^4}{8\eta 1 \times 32}$

The rate of flow, $Q' = \frac{Q}{32}$

Question 20

2 moles of a diatomic gas are mixed with 1 mole of a monoatomic gas. The ratio of two specific heats ($\gamma = C_p / C_V$) of the mixture will be

Options:

A. $\frac{7}{3}$

B. $\frac{5}{4}$

C. $\frac{19}{13}$

D. $\frac{15}{19}$

Answer: C

Solution:

Solution:

The ratio of specific heats $= \frac{C_p}{C_V} = \gamma_{mix}$

$$= \frac{\frac{\mu_1\gamma_1}{\gamma_1 - 1} + \frac{\mu_2\gamma_2}{\gamma_2 - 1}}{\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}}$$

For diatomic gas $\gamma_1 = 7/5$

and for a monoatomic gas $\gamma_2 = 5/3$

$$\therefore \quad \gamma_{mix} \quad \frac{\frac{2 \times \frac{7}{5}}{\frac{7}{5} - 1} + \frac{1 \times \frac{5}{3}}{(\frac{5}{3} - 1)}}{\frac{2}{(\frac{7}{5} - 1)} + \frac{1}{(\frac{5}{3} - 1)}}$$

$$= \frac{\frac{7}{1} + \frac{5}{2}}{\frac{10}{2} + \frac{3}{2}} = \frac{19}{13}$$

Question 21

If work is obtained from 1 cal heat, the amount of work obtained will be ($J = 4.18 J/cal$)

Options:

A. 4.18J

B. more than 4.18J

C. less than 4.18J

D. Nothing can be said

Answer: A

Solution:

Solution:

Work $W = JQ$

$= 4.18 \times 1$

$W = 4.18J$

Question 22

An ideal engine is working between temperatures 400K and 300K. It absorbs 600 cal heat from the source. The work obtained per cycle from the engine is

Options:

A. 630J

B. 630 cal

C. 2400 cal

D. zero

Answer: A

Solution:

Solution:

$\frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$

$\frac{W}{600} = 1 - \frac{300}{400}$

$W = 600[\frac{400-300}{400}] = \frac{600 \times 100}{400} = 150\,cal$

$W = 150 \times 4.2 = 630J$

Question 23

A particle is executing simple harmonic motion with a frequency f. The frequency of oscillations of its kinetic energy will be

Options:

A. $\frac{f}{2}$

B. f

C. 2f

D. zero

Answer: C

Solution:

Solution:

$y = a\,sin\,ft$

$v = \frac{dy}{dt} = af\cos ft$

Kinetic energy $K = \frac{1}{2}mv^2$

$= \frac{1}{2}ma^2f^2\cos^2 ft$

$= \frac{1}{2}ma^2f^2(1+\cos 2ft)$

Therefore, the frequency of oscillations of its kinetic energy will be 2f.

Question 24

The wavelength of light emitted from a star is $4320\mathring{A}$. If radius of star is 7×10^8m and period of its rotational motion is 22 days, the Doppler's displacement will be

Options:

A. $0.033\mathring{A}$

B. $0.33\mathring{A}$

C. $3.3\mathring{A}$

D. $33\mathring{A}$

Answer: C

Solution:

Solution:
Doppler's displacement
$\Delta\lambda = \lambda \cdot \frac{v}{c}$
$= \frac{\lambda}{c} \cdot r \times (\frac{2\pi}{T})$
$= \frac{4320 \times 7 \times 10^8 \times 2 \times 3.14}{3 \times 10^8 \times 22 \times 86400} = 0.033\mathring{A}$

Question 25

In double slit experiment with sodium light (λ = $5890\mathring{A}$), the angular width of interference fringes is 0.20°. The change in wavelength required to increase the angular width by 10% will be

Options:

A. increase of $589\mathring{A}$

B. decrease of $589\mathring{A}$

C. increase of $6479\mathring{A}$

D. decrease of $6479\mathring{A}$

Answer: A

Solution:

Solution:
Angular width of fringe $\theta = \frac{\lambda}{d}$
$\theta \propto \lambda$

$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2}$

When the angular width is increased by 1 % .

$\theta_2 = 0.20 + 0.20 \times \frac{10}{100} = 0.22$

$\therefore \quad \frac{0.20}{0.22} = \frac{5890}{\lambda_2}$

$\lambda_2 = \frac{5890 \times 0.22}{0.20}$

$\lambda = 6479 \mathring{A}$

Therefore change in wavelength

$\lambda = \lambda_2 - \lambda_1$

$= 6479 - 5890$

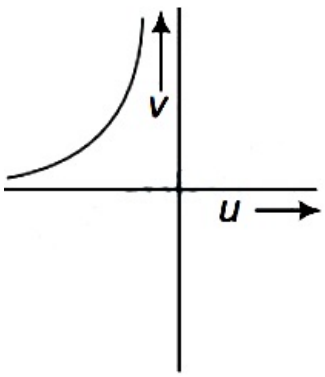
$= 589 \mathring{A}$ (increase)

Question 26

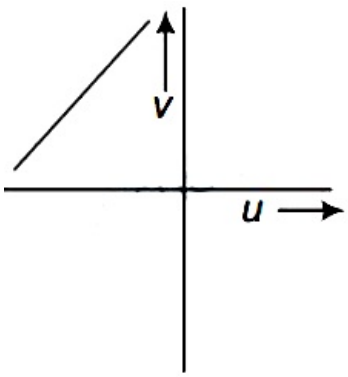
In experiment of finding the focal length of a convex lens by two-pin method, the u-v graph obtained by the student will be as

Options:

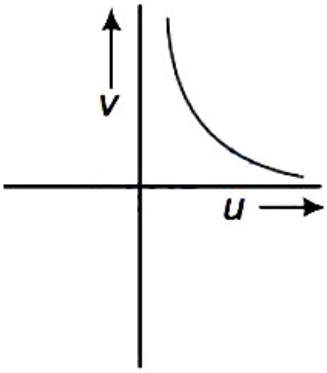
A.



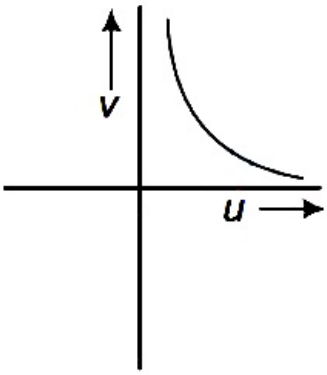
B.



C.



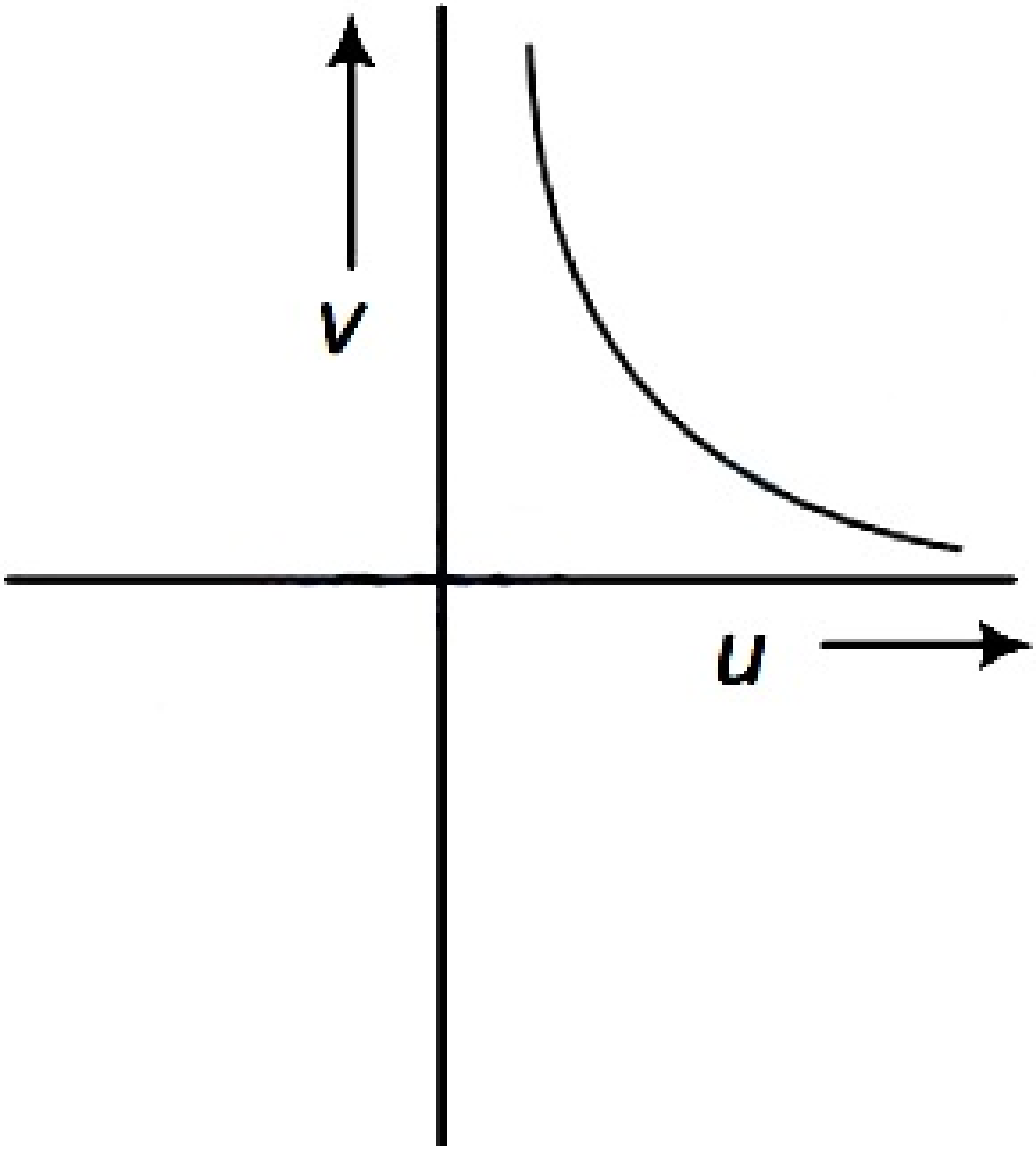
D.



Answer: C

Solution:

Solution:
When $u \to f, v \to \infty$ and when $u \to \infty, v \to f$



Question 27

For a glass prism of refractive index 1.5, the angle of minimum deviation is equal to the refracting angle of prism. The refracting angle of prism will be

Options:

A. 62°

B. 41°

C. 82°

D. 31°

Answer: C

Solution:

Solution:

$$\text{Refractive index } \mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

$$\mu = \frac{\sin\frac{2A}{2}}{\sin\frac{A}{2}} \quad (\because \delta_m = A)$$

$$\mu = \frac{\sin A}{\sin\frac{A}{2}}$$

$$1.5 = \frac{2\sin\frac{A}{2}\cdot\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\frac{1.5}{2} = \cos\frac{A}{2}$$

$$\cos\frac{A}{2} = \frac{3}{4}$$

$$\cos\frac{A}{2} = \cos 41.4$$

$$\frac{A}{2} = 41.4$$

$$A = 82.8^\circ$$

Question 28

In a compound microscope, length of the tube is 10 cm and focal lengths of objective and eye lens are respectively 0.5 cm and 1.0 cm. The magnifying power of microscope will be nearly

Options:

A. 5

B. 23

C. 166

D. 444

Answer: D

Solution:

Solution:
Given, L = 10 cm

$f_0 = 0.5\,cm$ and $f_c = 1.0\,cm$
Magnifying power of microscope
$M_D = \frac{L}{f_O}\left(\frac{D}{f_e}\right)$
$M_D = \frac{10}{0.5}(\frac{25}{1}) = 20(25) = 500$
Here the magnifying power of microscope will be nearly 444 .

Question 29

A straight wire carrying current / is bent in form of a circle. If length of wire is L, its magnetic moment will be

Options:

A. $\frac{IL}{4\pi}$

B. $\frac{IL^2}{4\pi}$

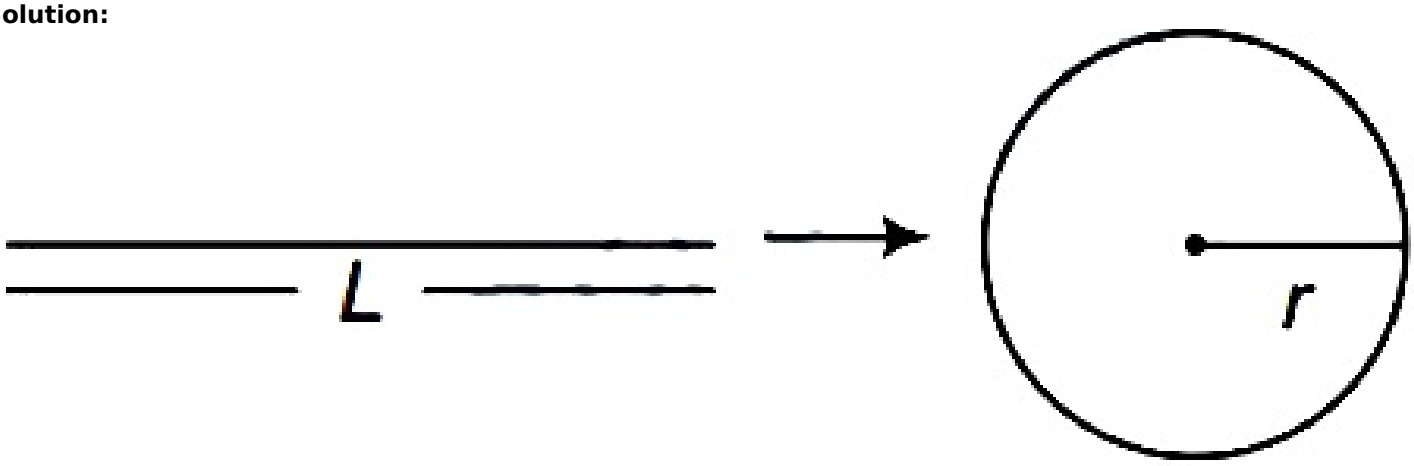
C. $\frac{I^2L^2}{4\pi}$

D. $\frac{LI^2}{4\pi}$

Answer: D

Solution:

Solution:



When a straight wire is bent in form of circle, Circumference of circle $= 2\pi r = L$
Area of circle $A = \pi r^2$
$A = \pi \frac{L^2}{4\pi^2}$ $[\because \gamma = \frac{L}{2\pi}]$
$A = \frac{L^2}{4\pi}$
Magnetic moment $M = IA = \frac{IL^2}{4\pi}$

Question 30

A condenser of capacity C is charged to a potential V. The electric flux passing through a closed surface surrounding the condenser will be

Options:

A. $\frac{CV}{\varepsilon_0}$

B. $\frac{2\,CV}{\varepsilon_0}$

C. $\frac{CV}{2\varepsilon_0}$

D. zero

Answer: A

Solution:

Solution:
Electric flux, $\varphi = \frac{q}{\varepsilon_0}$
$\varphi = \frac{CV}{\varepsilon_0}$ $(\because q = CV)$

Question 31

The electric potential at a point is $V = -5x + 3y + \sqrt{15}z$ volt, where x, y and z are in metres. The magnitude of electric field will be

Options:

A. $3\sqrt{2}\mathrm{Vm}^{-1}$

B. $4\sqrt{2}\mathrm{Vm}^{-1}$

C. $5\sqrt{2}\mathrm{Vm}^{-1}$

D. $7\mathrm{Vm}^{-1}$

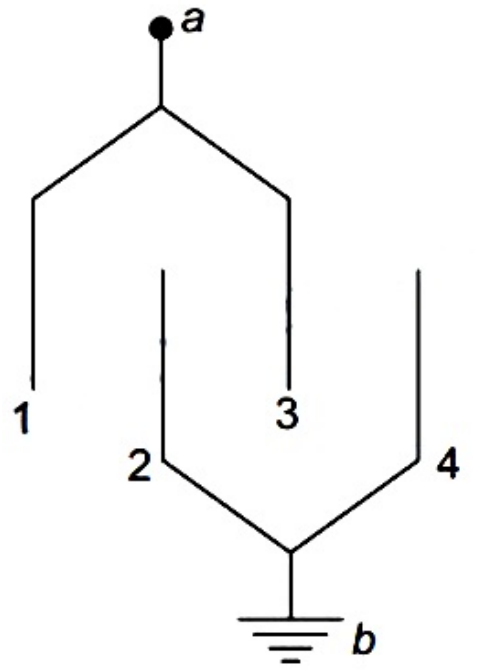
Answer: A

Solution:

Solution:
Electric potential $V = -5x + 3y + \sqrt{15}z$
Electric field $E_x = -\frac{dV}{dx} = -5$
$E_y = -\frac{dV}{dy} = -3$
$E_2 = -\frac{dV}{dz} = -\sqrt{15}$
$E = \sqrt{E_x^2 + E_y^2 + E_z^2}$
$= \sqrt{(-5)^2 + (-3)^2 + (-\sqrt{15})^2}$
$E = 7V/m$

Question 32

In the diagram below, area of each plate is A and separation between two consecutive plates is d, which is filled with air. The equivalent capacity between the points a and b is



Options:

A. $\frac{\varepsilon_0 A}{d}$

B. $\frac{2\varepsilon_0 A}{d}$

C. $\frac{3\varepsilon_0 A}{d}$

D. $\frac{4\varepsilon_0 A}{d}$

Answer: C

Solution:

Solution:
Given arrangement is equivalent to combination of three capacitors.
Therefore, capacity $= 3C = 3\frac{\varepsilon_0 A}{d}$

Question 33

Two wires A and B are of same lengths but different radii made up of copper and iron respectively. They carry same current under the same potential difference. If specific resistance of copper and iron is $1.7 \times 10^{-8} \Omega m$ and $1.0 \times 10^{-7} \Omega m$ respectively, the ratio of their radii r_B / r_A will be

Options:

A. 1.2

B. 2.4

C. 3.6

D. 4.8

Answer: B

Solution:

Solution:
Resistance of wire $R = \frac{\rho l}{A}$ or
$1 = \frac{RA}{\rho}$
$1 = \frac{AV}{\rho l}$ $[\because R = \frac{V}{I}]$
A and B have same length, same current and same potential difference.

