Chapter 5 Quadratic Functions

Ex 5.4

Answer 1e.

Rewrite the given expression using the replacement $u = 2x^3$.
$2u^2 + 5u - 3$

In general, we say an expression of the form $au^2 + bu + c$ to be in quadratic form, where u is any expression in x. This means that the given expression is quadratic in form.

Therefore, we can complete the statement given as "The expression $8x^6 + 10x^3 - 3$ is in **quadratic form** because it can be written as $2u^2 + 5u - 3$ where $u = 2x^3$."

Answer 1gp.

The monomial x is common among the terms in the given polynomial.

We can take out x from each term.
$x^3 - 7x^2 + 10x = x(x^2 - 7x + 10)$

Now, factor the trinomial $x^2 - 7x + 10$ so that the polynomial can be completely factored.
$x(x^2 - 7x + 10) = x(x - 5)(x - 2)$

Therefore, the given polynomial in completely factored form is $x(x - 5)(x - 2)$.

Answer 2e.

We need to say that in what condition the factorization of a polynomial must satisfy in order for the polynomial to be factored completely.

The factorization of a polynomial satisfies in order for the polynomial to be factored completely if it is written as a product of unfactorable polynomials with integer coefficients.

Answer 2gp.

Consider the polynomial:

$$3y^5 - 75y^3$$

We need to factor the polynomial (1) completely.

From (1), we have

$$3y^5 - 75y^3 = 3y^3\left(y^2 - 25\right) \quad \text{[Factor common monomial.]}$$
$$= 3y^3\left(y+5\right)\left(y-5\right) \quad \text{[Difference of two squares.]}$$

Therefore the factor of the polynomial $3y^5 - 75y^3$ is: $\boxed{3y^3\left(y+5\right)\left(y-5\right)}$.

Answer 3e.

The monomial $7x$ is common among the terms in the given polynomial.

Take out $7x$ from each term to factor the polynomial completely.
$14x^2 - 21x = 7x(2x - 3)$

Therefore, the given polynomial in completely factored form is $7x(2x - 3)$.

Answer 3gp.

The monomial $2b^3$ is common among the terms in the given polynomial.

We can take out $2b^3$ from each term.
$16b^5 + 686b^2 = 2b^2(8b^3 + 343)$

Rewrite $8b^3 + 343$ as the sum of two cubes.
$2b^2(8b^3 + 343) = 2b^2[(2b)^3 + 7^3]$

Factor using the pattern $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

$$2b^2\left[(2b)^3 + 7^3\right] = 2b^2\left\{(2b+7)\left[(2b)^2 - (2b)(7) + 7^2\right]\right\}$$
$$= 2b^2(2b+7)\left(4b^2 - 14b + 49\right)$$

Therefore, the given polynomial in completely factored form is
$2b^2(2b + 7)(4b^2 - 14b + 49)$.

Answer 4e.

Consider the polynomial:

$$30b^3 - 54b^2 \qquad \text{....... (1)}$$

We need to factor the polynomial (1) completely.

From (1), we have

$$30b^3 - 54b^2 = 6b^2\left(5b - 9\right) \quad \text{[Factor common monomial.]}$$

Therefore the factor of the polynomial $30b^3 - 54b^2$ is: $\boxed{6b^2\left(5b-9\right)}$.

Answer 4gp.

Consider the polynomial:

$$w^3 - 27$$

We need to factor the polynomial (1) completely.

From (1), we have

$$w^3 - 27 = w^3 - 3^3$$

$$= (w-3)(w^2 + 3w + 9) \qquad \left[\begin{array}{l}\text{Difference of two cubes:}\\ a^3 - b^3 = (a-b)(a^2 + ab + b^2).\end{array}\right]$$

Therefore the factor of the polynomial $w^3 - 27$ is: $\boxed{(w-3)(w^2+3w+9)}$.

Answer 5e.

The monomial c is common among the terms in the given polynomial.

We can take out c from each term.
$c^3 + 9c^2 + 18c = c(c^2 + 9c + 18)$

Now, factor the trinomial $c^2 + 9c + 18$ so that we can completely factor the polynomial.
$c(c^2 + 9c + 18) = c(c + 6)(c + 3)$

Therefore, the given polynomial in completely factored form is $c(c + 6)(c + 3)$.

Answer 5gp.

Apply the pattern for factoring by grouping.

Take out the common monomial x^2 from the first two terms, and 9 from the second two terms in the given polynomial.
$x^3 + 7x^2 - 9x - 63 = x^2(x + 7) - 9(x + 7)$

Now, use the distributive property.
$x^2(x + 7) - 9(x + 7) = (x + 7)(x^2 - 9)$

Factor the difference of squares $x^2 - 9$ using the pattern $a^2 - b^2 = (a + b)(a - b)$.
$(x + 7)(x^2 - 9) = (x + 7)(x + 3)(x - 3)$

Therefore, the given polynomial in completely factored form is $(x + 7)(x + 3)(x - 3)$.

Answer 6e.

Consider the polynomial:

$$z^3 - 6z^2 - 72z \quad \ldots\ldots (1)$$

We need to factor the polynomial (1) completely.

From (1), we have

$$z^3 - 6z^2 - 72z = z(z^2 - 6z - 72) \quad \text{[Factor common monomial.]}$$

$$= z(z+6)(z-12) \quad \text{[Factor trinomial.]}$$

Therefore the factor of the polynomial $z^3 - 6z^2 - 72z$ is: $\boxed{z(z+6)(z-12)}$.

Answer 6gp.

Consider the polynomial:

$$16g^4 - 625$$

We need to factor the polynomial (1) completely.

From (1), we have

$$16g^4 - 625 = (4g)^2 - (25)^2 \quad \text{[Difference of two squares.]}$$

$$= (4g^2 + 25)(4g^2 - 25) \quad \text{[Difference of two squares.]}$$

$$= (4g^2 + 25)(2g+5)(2g-5) \quad \text{[Difference of two squares.]}$$

Therefore the factor of the polynomial $16g^4 - 625$ is: $\boxed{(4g^2+25)(2g+5)(2g-5)}$.

Answer 7e.

The monomial $3y^3$ is common among the terms in the given polynomial.

We can take out $3y^3$ from each term.
$3y^5 - 48y^3 = 3y^3(y^2 - 16)$

Now, the expression $y^2 - 16$ is the difference of two squares, and can be factored using the pattern $a^2 - b^2 = (a+b)(a-b)$.
$3y^3(y^2 - 16) = 3y^3(y+4)(y-4)$

Therefore, the given polynomial in completely factored form is $3y^3(y+4)(y-4)$.

Answer 7gp.

Factor the common monomial $4t^2$ out from the terms in given polynomial.
$4t^6 - 20t^4 + 24t^2 = 4t^2(t^4 - 5t^2 + 6)$

The trinomial $t^4 - 5t^2 + 6$ is in quadratic form. As a result, we can replace t^2 with u.
$4t^2(t^4 - 5t^2 + 6) = 4u(u^2 - 5u + 6)$

Factor $u^2 - 5u + 6$.
$4u(u^2 - 5u + 6) = 4u(u-3)(u-2)$

Now, replace u with t^2.
$4u(u-3)(u-2) = 4t^2(t^2-3)(t^2-2)$

We can factor the difference of squares $t^2 - 3$ and $t^2 - 2$ using the pattern $a^2 - b^2 = (a+b)(a-b)$.
$4t^2\left(t^2-3\right)\left(t^2-2\right) = 4t^2\left(t+\sqrt{3}\right)\left(t-\sqrt{3}\right)\left(t+\sqrt{2}\right)\left(t-\sqrt{2}\right)$

Therefore, the polynomial in completely factored form is
$4t^2\left(t+\sqrt{3}\right)\left(t-\sqrt{3}\right)\left(t+\sqrt{2}\right)\left(t-\sqrt{2}\right)$.

Answer 8e.

Consider the polynomial:
$$54m^5 + 18m^4 + 9m^3 \qquad \ldots\ldots (1)$$

From (1), we have
$$54m^5 + 18m^4 + 9m^3 = 9m^3\left(6m^2 + 2m + 1\right) \qquad \text{[Factor common monomial.]}$$
Therefore the polynomial $54m^5 + 18z^4 + 9m^3$ is not factored completely.

Answer 8gp.

Consider the equation:
$$4x^5 - 40x^3 + 36x = 0 \qquad \ldots\ldots (1)$$
We need to find the real number solution of the equation (1).

From (1), we have
$$4x^5 - 40x^3 + 36x = 0$$
$$4x\left(x^4 - 10x^2 + 9\right) = 0 \qquad \text{[Fector common monomial.]}$$
$$4x\left(x^2 - 1\right)\left(x^2 - 9\right) = 0 \qquad \text{[Fector trinomial.]}$$
$$4x(x+1)(x-1)\left(x^2 - 9\right) = 0 \qquad \text{[Difference of two squares.]}$$
$$4x(x+1)(x-1)(x+3)(x-3) = 0 \qquad \text{[Difference of two squares.]}$$
Therefore the solution of the equation $4x^5 - 40x^3 + 36x = 0$ is:
$\boxed{x = 0,\ x = -1,\ x = 1,\ x = 3,\ x = -3}$.

Answer 9e.

The given polynomial contains the monomial $2x^3$ in common.

We can take out $2x^3$ from each term.
$2x^7 - 32x^3 = 2x^3(x^4 - 16)$

Now, the expression $x^4 - 16$ is the difference of two squares.
We can factor it as $(x^2+4)(x^2-4)$, which can be further factored as $(x^2+4)(x+2)(x-2)$.

$2x^3(x^4-16) = 2x^3(x^2+4)(x+2)(x-2)$

The completely factored form of the given polynomial is thus $2x^3(x^2+4)(x+2)(x-2)$, which matches with the expression given in choice **A**.

Answer 9gp.

Rewrite the given equation in standard form.
$2x^5 - 14x^3 + 24x = 0$

Take out the common monomial $2x$ from the terms in the expression on the left.
$2x(x^4 - 7x^2 + 12) = 0$

We can replace x^2 with u in $x^4 - 7x^2 + 12$ since it is in quadratic form.
$2x(u^2 - 7u + 12) = 0$

Now, factor $u^2 - 7u + 12$.
$2x(u-4)(u-3) = 0$

Substitute x^2 for u in the equation.
$2x(x^2-4)(x^2-3) = 0$

Use the pattern $a^2 - b^2 = (a+b)(a-b)$ for factoring the difference of squares.
$2x(x+2)(x-2)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right) = 0$

Solve for x by applying the Zero product property.
$2x = 0$ or $x+2 = 0$ or $x-2 = 0$ or $x+\sqrt{3} = 0$ or $x-\sqrt{3} = 0$
$x = 0$ $\quad x = -2$ $\quad x = 2$ $\quad x = -\sqrt{3}$ $\quad x = \sqrt{3}$

Therefore, the real-number solutions to the given equation are $0, -2, 2, -\sqrt{3}$, and $\sqrt{3}$.

Answer 10e.

Consider the polynomial:

$$x^3 + 8$$

We need to factor the polynomial (1) completely by using sum and difference of cubes.

From (1), we have

$$\begin{aligned} x^3 + 8 &= x^3 + (2)^3 \\ &= (x+2)\left(x^2 - 2x + 4\right) \quad \left[\begin{array}{l}\text{Sum of two cubes:} \\ a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\end{array}\right] \end{aligned}$$

Therefore the factor of the polynomial $x^3 + 8$ is: $\boxed{(x+2)\left(x^2-2x+4\right)}$.

Answer 10gp.

Consider the equation:

$$-27x^3+15x^2=-6x^4 \qquad \text{...... (1)}$$

We need to find the real number solution of the equation (1).

From (1), we have

$$-27x^3+15x^2=-6x^4$$

$$-27x^3+15x^2+6x^4=0 \qquad \left[\text{Adding } 6x^4 \text{in both sides.}\right]$$

$$3x^2\left(-9x+5+2x^2\right)=0 \qquad \left[\text{Factor common monomial.}\right]$$

$$3x^2\left(2x^2-9x+5\right)=0$$

Now,

$$3x^2=0$$

$$x=0$$

And

$$2x^2-9x+5=0$$

The solution of the equation $-27x^3+15x^2=-6x^4$ is:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-9\pm\sqrt{81-4\times2\times5}}{2\times2}$$

$$=\frac{-9\pm\sqrt{81-40}}{4}$$

$$=\frac{-9\pm\sqrt{41}}{4}$$

$$=\frac{-9\pm6.403}{4}$$

The solutions are:

$$\frac{-9+6.403}{4}=x$$

$$-0.649=x$$

And

$$\frac{-9-6.403}{4}=x$$

$$-3.851=x$$

Therefore the solutions of the equation

$-27x^3+15x^2=-6x^4$ is: $\boxed{x=0,\ x=-0.649,\ x=-3.851}$.

Answer 11e.

The given expression can be identified as the difference of two cubes.

Factor using the pattern $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$$y^3 - 64 = y^3 - 4^3$$
$$= (y - 4)(y^2 + 4y + 16)$$

Therefore, the given polynomial in completely factored form is $(y - 4)(y^2 + 4y + 16)$.

Answer 11gp.

The basin has a capacity of 40 cubic feet. The outer length of the basin is $6x$, width is $3x$, and the height is x.

The volume of the basin is the product of its length, width, and height.

Volume (cubic inches)	=	length (inches)	·	width (inches)	·	height (inches)
⇓		⇓		⇓		⇓
40	=	$6x$	·	$3x$	·	x

We get the equation $40 = 6x(3x)(x)$.

Rewrite the equation in standard form.

$$40 = 18x^3$$
$$0 = 18x^3 - 40$$

Now, factor $18x^3 - 40$.

$$0 = 2(9x^3 - 20)$$

Use the zero product property and solve for x.

$$9x^3 - 20 = 0$$
$$9x^3 = 20$$
$$x^3 = \frac{20}{9}$$
$$x \approx 1.30$$

The only real solution being 1.30, we get the outer height of the basin as about 1.30 feet. The outer length of the basin will then be about 6(1.30) or 7.8 feet, and the outer width will be about 3(1.3) or 3.9 feet.

Answer 12e.

Consider the polynomial:

$27m^3 + 1$ ...... (1)

We need to factor the polynomial (1) completely by using sum and difference of cubes.

From (1), we have

$$27m^3 + 1 = (3m)^3 + (1)^3$$

$$= (3m+1)(9m^2 - 3m + 1) \quad \left[\begin{array}{l}\text{Sum of two cubes:} \\ a^3 + b^3 = (a+b)(a^2 - ab + b^2)\end{array}\right]$$

Therefore the factor of the polynomial $27m^3 + 1$ is: $\boxed{(3m+1)(9m^2 - 3m + 1)}$.

Answer 13e.

The given expression can be identified as the sum of two cubes.

Factor using the pattern $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

$$125n^3 + 216 = (5n)^3 + 6^3$$

$$= (5n + 6)\left[(5n)^2 - (5n)(6) + 6^2\right]$$

$$= (5n + 6)(25n^2 - 30n + 36)$$

Therefore, the given polynomial in completely factored form is $(5n + 6)(25n^2 - 30n + 36)$.

Answer 14e.

Consider the polynomial:

$27a^3 - 1000$ ...... (1)

We need to factor the polynomial (1) completely by using sum and difference of cubes.

From (1), we have

$$27a^3 - 1000 = (3a)^3 - (10)^3$$

$$= (3a - 10)(9a^2 + 30a + 100) \quad \left[\begin{array}{l}\text{Difference of two cubes:} \\ a^3 - b^3 = (a-b)(a^2 + ab + b^2)\end{array}\right]$$

Therefore the factor of the polynomial $27a^3 - 1000$ is: $\boxed{(3a-10)(9a^2 + 30a + 100)}$.

Answer 15e.

The given expression can be identified as the sum of two cubes.

Factor using the pattern $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

$$\begin{aligned} 8c^3 + 343 &= (2c)^3 + 7^3 \\ &= (2c + 7)\left[(2c)^2 - (2c)(7) + 7^2\right] \\ &= (2c + 7)\left(4c^2 - 14c + 49\right) \end{aligned}$$

Therefore, the given polynomial in completely factored form is $(2c + 7)(4c^2 - 14c + 49)$.

Answer 16e.

Consider the polynomial:

$$192w^3 - 3 \qquad \text{...... (1)}$$

We need to factor the polynomial (1) completely by using sum and difference of cubes.

From (1), we have

$$\begin{aligned} 192w^3 - 3 &= 3\left[64w^3 - 1\right] && \left[\text{Fector common monomial}\right] \\ &= 3\left[(4w)^3 - 1^3\right] \\ &= 3(4w-1)\left(16w^2 + 4w + 1\right) && \left[\begin{array}{l}\text{Difference of two cubes:} \\ a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\end{array}\right] \end{aligned}$$

Therefore the factor of the polynomial $192w^3 - 3$ is: $\boxed{3(4w-1)\left(16w^2 + 4w + 1\right)}$.

Answer 17e.

The monomial -5 is common among the terms in the given polynomial.

We can take out -5 from each term.

$-5z^3 + 320 = -5(z^3 - 64)$

The expression $z^3 - 64$ can be identified as the difference of two cubes.

Factor using the pattern $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$$\begin{aligned} -5\left(z^3 - 64\right) &= -5\left(z^3 - 4^3\right) \\ &= -5(z - 4)\left(z^2 + 4z + 16\right) \end{aligned}$$

Therefore, the given polynomial in completely factored form is $-5(z - 4)(z^2 + 4z + 16)$.

Answer 18e.

Consider the polynomial:

$$x^3 + x^2 + x + 1 \qquad \text{...... (1)}$$

We need to factor the polynomial (1) completely by using factor by grouping.

From (1), we have

$$x^3+x^2+x+1=x^2(x+1)+1(x+1) \quad \text{[Fector by grouping]}$$
$$=(x^2+1)(x+1) \quad \text{[Distributive property]}$$

Therefore the factor of the polynomial x^3+x^2+x+1 is: $\boxed{(x^2+1)(x+1)}$.

Answer 19e.

Apply the pattern for factoring by grouping.

Take out the common monomial y^2 from the first two terms, and 4 from the second two terms in the given polynomial.

$$y^3-7y^2+4y-28 = y^2(y-7)+4(y-7)$$

Now, use the distributive property.

$$y^2(y-7)+4(y-7) = (y^2+4)(y-7)$$

Therefore, the given polynomial in completely factored form is $(y^2+4)(y-7)$.

Answer 20e.

Consider the polynomial:

$$x^3+5x^2-9x-45 \quad \text{...... (1)}$$

We need to factor the polynomial (1) completely by using factor by grouping.

From (1), we have

$$x^3+5x^2-9x-45=x^2(x+5)-9(x+5) \quad \text{[Fector by grouping.]}$$
$$=(x^2-9)(x+5) \quad \text{[Distributive property.]}$$
$$=(x+3)(x-3)(x+5) \quad \text{[Difference of two squares.]}$$

Therefore the factor of the polynomial $x^3+5x^2-9x-45$ is: $\boxed{(x+3)(x-3)(x+5)}$.

Answer 21e.

Apply the pattern for factoring by grouping.

Take out the common monomial m^2 from the first two terms, and 3 from the second two terms in the given polynomial.

$$3m^3-m^2+9m-3 = m^2(3m-1)+3(3m-1)$$

Now, use the distributive property.
$m^2(3m - 1) + 3(3m - 1) = (m^2 + 3)(3m - 1)$

Therefore, the given polynomial in completely factored form is $(m^2 + 3)(3m - 1)$.

Answer 22e.

Consider the polynomial:

$$25s^3 - 100s^2 - s + 4 \qquad \ldots\ldots (1)$$

We need to factor the polynomial (1) completely by using factor by grouping.

From (1), we have

$$\begin{aligned} 25s^3 - 100s^2 - s + 4 &= 25s^2(s-4) - 1(s-4) && \text{[Fector by grouping.]} \\ &= (25s^2 - 1)(s-4) && \text{[Distributive property.]} \\ &= (5s+1)(5s-1)(s-4) && \text{[Difference of two squares.]} \end{aligned}$$

Therefore the factor of the polynomial $25s^3 - 100s^2 - s + 4$ is: $\boxed{(5s+1)(5s-1)(s-4)}$.

Answer 23e.

Apply the pattern for factoring by grouping.

Take out the common monomial $4c^2$ from the first two terms, and 9 from the second two terms in the given polynomial.
$4c^3 + 8c^2 - 9c - 18 = 4c^2(c + 2) - 9(c + 2)$

Now, use the distributive property.
$4c^2(c + 2) - 9(c + 2) = (4c^2 - 9)(c + 2)$

The expression $4c^2 - 9$ is the difference of two squares. We can factor it as $(2c + 3)(2c - 3)$.

$(4c^2 - 9)(c + 2) = (2c + 3)(2c - 3)(c + 2)$

Therefore, the given polynomial in completely factored form is $(2c + 3)(2c - 3)(c + 2)$.

Answer 24e.

Consider the polynomial:

$$x^4 - 25 \qquad \ldots\ldots (1)$$

We need to factor the polynomial (1) completely by using quadratic form.

From (1), we have

$$x^4-25=\left(x^2\right)^2-5^2 \quad \text{[Difference of two squares.]}$$
$$=\left(x^2+5\right)\left(x^2-5\right) \quad \text{[Difference of two squares.]}$$
$$=\left(x^2+5\right)\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right) \quad \text{[Difference of two squares.]}$$

Therefore the factor of the polynomial x^4-25 is: $\boxed{\left(x^2+5\right)\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)}$.

Answer 25e.

Rewrite the given expression using u as a replacement for a^2.
u^2+7u+6

Now, factor the trinomial u^2+7u+6.
$u^2+7u+6 \;=\; (u+6)(u+1)$

Now, substitute a^2 for u.
$(u+6)(u+1)=(a^2+6)(a^2+1)$

Therefore, the given polynomial in completely factored form is $(a^2+6)(a^2+1)$.

Answer 26e.

Consider the polynomial:

$$3s^4-s^2-24 \qquad \ldots\ldots (1)$$

We need to factor the polynomial (1) completely by using quadratic form.

From (1), we have

$$3s^4-s^2-24=3s^4-9s^2+8s^2-24$$
$$=3s^2\left(s^2-3\right)+8\left(s^2-3\right)$$
$$=\left(3s^2+8\right)\left(s^2-3\right) \quad \text{[Distributive property]}$$
$$=\left(3s^2+8\right)\left(s+\sqrt{3}\right)\left(s-\sqrt{3}\right) \quad \text{[Difference of two squares]}$$

Therefore the factor of the polynomial $3s^4-s^2-24$ is: $\boxed{\left(3s^2+8\right)\left(s+\sqrt{3}\right)\left(s-\sqrt{3}\right)}$.

Answer 27e.

The monomial 2z is common among the terms in the given polynomial.

We can take out $2z$ from each term.
$32z^5-2z=2z(16z^4-1)$

The expression $16z^4 - 1$ is quadratic in form as it can rewritten as $u^2 - 1$, where $u = 4z^2$.
$2z(16z^4 - 1) = 2z(u^2 - 1)$

Factor $u^2 - 1$ using the pattern for the difference of two squares.
$2z(u^2 - 1) \ = \ 2z(u - 1)(u + 1)$

Now, substitute $4z^2$ for u.
$2z(u - 1)(u + 1) \ = \ 2z(4z^2 - 1)(4z^2 + 1)$

Again, factor $4z^2 - 1$ using the pattern for the difference of two squares.
$2z(4z^2 - 1)(4z^2 + 1) \ = \ 2z(2z + 1)(2z - 1)(4z^2 + 1)$

Therefore, the given polynomial in completely factored form is
$2z(2z + 1)(2z - 1)(4z^2 + 1)$.

Answer 28e.

Consider the polynomial:

$$36m^6 + 12m^4 + m^2 \qquad \ldots\ldots (1)$$

We need to factor the polynomial (1) completely by using quadratic form.

From (1), we have

$$\begin{aligned} 36m^6 + 12m^4 + m^2 &= m^2\left(36m^4 + 12m^2 + 1\right) && \text{[Factor common monomial]} \\ &= m^2\left(6m^2 + 1\right)^2 && \text{[Perfect square trinomial]} \end{aligned}$$

Therefore the factor of the polynomial $36m^6 + 12m^4 + m^2$ is: $\boxed{m^2\left(6m^2 + 1\right)^2}$.

Answer 29e.

The monomial $3x$ is common among the terms in the given polynomial.

We can take out $3x$ from each term.
$15x^5 - 72x^3 - 108x \ = \ 3x(5x^4 - 24x^2 - 36)$

The expression $5x^4 - 24x^2 - 36$ is quadratic in form as it can rewritten as $5u^2 - 24u - 36$, where $u = x^2$.
$3x(5x^4 - 24x^2 - 36) \ = \ 3x(5u^2 - 24u - 36)$

Now, factor the trinomial $5u^2 - 24u - 36$.
$3x(5u^2 - 24u - 36) \ = \ 3x(5u + 6)(u - 6)$

Substitute x^2 for u.
$3x(5u+6)(u-6) = 3x(5x^2+6)(x^2-6)$

Use the pattern $a^2-b^2=(a+b)(a-b)$ to factor x^2-6.
$3x\left(5x^2+6\right)\left(x^2-6\right) = 3x\left(5x^2+6\right)\left(x+\sqrt{6}\right)\left(x-\sqrt{6}\right)$

Therefore, the given polynomial in completely factored form is
$3x\left(5x^2+6\right)\left(x+\sqrt{6}\right)\left(x-\sqrt{6}\right)$.