$\therefore \quad 1_A = 1_B$

$$\frac{A_1 V}{\tau_A l} = \frac{A_2 V}{\tau_B l}$$

or

or

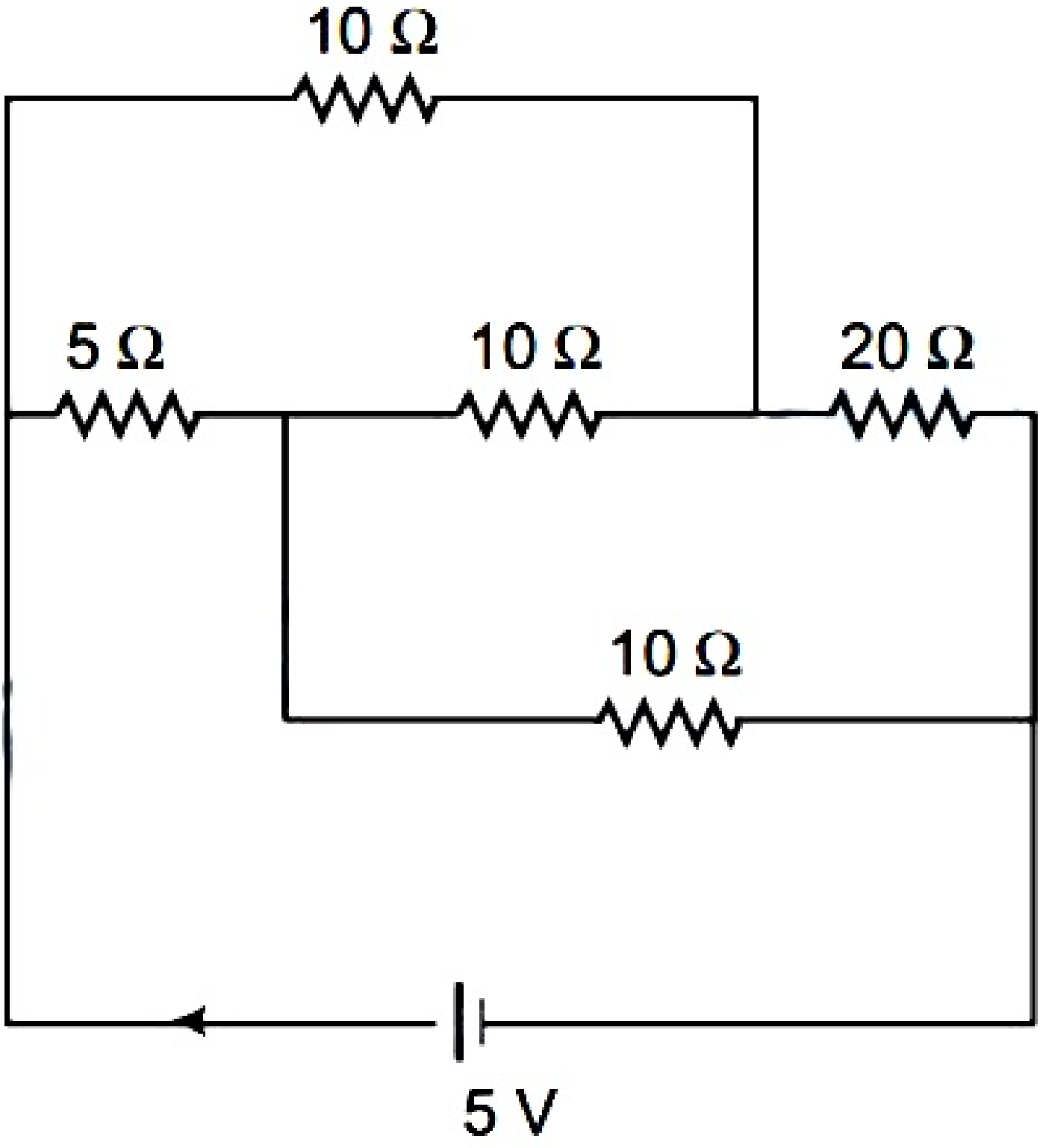
$$\frac{\pi r_A^2}{\tau_A} = \frac{\pi r_B^2}{\tau_B}$$

$$\frac{r_B^2}{r_A^2} = \frac{\tau_B}{\tau_A} = \frac{1.0 \times 10^{-7}}{1.7 \times 10^{-8}}$$

$$\frac{r_B}{r_A} = \sqrt{5.88} = 2.4$$

Question 34

In the diagram below, current / drawn from the battery is



Options:

A. 0.5A

B. 0.67A

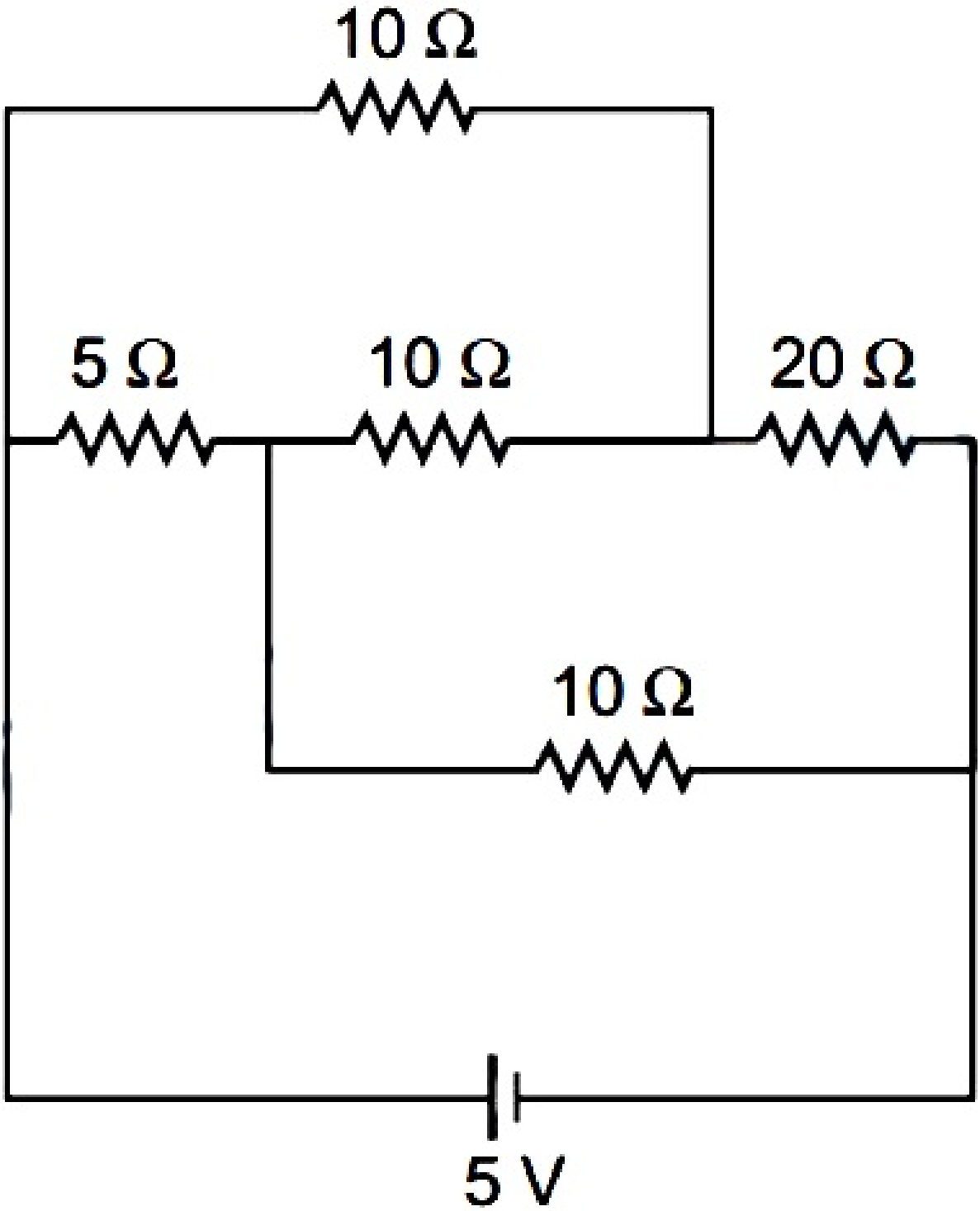
C. 0.17A

D. None of these

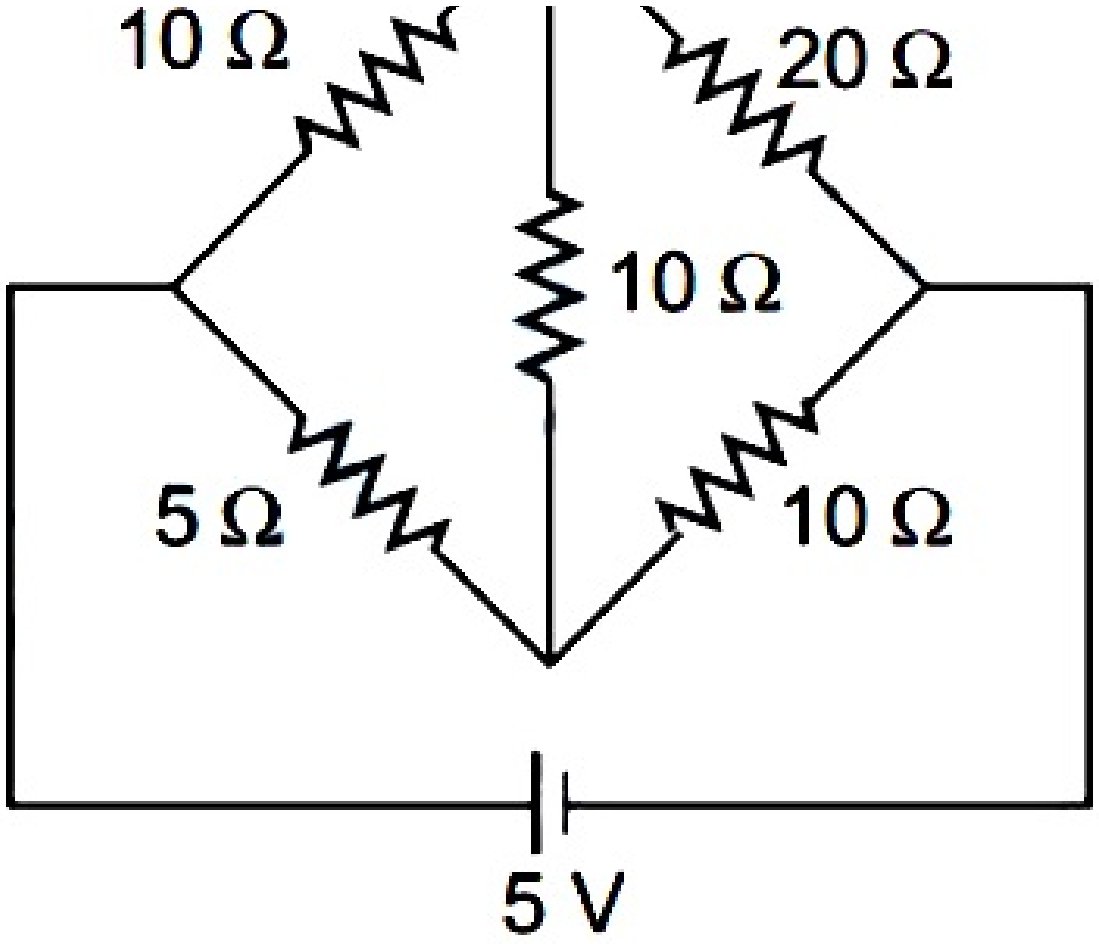
Answer: A

Solution:

Solution:



$\Rightarrow$



This circuit represents the Wheatstone bridge, therefore equivalent resistance

$R_{eq} = \frac{30 \times 15}{30 + 15}$

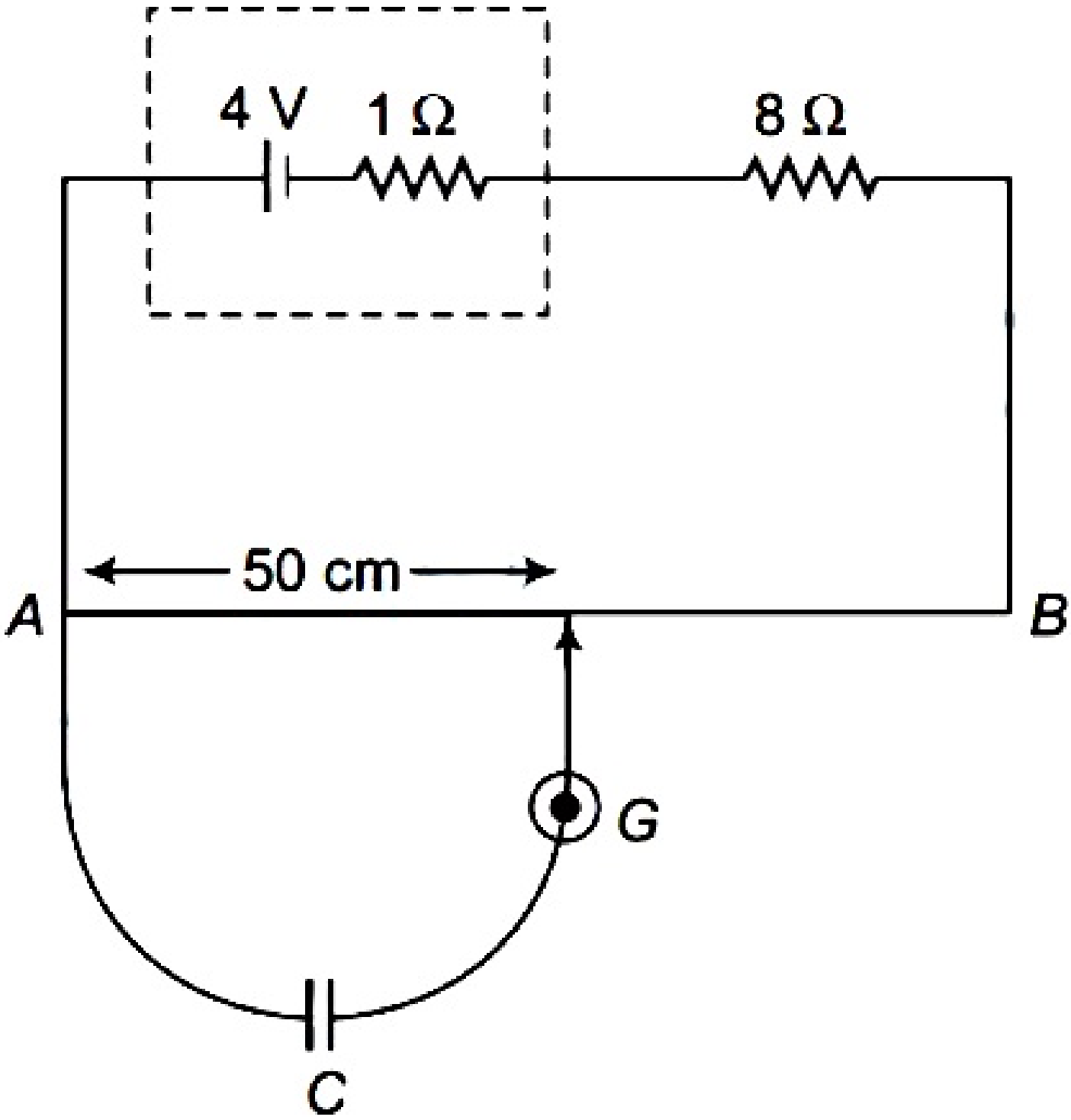
$= 10\Omega$

In this circuit, current $i = \frac{5}{10}$

$= 0.5A$

Question 35

In the diagram below, the wire AB is of length 100 cm and resistance 3Ω. The null point is obtained at 50 cm. The emf of cell C is



Options:

A. zero

B. 0.5V

C. 1.0V

D. 1.5V

Answer: B

Solution:

Solution:

Emf of cell $E = \frac{e}{(R + R_h + r)} \times \frac{R}{L} \times 1$

$= \frac{4}{(3+8+1)} \times \frac{3}{1} \times 0.5$

$E = \frac{12 \times 0.5}{12} = 0.5V$

Question 36

The current flow in electrolytes is due to motion of

Options:

A. only positive ions

B. only negative ions

C. free electrons

D. both positive and negative ions

Answer: D

Solution:

Solution:
The current flow in electrolytes is due to motion both positive and negative ions.

Question 37

1 kWh is equal to

Options:

A. 3.6×10^6J

B. 3.6×10^4J

C. 3.6×10^3J

D. 6×10^{-4}J

Answer: A

Solution:

Solution:

Question 38

An ideal voltmeter has resistance

Options:

A. zero

B. very low

C. very high

D. infinite

Answer: D

Solution:

Solution:
An ideal voltmeter has resistance infinite.

Question 39

The SI unit of magnetic dipole moment is

Options:

A. Am^{-1}

B. Am^{2}

C. $\mathrm{mA}^{-1}s$

D. $\mathrm{mA}^{-2}s$

Answer: B

Solution:

Solution:
Magnetic dipole moment $= \mathrm{NiA}$ where i is current and A is area of coil.
$\therefore$ The SI unit of magnetic moment is Am^{2}.

Question 40

The emf induced in a coil of area A due to change in magnetic flux in a magnetic field B is given as

Options:

A. $e = -A \cdot \frac{dB}{dt}$

B. $e = -B \cdot \frac{dA}{dt}$

C. $e = -\frac{d}{dt}(B \cdot A)$

D. $e = -\frac{d}{dt}(A \times B)$

Answer: B

Solution:

Solution:
The emf induced in a coil of area A due to change in magnetic flux in a magnetic field B is given as
$E = -B\frac{dA}{dt}$

Question 41

Lenz's law is based on

Options:

A. conservation of energy

B. conservation of momentum

C. conservation of mass

D. conservation of charge

Answer: A

Solution:

Solution:
Lenz's law is based on conservation of energy.

Question 42

On an AC circuit, the hot wire ammeter reads current 10A. Its peak value is

Options:

A. 10A

B. 20A

C. 14.14A

D. 7.07A

Answer: C

Solution:

Solution:

$I_{rms} = \frac{I_0}{\sqrt{2}}$

$I_0 = \sqrt{2} I_{rms}$

$= \sqrt{2} \times 10$

$= 14.14A$

Question 43

In an AC circuit with pure capacitance

Options:

A. emf leads ahead of current by $\pi / 2$

B. current leads ahead of voltage by $\pi / 2$

C. current lags behind the voltage by π

D. voltage lags behind the current by π

Answer: B

Solution:

Solution:
In an AC circuit with pure capacitance, current leads ahead of voltage by $\pi / 2$.

Question 44

Light rays of frequency v are made incident on a substance of threshold frequency v_0. The kinetic energy of photoelectron emitted can be

Options:

A. hv

B. hv_0

C. $hv - hv_0$

D. $hv + hv_0$

Answer: C

Solution:

Solution:
Kinetic energy of photoelectron
$K_{max} = h(v - v_0)$

Question 45

The specific charge of electron is

Options:

A. $1.76 \times 10^{11} C kg^{-1}$

B. $1.6 \times 10^{-19} C kg^{-1}$

C. $9.1 \times 10^{-31} C kg^{-1}$

D. $1.76 \times 10^{-11} C kg^{-1}$

Answer: A

Solution:

Solution:

Question 46

The penetrating power of X-rays can be increased by

Options:

A. increasing the filament current

B. decreasing the filament current

C. increasing the potential on anode

D. decreasing the potential on anode

Answer: C

Solution:

Solution:

The penetrating power of X-rays can be increased by increasing the potential on anode.

Question 47

The source of sun's energy is

Options:

A. nuclear fission

B. ion exchange process

C. photoelectric process

D. nuclear fusion

Answer: D

Solution:

Solution:
The source of sun's energy is nuclear fusion.