Answer 30e.

Consider error analysis:

$$8x^3-27=0$$
$$\left(2x+3\right)\left(4x^2+6x+9\right)=0 \qquad \text{...... (1)}$$
$$x=-\frac{3}{2}$$

We need to describe and correct the error in finding all real number solutions.

From (1), we see that it use difference of two cubes: $a^3+b^3=(a+b)\left(a^2+ab+b^2\right)$.

But here we consider the equation $8x^3-27=0$.

So, here we use the sum of two squares: $a^3-b^3=(a-b)\left(a^2-ab+b^2\right)$.

Now we correct the error in finding all real number solutions.

$$8x^3-27=0$$
$$\left(2x-3\right)\left(4x^2-6x+9\right)=0$$
$$x=\frac{3}{2}$$

Therefore the solution of the equation $8x^3-27=0$ is: $\boxed{\frac{3}{2}}$.

Answer 31e.

The terms in the expression on the left side of the given equation has the monomial $3x$ in common.

Let us first take out $3x$ from each term.
$3x(x^2-16)=0$

Factor x^2-16 using the pattern for the difference of two squares.
$3x(x+4)(x-4)=0$

Now, we can apply the Zero product property.
$3x=0$ or $x+4=0$ or $x-4=0$

Solve for x.
$x = 0$ or $x = -4$ or $x = 4$

In the method shown, the factor $3x$ has not been set equal to 0, and thus, the solutions are not correct.

The correct real solutions of the given equation are $x = 0$, $x = 4$, or $x = -4$.

Answer 32e.

Consider the equation:

$$y^3 - 5y^2 = 0 \qquad \ldots\ldots (1)$$

We need to find the real number solution of the equation (1).

From (1), we have

$$y^3 - 5y^2 = 0$$
$$y^2(y-5) = 0 \qquad \text{[Factor common monomial.]}$$

Therefore the solution of the equation $y^3 - 5y^2 = 0$ is: $\boxed{y = 0,\ y = 5}$.

Answer 33e.

Rewrite the given equation in standard form.
$18s^3 - 50s = 0$

The terms in the expression on the left contains the common monomial $2s$.
Take out $2s$ from each term.
$2s(9s^2 - 25) = 0$

Factor $9s^2 - 25$ using the pattern for the difference of two squares.
$2s(3s + 5)(3s - 5) = 0$

Now, we can apply the Zero product property. Set each factor equal to 0.
$2s = 0$ or $3s + 5 = 0$ or $3s - 5 = 0$

Solve for s.
$s = 0$ or $3s = -5$ or $3s = 5$

$s = -\frac{5}{3}$ $\qquad$ $s = \frac{5}{3}$

Therefore, the real-number solutions of the given equation are $0, -\frac{5}{3}$, and $\frac{5}{3}$.

Answer 34e.

Consider the equation:

$$g^3 + 3g^2 - g - 3 = 0 \quad \text{......} (1)$$

We need to find the real number solution of the equation (1).

From (1), we have

$$g^3 + 3g^2 - g - 3 = 0$$
$$g^2(g+3) - 1(g+3) = 0$$
$$(g^2 - 1)(g+3) = 0 \quad \text{[Factor trinomial.]}$$
$$(g+1)(g-1)(g+3) = 0 \quad \text{[Difference of two squares.]}$$

Therefore the solution of the equation $g^3 + 3g^2 - g - 3 = 0$ is: $\boxed{g = -1,\ g = 1,\ g = -3}$.

Answer 35e.

We can factor the expression on the left side of the given equation by grouping.

$m^3 + 6m^2 - 4m - 24 = 0$

Take out the common monomial m^2 from the first two terms, and 4 from the second two terms in the given polynomial.

$m^2(m + 6) - 4(m + 6) = 0$

Now, use the distributive property.

$(m + 6)(m^2 - 4) = 0$

Factor the difference of two squares $m^2 - 4$.

$(m + 6)(m + 2)(m - 2) = 0$

Apply the zero product property. For this, set each factor equal to 0.

$m + 6 = 0$ or $m + 2 = 0$ or $m - 2 = 0$

Solve for m.

$m = -6$ or $m + 2 = 0$ or $m - 2 = 0$

$m = -2 \qquad m = 2$

Therefore, the real-number solutions of the given equation are −6, −2, or 2.

Answer 36e.

Consider the equation:

$$4w^4 + 40w^2 - 44 = 0 \quad \text{......} (1)$$

We need to find the real number solution of the equation (1).

From (1), we have

$$4w^4+40w^2-44=0$$
$$4(w^4+10w^2-11)=0 \quad \text{[Factor common monomial.]}$$
$$4(w^2-1)(w^2+11)=0 \quad \text{[Factor trinomial.]}$$
$$4(w+1)(w-1)(w+11)=0 \quad \text{[Difference of two squares.]}$$

Therefore the solution of the equation $4w^4+40w^2-44=0$ is: $\boxed{w=-1,\ w=1,\ w=-11}$.

Answer 37e.

Rewrite the given equation in standard form.
$4z^5-84z^3=0$

Take out the common monomial $4z^3$ from the terms in the expression on the left.
$4z^3(z^2-21)=0$

Use the pattern $a^2-b^2=(a+b)(a-b)$ to factor z^2-21.
$4z^3\left(z+\sqrt{21}\right)\left(z-\sqrt{21}\right) = 0$

Apply the Zero product property. For this, set each factor equal to 0 and solve for z.

$4z^3 = 0$ or $z+\sqrt{21} = 0$ or $z-\sqrt{21} = 0$

$z = 0$ $\quad z = -\sqrt{21}$ $\quad z = \sqrt{21}$

Therefore, the real-number solutions of the given equation are $0, -\sqrt{21}$, and $\sqrt{21}$.

Answer 38e.

Consider the equation:

$$5b^3+15b^2+12b=-36 \quad \text{...... (1)}$$

We need to find the real number solution of the equation (1).

From (1), we have

$$5b^3+15b^2+12b=-36$$
$$5b^3+15b^2+12b+36=0 \quad \text{[Adding 36 in both sides]}$$
$$5b^2(b+3)+12(b+3)=0 \quad \text{[Factor common monomial.]}$$
$$(5b^2+12)(b+3)=0 \quad \text{[Factor trinomial.]}$$

Therefore the solution of the equation $5b^3+15b^2+12b=-36$ is:

$\boxed{b=-\frac{12}{5},\ b=\frac{12}{5},\ b=-3}$.

Answer 39e.

We can factor the expression on the left side of the given equation by grouping.

Take out the common monomial x^4 from the first two terms, and 9 from the second two terms in the given polynomial.

$x^4(x^2-4)-9(x^2-4) = 0$

Now, use the distributive property.

$(x^4-9)(x^2-4)=0$

Factor x^4-9 and x^2-4 using the pattern for the difference of two squares.

$(x^2-3)(x^2+3)(x+2)(x-2)=0$

Again, we can factor x^2-3 as $\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$.

$\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)\left(x^2+3\right)(x+2)(x-2) = 0$

Apply the zero product property. For this, set each factor equal to 0 and solve for x.

$x+\sqrt{3}=0$ or $x-\sqrt{3}=0$ or $x^2+3=0$ or $x+2=0$ or $x-2=0$

$x=-\sqrt{3}$ $\quad x=\sqrt{3}$ $\quad x=-2$ $\quad x=2$

Therefore, the real-number solutions of the given equation are $-\sqrt{3}, \sqrt{3}, -2$, and 2.

Answer 40e.

Consider the equation:

$$48p^5=27p^3 \quad \text{......} (1)$$

We need to find the real number solution of the equation (1).

From (1), we have

$$48p^5=27p^3$$

$48p^5-27p^3=0$ [Substracting $27p^3$ from both sides]

$3p^3\left(16p^2-9\right)=0$ [Factor common monomial.]

$3p^3(4p+3)(4p-3)=0$ [Factor trinomial.]

Therefore the solution of the equation $48p^5=27p^3$ is: $\boxed{p=0,\ p=-\frac{3}{4},\ p=\frac{3}{4}}$.

Answer 41e.

Let us first rewrite the given equation in standard form.

$3x^4-x^3-27x^2+9x=0$

Factor the expression on the left side by grouping. For this, take out the common monomial x^3 from the first two terms, and $9x$ from the second two terms.

$x^3(3x-1)-9x(3x-1) = 0$

Now, use the distributive property.
$(x^3 - 9x)(3x - 1) = 0$

We can factor out the common monomial x from $x^3 - 9x$.
$x(x^2 - 9)(3x - 1) = 0$

Factor $x^2 - 9$ using the pattern $a^2 - b^2 = (a + b)(a - b)$.
$x(x + 3)(x - 3)(3x - 1) = 0$

Apply the Zero product property. For this, set each factor equal to 0 and solve for x.

$x = 0$ or $x + 3 = 0$ or $x - 3 = 0$ or $3x - 1 = 0$

$x = -3$ $\quad x = 3$ $\quad x = \frac{1}{3}$

The real-number solutions of the given equation are thus -3, 0, 3, and $\frac{1}{3}$, which matches with the choice **C**.

Answer 42e.

Consider the polynomial:

$$16x^3 - 44x^2 - 42x \qquad \text{...... (1)}$$

We need to factor the polynomial (1) complicity.

From (1), we have

$$16x^3 - 44x^2 - 42x = 2x(8x^2 - 22x - 21) \qquad \text{[Factor common monomial.]}$$
$$= 2x(4x - 3)(2x - 7) \qquad \text{[Factor trinomial.]}$$

Therefore the factor of the polynomial $16x^3 - 44x^2 - 42x$ is: $\boxed{2x(4x-3)(2x-7)}$.

Answer 43e.

Rewrite the given expression using the replacement $u = n^2$.
$u^2 - 4u - 60$

Now, factor the perfect square trinomial $u^2 - 4u - 60$.
$u^2 - 4u - 60 = (u - 10)(u + 6)$

Substitute n^2 for u.
$(u - 10)(u + 6) = (n^2 - 10)(n^2 + 6)$

Therefore, the given polynomial in completely factored form is $(n^2 - 10)(n^2 + 6)$.

Answer 44e.

Consider the polynomial:

$$-4b^4 - 500b \quad \text{...... (1)}$$

We need to factor the polynomial (1) complicity.

From (1), we have

$$-4b^4 - 500b = -4b\left(b^3 + 125\right) \quad \text{[Factor common monomial]}$$

$$= -4b\left(b^3 + 5^3\right) \quad \text{[Sum of two squares]}$$

$$= -4b(b+5)\left(b^2 - 5b + 25\right) \quad \left[\text{Sum of two squares: } a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right]$$

Therefore the factor of the polynomial $-4b^4 - 500b$ is: $\boxed{-4b(b+5)\left(b^2 - 5b + 25\right)}$.

Answer 45e.

We can factor the given expression by grouping.
Take out the common monomial $3a^2$ from the first two terms and 12 from the last two terms.

$36a^3 - 15a^2 + 84a - 35 = 3a^2(12a - 5) + 7(12a - 5)$

Now, use the distributive property.

$3a^2(12a - 5) + 7(12a - 5) = (12a - 5)(3a^2 + 7)$

Therefore, the given expression in completely factored form is $(12a - 5)(3a^2 + 7)$.

Answer 46e.

Consider the polynomial:

$$18c^4 + 57c^3 - 10c^2 \quad \text{...... (1)}$$

We need to factor the polynomial (1) complicity.

From (1), we have

$$18c^4 + 57c^3 - 10c^2 = c^2\left(18c^2 + 57c - 10\right) \quad \text{[Factor common monomial]}$$

$$= c^2(6c - 1)(3c + 10) \quad \text{[Factor trinomial]}$$

Therefore the factor of the polynomial $18c^4 + 57c^3 - 10c^2$ is: $\boxed{c^2(6c - 1)(3c + 10)}$.

Answer 47e.

Rewrite the given expression using the replacement $u = d^2$.
$2u^2 - 13u - 45$

Factor the trinomial $2u^2 - 13u - 45$.
$(u - 9)(2u + 5)$

Now, replace u with d^2.
$(d^2 - 9)(2d^2 + 5)$

Factor $d^2 - 9$ using the pattern for the difference of two squares.
$(d + 3)(d - 3)(2d^2 + 5)$

Therefore, the given expression in completely factored form is $(d + 3)(d - 3)(2d^2 + 5)$.

Answer 48e.

Consider the polynomial:

$$32x^5 - 108x^2 \qquad \text{...... (1)}$$

We need to factor the polynomial (1) complicity.

From (1), we have

$$32x^5 - 108x^2 = 4x^2\left(8x^3 - 27\right) \qquad \text{[Fector common monomial]}$$

$$= 4x^2\left\{(2x)^3 - 3^3\right\} \qquad \text{[Difference of two squares]}$$

$$= 4x^2(2x-3)\left(4x^2+6x+9\right) \qquad \left[\text{Difference of two squares: } a^3 - b^3 = (a-b)\left(a^2+ab+b^2\right)\right]$$

Therefore the factor of the polynomial $32x^5 - 108x^2$ is: $\boxed{4x^2(2x-3)\left(4x^2+6x+9\right)}$.

Answer 49e.

The given expression contains a common monomial $2y^2$.
$8y^6 - 38y^4 - 10y^2 = 2y^2(4y^4 - 19y^2 - 5)$

Now, rewrite the expression using the replacement $u = y^2$.
$2u(4u^2 - 19u - 5)$

Factor the trinomial $4u^2 - 19u - 5$.
$2u(4u + 1)(u - 5)$

Now, replace u with y^2.
$2y^2(4u^2 + 1)(y^2 - 5)$

Therefore, the given expression in completely factored form is $2y^2(4u^2 + 1)(y^2 - 5)$.

Answer 50e.

Consider the polynomial:

$$z^5 - 3z^4 - 16z + 48 \qquad \ldots\ldots (1)$$

We need to find the real number solution of the equation (1).

From (1), we have

$$\begin{aligned} z^5 - 3z^4 - 16z + 48 &= z^4(z-3) - 16(z-3) && \text{[Factor by grouping]} \\ &= (z^4 - 16)(z-3) && \text{[Distributive property]} \\ &= (z^2+4)(z^2-4)(z-3) && \left[\begin{array}{l}\text{Difference of two squares,} \\ a^2 - b^2 = (a+b)(a-b)\end{array}\right] \\ &= (z^2+4)(z+2)(z-2)(z-3) && \left[\begin{array}{l}\text{Difference of two squares,} \\ a^2 - b^2 = (a+b)(a-b)\end{array}\right] \end{aligned}$$

Therefore the factor of the polynomial $z^5 - 3z^4 - 16z + 48$ is:

$\boxed{(z^2+4)(z+2)(z-2)(z-3)}$.

Answer 51e.

The area of a rectangular region is given by the product of its length and width. It is given that the length of the region is $3x + 2$, the width is $x + 4$ and the area is 48.

$$\begin{array}{ccccc} \text{Area} & = & \text{length} & \cdot & \text{width} \\ \Downarrow & & \Downarrow & & \Downarrow \\ 48 & = & (3x+2) & \cdot & (x+4) \end{array}$$

We get the equation $48 = (3x + 2)(x + 4)$.

Rewrite the equation in standard form.

$$48 = 3x^2 + 14x + 8$$

$$0 = 3x^2 + 14x - 40$$

Now, factor the trinomial $3x^2 + 14x - 40$.

$0 = (x - 2)(3x + 20)$

Apply the Zero product property and solve for x.

$$x - 2 = 0 \quad \text{or} \quad 3x + 20 = 0$$

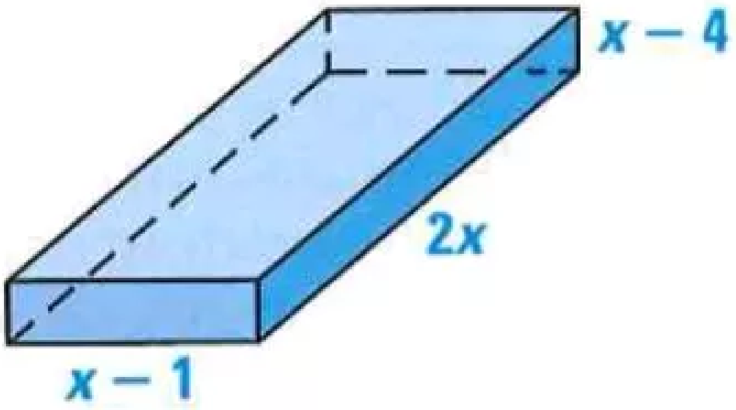
$$x = 2 \qquad\qquad x = -\frac{20}{3}$$

Since length cannot be negative, we only consider the value 2.

Therefore, the possible value of x is 2.

Answer 52e.

Consider the rectangle as shown below:



The volume is: 40 and we need to find the possible value of x.

The volume is calculated by

$$\text{Volume} = \text{width} \cdot \text{length} \cdot \text{height}$$

From the diagram, we have

$$\text{Volume} = \text{width} \cdot \text{length} \cdot \text{height}$$

$40 = (x-1)\cdot(2x)\cdot(x-4)$ [Writting the equation]

$40 = (2x^2 - 2x)\cdot(x-4)$ [Multiplying first two terms]

$40 = 2x^3 - 8x^2 - 2x^2 + 8x$ [Writing in standard form]

$0 = 2x^3 - 10x^2 + 8x - 40$ [Substracting 40 from both sides]

$0 = 2x^2(x-5) + 8(x-5)$ [Factor by grouping]

$0 = (2x^2 + 8)(x-5)$ [Distributive property]

The only real solution is $x = 5$.

Therefore the rectangle is 10 units length, 4 units width and 1 units height.

Answer 53e.

The volume of a cone is given by $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height. It is given that the radius of the cone is $2x - 5$, the height is $3x$ and the volume is 125π.

$$125\pi = \frac{1}{3}\pi(2x-5)^2(3x)$$

Simplify.

$125 = (2x - 5)^2 x$

Rewrite the equation in standard form.

$125 = 4x^3 - 20x^2 + 25x$

$0 = 4x^3 - 20x^2 + 25x - 125$

Now, factor $4x^3 - 20x^2 + 25x - 125$ by grouping.

$0 = (x - 5)(4x^2 + 25)$

Apply the Zero product property and solve for x.

$x - 5 = 0$ or $4x^2 + 25 = 0$

$x = 5$ $\quad x = \sqrt{-\frac{25}{4}}$

Since the only real solution is 5, the only possible value for x is 5.

Answer 54e.

Consider the polynomial:

$$x^3y^6 - 27 \qquad \ldots\ldots (1)$$

We need to factor the polynomial (1) complicity.

From (1), we have

$$x^3y^6 - 27 = \left(\left(xy^2\right)^3 - 3^3\right) \qquad \text{[Difference of two squares]}$$

$$= \left(xy^2 - 3\right)\left(x^2y^4 + 3xy^2 + 9\right) \qquad \left[\begin{array}{l}\text{Difference of two squares: } a^3 - b^3 = \\ (a-b)\left(a^2 + ab + b^2\right)\end{array}\right]$$

Therefore the factor of the polynomial $x^3y^6 - 27$ is: $\boxed{\left(xy^2 - 3\right)\left(x^2y^4 - 3xy^2 + 9\right)}$.

Answer 55e.

We can factor the given polynomial by grouping.

Take out the common monomial c^2 from the first two terms, and d^2 from the last two terms.

$7ac^2 + bc^2 - 7ad^2 - bd^2 = c^2(7a + b) - d^2(7a + b)$

Now, use the distributive property.

$c^2(7a + b) - d^2(7a + b) = (c^2 - d^2)(7a + b)$

Factor $c^2 - d^2$ using the pattern for the difference of two squares.

$(c^2 - d^2)(7a + b) = (c + d)(c - d)(7a + b)$

Therefore, the given expression in completely factored form is $(c + d)(c - d)(7a + b)$.

Answer 56e.

Consider the polynomial:

$$x^{2n} - 2x^n + 1 \qquad \ldots\ldots (1)$$

We need to factor the polynomial (1) complicity.

From (1), we have

$$x^{2n} - 2x^n + 1 = \left(x^n\right)^2 - 2x^n + 1^2$$

$$= \left(x^n - 1\right)^2 \qquad \left[\text{Perfect square: } (a-b)^2 = a^2 - 2ab + b^2\right]$$

Therefore the factor of the polynomial $x^{2n} - 2x^n + 1$ is: $\boxed{\left(x^n - 1\right)^2}$.

Answer 57e.

We can factor the given polynomial by grouping.
Take out the common monomial a^2b^2 from the first two terms, and $2ab$ from the next two terms.

$$a^5b^2 - a^2b^4 + 2a^4b - 2ab^3 + a^3 - b^2 = a^2b^2(a^3 - b^2) + 2ab(a^3 - b^2) + a^3 - b^2$$

Now, use the distributive property.

$$a^2b^2(a^3 - b^2) + 2ab(a^3 - b^2) + a^3 - b^2 = (a^2b^2 + 2ab + 1)(a^3 - b^2)$$

Therefore, the given expression in completely factored form is $(a^2b^2 + 2ab + 1)(a^3 - b^2)$.

Answer 58e.

Suppose at the ruins of Caesarea, archaeologists discovered a huge hydraulic concrete block with a volume of 945 cubic meters. The block's diameters are x meters high by $12x-15$ meters long by $12x-21$ meters wide. We need to find the height of the block.

The volume can be calculated by

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}$$

Now,

$$\text{volume} = \text{length} \cdot \text{width} \cdot \text{height}$$

$$945 = (12x-15)\cdot(12x-21)\cdot(x) \quad \text{[Writing the equation]}$$

$$945 = (12x-15)\cdot(12x^2-21x) \quad \text{[Multiplying first two terms]}$$

$$945 = 144x^3 - 252x^2 - 180x^2 + 315x \quad \text{[Writting in standard form]}$$

$$0 = 144x^3 - 432x^2 + 315x - 945 \quad \text{[Substracting 945 from both sides]}$$

$$0 = 144x^2(x-3) + 315(x-3) \quad \text{[Factor by grouping]}$$

$$0 = (144x^2 + 315)(x-3) \quad \text{[Distributive property]}$$

The only real solution is $x = 3$.

Therefore the height of the block is 3 meters.

Answer 59e.

Considering a chocolate mold shaped like a rectangular prism for a candy manufacturer wherein the molds outer dimensions should be in the ratio 1:3:6.

In order to find the outer dimensions if it can hold 112 cubic centimeters of the chocolate if the mold have a thickness of 1cm in all directions.

Since the outer dimension's ratio is 1:3:6, therefore let us consider the dimension as

$$x, 3x, 6x$$

Again, as the thickness is 1 cm in all directions, therefore 2 must be subtracted from the dimensions of length, breadth and height, $(x-2),(3x-2),(6x-2)$

The volume can be calculated by

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}$$

Now,

$$\text{volume} = \text{length} \cdot \text{width} \cdot \text{height}$$

$$112 = (x-2)\cdot(3x-2)\cdot(6x-2)$$

$$112 = (3x^2 - 8x + 4)\cdot(6x-2)$$

$$112 = 18x^3 - 6x^2 - 48x^2 + 16x + 24x - 8$$

$$0 = 18x^3 - 6x^2 - 48x^2 + 16x + 24x - 120 \quad \text{Standard form.}$$

$$0 = 18x^3 - 54x^2 + 40x - 120 \quad \text{Combine like terms.}$$

$$0 = 18x^2(x-3) + 40(x-3) \quad \text{Factor.}$$

$$0 = (18x^2 + 40)(x-3)$$

The only real solution is $x = 3$.

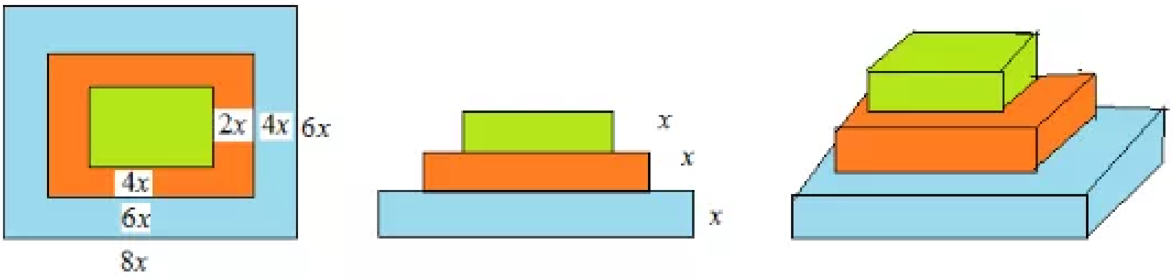
Therefore the dimensions of length, breadth and height are respectively $x, 3x, 6x$

That is $\boxed{\text{3 cm, 6cm, 18cm}}$

Answer 60e.

(a)

Consider the platform has the dimensions shown in the following figures, and has a total volume of 1250 cubic feet.