Question 48

In a Bohr orbit of hydrogen atom, the ratio of kinetic energy of electron to its potential energy is

Options:

A. $1:2$

B. $2:1$

C. $-1:2$

D. $-2:1$

Answer: A

Solution:

Solution:
Potential energy of electron $U = \frac{kZ\,e^2}{r_n}$

Kinetic energy of electron $K = \frac{kZe^2}{2r_n}$

$\therefore \quad \frac{K}{U} = \frac{1}{2}$

Question 49

In the following diagram, current in 100Ω, resistor will be

Options:

A. zero

B. 10 mA

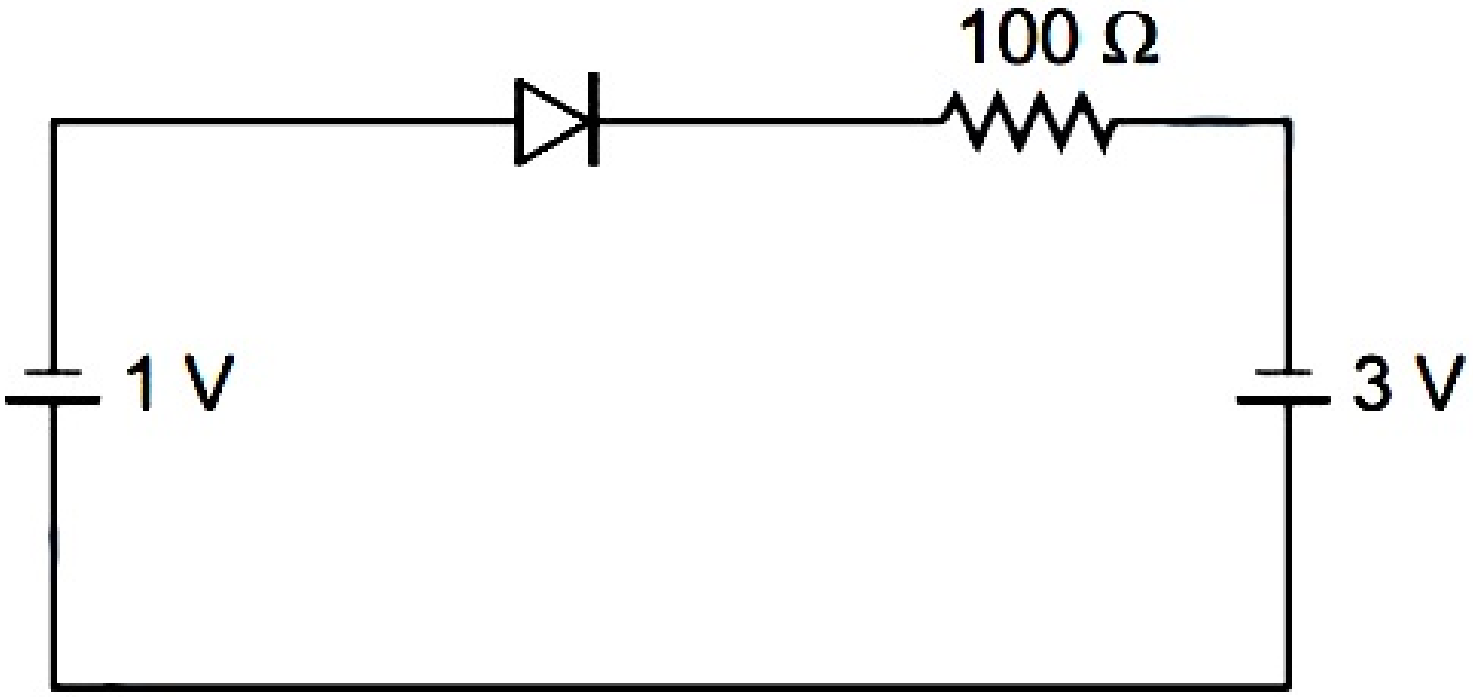
C. 20 mA

D. 50 mA

Answer: C

Solution:

Solution:



Potential difference $V = 3 - 1 = 2V$

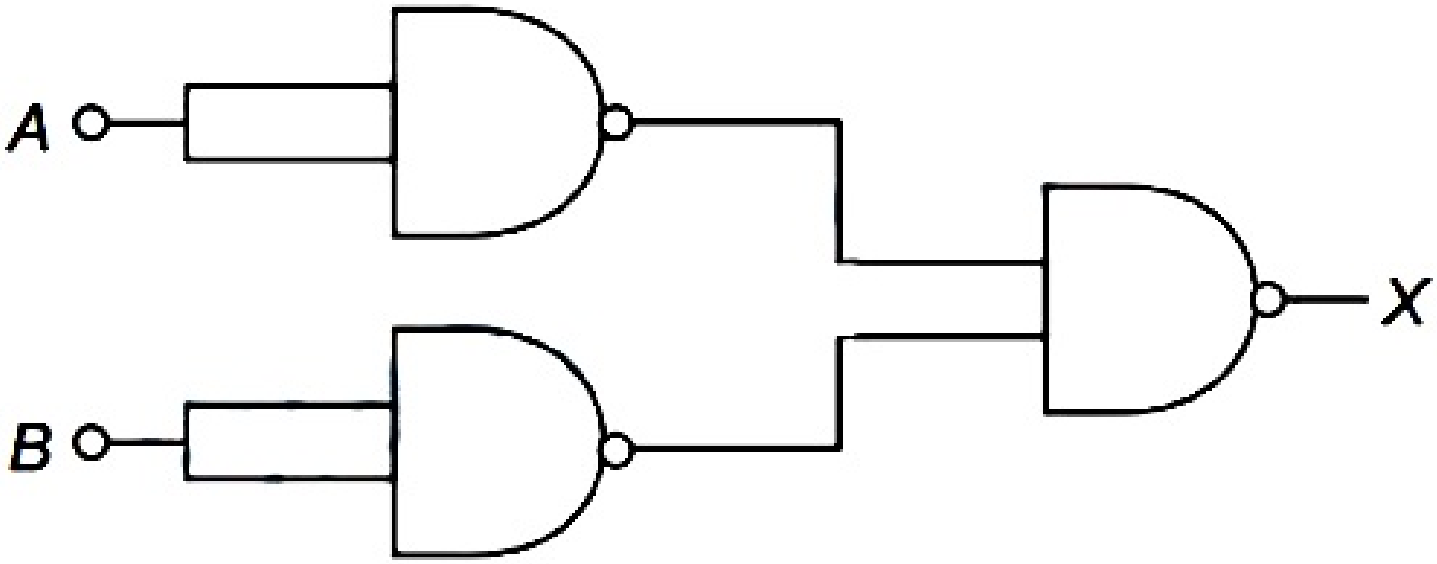
Current in $100\Omega, i = \frac{V}{R}$

$= \frac{2}{100}$

$= 20 \times 10^{-3} A = 20\,mA$

Question 50

The following combination of gates is equivalent to



Options:

A. OR gate

B. NOT gate

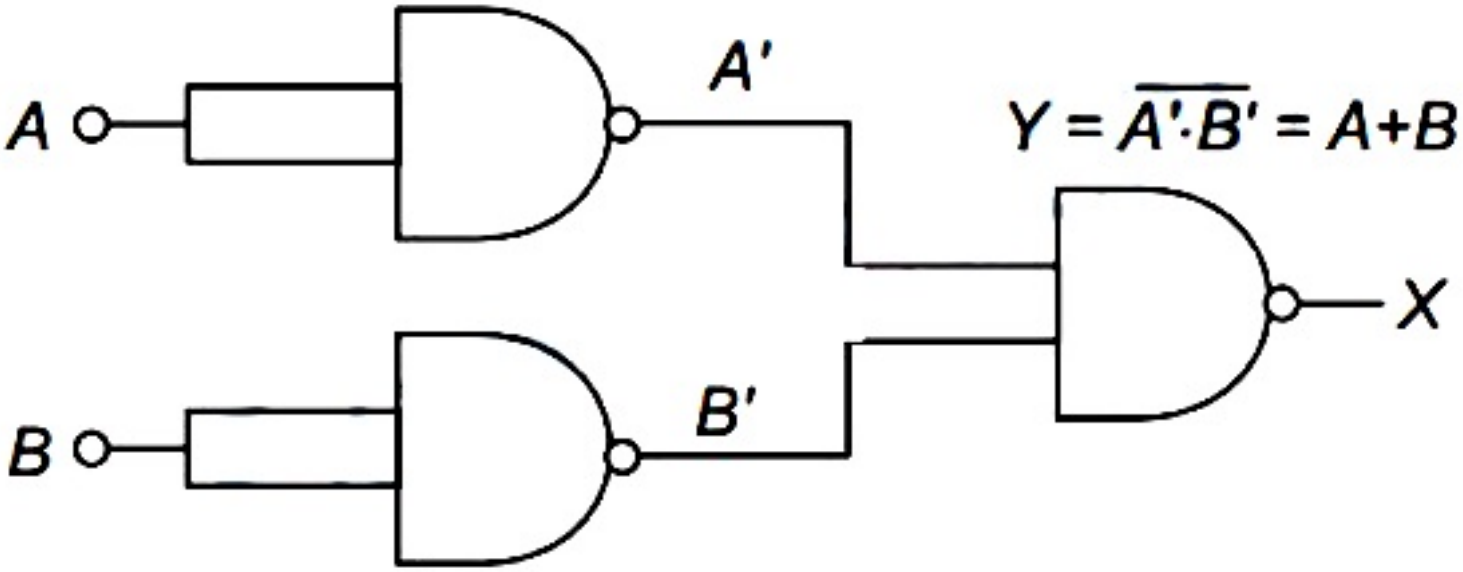
C. XOR gate

D. NAND gate

Answer: A

Solution:

Solution:



The following combination of gates is equivalent to OR gate.

Question 51

Among the following, the group of molecules that undergoes rapid hydrolysis is

Options:

A. $SF_6, Al_2Cl_6, SiMe_4$

B. $BCl_3, SF_6, SiCl_4$

C. $BCl_3, SiCl_4, PCl_5$

D. $SF_6, Al_2Cl_6, SiCl_4$

Answer: C

Solution:

Solution:
$BCl_3, SiCl_4$ and PCl_5 molecules undergo rapid hydrolysis.

$$BCl_3 + 3H_2O \longrightarrow \underset{\text{boric acid}}{H_3BO_3} + 3\,HCl$$

$$3SiF_4 + 4H_2O \longrightarrow H_4SiO_4 + \underset{\text{fluorosilicic acid}}{2H_2SiF_6}$$

$$PCl_5 + 4H_2O \longrightarrow H_3PO_4 + \underset{\text{orthophosphoric acid}}{5\,HCl}$$

Question 52

Nitric acid ionises slightly in HF, $HNO_3 + \underset{(\text{Solvent})}{HF} \rightleftharpoons H_2NO_3^+ + F^-$
Then

Options:

A. HNO_3 and $H_2NO_3^+$ are bases

B. HF and F^- are bases

C. HNO_3 and F^- are bases

D. only HNO_3 is base

Answer: C

Solution:

Solution:
According to Bronsted-Lowry acid is proton donor while base is proton acceptor.

$$\underset{\text{base}}{HNO_3} + \underset{\text{acid}}{HF} \rightleftharpoons \underset{\text{acid}}{H_2NO_3^+} + \underset{\text{base}}{F^-}$$

Question 53

For the reaction,
$2\,NO(g) + Cl_2(g) \rightleftharpoons 2\,NOCl(g)$
which relation is true ?

Options:

A. $K_p = K_c \times RT$

B. $K_p = \frac{K_c}{RT}$

C. $K_p = K_c(RT)^2$

D. $K_p = \frac{K_c}{(RT)^2}$

Answer: B

Solution:

Solution:
$K_p = K_c(RT)^{\Delta n_g}$
$\Delta n_g = (n_p - n_r)_{\text{gaseous}}$
$2NO(g) + Cl_2(g) \rightleftharpoons 2NOCl(g)$
$\Delta n_g = 2 - 3 = -1$
$K_p = K_c(RT)^{-1}$
$K_p = \frac{K_c}{RT}$

Question 54

The following compounds have identical molecular weight. Which would have the lowest boiling point?

Options:

A. 2-butanol

B. 2-methyl-1-propanol

C. 1, 1-dimethyl ethanol

D. 1-methoxypropane

Answer: D

Solution:

Solution:
For isomeric alcohols, the boiling points decrease in the order
Primary > secondary > tertiary
It is due to the fact that surface area decreases with branching and therefore van der Waals' force decreases.
Further, ethers have low boiling point, lower than that of isomeric alcohols. Hence, 1-methoxypropane has the lowest boiling point among the given compounds.

Question 55

Which pair of the following carbonyl compounds can be differentiated by $I_2 / NaOH$?

Options:

A. $C_6H_5 - CHO$ and $C_6H_5 - \overset{O}{\overset{\|}{C}} - CH_2 - CH_3$

B. $C_6H_5 - C - CH_3$ and $CH_3 - CH_2 - \overset{O}{\overset{\|}{C}} - CH_3$

C. $CH_3 - CH_2 - C - CH_2 - CH_3$ and $C_6H_5 - \overset{O}{\overset{\|}{C}} - CH_2 - CH_3$

D. $C_6H_5 - \overset{O}{\overset{\|}{C}} - \underset{\underset{CH_3}{\|}}{CH} - CH_3$ and $C_6H_5 - \overset{O}{\overset{\|}{C}} - CH_3$

Answer: D

Solution:

Solution:

Compounds containing $-\overset{O}{\overset{\|}{C}} - CH_3$ (methyl carbonyl species) give a positive iodoform test with $I_2 / NaOH$.

$$C_6H_5COCH_3 + 4\,NaOH + 3I_2 \xrightarrow[\text{acetophenone}]{} C_6H_5COONa + \underset{\text{yellow ppt.}}{CHI_3} + 3H_2O + 3\,NaI$$

$$C_6H_5 - \overset{O}{\overset{\|}{C}} - CH - CH_3 \xrightarrow{I_2 / NaOH}$$

No yellow ppt.

Question 56

In the reaction sequence,

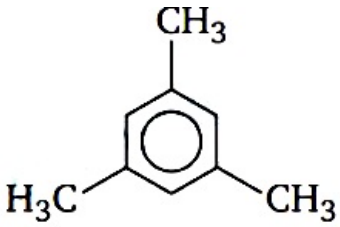
$$CH_3CN \xrightarrow[H_3O^+]{CH_3MgBr\ (2\ mol)} [A] \xrightarrow{\text{Conc. } H_2SO_4} [B]$$

The products [*A*] and [*B*] are

Options:

A.

CH_3COCH_3,



B. $(CH_3)_3COH, (CH_3)_2C = = CH_2$

C. $(CH_3)_2CHOH, CH_3CH = = CH_2$

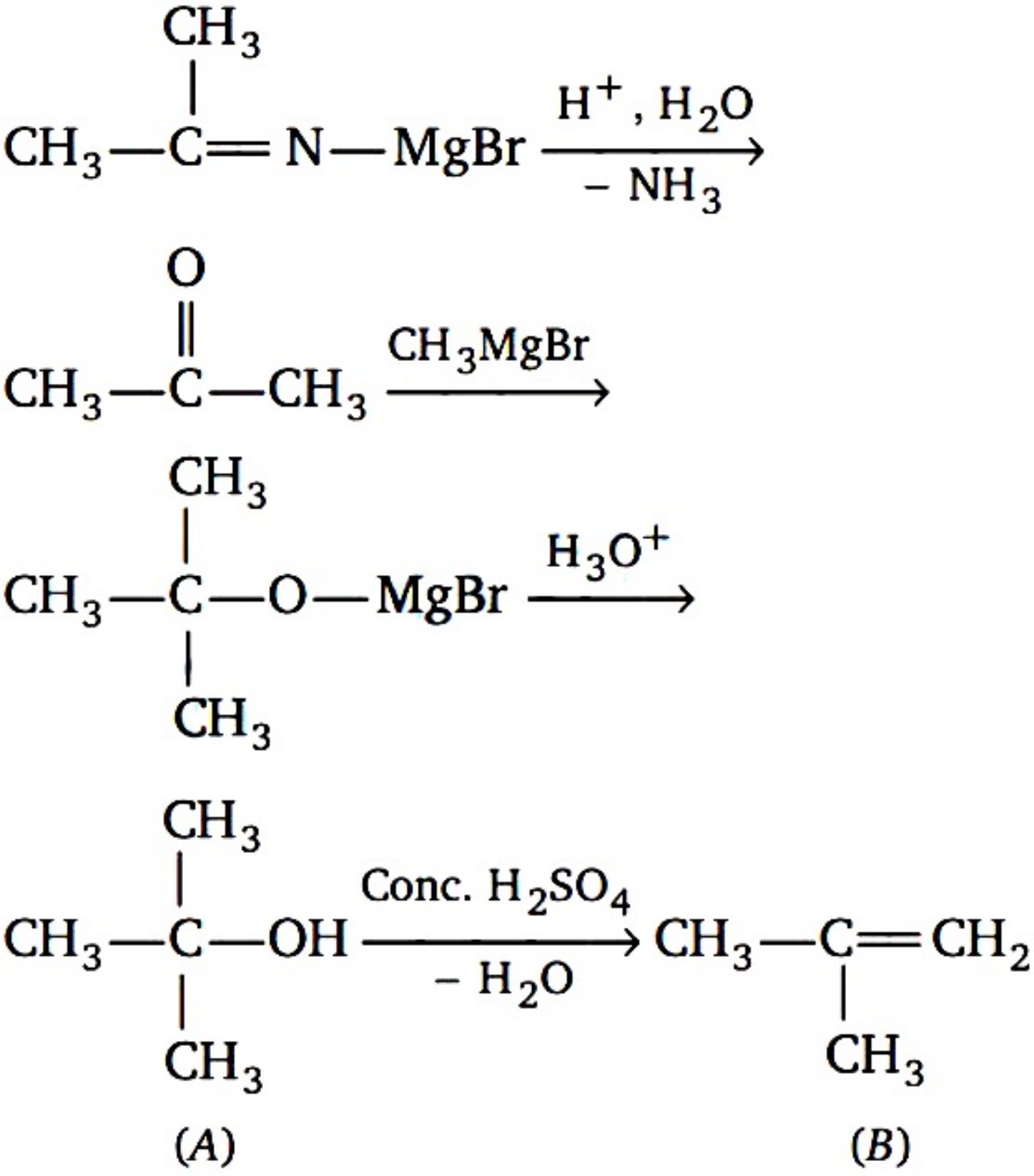
D. $CH_3COCH_3, CH_3CH = = CH_2$

Answer: B

Solution:

Solution:

$$CH_3 - C \equiv = N + CH_3MgBr \xrightarrow{\text{Dry ether}}$$



Question 57

Which one of the following is both nucleophilic and electrophilic?

Options:

A. CH_4

B. CH_3CN

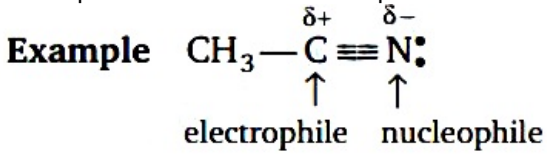
C. H_2O

D. CH_3OH

Answer: B

Solution:

Solution:
Organic compounds containing a multiple bond between carbon and a more electronegative atom can act both as an electrophile as well as a nucleophile.



Question 58

Match List I with List II and select the correct answer using the codes given below the lists.

List I(Metals)	List II (Name of refining process)
(P) Ni	(1) Distillation
(Q) Cu	(2) Electrolysis
(R) Cr	(3) Mond process
(S) Hg	(4) Aluminothermic process

Options:

A. (P-3), (Q-2), (R-4), (S-1)

B. (P-2), (Q-3), (R-1), (S-4)

C. (P-4), (Q-1), (R-2), (S-3)

D. (P-1), (Q-4), (R-3), (S-2)

Answer: A

Solution:

Solution:
Ni metal is refined by Mond process.

$$\underset{\text{impure}}{Ni} + 4\,CO \xrightarrow{80^\circ C} Ni(CO)_4$$

$$Ni(CO)_4 \xrightarrow{200^\circ C} \underset{\text{Pure}}{Ni} + 4\,CO \uparrow$$

Cu metal is refined by electrolysis process.
Cr metal is refined by aluminothermic process.
Hg metal is refined by distillation process.

Question 59

Which of the following two ions have the same number of unpaired electrons ?