The volume is calculated by

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}$$

For the larger platform, the volume is given by

$$V_1 = 8x \times 6x \times x$$

$$V_1 = 48x^3 \quad \ldots\ldots (1)$$

For the middle platform, the volume is given by

$$V_2 = 6x \times 4x \times x$$

$$V_2 = 24x^3 \quad \ldots\ldots (2)$$

For the smaller platform, the volume is given by

$$V_3 = 4x \times 2x \times x$$

$$V_3 = 8x^3 \quad \ldots\ldots (3)$$

Therefore, the total volume from equation (1), (2) and (3).

$$\text{Volume} = V_1 + V_2 + V_3$$

$$= 48x^3 + 24x^3 + 8x^3$$

$$\text{Volume} = \boxed{80x^3} \quad \ldots\ldots (4)$$

(b)

From the hypothesis, the total volume is 1250 cubic feet must be equal to the total Volume as calculated from the above equation (4)

$$\boxed{80x^3 = 1250} \quad \ldots\ldots (5)$$

(c)
To calculate the dimensions each of the three levels of the platform.
From equation (5) we have

$$80x^3 = 1250$$
$$x^3 = \frac{1250}{80} \quad \text{Dividing both sides by } 80.$$
$$x^3 = 15.625$$
$$x = 2.5 \quad \text{Taking cube root of } 15.625$$

For larger platform,

$$\text{Length} = 8 \times 2.5$$
$$= \boxed{20 \text{ feet}}$$
$$\text{Breadth} = 6 \times 2.5$$
$$= \boxed{15 \text{ feet}}$$
$$\text{Height} = \boxed{2.5 \text{ feet}}$$

For middle platform,

$$\text{Length} = 6 \times 2.5$$
$$= \boxed{15 \text{feet}}$$
$$\text{Breadth} = 4 \times 2.5$$
$$= \boxed{10 \text{ feet}}$$
$$\text{Height} = \boxed{2.5 \text{ feet}}$$

For smaller platform,

$$\text{length} = 4 \times 2.5$$
$$= \boxed{10 \text{ feet}}$$
$$\text{Breadth} = 2 \times 2.5$$
$$= \boxed{5 \text{ feet}}$$
$$\text{Height} = \boxed{2.5 \text{ feet}}$$

Answer 61e.

Let the length of the rectangular prism be x.

It is given that the width and height are 5 inches less than the length. From this, we have width = height = $(x - 5)$ inches.

The volume of the prism, which is given as 250 cubic inches, is the product of its length, width, and height.

$$\begin{array}{ccccccc} \text{Volume} \\ \text{(cubic inches)} & = & \text{length (inches)} & \cdot & \text{width (inches)} & \cdot & \text{height (inches)} \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ 250 & = & x & \cdot & (x-5) & \cdot & (x-5) \end{array}$$

We get the equation $250 = x(x-5)(x-5)$.

Rewrite the equation in standard form.

$$250 = x^3 - 10x^2 + 25x$$
$$0 = x^3 - 10x^2 + 25x - 250$$

Now, factor $x^3 - 10x^2 + 25x - 250$ by grouping.

$0 = x^2(x-10) + 25(x-10)$

Apply the distributive property.

$0 = (x-10)(x^2+25)$

Use the zero product property and solve for x.

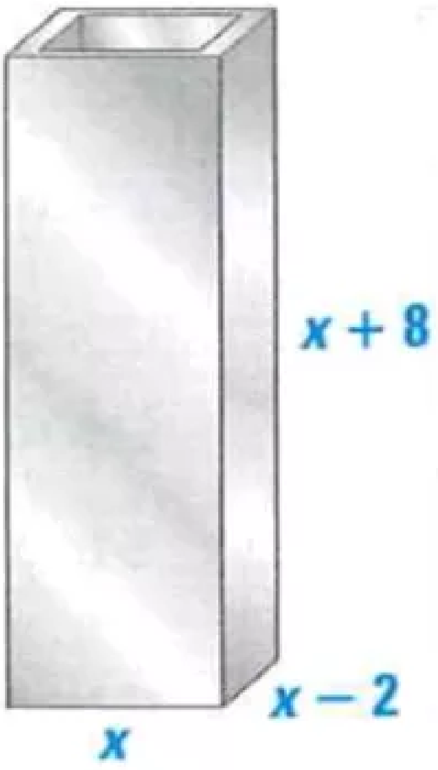
$$x - 10 = 0 \quad \text{or} \quad x^2 + 25 = 0$$
$$x = 10 \qquad\qquad x^2 = -25$$

The only real solution being 10, we get the length of the prism as 10 inches. The width and height of the prism will then be 10 – 5 or 5 inches.

Therefore, the dimensions of the prism are 10 inches by 5 inches by 5 inches.

Answer 62e.

Suppose a manufacturer wants to build a rectangular stainless steel tank with a holding capacity of 670 gallon, or about 89.58cubic feet. The tank's walls will be one half inch thick, and about 6.24 cubic feet of steel will be used for the tank. The manufacturer wants the outer diameters of the tank to be related as follows:
The width should be 2 feet less than the length.
And the height should be 8 feet more than the length.



We need to find the outer diameter of the tank.

Now,
$\text{volume} = \text{length} \cdot \text{width} \cdot \text{height}$

$89.58 = (x-2-0.0833)\cdot(x-0.0833)\cdot(x+8-0.0417)$ [Writing the equation]

$89.58 = (x-2.0833)\cdot(x-0.0833)\cdot(x+7.9166)$ [Multiplying first two terms]

$89.58 = (x^2-2.0833x-0.0833x+0.17354)\cdot(x+7.9166)$ [Writing in standard form]

$0 = (x^2-2.1666x+0.17354)\cdot(x+7.9166)$

$0 = x^3+5.76x^2-16.978x-88.21$ [Substracting 89.58 from both sides]

$0 = x^2(x+5.76)-16.978(x+5.76)$

$0 = (x^2-16.978)(x+5.76)$ [Distributive property]

$0 = (x+4.12)(x-4.12)(x+5.76)$ [Difference of two squares]

The only real solution is $x = 4.12$.
Therefore the tank is 12.12 meters height, 2.12 meters width and 4.12 meters length.

Answer 63e.

The volume of the platform is the product of its length, width and height.
Volume $= (x-2)(3-2x)(3x+4)$

Let us find the value of x when the volume is $\frac{7}{3}$ cubic feet.

$$(x-2)(3-2x)(3x+4) = \frac{7}{3}$$

Simplify and rewrite the equation in standard form.

$$-6x^3 + 13x^2 + 10x - 24 = \frac{7}{3}$$
$$-18x^3 + 39x^2 + 30x - 72 = 7$$
$$18x^3 - 39x^2 - 30x + 79 = 0$$

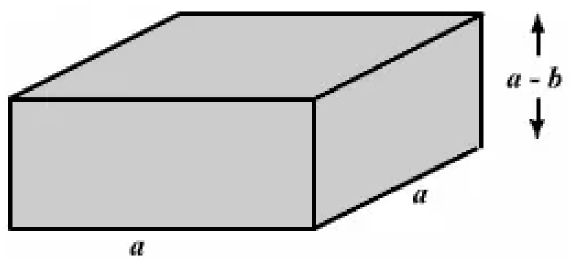
On solving $18x^3 - 39x^2 - 30x + 79 = 0$, we get $x \approx -1.37$.

It can be seen that the only value of x that corresponds to the volume $\frac{7}{3}$ is about -1.37, which is negative.

Therefore, the volume of the platform cannot be $\frac{7}{3}$ cubic feet.

Answer 65e.

a. Let us calculate the volumes of the three solids separately.
Begin with solid I. Make a sketch of it.

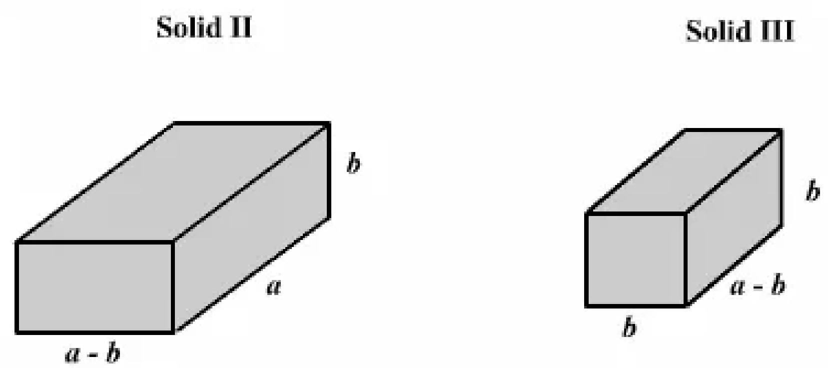


The length and width of the solid is a, and the height is $a - b$.

We know that the volume of a solid is the product of its length, width and height.

$$\text{Volume of solid I} = a(a)(a-b) = a^2(a-b) = a^3 - a^2b$$

In a similar way, find the volumes of solid II and solid III.



Volume of solid II $= (a-b)ab$
$= a^2b - ab^2$

Volume of solid III $= b(a-b)b$
$= ab^2 - b^3$

Add the volumes of the three solids.
$a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3 = a^3 - b^3$

Therefore, the sum of the volumes of solid 1, solid II, and solid III is $a^3 - b^3$.

b. The volumes of the three solids calculated in part **a** are as follows:

Volume of solid I $= a^2(a-b)$

Volume of solid II $= ab(a-b)$

Volume of solid III $= b^2(a-b)$

c. We know that the factoring pattern used for the difference of two cubes $a^3 - b^3$ is $(a-b)(a^2 + ab + b^2)$.

By using the results from parts **a** and **b**, we can write
$a^3 - b^3 = a^2(a-b) + ab(a-b) + b^2(a-b)$.

It can be seen that the expression on the right contains the common factor $a - b$.
Take out $a - b$ from each term.
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

The equation that we now obtained is same as the pattern used to factor the difference of two cubes.

Answer 66e.

Consider the function:

$$f(x) = -2|x-3| + 5 \qquad \text{...... (1)}$$

We need to graph the function (1).

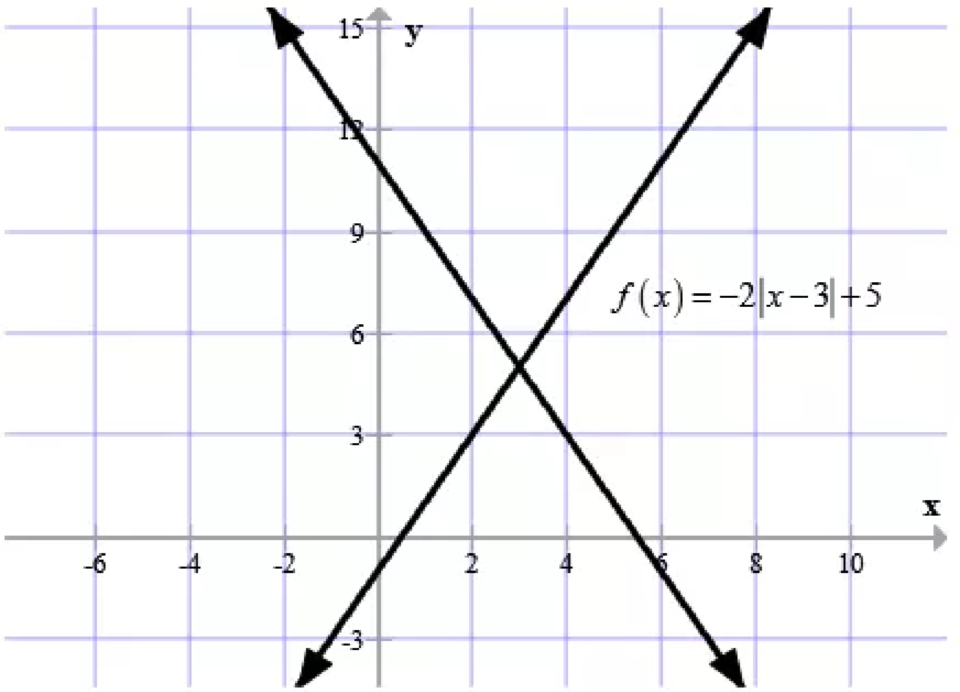
From (1), we have

$$f(x) = -2(x-3)+5$$

And

$$f(x) = -2(-(x-3))+5$$
$$f(x) = 2(x-3)+5$$

Therefore the graph of the function $f(x) = -2|x-3|+5$ is shown below:



Answer 67e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$. On comparing, we have a as $\frac{1}{2}$, b as 4, and c as 5. Since $a = \frac{1}{2} > 0$, the graph opens up.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute $\frac{1}{2}$ for a, and 4 for b and evaluate.

$$-\frac{b}{2a} = -\frac{4}{2\left(\frac{1}{2}\right)}$$
$$= -\frac{4}{1}$$
$$= -4$$

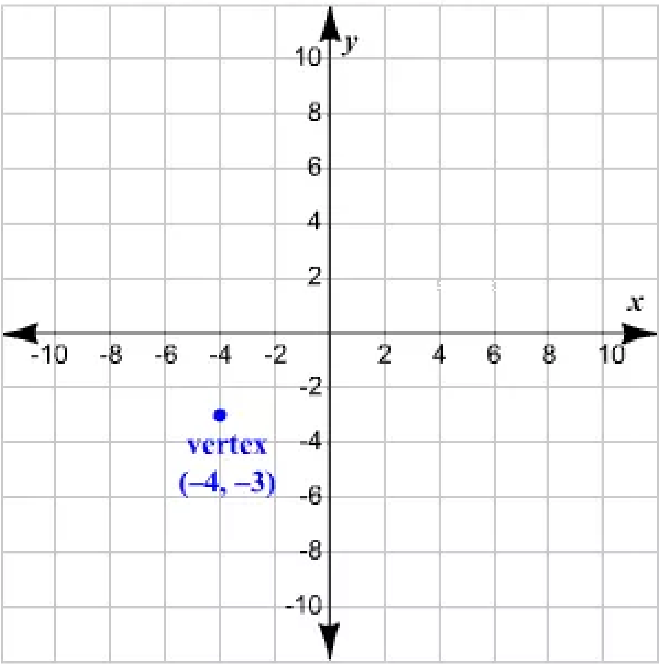
The x-coordinate of the vertex is -4.

Substitute −4 for x in the given function to find the y-coordinate.

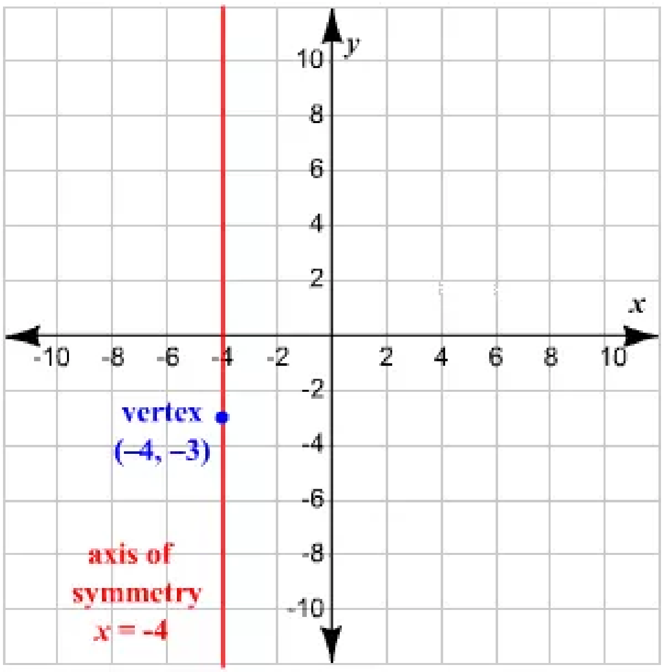
$$\begin{aligned} y &= \frac{1}{2}(-4)^2 + 4(-4) + 5 \\ &= 8 - 16 + 5 \\ &= -3 \end{aligned}$$

Thus, the vertex of the graph of the given function is (−4, −3).

Plot the vertex on a coordinate plane.

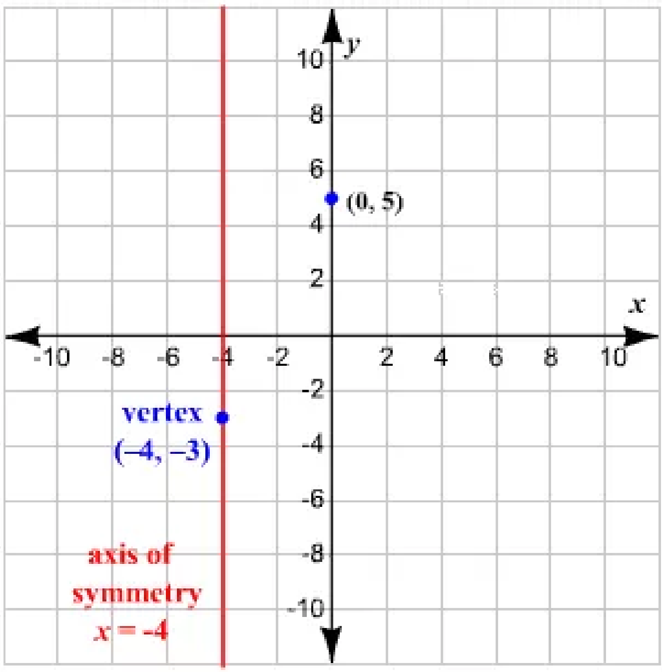


STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$. The axis of symmetry of the given function is the line $x = -4$. Now, draw the axis of symmetry $x = -4$.

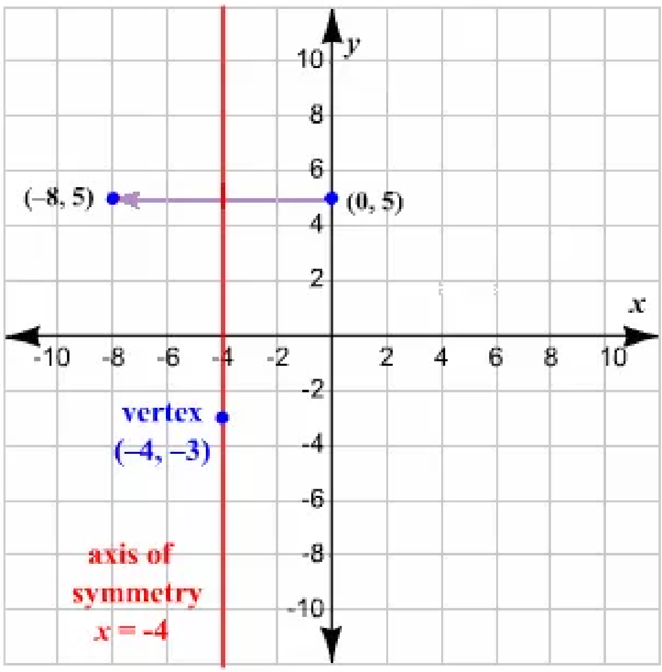


STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.

Thus, the y-intercept of the given function is 5 and (0, 5) is on the parabola.
Plot the point (0, 5) on the same coordinate plane.



Now, reflect the point (0, 5) in the axis of symmetry to get another point.

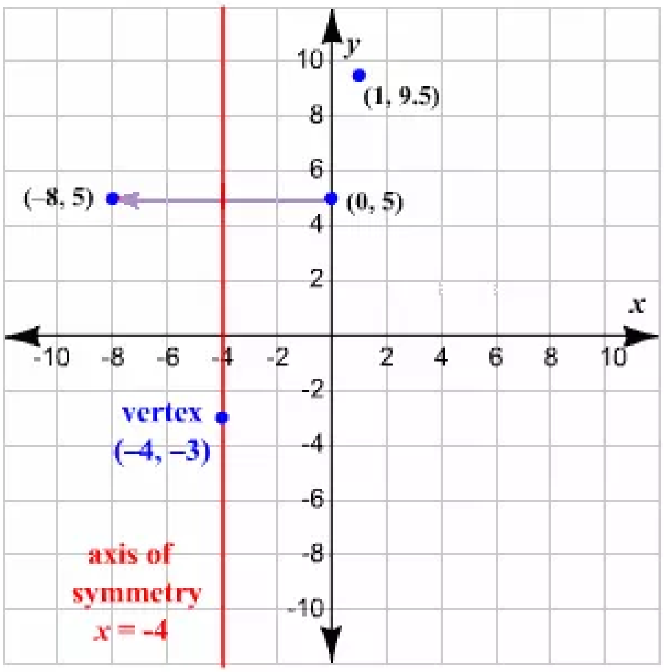


STEP 5 Evaluate the given function for another value of x, say, 1.

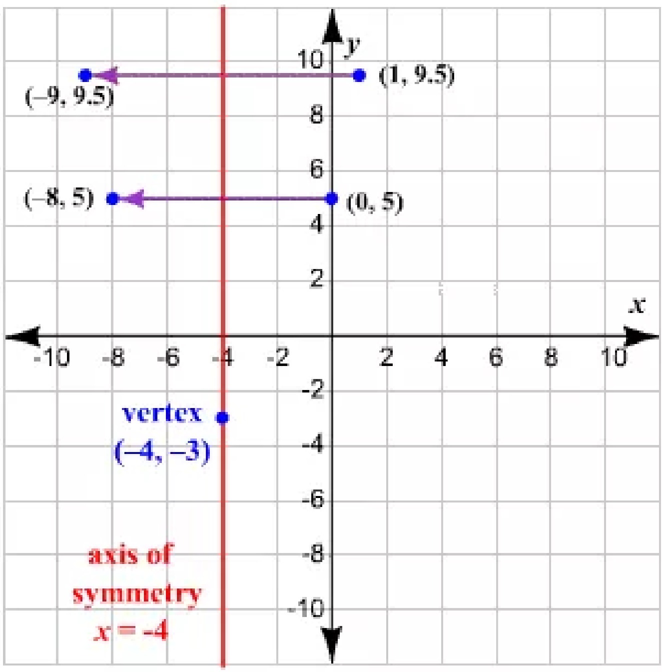
Substitute 1 for x in the function and simplify.

$$y = \frac{1}{2}(1)^2 + 4(1) + 5$$
$$= \frac{1}{2} + 4 + 5$$
$$\approx 9.5$$

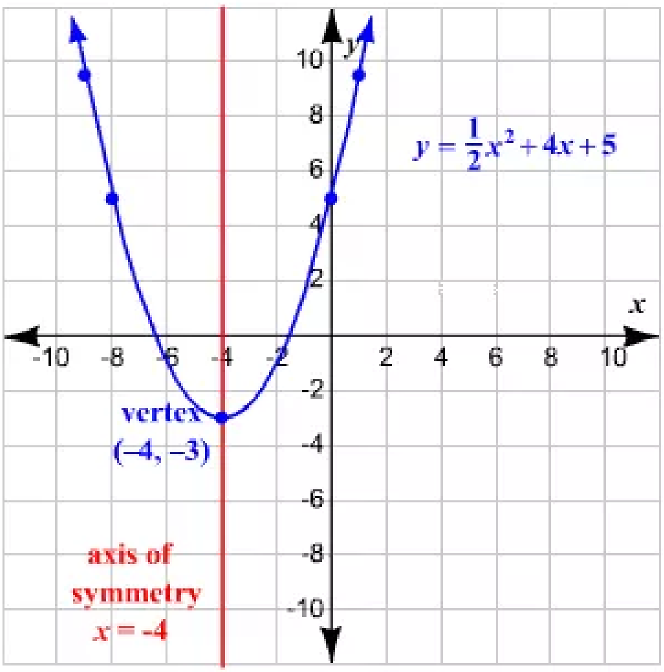
Thus, the point (1, 9.5) lies on the graph. Plot the point on the coordinate plane.



Reflect the point (1, 9.5) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



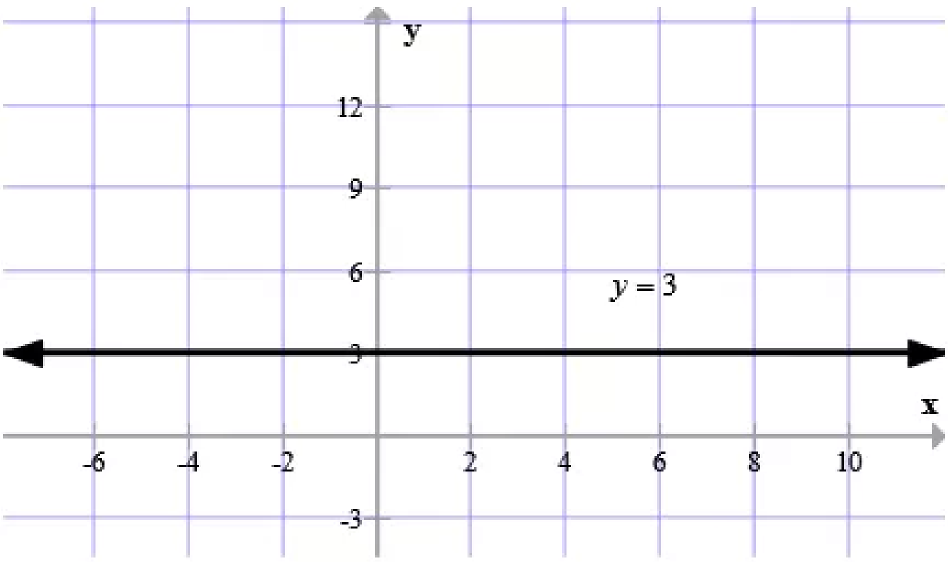
Answer 68e.