(1) Mn^{3+}
(2) Fe^{3+}
(3) Cr^{3+}
(4) Co^{3+}

Options:

A. (2) and (3)

B. (3) and (4)

C. (1) and (2)

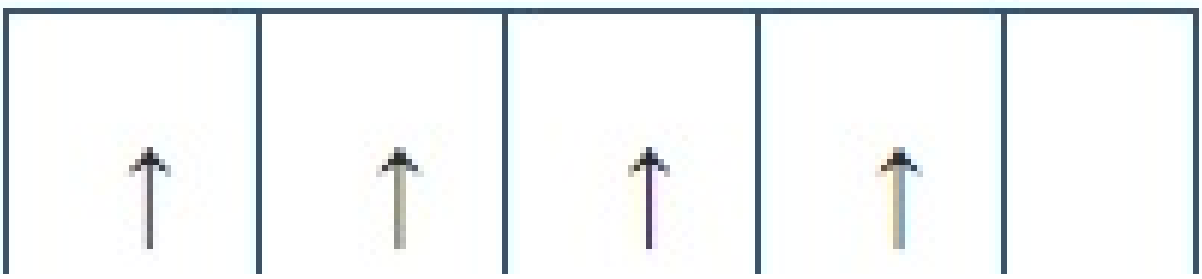
D. (1) and (4)

Answer: D

Solution:

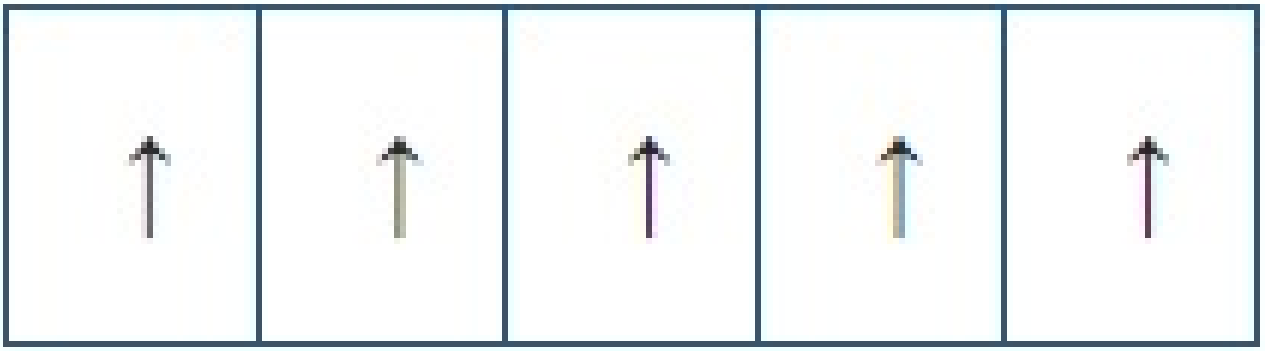
Solution:

$$Mn^{3+} : [Ar]\,3d^4$$



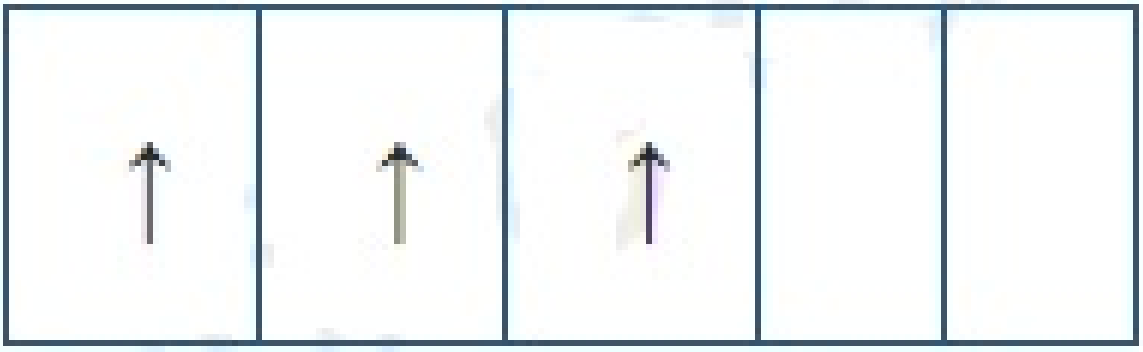
4 unpaired e^-

$Fe^{3+}: [Ar]3d^5$



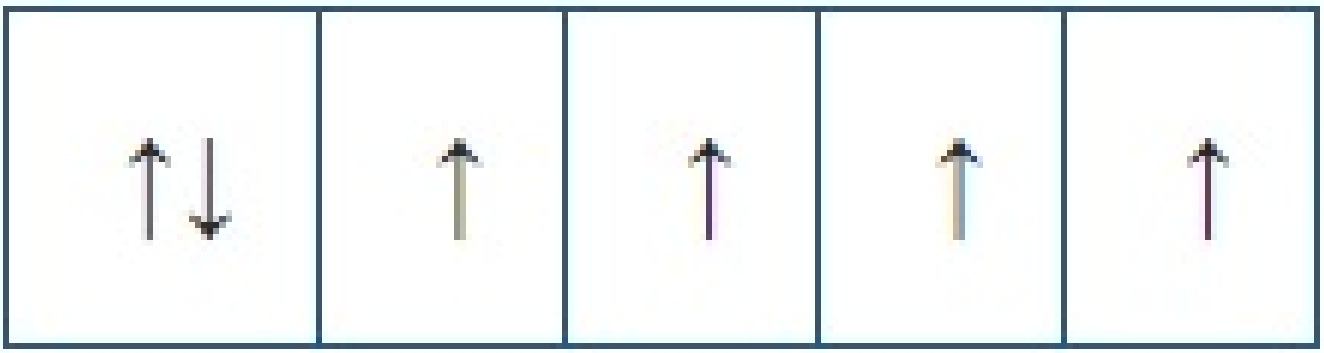
5 unpaired e^-

$Cr^{3+}: [Ar]3d^3$



3 unpaired e^-

$$Co^{3+} : [Ar]3d^6$$



$$4 \text{ unpaired } e^-$$

Hence, Mn^{3+} and Co^{3+} ions have the same number of unpaired electrons.

Question 60

The correct IUPAC name of $[Pt(NH_3)_3Br(NO_2)Cl]Cl$ is

Options:

A. triammine chlorobromonitroplatinum (IV) chloride

B. triamminebromochloronitroplatinate (IV) chloride

C. triamminebromochloronitroplatinum (IV) chloride

D. triamminebromochloronitroplatinum (II) chloride

Answer: C

Solution:

Solution:
$[Pt(NH_3)_3Br(NO_2)Cl]Cl$
Let the oxidation number of Pt is x.
$x + 3(0) - 1 - 1 - 1 - 1 = 0$
$x = +4$
Hence, its IUPAC name is triammine bromochloronitroplatinum (IV) chloride.

Question 61

The pair having the similar shape is

Options:

A. BF_3 and NF_3

B. BF_4^- and NH_4^+

C. $SiCl_4$ and SCl_4

D. CH_3^+ and H_3O^+

Answer: B

Solution:

Solution:
In BF_4^- and NH_4^+ both central atom is sp^3 hybridised hence, both BF_4^- and NH_4^+ have similar tetrahedral structure.

Question 62

The wavelength of an electron having kinetic energy equal to 4.55×10^{-25}J is (h $= 6.6 \times 10^{-34}$kgm$^2 s^{-1}$., mass of electron $= 9.1 \times 10^{-31}$kg)

Options:

A. 7.25×10^{-7}nm

B. 725m

C. 7.25×10^{-7}m

D. 7.25×10^{7}m

Answer: C

Solution:

Solution:

$$KE = \frac{1}{2}mv^2$$

$$4.55 \times 10^{-25} = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$$

$$v^2 = \frac{2 \times 4.55 \times 10^{-25}}{9.1 \times 10^{-31}}$$

$$v = 10^3 m/s$$

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^3}$$

$$\lambda = 7.25 \times 10^{-7}m$$

Question 63

In the carbylamine reaction, $R-X$ is converted to $R-Y$ via the intermediate Z. $R-X, R-Y$ and Z, respectively, are

Options:

A. RNH_2, RNC, carbene

B. RNH_2, RNC, nitrene

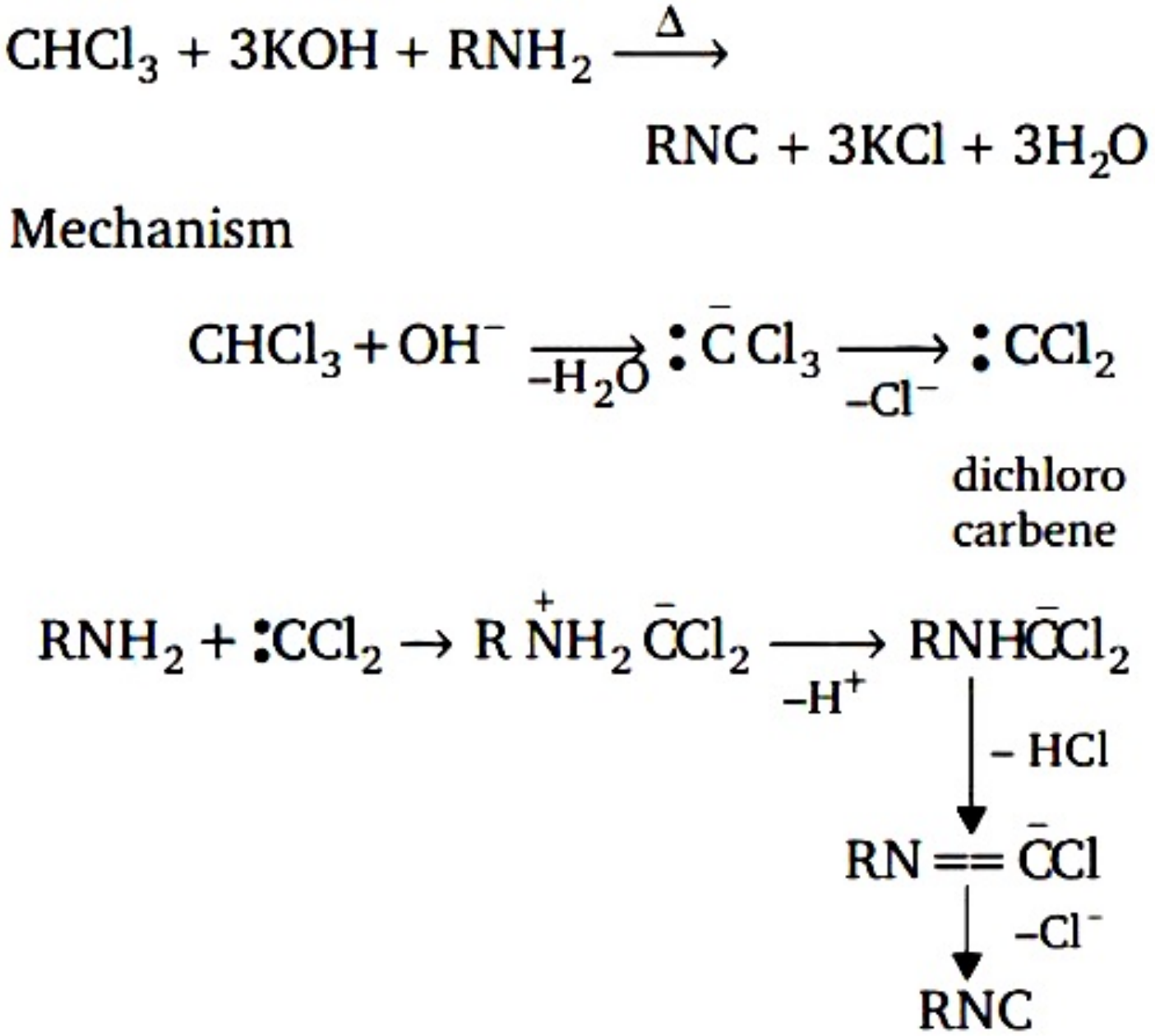
C. RNC, RNH_2, carbene

D. ROH, RNC, nitrene

Answer: A

Solution:

Solution:
In carbylamine reaction, a primary amine (aliphatic or aromatic) is warmed with chloroform and alcoholic KOH. It forms an isocyanide or carbylamine having extremely offensive smell. In this reaction intermediate carbene is formed.



Question 64

Which one of the following molecules has zero dipole moment?

Options:

A. NO_2^+

B. H_2O

C. NH_3

D. CO

Answer: A

Solution:

Solution:
NO_2^+ has linear structure with bond angle 180°. Dipole moment of symmetrical molecule is always zero because all the bond moments are cancelled. Hence, the dipole moment of NO_2^+ is zero. Its structure is as
$:\ddot{O} = N - \ddot{O}:$
Molecule NO_2^+ H_2O NH_3 CO
Dipole moment (D) zero 1.84 1.46 2.1

Question 65

A proton is converted to a neutron by
1. β^- emission
2. β^+ emission
3. Electron capture
Which of the statements given above are correct ?

Options:

A. (1) and (2) only

B. (2) and (3) only

C. (1) and (3) only

D. (1), (2) and (3)

Answer: B

Solution:

Solution:
A proton is converted to a neutron by β^+ emission or electron capture.
$^1_1H \longrightarrow {}^1_0n + {}^0_{+1}e(\beta^+)$
$^1_1H + {}^0_{-1}e \longrightarrow {}^1_0n$

Question 66

The temperature of maximum density of H_2O and D_2O respectively are

Options:

A. $0^\circ C$ and $11.6^\circ C$

B. $4^\circ C$ and $0^\circ C$

C. $4^\circ C$ and $11.6^\circ C$

D. $4^\circ C$ and $12.5^\circ C$

Answer: C

Solution:

Solution:
The temperature of maximum density of H_2O and D_2O respectively are $4^\circ C$ and $11.6^\circ C$.

Question 67

Consider the following statements. The role of catalyst is to
(1) reduce the activation energy
(2) increase the activation energy
(3) increase the rate of attainment of equilibrium
(4) decrease the rate of attainment of equilibrium
Which of the statements given above are correct?

Options:

A. (2) and (4)

B. (1) and (4)

C. (1) and (3)

D. (2) and (3)

Answer: C

Solution:

Solution:
A catalyst decreases the activation energy of the reaction and hence, increases the rate of reaction. It increases the rate of attainment of equilibrium.

Question 68

A silver iodide sol has been prepared by adding slight excess of KI solution to $AgNO_3$ solution having the same concentration as that of KI solution. The silver iodide sol particles are

Options:

A. positively charged

B. negatively charged

C. neutral

D. partially positively and partially negatively charged

Answer: B

Solution:

Solution:
When KI solution and $AgNO_3$ solution of equal concentration are mixed, silver iodide sol is formed.
Since KI is in excess, the sol formed adsorbed I^- ions. Thus, it is negatively charged.
$AgNO_3 + KI \longrightarrow AgI + KNO_3$
$AgI + KI \longrightarrow \underset{\text{negatively charged sol}}{[AgI]x^- : K^+}$

Question 69

P_4O_{10} on reacting with water does not form

Options:

A. tetrametaphosphoric acid

B. phosphorus acid

C. orthophosphoric acid

D. pyrophosphoric acid

Answer: B

Solution:

Solution:
Phosphorus pentoxide reacts with water to form metaphosphoric acid which reacts with excess of water to give pyrophosphoric acid and orthophosphoric acid.

$$\underset{\text{phosphorus pentoxide}}{P_4O_{10}} \xrightarrow{2H_2O} \underset{\text{metaphosphoric acid}}{4HPO_3} \xrightarrow{2H_2O}$$

$$\underset{\text{pyrophosphoric acid}}{2H_4P_2O_7} \xrightarrow{2H_2O} \underset{\text{orthophosphoric acid}}{2H_3PO_4}$$

Question 70

If the molar solubility of $X_3B_3(AlF_6)_2$ at 298K is x, the solubility product K_{sp} is

Options:

A. $18x^3$

B. $27x^4$

C. $27x^8$

D. $2916x^8$

Answer: D

Solution:

Solution:
$X_3B_3(AlF_6)_2 \rightleftharpoons 3X + 3B + 2AlF_6$
$3x \quad 3x \quad 2x$
$K_{sp} = [X]^3[B]^3[AlF_6]^2$
$= (3x)^2(3x)^2(2x)^2$
$= (27x^3)(27x^3)(4x^2) = 2916x^8$

Question 71

In Neptunium series $^{241}_{94}Pu \rightarrow Am \rightarrow Np \rightarrow Pa \rightarrow {}^{238}_{92}U$, the order of radiation is

Options:

A. $\beta, \alpha, \alpha, \beta$

B. $\beta, \beta, \alpha, \alpha$

C. $\alpha, \beta, \alpha, \beta$

D. $\alpha, \alpha, \beta, \beta$

Answer: A

Solution:

Solution:
$^{241}_{94}Pu \xrightarrow{-\beta} {}^{241}_{95}Am \xrightarrow{-\alpha} {}^{237}_{93}Np$
$\xrightarrow{-\alpha} {}^{233}_{91}Pa \xrightarrow{-\beta} {}^{233}_{92}U$
Hence, In Neptunium series the order of radiation is $\beta, \alpha, \alpha, \beta$.

Question 72

Which of the following is not a biopolyme?

Options:

A. Polysaccharide

B. Protein

C. Lipid

D. Nucleic acid

Answer: C

Solution:

Solution:
Polysaccharides are neutral polymeric compounds in which monosaccharide units are joined by glycosidic linkages.
Proteins are long polymers of amino acids linked by peptide bonds.
Nucleic acids are regarded as polynucleotides. These are the polymer of nucleotides.
Lipids are oily, fatty or waxy substances present in living organisms. Chemically, lipids are esters of long chain fatty acids and alcohols.

Question 73

The boiling points of three isomeric pentanes 1,2 and 3 are
(1) 9.5 °C
(2) 28 °C
(3) 36 °C
1,2 and 3 are respectively

Options:

A. n-pentane, iso-pentane, neo-pentane

B. iso-pentane, neo-pentane, n-pentane

C. n-pentane, neo-pentane, iso-pentane

D. neo-pentane, iso-pentane, n-pentane

Answer: D

Solution:

Solution:
In a group of isomeric compounds, the normal compound always has the highest boiling point and generally, the more the branching, lower is the boiling point.

Com-pound :	n-pentane	iso-pentane (2-methyl butane)	neo-pentane (2,2-dimethyl propane)
Boiling point :	36°C	28°C	9.5°C

Question 74

Indicate the pair whose one member has the highest and other has the lowest electronegativity

Options:

A. I and F

B. Fr and Li

C. K and Cs

D. F and Fr

Answer: D

Solution:

Solution:
Electronegativity decreases on moving down the group and increases along a period. Hence, *F* has the highest and Fr has the lowest electronegativity.

Question 75

Atomic numbers of elements X, Y and Z are 50,78 and 60 respectively. These elements are placed in modern long form of Periodic Table respectively in

Options:

A. p-block, d-block and f-block

B. p-block, d-block and s-block

C. s-block, p-block and d-block

D. s-block, d-block and f-block

Answer: A

Solution:

Solution:
The electronic configurations of X, Y and Z are as
$X(50):[Kr]4d^{10},5s^2 5p^2$
$Y(78):[Xe]4f^{14},5d^9,6s^1$
$Z(60):[Xe]4f^4,6s^2$
Hence, these elements are placed in long form of Periodic Table respectively in p-block, d-block and f-block.