Consider the function:

$$y = 3(x+4)^2 + 7 \quad \ldots\ldots (1)$$

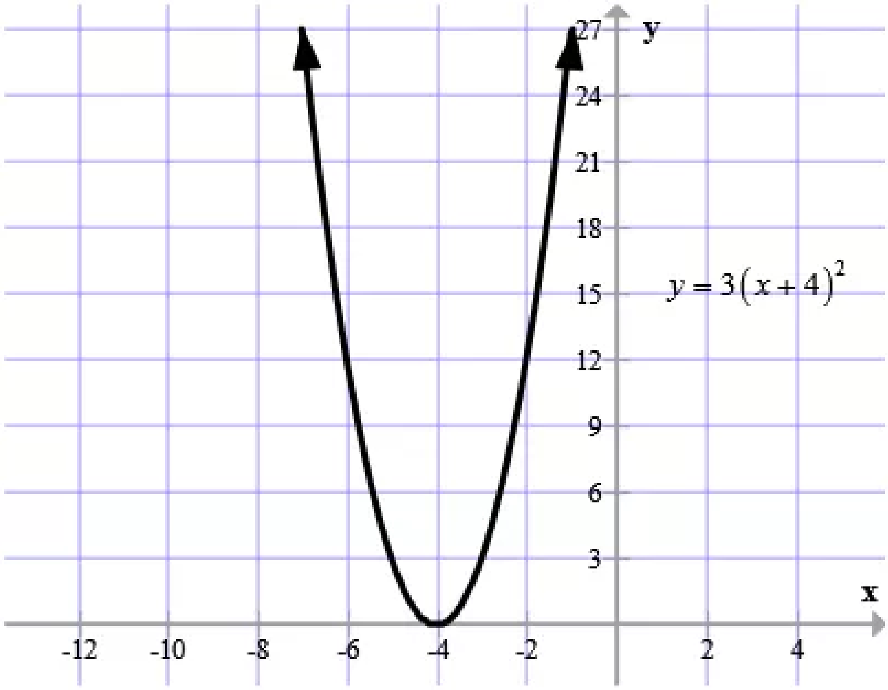
We need to graph the function (1).

First we graph the function $y = 3$ and the graph is shown below:



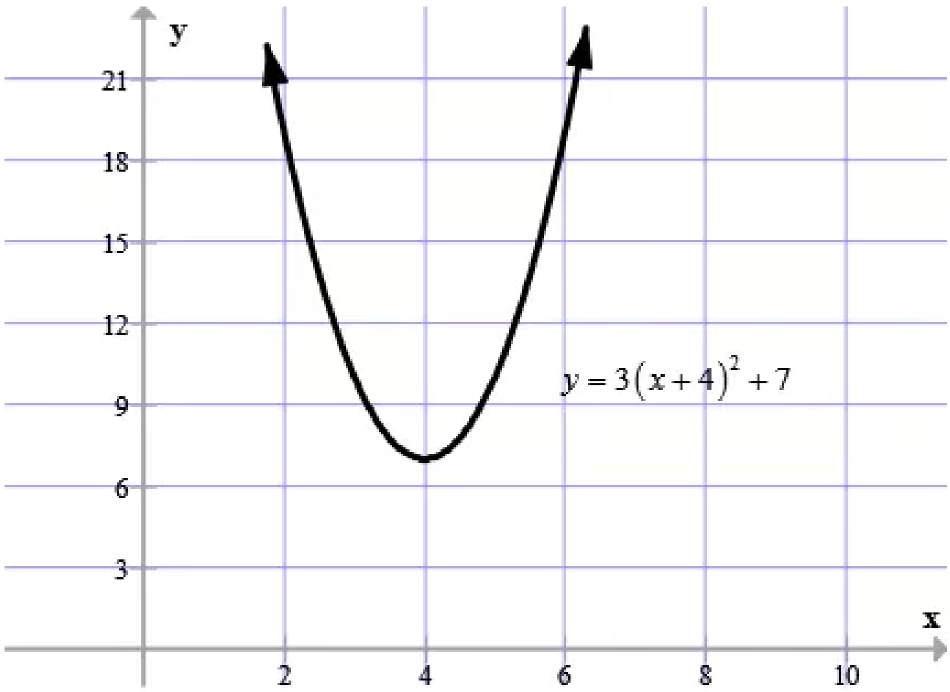
We see that the graph is a straight line.

Now, we graph the function $y=3(x+4)^2$ and the graph is shown below:



We see that the graph of the function $y=3(x+4)^2$ is a parabola shape.

Finally the graph of the function $y=3(x+4)^2+7$ is shown below:



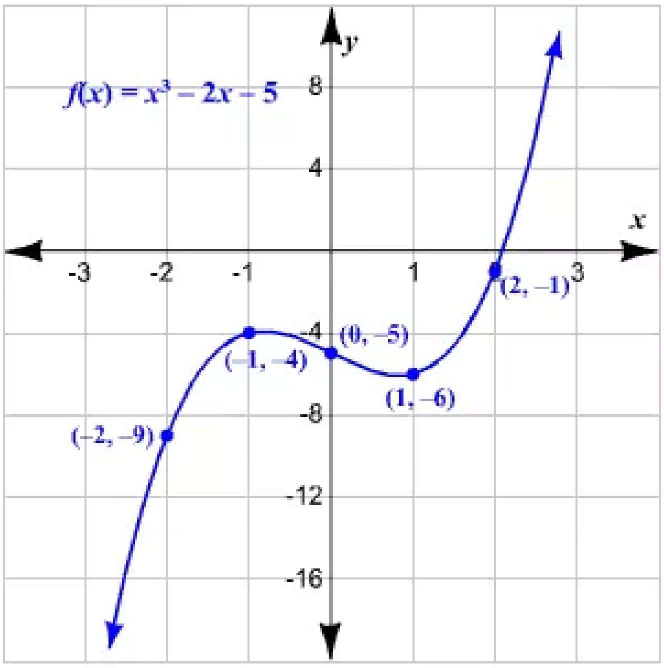
Therefore the graph of the function $y=3(x+4)^2+7$ is a upward parabola.

Answer 69e.

Make a table of values that satisfies the given function. For this, select some values for x and find the corresponding values of $f(x)$ or y.

x	-2	-1	0	1	2
$f(x)$	-9	-4	-5	-6	-1

Plot the points and connect them using a smooth curve.



Answer 70e.

Consider the inequality:

$$y \le 2x - 3 \qquad \ldots\ldots (1)$$

We need to graph the inequality (1) in a coordinate plane.

The boundary equation is:

$$y = 2x - 3.$$

Now, we consider the point $(0,0)$ and we see whether this point satisfied the condition or not.

$$y \le 2x - 3$$
$$0 \le 2 \cdot 0 - 3$$
$$0 \le -3 \qquad [\text{False}]$$

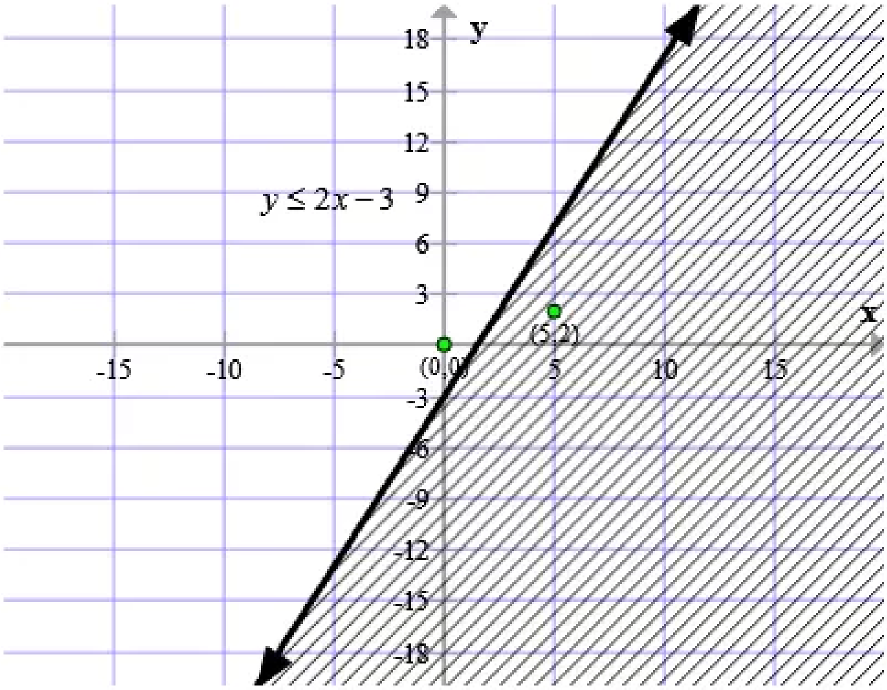
So the point $(0,0)$ is not satisfied the condition.

Again we consider a point $(5,2)$ and we see whether this point satisfied the condition or not.

$$y \le 2x - 3$$
$$2 \le 2 \cdot 5 - 3$$
$$2 \le 10 - 3$$
$$2 \le 7 \qquad [\text{True}]$$

So the point $(5,2)$ is satisfied the condition.

Therefore the graph of the inequality $y \le 2x - 3$ is shown below:



Answer 71e.

STEP 1 **Graph the boundary line of the inequality.**

In order to obtain the boundary line, replace the inequality sign with "=". Thus, we get an equation of the form $y = -5 - x$.

Substitute 0 for y in the equation and solve for x.

$$0 = -5 - x$$

$$x = -5$$

The x-intercept is -5. A point that can be plotted on the graph is $(-5, 0)$.

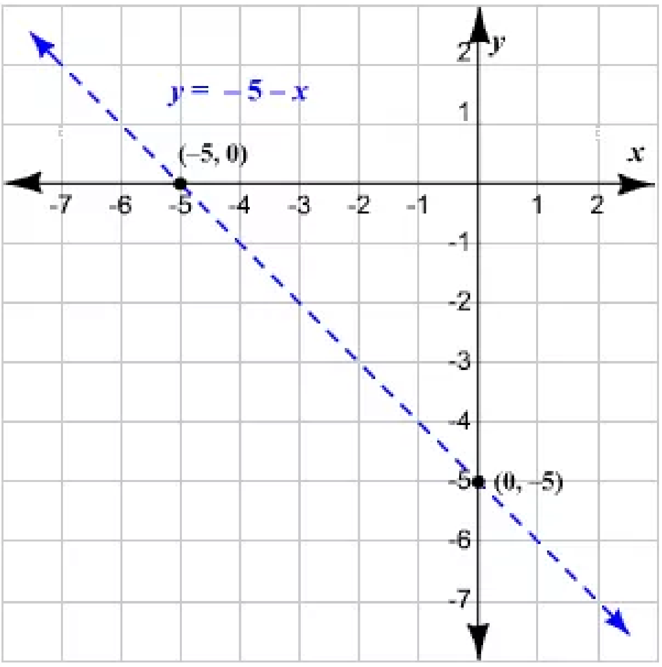
Next, replace x with 0 and solve for y.

$$y = -5 - 0$$

$$= -5$$

Since the y-intercept is -5, another point that can be plotted on the graph is $(0, -5)$.

Plot (−5, 0) and (0, −5) on the graph and draw a line passing through them.
Since > is the inequality sign used, draw a dashed line.



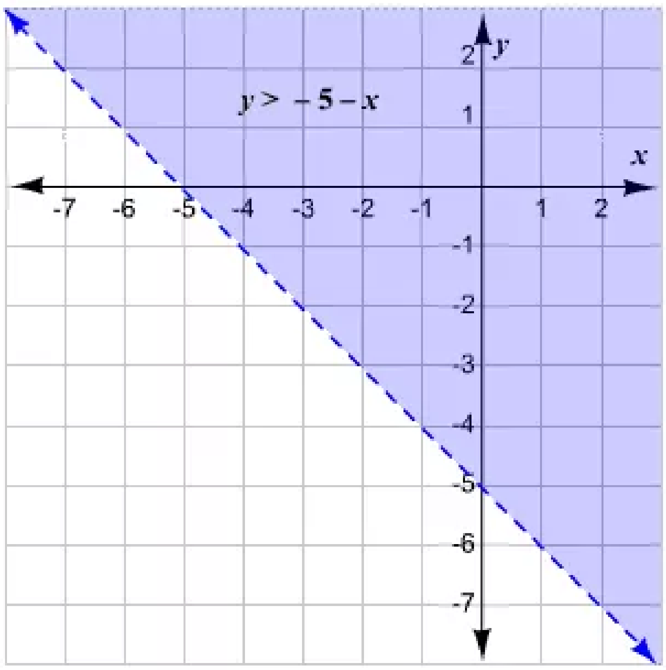
STEP 2 **Test a point.**

Let us take a test point that does not lie on the boundary line, say, (0, 0). Substitute 0 for y, and 0 for x. Check if the test point satisfies the given inequality.

$$0 \overset{?}{>} -5 - 0$$

$$0 > -5 \qquad \text{TRUE}$$

The test point is a solution to the inequality. Shade the half-plane that contains (0, 0).