Question 76

Consider the following statements.
(1) Fuel cells are voltaic cell which convert electrical energy of fuel into chemical energy.
(2) Fuel cells are galvanic cells which convert chemical energy of fuel into electrical energy.
(3) The efficiency of $H_2 - O_2$ fuel cell is approximately 25%.
(4) Fuel cells do not cause pollution problem.
Among these, the correct statements are

Options:

A. (2) and (3) only

B. (1) and (4) only

C. (2) and (4) only

D. (2), (3) and (4) only

Answer: C

Solution:

Solution:
A fuel cell is a galvanic cell in which one of the reactants is a traditional fuel such as CH_4 or H_2. Fuel cells convert chemical energy of fuel into electrical energy. Fuel cells do not cause pollution problems. These cells converts 74% of chemical energy into electrical energy.

Question 77

Which of the following relationships is correct?

Options:

A. $k = \frac{T\Delta S^\circ - \Delta H^\circ}{RT}$

B. $k = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

C. $\ln k = \frac{T\Delta S^\circ - \Delta H^\circ}{RT}$

D. $\ln k = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

Answer: C

Solution:

Solution:
$\Delta G^\circ = -RT \ln k$
$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$
$-RT \ln k = \Delta H^\circ - T\Delta S^\circ$
$RT \ln k = T\Delta S^\circ - \Delta H^\circ$
$\ln k = \frac{T\Delta S^\circ - \Delta H^\circ}{RT}$

Question 78

The thermal decomposition of a compound is of first order. If a sample of the compound decomposes 50% in $120\,\text{min}$, what time will it take to undergo 90% decomposition?

Options:

A. Nearly $400\,\text{min}$

B. Nearly $45\,\text{min}$

C. Nearly 480 min

D. Nearly 240 min

Answer: A

Solution:

Solution:

$$k = \frac{2.303}{t}\log\frac{[A]_0}{[A]}$$

Let $[A]_0 = 100, [A] = 100 - 50 = 50$

$$k = \frac{2.303}{120}\log\frac{100}{50}$$

$$k = \frac{2.303}{120}\log 2$$

$$= 0.00578\text{min}^{-1}$$

If $[A] = 100 - 90 = 10$ then

$$t = \frac{2.303}{k}\log\frac{[A]_0}{[A]}$$

$$= \frac{2.303}{0.00578}\log\frac{100}{10}$$

$$= 398.44\,\text{min} \approx 400\,\text{min}$$

Question 79

What is the half-life of $\text{Na} - 24$, if $2 \times 10^{-4}\text{g}$ sample of it disintegrates at the rate of 7.0×10^{12} atoms per second?

Options:

A. $4.97 \times 10^6 s$

B. $4.97 \times 10^5 s$

C. $0.497 \times 10^5 s$

D. $4.97 \times 10^4 s$

Answer: B

Solution:

Solution:

$$N = \frac{6.02 \times 10^{23} \times 2 \times 10^{-4}}{24}$$

$$= 5.02 \times 10^{18}\,\text{atoms}$$

$$-\frac{dN}{dt} = kN$$

$$7.0 \times 10^{12} = k \times 5.02 \times 10^{18}$$

$$k = \frac{7.0 \times 10^{12}}{5.02 \times 10^{18}} = 1.4 \times 10^{-6}$$

$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{1.4 \times 10^{-6}} = 4.97 \times 10^{5} s$

Question 80

Energy of activation of an endothermic reaction is

Options:

A. negative

B. positive

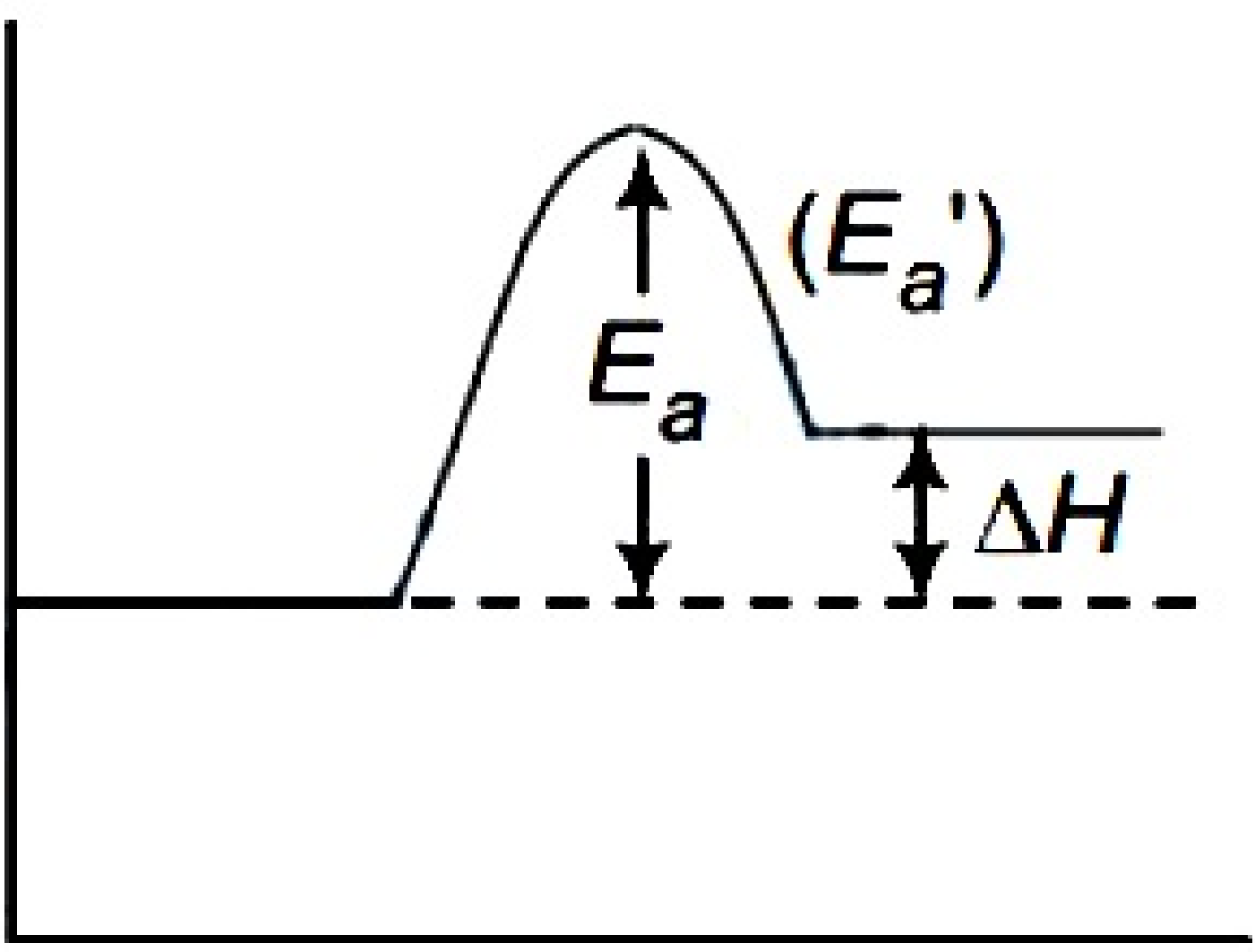
C. zero

D. Cannot be predicted

Answer: B

Solution:

Solution:
For endothermic reaction



$E_a = E_a' + \Delta H$

ie, $E_a > \Delta H$
Hence, energy of activation of an endothermic reaction is positive.

Question 81

Chlorine reacts with carbon disulphide in presence of I_2 catalyst to form

Options:

A. $CHCl_3$

B. CHI_3

C. CCl_4

D. C_2H_5Cl

Answer: C

Solution:

Solution:
Chlorine reacts with carbon disulphide in the presence of I_2 catalyst to form carbon tetrachloride.

$$CS_2 + 3Cl_2 \xrightarrow{I_2} CCl_4 + \underset{\substack{\text{Sulphur}\\\text{monochloride}}}{S_2Cl_2}$$

S_2Cl_2 formed further reacts with CS_2 to form more CCl_4.

Question 82

The sample with largest number of atoms is

Options:

A. $1g\,O_2(g)$

B. $1g\,Ni(s)$

C. $1g\,B(s)$

D. $1g\,N_2(g)$

Answer: C

Solution:

Solution:
No. of atoms in $1g$ of
$O_2(g) = 2 \times \frac{1}{32} \times 6.023 \times 10^{23}$
$= 0.38 \times 10^{23}$
No. of atoms in $1g$ of

$\mathrm{Ni}(s) = \frac{1}{58.2} \times 6.023 \times 10^{23}$
$= 0.10 \times 10^{23}$
No. atoms in 1g of
$\mathrm{B}(s) = \frac{1}{10.8} \times 6.023 \times 10^{23}$
$= 0.56 \times 10^{23}$
No. of atoms in 1g of
$\mathrm{N_2(g)} = 2 \times \frac{1}{28} \times 6.023 \times 10^{23}$
$= 0.43 \times 10^{23}$
Thus, 1g $B(s)$ contains the largest number of atoms.

Question 83

The electronic configuration of P in $\mathrm{H_3PO_4}$

Options:

A. $1s^2 2s^2, 2p^6, 3s^2 3p^6$

B. $1s^2, 2s^2, 2\mathrm{p}^6 3s^2$

C. $1s^2, 2s^2 2p^6$

D. $1s^2, 2s^2, 2p^6, 3s^2 3p^3$

Answer: C

Solution:

Solution:
In $\mathrm{H_3PO_4}$, P is present as P^{5+}
The electric configuration of P atom
$\mathrm{P}(15): 1s^2, 2s^2, 2\mathrm{p}^6, 3s^2 3\mathrm{p}^3$
$\mathrm{P}^{5+} = 1s^2, 2s^2, 2\mathrm{p}^6$
Thus, the electronic configuration of P in $\mathrm{H_3PO_4}$ is $1s^2, 2s^2, 2\mathrm{p}^6$.

Question 84

Pick out the isoelectronic structures from the following

$\mathrm{CH_3^+}$; $\mathrm{H_3O^+}$; $\mathrm{NH_3}$; $\mathrm{CH_3^-}$
I II III IV

Options:

A. I and II

B. I and III

C. I and IV

D. II, III and IV

Answer: D

Solution:

Solution:
Isoelectronic structures have same number of electrons.
Number of electrons in $CH_3^+ = 6 + 3 - 1 = 8$
Number of electrons in $H_3O^+ = 3 + 8 - 1 = 10$
Number of electrons in $NH_3 = 7 + 3 = 10$
Number of electrons in $CH_3^- = 6 + 3 + 1 = 10$
Hence, H_3O^+, NH_3 and CH_3^- are isoelectronic structures.

Question 85

Doping of silicon with boron leads to

Options:

A. n-type semiconductor

B. p-type semiconductor

C. superconductor

D. insulator

Answer: B

Solution:

Solution:
Doping of silicon with group- 13 elements such as B, Al, or Ga gives p-type semiconductors.

Question 86

The flame colours of metal ions are due to

Options:

A. Schottky defect

B. Frenkel defect

C. metal excess defect

D. metal deficiency defect

Answer: C

Solution:

Solution:
The flame colours of metal ions are due to
metal excess defect.
Example
non-stoichiometric sodium chloride is yellow, non-stoichiometric potassium chloride is violet.

Question 87

The percentage composition by weight of an aqueous solution of a solute (molar mass 150) which boils at 373.26K (k_b = 0.52) is

Options:

A. 7

B. 6

C. 9

D. 15

Answer: A

Solution:

Solution:
$\Delta T_b = K_b \times m$
$0.26 = 0.52m$
$m = 0.5$
0.5 mole is present in 1000g of the solvent. or $0.5 \times 150g = 75g$ is present in 1000g of the solvent.
Hence, weight of solution = 1075g
$\%$ by weight $= \frac{75}{1075} \times 100 \approx 7\%$

Question 88

Solutions P, Q, R and S are respectively 0.1M glucose, 0.05M NaCl, 0.05M $BaCl_2$ and 0.1M $AlCl_3$. Which one of the following pairs is isotonic?

Options:

A. P and Q

B. Q and R

C. P and S

D. P and R

Answer: A

Solution:

Solution:
Isotonic solutions have equal concentrations of particles. For isotonic solution
$\pi = iC$
For glucose, $\pi = 1 \times 0.1 = 0.1$
For NaCl, $\pi = 2 \times 0.05 = 0.1$
Hence, $0.1M$ glucose and $0.05M$ NaCl solutions are isotonic.

Question 89

In which one of the following reactions, nitrogen is not reduced?

Options:

A. $NO_2 \rightarrow NO_2^-$

B. $NO_3^- \rightarrow NO$

C. $NO_3^- \rightarrow NH_4^+$

D. $NH_4^+ \rightarrow N_2$

Answer: D

Solution:

Solution:
Increase in oxidation number is defined as oxidation while decrease in oxidation number is defined as reduction.
$\overset{+4}{N}O_2 \longrightarrow \overset{+3}{N}O_2^-$ (Reduction)
$\overset{+5}{N}O_3^- \longrightarrow \overset{+2}{N}O$ (Reduction)
$\overset{+5}{N}O_3^- \longrightarrow \overset{-3}{N}H_4^+$ (Reduction)
$\overset{-3}{N}H_4^+ \longrightarrow \overset{0}{N}_2$ (Oxidation)

Question 90

Which one of the following is the strongest base?

Options:

A. 2, 4, 6-trinitroaniline

B. 2, 4, 6-trinitro-N, N-dimethyl aniline

C. N, N-dimethyl aniline

D. Aniline

Answer: B

Solution:

Solution:
In 2, 4, 6-trinitro-N, N-dimethylaniline, the bulky nitro substituents at o-position throw the p-orbital of N-atom containing the lone pair of electrons out of the plane of the p-orbitals of the benzene ring. As a result delocalisation of nitrogen electrons on the ring cannot occur. In other words, these electrons are readily available for protonation and hence, it is much stronger base than 2,4 , 6-trinitroaniline, aniline and N, N-dimethylaniline where delocalisation is not inhibited by resonance.

Question 91

Which one of the following is the best reagent to accomplish the following conversion ?

$$CH_3CH_2Br \xrightarrow{?} CH_3 - CH_3$$

Options:

A. $NaBH_4$

B. $Na \mid$ ether

C. $Zn \mid C_2H_5OH$

D. Mg followed by H_3O^+

Answer: C

Solution:

Solution:
Reduction of alkyl halides by dissolving metals e.g., zinc and acetic acid or dil. HCl, zinc and NaOH, zinc-copper couple and ethanol gives the alkane.

$$CH_3CH_2Br \xrightarrow{Zn - Cu / C_2H_5OH} CH_3 - CH_3$$

Mechanism

$$Zn \longrightarrow Zn^{2+} + 2e^-$$
$$CH_3CH_2 - Br + e^- \longrightarrow CH_3CH_2^{\bullet} + Br^-$$
$$CH_3CH_2^{\bullet} + e^- \longrightarrow CH_2CH_2 : -$$
$$CH_3CH_2 : - + C_2H_5OH \longrightarrow$$
$$CH_3 - CH_3 + C_2H_5O^-$$

Question 92

Which of the following statements is true?

Options:

A. All proteins act as biocatalyst

B. Denaturation of protein changes the primary structure of protein

C. C-terminal amino acid in proteins is determined by Edman degradation

D. The pleated sheet structure of proteins was determined by Pauling

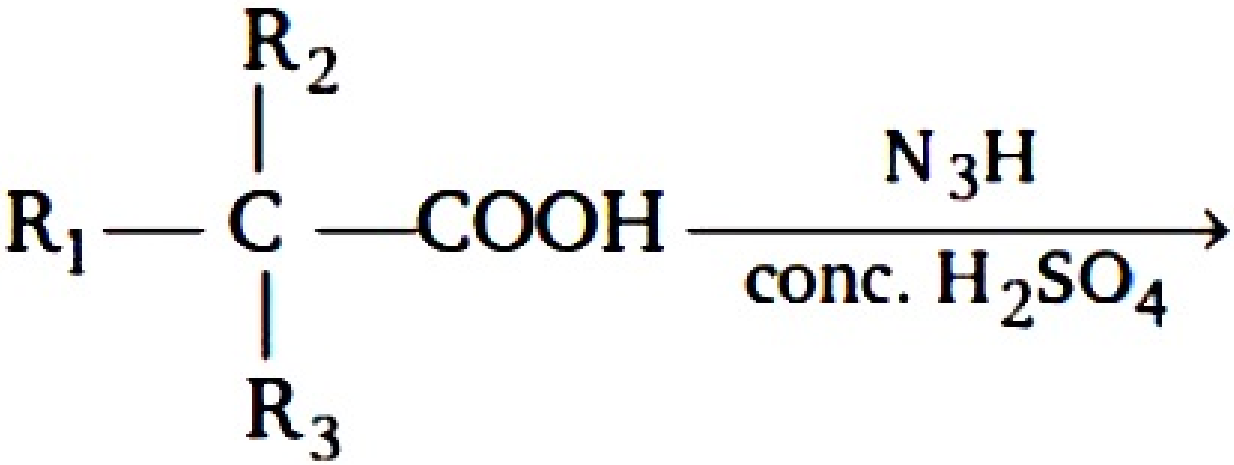
Answer: D

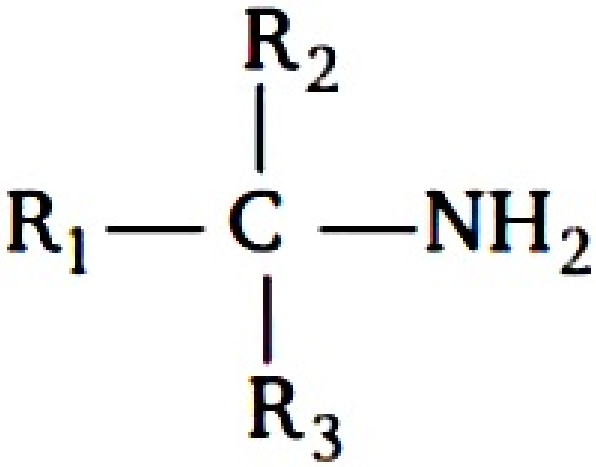
Solution:

Solution:
All proteins are not biocatalysts. Denaturation of proteins affects only secondary and fertiary structures. The most commonly used method for determining the C-terminal amino acid residue in a protein is hydrazinolysis. In this method, protein is treated with anhydrous hydrazine at 373K when all amino acid residues except C-terminal one is converted into amino acid hydrazides. The mixture of products is subjected to chromatography. On elution the strongly basic hydrazides are retained, but the free amino acid is eluted. By identifying the free amino acid the C-terminal amino acid residue of a protein can be determined.
The first β-pleated sheet structure was proposed by ω. Astburry while a refined version was proposed by Linus pauling and Robert corey.

Question 93

The given reaction is called as





Options:

A. Schmidt rearrangement

B. Curtius rearrangement

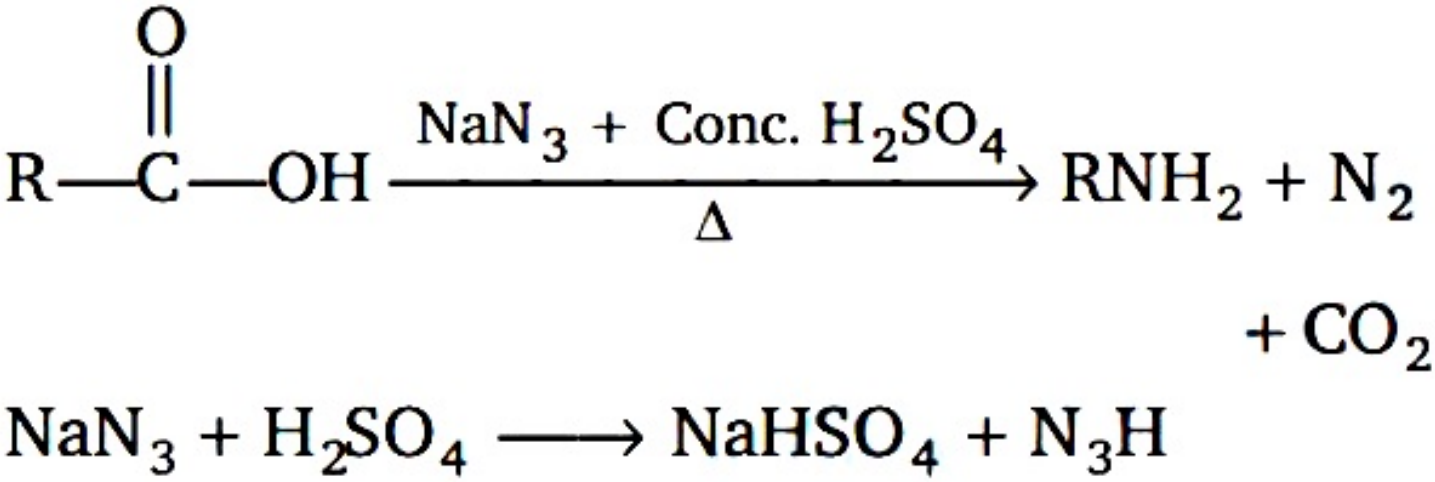
C. Hofmann rearrangement

D. Lossen rearrangement

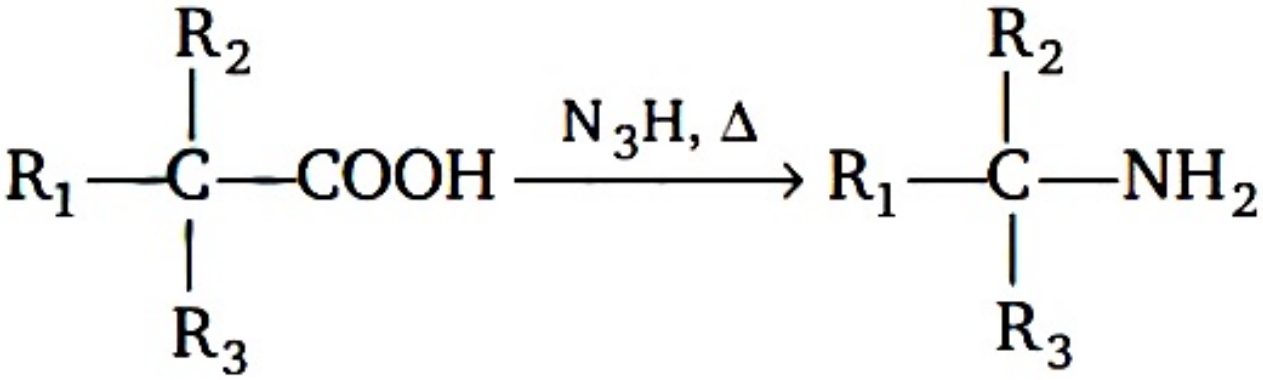
Answer: A

Solution:

Solution:
Schmidt reaction converts RCOOH into RNH_2



$$NaN_3 + H_2SO_4 \longrightarrow NaHSO_4 + N_3H$$



Question 94

Which of the following contains thymine?

Options:

A. m-RNA

B. r-RNA

C. t-RNA

D. None of the above

Answer: D

Solution:

Solution:
RNA contains cytosine and uracil as pyrimidine bases and guanine and adenine as purine bases. Thymine is not present

in RNA.

Question 95

The functional groups present in aspirin are

Options:

A. $OH, NHCOCH_3$

B. $OC_2H_5, COOH$

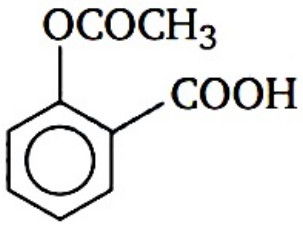
C. $COOH, OCOCH_3$

D. $OH, OCOCH_3$

Answer: C

Solution:

Solution:
Acetyl salicylic acid or 2-acetoxy benzoic acid is known as aspirin. It contains - $COOH$ and $-OCOCH_3$ group. Its structure is as



Question 96

The coordination number and oxidation state of Cr in $K_3[Cr(C_2O_4)_3]$ are respectively

Options:

A. 6 and +3

B. 3 and 0

C. 4 and +2

D. 3 and +3

Answer: A

Solution:

Solution:
Coordination number is the number of ligands in the coordination sphere of the complex compound. Oxalate ion $(C_2O_4^{2-})$ is a bidentate ligand, hence coordination number of Cr in $K_3[Cr(C_2O_4)_3]$ is 6 .
Let the oxidation number of Cr is x .
$3 + x + 3(-2) = 0$
$3 + x - 6 = 0$
$x - 3 = 0$
$x = 3$

Question 97

The equivalent weight of $MnSO_4$ is equal to its molecular weight when it is converted to

Options:

A. Mn_2O_3

B. MnO_2

C. MnO_4^-

D. MnO_4^{2-}

Answer: A

Solution:

Solution:
Oxidation number of Mn in $MnSO_4$ is $+2$.
Oxidation number of Mn in Mn_2O_3 is $+3$.
Change in oxidation number $= 3 - 2 = 1$
Equivalent weight

$$= \frac{\text{molecular weight}}{\text{total change in oxidation number of Mn}}$$

$$\text{Equivalent weight} = \frac{\text{molecular weight}}{1}$$

$$\text{Equivalent weight} = \text{molecular weight}$$

Question 98

The mineral from which potassium permanganate is manufactured

Options:

A. manganite, $Mn_2O_3 \cdot H_2O$

B. pyrolusite, MnO_2

C. Both (a) and (b)

D. None of the above

Answer: B

Solution:

Solution:
Potassium permanganate is prepared from mineral pyrolusite (MnO_2).

$$2MnO_2 + 4KOH + O_2 \xrightarrow{\text{Heat}} \underset{\substack{\text{potassium manganate} \\ \text{(green)}}}{2K_2MnO_4} + 2H_2O$$

$$2K_2MnO_4 + Cl_2 \longrightarrow 2KCl + \underset{\substack{\text{potassium} \\ \text{permanganate} \\ \text{(purple)}}}{2KMnO_4}$$

Question 99

Which one of the following hydrocarbons has octane number 100 ?

Options:

A. 2, 2, 3-trimethylpentane

B. 2, 3, 3-trimethylpentane

C. 2, 2, 4trimethylpentane

D. 2, 3, 4-trimethylpentane

Answer: C

Solution:

Solution:
The octane number of iso-octane (2, 2, 4-trimethyl pentane) is 100 . The quality of a petrol sample can be expressed in terms of octane number.

Question 100

Which one of the following gases has both oxidising as well as reducing property?

Options:

A. CO

B. SO_2

C. H_2S

D. PH_3

Answer: B

Solution:

Solution:
Sulphur dioxide (SO_2) gas has both oxidising as well as reducing property. It reduces acidified $Cr_2O_7^{2-}$ into green Cr^{3+}.
$K_2Cr_2O_7 + 3SO_2 + H_2SO_4 \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O$
It oxidises H_2S to S.
$2H_2S + SO_2 \longrightarrow 2H_2O + S$

Question 101

Evaluate

$$\sum_{k=1}^{6}(sin\ \frac{2k\pi}{7} - i\cos\frac{2k\pi}{7})$$

Options:

A. 2i

B. $-$i

C. i

D. $-$2i

Answer: C

Solution:

Solution:

$$\sum_{k=1}^{6}(sin\ \frac{2k\pi}{7} - i\cos\frac{2k\pi}{7})$$

$$= -i\left[\sum_{k=1}^{6}(\cos\frac{2k\pi}{7} + isin\ \frac{2k\pi}{7})\right]$$

$= -i[(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7})$

$+(\cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7})$
$+ \ldots + (\cos\frac{12\pi}{7} + i\sin\frac{12\pi}{7})]$
$= -i[\cos(\frac{2\pi}{7} + \frac{4\pi}{7} + \frac{6\pi}{7} + \frac{8\pi}{7} + \frac{10\pi}{7} + \frac{12\pi}{7}) .$
$+ i\sin(\frac{2\pi}{7} + \frac{4\pi}{7} + \frac{6\pi}{7} + \frac{8\pi}{7} + \frac{10\pi}{7} + \frac{12\pi}{7})]$
$= -i[\cos\frac{42\pi}{7} + i\sin\frac{42\pi}{7})$
$= -i[\cos 6\pi + i\sin 6\pi]$
$= -i[\cos 6\pi] = -i[-1] = i$

Question 102

If a, b, c are in HP, then the value of $\frac{b+a}{b-a} + \frac{b+c}{b-c}$ is

Options:

A. 0

B. 1

C. 2

D. 3

Answer: C

Solution:

Solution:
Since a, b, c are in HP.
$\therefore \quad b = \frac{2ac}{a+c}$
Now, $\frac{b+a}{b-a} + \frac{b+c}{b-c}$
$= \frac{\frac{2ac}{a+c} + a}{\frac{2ac}{a+c} - a} + \frac{\frac{2ac}{a+c} + c}{\frac{2ac}{a+c} - c}$
$= \frac{3ac + a^2}{ac - a^2} + \frac{3ac + c^2}{ac - c^2}$
$= \frac{3c + a}{c - a} + \frac{3a + c}{a - c}$
$= \frac{3c + a - 3a - c}{c - a} = \frac{2(c-a)}{c-a} = 2$

Question 103

If $x^2 - 4x + \log_{1/2} a = 0$ does not have two distinct real roots, then maximum value of a is

Options:

A. $-\frac{1}{4}$

B. $\frac{1}{16}$

C. $\frac{1}{4}$

D. None of these

Answer: B

Solution:

Solution:
Since, given equation does not have two distinct real roots. Therefore,
$D < 0$
$16 - 4\log_{1/2} a < 0$
$\Rightarrow 4\log_{1/2} a > 16$
$\Rightarrow \log_{1/2} a > 4$
$\Rightarrow a < (\frac{1}{2})^4$
$\Rightarrow a < \frac{1}{16}$

Question 104

A polygon has 44 diagonals. Find the number of sides.

Options:

A. 8

B. 10

C. 11

D. 13

Answer: C

Solution:

Solution:
Number of diagonals in a polygon of n sides is
$\frac{n(n-3)}{2}$
$\therefore \frac{n(n-3)}{2} = 44$
$\Rightarrow n(\frac{n}{2} - 3) = 88 = 11 \times 8$
$\therefore n = 11$

Question 105

The coefficient of x^4 in the expansion of $\log(1 + 3x + 2x^2)$ is

Options:

A. $\frac{16}{3}$

B. $-\frac{16}{3}$

C. $\frac{17}{4}$

D. $-\frac{17}{4}$

Answer: D

Solution:

Solution:

$\log(1+3x+2x^2)$

$= (3x+2x^2) - \frac{(3x+2x^2)^2}{2} + \frac{(3x+2x^2)^3}{3}$

$- \frac{(3x+2x^2)^4}{4} + \ldots$

In this expansion, the coefficient of x^4

$= -\frac{1}{2}(4) + \frac{1}{3}(54) - \frac{1}{4}(81)$

$= -2 + 18 - \frac{81}{4}$

$= 16 - \frac{81}{4}$

$= -\frac{17}{4}$

Question 106

If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then the value of $|A^T A^{-1}|$ is

Options:

A. $\cos 4x$

B. $sec^2 x$

C. $-\cos 4x$

D. 1

Answer: B

Solution:

Solution:

We have,

$$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$$

$$\text{Now, } A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$$

$$\text{and } A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{sec^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

Now, $|A^T A^{-1}|$

$$= \left| \frac{1}{sec^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \right|$$

$$= \left| \frac{1}{sec^2 x}\begin{bmatrix} 1-\tan^2 x & -\tan x - \tan x \\ \tan x + \tan x & 1-\tan^2 x \end{bmatrix} \right|$$

$$= \frac{1}{sec^2 x}\begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{bmatrix}$$

$$= \frac{(1-\tan^2 x)^2 + 4\tan^2 x}{sec^2 x}$$

$$= \frac{(1+\tan^2 x)^2}{sec^2 x}$$

$$= sec^2 x$$

Question 107

In a $\triangle ABC$, if $\cot A \cot B \cot C > 0$, then the triangle is

Options:

A. acute angled

B. right angled

C. obtuse angled

D. does not exist

Answer: A

Solution:

Solution:
Since, $\cot A \cot B \cot C > 0$
$\therefore \cot A, \cot B$ and $\cot C$ are positive.
$\Rightarrow$ Triangle is acute angled.

Question 108

If $\left(\frac{1+sin\,\theta-\cos\theta}{1+sin\,\theta+\cos\theta}\right)^2 = \lambda\left(\frac{1-\cos\theta}{1+\cos\theta}\right)$, then λ equals

Options:

A. -1

B. 1

C. 2

D. -2

Answer: B

Solution:

Solution:

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\lambda\left(\frac{1-\cos\theta}{1+\cos\theta}\right)$$

$$\Rightarrow\lambda=\frac{[\,(1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta)(1+\cos\theta)\,]}{[(1+\sin^2\theta+\cos^2\theta+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta)(1-\cos\theta)]}$$

$$\Rightarrow\lambda=\frac{\begin{bmatrix}2(1+\sin\theta-\cos\theta-\sin\theta\cos\theta)\\(1+\cos\theta)\end{bmatrix}}{[2(1+\sin\theta+\cos\theta+\sin\theta\cos\theta);(1-\cos\theta)]}$$

$$\Rightarrow\lambda=\frac{\begin{bmatrix}1+\sin\theta-\cos\theta-\sin\theta\cos\theta+\cos\theta\\+\sin\theta\cos\theta-\cos^2\theta-\sin\theta\cos^2\theta\end{bmatrix}}{[1+\sin\theta+\cos\theta+\sin\theta\cos\theta-\cos\theta;-\sin\theta\cos\theta-\cos^2\theta-\sin\theta\cos^2\theta]}$$

$$\Rightarrow\lambda=\frac{1+\sin\theta-\cos^2\theta-\sin\theta\cos^2\theta}{1+\sin\theta-\cos^2\theta-\sin\theta\cos^2\theta}$$

$$\Rightarrow\lambda=1$$

Question 109

If the sides of a $\triangle ABC$ are in AP and a is the smallest side, then $\cos A$ equals

Options:

A. $\frac{3c-4b}{2c}$

B. $\frac{3c-4b}{2b}$

C. $\frac{4c-3b}{2c}$

D. $\frac{4c-3b}{2b}$

Answer: C

Solution:

Solution:

Given, a, b, c are in AP where a is the smallest side.

$\therefore\ 2b = a + c$

$\Rightarrow\ a = 2b - c$

Now, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$$= \frac{b^2 + c^2 - (2b - c)^2}{2bc}$$
$$= \frac{b^2 + c^2 - 4b^2 - c^2 + 4bc}{2bc}$$
$$= \frac{-3b^2 + 4bc}{2bc}$$
$$\Rightarrow \cos A = \frac{4c - 3b}{2c}$$

Question 110

The number of common tangents that can be drawn to the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$ is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Solution:
Given circles are
$x^2 + y^2 - 4x - 6y - 3 = 0$ . . . (i)
and $x^2 + y^2 + 2x + 2y + 1 = 0$ . . . (ii)
For circle (i), $g_1 = -2, f_1 = -3, c_1 = -3$
$\therefore$ Centre $C_1(2, 3)$ and $r_1 = \sqrt{4 + 9 + 3} = 4$
and for circle (ii), $g_2 = 1, f_2 = 1, c_2 = 1$
$\therefore$ Centre $C_2(-1, -1)$ and $r_2 = \sqrt{1 + 1 - 1} = 1$
Now, $C_1C_2 = \sqrt{(2 + 1)^2 + (3 + 1)^2} = \sqrt{9 + 16} = 5$
and $r_1 + r_2 = 4 + 1 = 5$
So, $C_1C_2 = r_1 + r_2$
Thus, both the circles touch each other externally.
Hence, number of common tangents = 3

Question 111

If two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

Options:

A. $2 < r < 8$

B. $r < 2$

C. $r = 2$

D. $r > 2$

Answer: A

Solution:

Solution:
Given circles are
$(x-1)^2 + (y-3)^2 = r^2$
and $x^2 + y^2 - 8x + 2y + 8 = 0$
For circle (i),
$C_1 = (1,3), r_1 = r$
For circle (ii)
$C_2 = (4,-1), r_2 = \sqrt{16+1-8} = 3$
Since, both the circles intersect in two distinct points, therefore
$r_1 - r_2 < C_1C_2 < r_1 + r_2$
$\Rightarrow r - 3 < \sqrt{9+16} < r + 3$
$\Rightarrow r - 3 < 5 < r + 3$
$\Rightarrow r < 8$ and $2 < r$
$\Rightarrow 2 < r < 8$

Question 112

The equation of the tangent at the vertex of the parabola $x^2 + 4x + 2y = 0$, is

Options:

A. $x = -2$

B. $x = 2$

C. $y = -2$

D. $y = 2$

Answer: D

Solution:

Solution:
Given, equation of parabola
$x^2 + 4x + 2y = 0$
$\Rightarrow x^2 + 4x + 4 = -2y + 4$
$\Rightarrow (x+2)^2 = -2(y-2)$
$\Rightarrow X^2 = -2Y$
where $X = x + 2$ and $Y = y - 2$
Vertex of this parabola
$X = 0, Y = 0$
$\Rightarrow x + 2 = 0, y - 2 = 0$
$\Rightarrow (-2, 2)$
Now, equation of tangent at the vertex $(-2, 2)$ of the given parabola is
$x(-2) + 2(x-2) + y + 2 = 0$
$\Rightarrow -2x + 2x - 4 + y + 2 = 0$
$\Rightarrow y = 2$

Question 113

The distance between the directrices of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is

Options:

A. $\frac{9}{\sqrt{5}}$

B. $\frac{18}{\sqrt{5}}$

C. $\frac{24}{\sqrt{5}}$

D. None of these

Answer: B

Solution:

Solution:
We have, equation of ellipse
$\frac{x^2}{4} + \frac{y^2}{9} = 1$
Eccentricity of the ellipse
$e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$
Now, distance between the directries
$= \frac{2b}{e} = \frac{2 \cdot 3}{\sqrt{5}/3} = \frac{18}{\sqrt{5}}$

Question 114

The ratio in which yz-plane divides the line joining $(2, 4, 5)$ and $(3, 5, 7)$ is

Options:

A. $-2:3$

B. $2:3$

C. $3:2$

D. $-3:2$

Answer: A

Solution:

Solution:
Let the required ratio be $m_1 : m_2$.
$\therefore \quad \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = 0$

$\Rightarrow \left(\because x \text{-coordinate on yz-plane is zero} \right)$
$\Rightarrow m_1 \times 3 + m_2 \times 2 = 0$
$\Rightarrow 3m_1 = -2m_2$
$\frac{m_1}{m_2} = \frac{-2}{3}$

Question 115

If a line makes angles α, β, γ with x-axis, y-axis and z-axis respectively, then $sin^2\alpha + sin^2\beta + sin^2\gamma$ equals

Options:

A. 1

B. 2

C. 3

D. -1

Answer: B

Solution:

Solution:
Since, the line makes angles α, β, γ with x, y and z-axis respectively, therefore
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow 1 - sin^2\alpha + 1 - sin^2\beta + 1 - sin^2\gamma = 1$
$\Rightarrow sin^2\alpha + sin^2\beta + sin^2\gamma = 2$

Question 116

The length of the perpendicular from the point $(1, 2, 3)$ on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

Options:

A. 3 units

B. 4 units

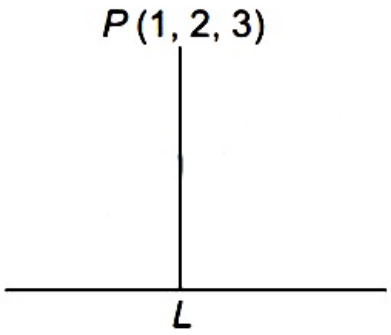
C. 5 units

D. 7 units

Answer: D

Solution:

Solution:
Let L be the foot of perpendicular drawn from the point $P(1,2,3)$ to the given line. The coordinate of a general point on $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=\lambda$ are given by $(3\lambda+6, 2\lambda+7, -2\lambda+7)$
Let this point be L.



Now, direction ratios of PL are
$3\lambda+6-1, 2\lambda+7-2, -2\lambda+7-3$
i.e., $3\lambda+5, 2\lambda+5, -2\lambda+4$
and direction cosines of given line are $3, 2, -2$.
$\because$ PL is perpendicular to the given line.
$\therefore$
$3(3\lambda+5)+2(2\lambda+5)+(-2)(-2\lambda+4)=0$
$\Rightarrow \lambda=-1$
$\therefore$ L$(3\times-1+6, 2\times-1+7, -2\times-1+7)$
= L$(3,5,9)$
$\therefore$ PL $=\sqrt{(3-1)^2+(5-2)^2+(9-3)^2}$
= 7 units

Question 117

The equation of the plane passing through the intersection of the planes $2x-3y+z-4=0$ and $x-y+z+1=0$ and perpendicular to the plane $x+2y-3z+6=0$ is

Options:

A. $x-5y+3z-23=0$

B. $x-5y-3z-23=0$

C. $x+5y-3z+23=0$

D. $x-5y+3z+23=0$

Answer: B

Solution:

Solution:
The equation of a plane passing through the intersection of the planes $2x-3y+z-4=0$ and $x-y+z+1=0$ is
$(2x-3y+z-4)+\lambda(x-y+z+1)=0$
$\Rightarrow(2+\lambda)x+(-3-\lambda)y+(1+\lambda)z\backslash+\lambda-4=0$ . . . (i)
But plane (i) is perpendicular to the plane
$x+2y-3z+6=0$
$\therefore(2+\lambda)1+(-3-\lambda)2+(1+\lambda)(-3)=0$
$\Rightarrow 2+\lambda-6-2\lambda-3-3\lambda=0$
$\Rightarrow -4\lambda-7=0$
$\Rightarrow \lambda=\frac{-7}{4}$
Putting this value of λ in Eq. (i), we get the required equation of plane
$(2-\frac{7}{4})x+(-3+\frac{7}{4})y+(1-\frac{7}{4})z$
$+(-4-\frac{7}{4})=0$

$\Rightarrow x - 5y - 3z - 23 = 0$

Question 118

The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{y+3}{3} = \frac{z+5}{2}$ is

Options:

A. $\cos^{-1}(\frac{-3}{\sqrt{182}})$

B. $\cos^{-1}(\frac{5}{\sqrt{182}})$

C. $\cos^{-1}(\frac{3}{\sqrt{182}})$

D. $\cos^{-1}(\frac{-5}{\sqrt{182}})$

Answer: A

Solution:

Solution:
Given lines are
$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ . . . (i)
and $\frac{x-1}{1} = \frac{y+3}{3} = \frac{z+5}{2}$ . . . (ii)
Here, $a_1 = 3, b_1 = -2, c_1 = 0$
If θ be the angle between both the lines, then
$a_2 = 1, b_2 = 3, c_2 = 2$
$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$
$\Rightarrow \theta = \cos^{-1}(\frac{(3)(1) + (-2)(3) + (0)(2)}{\sqrt{9+4+0}\sqrt{1+9+4}})$
$\Rightarrow \theta = \cos^{-1}(\frac{-3}{\sqrt{13}\sqrt{14}})$
$\Rightarrow \theta = \cos^{-1}(\frac{-3}{\sqrt{182}})$

Question 119

The scalar $A \cdot \{(B + C) \times (A + B + C)\}$ equals

Options:

A. $[ABC][BCA]$

B. $[ABC]$

C. 0

D. None of these

Answer: C

Solution:

Solution:
$A \cdot \{(B+C) \times (A+B+C)\}$
$= A \cdot \{(B+C) \times A + (B+C) \times (B+C)\}$
$= A \cdot \{(B+C) \times A + 0\}$
$= A \cdot \{(B+C) \times A\} = [A(B+C)A]$
$= 0$

Question 120

The points with position vectors $60i + 3j, 40i - 8j$ and ai $-52j$ are collinear, if

Options:

A. a $= -40$

B. a $= 40$

C. a $= -20$

D. a $= 20$

Answer: A

Solution:

Solution:
Given that the points with position vectors $60i + 3j, 40i - 8j$ and $i - 52j$ are collinear, then there exist three scalars $1, x$ and y such that
$1 + x + y = 0$ . . . (i)
and $(60i + 3j) \cdot 1 + (40i - 8j) \cdot x$
$+ (ai - 52j) \cdot y = 0$
$\Rightarrow \quad (60 + 40x + ay)\, i + (3 - 8x - 52y)\, j = 0$
$= 0i + 0j$

Comparing both sides, we get
$60 + 40x + ay = 0$ . . . (ii)
and $3 - 8x - 52y = 0$ . . . (iii)
Solving Eqs. (i) and (iii), we get
$x = \frac{-5}{4}, y = \frac{1}{4}$
Then, from Eq. (ii),
$60 + 40(\frac{-5}{4}) + a(\frac{1}{4}) = 0$
$\Rightarrow \quad \frac{a}{4} = -10 \Rightarrow a = -40$

Question 121

$\lim_{x \to 5} \frac{x-5}{|x-5|}$ **equals**

Options:

A. 2

B. 0

C. -2

D. None of these

Answer: D

Solution:

Solution:

$\lim_{x \to 5} \frac{x-5}{|x-5|}$

Taking LHL, we get

$\lim_{x \to 5^-} \frac{x-5}{|x-5|}$

$= \lim_{h \to 0^-} \frac{5-h-5}{|5-h-5|}$

$= \lim_{h \to 0^-} \frac{-h}{|-h|}$

$= \lim_{h \to 0^-} \frac{-h}{h} = -1$

and RHL, $\lim_{x \to 5^+} \frac{x-5}{|x-5|}$

$= \lim_{h \to 0^+} \frac{5+h-5}{|5+h-5|}$

$= \lim_{h \to 0^+} \frac{h}{|h|}$

$= \lim_{h \to 0^+} \frac{h}{h} = 1$

Since, LHL $\neq$ RHL

$\therefore$ Limit does not exist.

Question 122

$\lim_{x \to \frac{\pi}{3}} \frac{sin\,(\frac{\pi}{3}-x)}{2\cos x - 1}$ **is equal to**

Options:

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{-1}{\sqrt{3}}$

D. $\frac{2}{\sqrt{3}}$

Answer: B

Solution:

Solution:

$$\lim_{x \to \frac{\pi}{3}} \frac{sin\,(\frac{\pi}{3} - x)}{2\cos x - 1}$$
$$= \lim_{x \to \frac{\pi}{3}} \frac{\cos(\frac{\pi}{3} - x)(-1)}{-2sin\,x}$$
$$= \lim_{x \to \frac{\pi}{3}} \frac{\cos(\frac{\pi}{3} - x)}{2sin\,x}$$
$$= \frac{\cos(\frac{\pi}{3} - \frac{\pi}{3})}{2sin\,\frac{\pi}{3}}$$
$$= \frac{1}{2 \cdot \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Question 123

$sin^{-1}(\frac{1+x^2}{2x})$ is

Options:

A. continuous but not differentiable at x = 1

B. differentiable at x = 1

C. neither continuous nor differentiable atx = 1

D. continuous everywhere

Answer: A

Solution:

Solution:

Given function $sin^{-1}(\frac{1+x^2}{2x})$

At $x = 1$, LHL

$$\lim_{h \to 0^-} sin^{-1}\left[\frac{1+(1-h)^2}{2(1-h)}\right]$$
$$= sin^{-1}(\frac{1+1}{2}) = \frac{\pi}{2}$$

and RHL

$$\lim_{h \to 0^+} sin^{-1}\left[\frac{1+(1+h)^2}{2(1+h)}\right]$$
$$= sin^{-1}(\frac{2}{2}) = \frac{\pi}{2}$$

and $f(1) = sin^{-1}(\frac{2}{2}) = \frac{\pi}{2}$

Thus, given function is continuous at x = 1.

Now, $\frac{d}{dx}\{sin^{-1}(\frac{1+x^2}{2x})\}$

$= \frac{1}{\sqrt{1-(\frac{1+x^2}{2x})^2}} \cdot \left[\frac{2x(2x)-(1+x^2)\cdot 2}{(2x)^2}\right]$

$= \frac{2x}{\sqrt{4x^2-(1+x^4+2x^2)}} \cdot \frac{2x^2-1}{(2x)^2}$

$= \frac{2x^2-1}{\sqrt{-(1+x^4-2x^2)\cdot(2x)}}$

$= \frac{2x^2-1}{2x\sqrt{-(1-x^2)^2}}$

which does not exist at x = 1.
Hence, given function is not differentiable at $x = 1$.

Question 124

If $\sqrt{x^2+y^2} = a\tan^{-1}(\frac{y}{x})$, $a > 0$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is

Options:

A. 0

B. $\frac{2}{a}e^{-\frac{\pi}{2}}$

C. $-\frac{2}{a}e^{-\frac{\pi}{2}}$

D. None of these

Answer: D

Solution:

Solution:

$\sqrt{x^2+y^2} = a\tan^{-1}(\frac{y}{x})$ . . . (i)

Putting $x = 0$, we get

$y = a\tan^{-1}(\infty)$

$y = \frac{a\pi}{2}$

Now, differentiating Eq. (i) w.r.t. x, we get

$\frac{(2x+2y\frac{dy}{dx}\cdot}{2\sqrt{x^2+y^2}} = a \cdot \frac{1}{1+\frac{y^2}{x^2}} \cdot \left\{\frac{x\frac{dy}{dx}-y\cdot 1}{x^2}\right\}$

$\Rightarrow \frac{x+y\frac{dy}{dx}}{\sqrt{x^2+y^2}} = \frac{ax^2}{x^2+y^2}\left\{\frac{x\frac{dy}{dx}-y}{x^2}\right\}$

$\Rightarrow \sqrt{x^2+y^2}(x+y\frac{dy}{dx}) = a(x\frac{dy}{dx}-y) \ldots (ii)$

At $x = 0, y = \frac{a\pi}{2}$,

$\frac{a\pi}{2}[\frac{a\pi}{2}\cdot\frac{dy}{dx}] = a[-\frac{a\pi}{2}]$

$\frac{\pi}{2}\cdot\frac{dy}{dx} = -1$

$\frac{dy}{dx} = -\frac{2}{\pi}$

Again, differentiating Eq. (ii), w.r.t. x, we get

$$\frac{(2x+2y\frac{dy}{dx})}{2\sqrt{x^2+y^2}}\cdot(x+y\frac{dy}{dx})$$
$$+\sqrt{x^2+y^2}\left[1+y\frac{d^2y}{dx^2}+(\frac{dy}{dx})^2\right]$$
$$=a\left[x\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}\right]$$
$$\Rightarrow \frac{(x+y\frac{dy}{dx})^2}{\sqrt{x^2+y^2}}+\sqrt{x^2+y^2}$$
$$\left[1+y\frac{d^2y}{dx^2}+(\frac{dy}{dx})^2\right]=ax\frac{d^2y}{d^2}$$
Putting $x=0, y=\frac{a\pi}{2}, \frac{dy}{dx}=-\frac{2}{\pi}$
$$\frac{a\pi}{2}(-\frac{2}{\pi})^2+\frac{a\pi}{2}\left[1+\frac{a\pi}{2}\left(\frac{d^2y}{dx^2}\right)+(-\frac{2}{\pi})^2\right]=0$$
$$\Rightarrow \frac{2a}{\pi}+\frac{a\pi}{2}+\frac{a^2\pi^2}{4}\cdot\frac{d^2y}{dx^2}+\frac{4}{\pi^2}\cdot\frac{a\pi}{2}=0$$
$$\Rightarrow \frac{4a}{\pi}+\frac{a\pi}{2}+\frac{a^2\pi^2}{4}\cdot\frac{d^2y}{dx^2}=0$$
$$\Rightarrow \frac{4}{\pi}+\frac{\pi}{2}+\frac{a\pi^2}{4}\cdot\frac{d^2y}{dx^2}=0$$
$$\Rightarrow \frac{d^2y}{dx\,x^2}=\frac{-\frac{4}{\pi}-\frac{\pi}{2}}{\frac{a\pi^2}{4}}=\frac{-2(8+\pi^2)}{a\pi^3}$$

Question 125

Find C of Lagrange's mean value theorem for the function $f(x)=3x^2+5x+7$ in the interval $[1,3]$.

Options:

A. $\frac{7}{3}$

B. 2

C. $\frac{3}{2}$

D. $\frac{4}{3}$

Answer: B

Solution:

Solution:
Given, $f(x)=3x^2+5x+7$ and interval $[1,3]$
Now, $f(c)=3c^2+5c+7$
$\Rightarrow f'(c)=6c+5$
But $f'(c)=\frac{f(3)-f(1)}{3-1}$
$=\frac{49-15}{2}=\frac{34}{2}=17$
$\therefore 6c+5=17$
$\Rightarrow 6c=12$
$\Rightarrow c=2$

Question 126

A man 2m tall walks at a uniform speed of 5 km / h away from a lamp post 6m high. The rate at which the length of his shadow increases, is

Options:

A. 2.5 km / h

B. 5 km / h

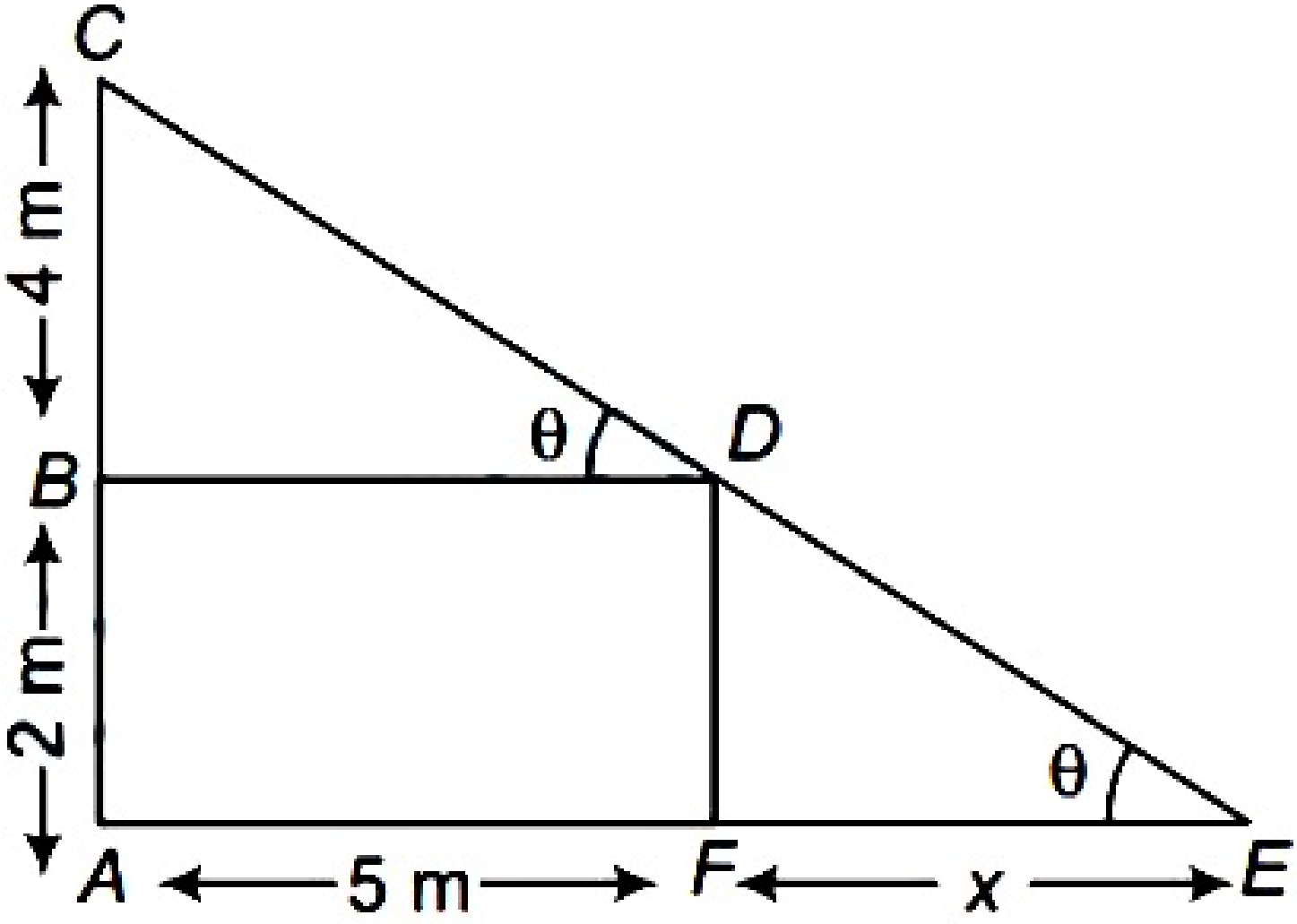
C. 15 km / h

D. $\frac{5}{3}$ km / h

Answer: A

Solution:

Solution:
Let the person reaches F from the lamp post A and let his shadow is EF.



Now, in $\triangle$ BCD,
$\tan\theta = \frac{4}{5}$
In ΔDEF
$\tan\theta = \frac{2}{x}$
$\therefore \frac{2}{x} = \frac{4}{5}$

$\Rightarrow \ x = \frac{10}{4} = 2.5\,\text{km}$

Hence, his shadow increases at the rate of $2.5\,\text{km/h}$

Question 127

$\int \frac{dx}{x(x^n+1)}$ is equal to

Options:

A. $\frac{1}{n}\log_e\left(\frac{x^n}{x^n+1}\right)+C$

B. $-\frac{1}{n}\log_e\left(\frac{x^n+1}{x^n}\right)+C$

C. $\log_e\left(\frac{x^n}{x^n+1}\right)+C$

D. None of the above

Answer: A

Solution:

Solution:

Let $I=\int \frac{dx}{x(x^n+1)}$

Put $x^n=t$

$\Rightarrow \ nx^{n-1}dx=dt$

$\therefore \ I=\int \frac{dt}{nx\,n^{n-1}\cdot x(t+1)}=\int \frac{dt}{nt(t+1)}$

$=\frac{1}{n}\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$

$=\frac{1}{n}[\log_e t-\log_e(t+1)]+C$

$\Rightarrow \ I=\frac{1}{n}\log_e\left(\frac{t}{t+1}\right)+C$

$\Rightarrow \ I=\frac{1}{n}\log_e\left(\frac{x^n}{x^n+1}\right)+C$

Question 128

Evaluate $\int \frac{dx}{x(x^5+2)}$

Options:

A. $\frac{1}{10}\log\left|\frac{x^5+1}{x^5+2}\right|+C$

B. $\frac{1}{5}\log\left|\frac{x^5}{x^5+2}\right|+C$

C. $\frac{1}{10}\log\left|\frac{x^5}{x^5+2}\right|+C$

D. $\frac{1}{5}\log\left|\frac{x^5+1}{x^5+2}\right|+C$

Answer: C

Solution:

Solution:

Let $I=\int\frac{dx}{x(x^5+2)}$

Put $x^5+2=t$

Then, $5x^4dx=dt$

$\therefore\ I=\int\frac{dt}{5x^4\cdot x(t)}$

$=\int\frac{dt}{5(t-2)t}$

$\Rightarrow\ I=\frac{1}{5}\int\frac{1}{2}\cdot\frac{2}{t(t-2)}dt$

$=\frac{1}{10}\int(\frac{1}{t-2}-\frac{1}{t})\,dt$

$=\frac{1}{10}\log(\frac{t-2}{t})+C$

$\Rightarrow\ I=\frac{1}{10}\log(\frac{x^5}{x^5+2})+C$

Question 129

$\int_2^3\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is equal to

Options:

A. $\frac{1}{4}$

B. 1

C. $\frac{3}{2}$

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:

Let $I=\int_2^3\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ . . . (i)

$\Rightarrow I=\int_2^3\frac{\sqrt{2+3-x}}{\sqrt{2+3-(5-x)}+\sqrt{2+3-x}}dx$

$\Rightarrow I=\int_2^3\frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$ . . . (ii)

Adding Eqs. (i) and (ii), we get

$$2I = \int_2^3 \left(\frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} \right) dx$$
$$= \int_2^3 1 \, dx = [x]_2^3 = 1$$
$$\Rightarrow I = \frac{1}{2}$$

Question 130

If $I = \int_{-1}^{1} \left(\frac{x^2 + sin\, x}{1 + x^2} \right) dx$, then its value is

Options:

A. 0

B. $2 + \frac{\pi}{2}$

C. $2 - \frac{\pi}{2}$

D. $\frac{\pi}{2} - 2$

Answer: C

Solution:

Solution:

$$I = \int_{-1}^{1} \left(\frac{x^2 + sin\, x}{1 + x^2} \right) dx$$
$$= \int_{-1}^{1} \frac{x^2}{1 + x^2} dx + \int_{-1}^{1} \frac{sin\, x}{1 + x^2} dx$$
$$= 2 \int_0^1 \frac{x^2}{1 + x^2} dx + 0$$
$$= 2 \int_0^1 \left(1 - \frac{1}{1 + x^2} \right) dx$$
$$= 2[x - \tan^{-1} x]_0^1$$
$$= 2[1 - \tan^{-1}(1) - 0 + \tan^{-1}(0)]$$
$$= 2[1 - \frac{\pi}{4}]$$
$$= 2 - \frac{\pi}{2}$$

Question 131

Find the area of the region $\{(x, y) : x^2 \leq y \leq x|\}$

Options:

A. $\frac{1}{3}$ sq units

B. $\frac{2}{3}$ sq units

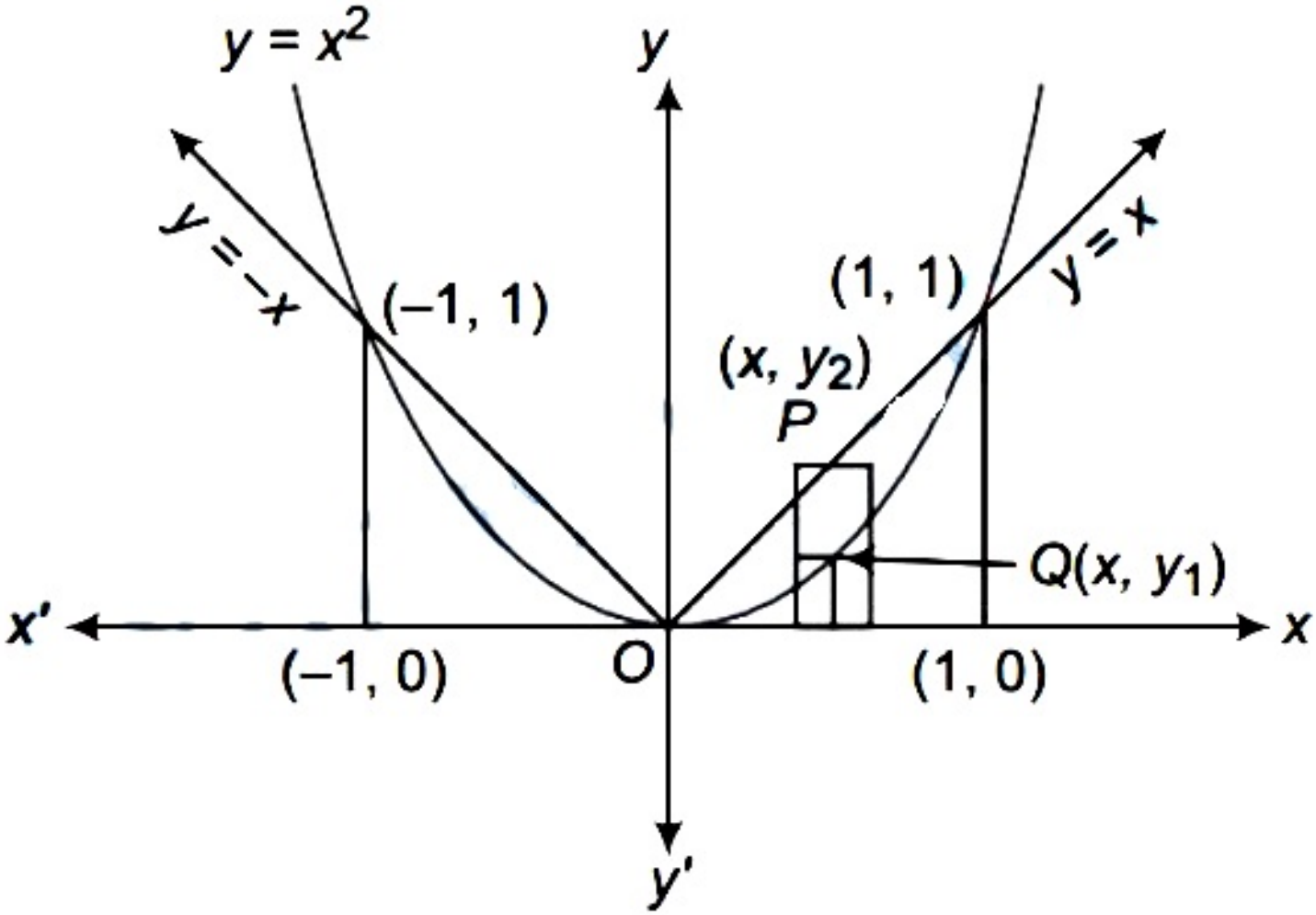
C. $\frac{4}{3}$ sq units

D. None of these

Answer: A

Solution:

Solution:
Let $R = \{(x, y) : x^2 \leq y \leq |x|\}$



$=\{(x,y):x2\leq y\}\cap\{(x,y):y\leq|x|\}$
$= \{(x, y) : x^2 \leq y\} \cap \{(x, y) : -x \leq y \leq x\}$
Required area $= 2$ (shaded area in first quadrant)
$= 2\int_0^1 (x - x^2)\, dx = 2 \times \frac{1}{6} = \frac{1}{3}$ sq units

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Question 132

Determine the area included between the curve $y = \cos^2 x, 0 \leq x \leq \frac{\pi}{2}$ and the axes.

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{\pi}{4}$

Answer: D

Solution:

Solution:
Required area

$= \int_0^{\pi/2} \cos^2 x \, dx$

$= \int_0^{\pi/2} \left(\frac{1+\cos 2x}{2} \right) dx$

$= \left[\frac{1}{2} \cdot x + \frac{1}{2} sin\, 2x \cdot \frac{1}{2} \right]_0^{\pi/2}$

$= \frac{1}{2} \cdot \frac{\pi}{2} + 0 + 0 - 0$

$= \frac{\pi}{4}$

Question 133

The differential equation whose solution represents the family $y = ae^{3x} + be^{x}$ is given by

Options:

A. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 3y = 0$

B. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 3y = 0$

C. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$

D. None of the above

Answer: C

Solution:

Solution:
We have, $y = ae^{3x} + be^{x}$ . . . (i)

$\Rightarrow \frac{dy}{dx} = 3a^{3x} + be^{x}$ . . . (ii)

and $\frac{d^2y}{dx^2} = 9a^{3x} + be^{x}$ . . . (ii)

Now, $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 3y$

$= 9ae^{3x} + be^x - 12ae^{3x} - 4e^x$

$\Rightarrow \frac{d^2y}{dx^2} - \frac{4dy}{dx} + 3y = 0 \quad + 3e^{3x} + 3be^x$

Which is the required differential equation.

Question 134

Solve $\frac{2dy}{dx} = \frac{y}{x} + \frac{y}{x^2}$

Options:

A. $y = x + C\sqrt{xy}$

B. $y = x - C\sqrt{xy}$

C. $y = x + Cy\sqrt{x}$

D. $y = x + C\sqrt{y}$

Answer: C

Solution:

Solution:

$2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$ . . . (i)

Put $\frac{y}{x} = v$

$\Rightarrow$ y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Then, Eq. (i) becomes,

$2v + 2x\frac{dv}{dx} = v^2 + v$

$\Rightarrow 2x\frac{dv}{dx} = v^2 - v$

$\Rightarrow 2 \cdot \frac{dv}{v^2 - v} = \frac{dx}{x}$

$\Rightarrow \int \frac{dv}{v^2 - v} = \frac{1}{2}\int \frac{dx}{x}$

$\Rightarrow \int (\frac{1}{v-1} - \frac{1}{v})\, dv = \frac{1}{2}\int \frac{dx}{x}$

$\Rightarrow \log(\frac{v-1}{v}) = \frac{1}{2}\log x + \log C$

$\Rightarrow \log\frac{v-1}{v} = \log \sqrt{x}\, C$

$\Rightarrow \frac{v-1}{v} = \sqrt{x}C$

$\Rightarrow \frac{y-x}{y} = C\sqrt{x}$

$\Rightarrow y = x + yC\sqrt{x}$

Question 135

Seven weddings occur in a week. What is the probability that they happen on the same day?

Options:

A. $\frac{1}{7}$

B. $\frac{1}{7^4}$

C. $\frac{1}{7^6}$

D. None of these

Answer: C

Solution:

Solution:
First wedding can be occured in a week in 7 ways. Similarly, second, third , ..., seventh wedding can also be occured in 7 ways.
$\therefore$ Total number of cases of occuring the weddings $= 7^7$
Number of cases that these weddings occur on the same day $= 7$
$\therefore$ Required probability $= \frac{7}{7^7} = \frac{1}{7^6}$

Question 136

Two small squares on a chess board are chosen at random. Probability that they have a common side is

Options:

A. $\frac{1}{3}$

B. $\frac{1}{9}$

C. $\frac{1}{15}$

D. $\frac{1}{18}$

Answer: D

Solution:

Solution:
Two squares can be chosen in a single row by 7 ways as there are 8 squares in each row. But there are 8 rows. So, number of ways to choose two squares in any of the row $= 7 \times 8 = 56$. Similarly, number of ways to choose two squares in any of the column $= 56$
$\therefore$ Total number of favourable cases $= 56 + 56$
$= 112$ and total number of cases $= {}^{64}C_2 = \frac{64 \times 63}{2} = 32 \times 63$
$\therefore$ Required probability $= \frac{112}{32 \times 63} = \frac{1}{18}$

Question 137

A die is thrown 7 times. What is the chance that an odd number turns up exactly 4 times?

Options:

A. $\frac{35}{128}$

B. $\frac{37}{128}$

C. $\frac{4}{7}$

D. $\frac{43}{128}$

Answer: A

Solution:

Solution:
Required probability is given by
${}^{n}C_{r}p^{r}q^{n-r}$
where $n = 7, r = 4, p = q = \frac{1}{2}$
$\therefore$ Required probability $= {}^{7}C_{4}(\frac{1}{2})^{4}(\frac{1}{2})^{3}$
$= \frac{7\times6\times5\times4}{4\times3\times2\times1}\cdot\frac{1}{2^{7}} = \frac{35}{128}$

Question 138

Find the regression coefficient b_{xy} for the data $\Sigma x = 32$, $\Sigma y = 24$, $\Sigma xy = 218$, $\Sigma x^2 = 216$, $\Sigma y^2 = 246$ and $n = 8$

Options:

A. 0.3

B. 0.7

C. 0.8

D. 0.6

Answer: B

Solution:

Solution:
Regression coefficient $b_{xy} = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$

$$\Rightarrow \quad b_{xy} = \frac{8 \times 218 - 32 \times 24}{8 \times 246 - (24)^2}$$
$$\Rightarrow \quad b_{xy} = \frac{1744 - 768}{1968 - 576}$$
$$\Rightarrow \quad b_{xy} = \frac{976}{1392} = 0.7$$

Question 139

If two regression coefficients are found to be -0.6 and -1.4, the correlation coefficient will be

Options:

A. 0.92

B. 0.43

C. -0.43

D. -0.92

Answer: D

Solution:

Solution:
We known that the correlation coefficient is the GM of regression coefficient.
$\therefore$ Required correlation coefficient
$= -\sqrt{(-0.6)(-1.4)} = -\sqrt{0.84} = -0.916$
≈ -0.92

Question 140

The iteration formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ is commonly known as

Options:

A. Bisection method

B. Newton-Raphson method

C. False-positions method

D. None of the above

Answer: B

Solution:

Solution:
The iteration formula
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
is commonly known as Newton-Raphson method.

Question 141

Which of the following methods is surely convergent?

Options:

A. Regula-Falsi method

B. Bolzano method

C. Both (a) and (b)

D. Neither (a) nor (b)

Answer: C

Solution:

Solution:
Both Regula-falsi method and Bolzano method are convergent.

Question 142

A curve is drawn to pass through the points given by the following table.

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Using Simpson's 1/3rd rule, estimate the area bounded by the curve, the x-axis and the lines x = 1, x = 4

Options:

A. 7.74 sq units

B. 7.76 sq units

C. 7.78 sq units

D. 7.82 sq units

Answer: C

Solution:

Solution:
We have,

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Here, $h = 1.5 - 1 = 0.5, n = 6$
Simpson's $\frac{1}{3}$ rd rule is

$$\int_a^b f(x)\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots)]$$

$$\therefore \int_1^4 f(x)\,dx = \frac{0.5}{3}[(2 + 2.1) + 4(2.4 + 2.8 + 2.6) + 2(2.7 + 3)]$$

$$= \frac{0.5}{3}[4.1 + 4(7.8) + 2(5.7)]$$

$$= \frac{0.5}{3}[4.1 + 31.2 + 11.4]$$

$$= 7.78 \text{ squnits}$$

Question 143

Which of these methods for numerical integration is also called as parabolic formula?

Options:

A. Simpson's one-third rule

B. Simpson's three-eighth's rule

C. Trapezoidal rule

D. None of the above

Answer: A

Solution:

Solution:
Simpson's One third rule is called parabolic formula.

Question 144

Calculate by Trapezoidal rule an approximate value of $\int_{-3}^{3} x^4 \, dx$ by taking seven equidistant ordinates

Options:

A. 98

B. 97.2

C. 100

D. 115

Answer: D

Solution:

Solution:
We know the trapezoidal rule

$$\int_a^b f(x)\,dx = h\left[\frac{1}{2}(y_0 + y_n) + (y_1 + y_2 + \ldots + y_{n-1})\right]$$

For seven ordinates, $n = 6$

$$\therefore \ h = \frac{3-(-3)}{6} = \frac{6}{6} = 1$$

Now,

x	-3	-2	-1	0	1	2	3
x^4	81	16	1	0	1	16	81

$$\therefore \int_{-3}^{3} x^4\,dx$$
$$= 1\left[\frac{1}{2}(81 + 81) + (16 + 1 + 0 + 1 + 16)\right]$$
$$= 81 + 34 = 115$$

Question 145

A gigabyte (GB) refers to

Options:

A. 2^{10} bytes

B. 2^{20} bytes

C. 2^{30} bytes

D. 2^{40} bytes

Answer: C

Solution:

Solution:
1 GB = 2^{30} bytes.

Question 146

Which of the following is a peripheral device?

Options:

A. Visual display unit

B. Hard disk drive

C. Floppy disk drive

D. All of the above

Answer: D

Solution:

Solution:
Visual display unit, Hard disk drive and floppy disk drive, all are peripheral device.

Question 147

Which of the following is not a general purpose application software?

Options:

A. Word processors

B. Programs for playing games

C. Spread-sheets

D. Data communication software

Answer: B

Solution:

Solution:
Programs for playing games is not a general purpose application software.

Question 148

The value of $(1+i)^3+(1-i)^6$ is

Options:

A. i

B. $2(-1+5i)$

C. $1-5i$

D. $2+1-5i$

Answer: B

Solution:

Solution:
$$\begin{aligned}&(1+i)^3+(1-i)^6\\&=(1+i)^3+[(1-i)^3]^2\\&=1+i^3+3i(1+i)+[1-i^3-3i(1-i)]^2\\&=1-i+3i-3+(1+i-3i-3)^2\\&=-2+2i+(-2-2i)^2\\&=-2+2i+4+4i^2+8i\\&=-2+2i+4-4+8i\\&=10i-2=2(-1+5i)\end{aligned}$$

Question 149

The equation of the normal to the hyperbola $x^2-16y^2-2x-64y-72=0$ at the point $(-4,-3)$ is

Options:

A. 5x + 16y + 79 = 0

B. $16x+5y+97=0$

C. $16x+5y+79=0$

D. $5x + 16y + 97 = 0$

Answer: C

Solution:

Solution:
First, we find the equation of tangent to the hyperbola
$x^2 - 16y^2 - 2x - 64y - 72 = 0$ . . . (i)
at the point $(-4, -3)$, which is given by
$-4x + 48y - (x - 4) - 32(y - 3) - 72 = 0$
$\Rightarrow -4x + 48y - x + 4 - 32y + 96 - 72 = 0$
$\Rightarrow -5x + 16y + 28 = 0$ . . . (ii)
Required normal is the normal to the tangent (ii), which is given by
$16x + 5y + c = 0$
But it passes through $(-4, -3)$.
$\therefore \ -64 - 15 + c = 0$ . . . (iii)
$\Rightarrow c = 79$
Putting this value in Eq. (iii), we get
$16x + 5y + 79 = 0$
Which is the required normal.

Question 150

Solve $(x + 2y^3)\frac{dy}{dx} = y, y > 0$

Options:

A. $y = x^3 + Cy$

B. $x = y^3 + Cy$

C. $y = x^3 - Cy$

D. $x = y^3 - Cy$

Answer: D

Solution:

Solution:
$(x + 2y^3)\frac{dy}{dx} = y, y > 0$
$\Rightarrow \ (x + 2y^3)dy = y\,dx$
$\Rightarrow \ 2y^3 dy = y\,dx - x dy$
$\Rightarrow \ 2y dy = \frac{y\,dx - x dy}{y^2}$
On integrating,
$\frac{2y^2}{2} = \frac{x}{y} + C$
$y^3 = \frac{x}{y} + C$
$\Rightarrow \ y^3 = x + Cy$
$\Rightarrow \ x = y^3 - Cy$