Answer 72e.

Consider the inequality:

$y < 0.5x + 5$ ...... (1)

We need to graph the inequality (1) in a coordinate plane.

The boundary equation is:

$y = 0.5x + 5$.

Now, we consider the point $(0,0)$ and we see whether this point satisfied the condition or not.

$y < 0.5x + 5$

$0 < 0.5 \times 0 + 5$

$0 < 5$ [True]

So the point $(0,0)$ is satisfied the condition.

Again we consider a point $(2,7)$ and we see whether this point satisfied the condition or not.

$y < 0.5x +$

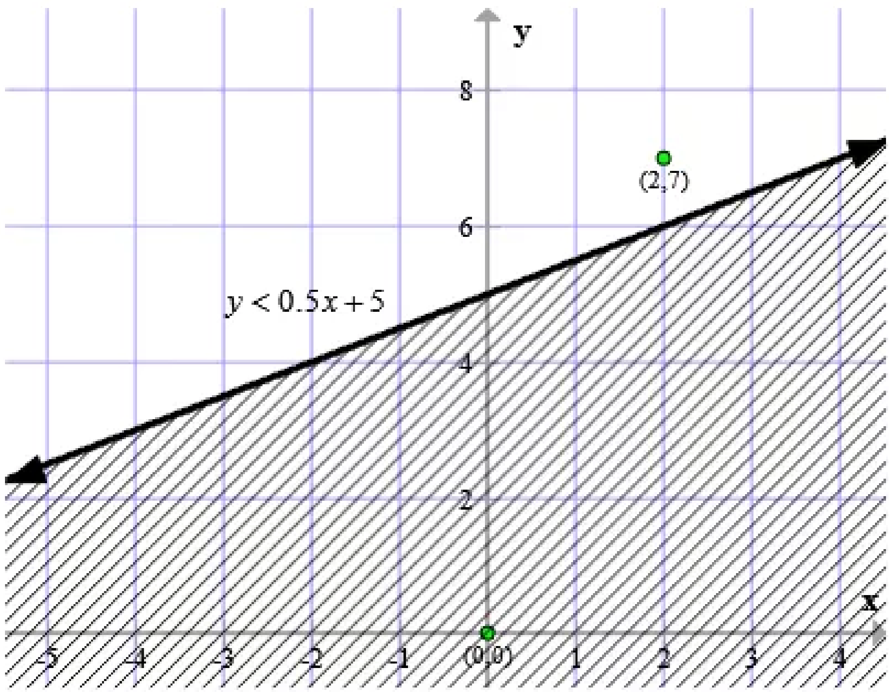
$7 < 0.5 \times 2 + 5$

$7 < 1 + 5$

$7 < 6$ [False]

So the point $(2,7)$ is not satisfied the condition.

Therefore the graph of the inequality $y < 0.5x + 5$ is shown below:



Answer 73e.

STEP 1 **Graph the boundary line of the inequality.**

In order to obtain the boundary line, replace the inequality sign with "=". Thus, we get an equation of the form $4x + 2y = 4$.

Substitute 0 for y in the equation and solve for x.

$$\begin{aligned} 4x + 0 &= 4 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

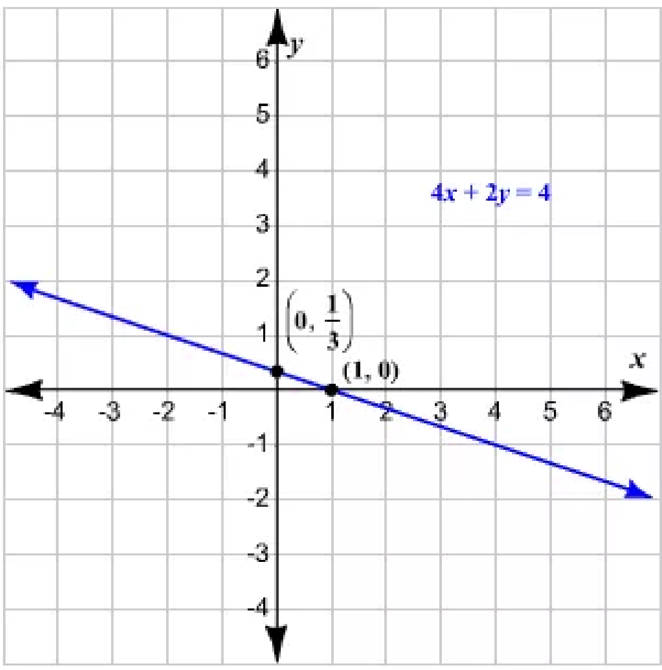
The x-intercept is 1. A point that can be plotted on the graph is (1, 0).

Next, replace x with 0 and solve for y.

$$\begin{aligned} 4(0) + 12y &= 4 \\ 12y &= 1 \\ y &= \frac{1}{3} \end{aligned}$$

Since the y-intercept is $\frac{1}{3}$, another point that can be plotted on the graph is $\left(0, \frac{1}{3}\right)$.

Plot the points $\left(0, \frac{1}{3}\right)$, and (1, 0) on the graph and draw a line passing through them. Since $\leq$ is the inequality sign used, draw a solid line.



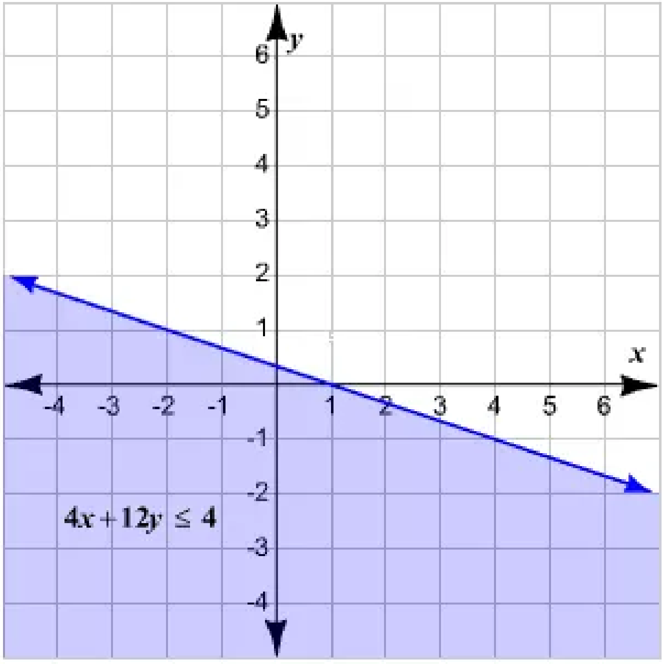
STEP 2 **Test a point.**

Let us take a test point (0, 0) which does not lie on the boundary line. Substitute 0 for y, and 0 for x. Check if the test point satisfies the given inequality.

$$4(0) + 12(0) \overset{?}{\leq} 4$$

$$0 \leq 4 \qquad \text{TRUE}$$

The test point is a solution to the inequality. Shade the half-plane that contains (0, 0).



Answer 74e.

Consider the inequality:

$$9x - 9y \geq 27 \qquad \ldots\ldots (1)$$

We need to graph the inequality (1) in a coordinate plane.

The boundary equation is:

$$9x - 9y = 27.$$

Now, we consider the point $(0,0)$ and we see whether this point satisfied the condition or not.

$$9x - 9y \geq 27$$

$$9 \times 0 - 9 \times 0 \geq 27$$

$$0 \geq 27 \qquad [\text{False}]$$

So the point $(0,0)$ is not satisfied the condition.

Again we consider a point $(5,1)$ and we see whether this point satisfied the condition or not.

$$9x - 9y \geq 27$$

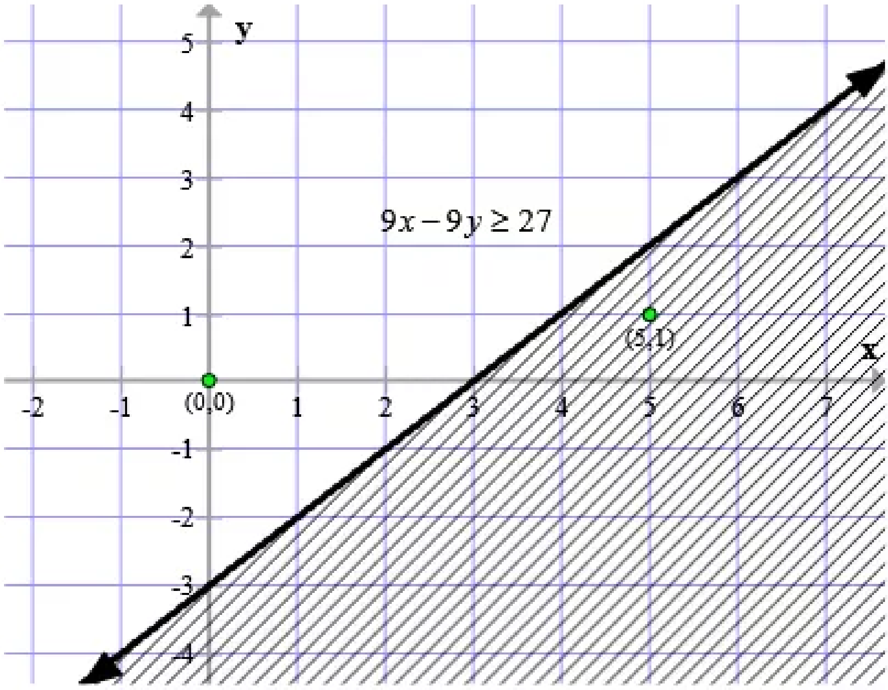
$$9 \times 5 - 9 \times 1 \geq 27$$

$$45 - 9 \geq 27$$

$$36 \geq 27 \qquad [\text{True}]$$

So the point $(5,1)$ is satisfied the condition.

Therefore the graph of the inequality $9x-9y\geq 27$ is shown below:



Answer 75e.

STEP 1 **Graph the boundary line of the inequality.**

In order to obtain the boundary line, replace the inequality sign with "=".

Thus, we get an equation of the form $\frac{2}{5}x+\frac{5}{2}y = 5$.

Substitute 0 for y in the equation and solve for x.

$$\frac{2}{5}x+\frac{5}{2}(0) = 5$$
$$\frac{2}{5}x = 5$$
$$x = \frac{25}{2}$$

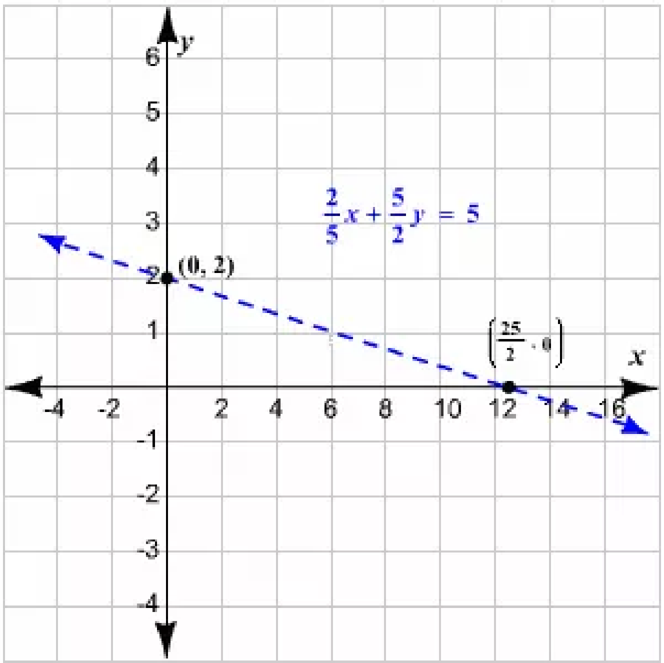
Since the x-intercept is $\frac{25}{2}$, plot the point $\left(\frac{25}{2}, 0\right)$ on the graph.

Next, replace x with 0 and solve for y.

$$\frac{2}{5}(0)+\frac{5}{2}y = 5$$
$$\frac{5}{2}y = 5$$
$$y = 2$$

Since the y-intercept is 2, another point that can be plotted on the graph is (0, 2).

Plot $\left(\frac{25}{2}, 0\right)$ and (0, 2) on the graph and draw a line passing through them. Since > is the inequality sign used, draw a dashed line.



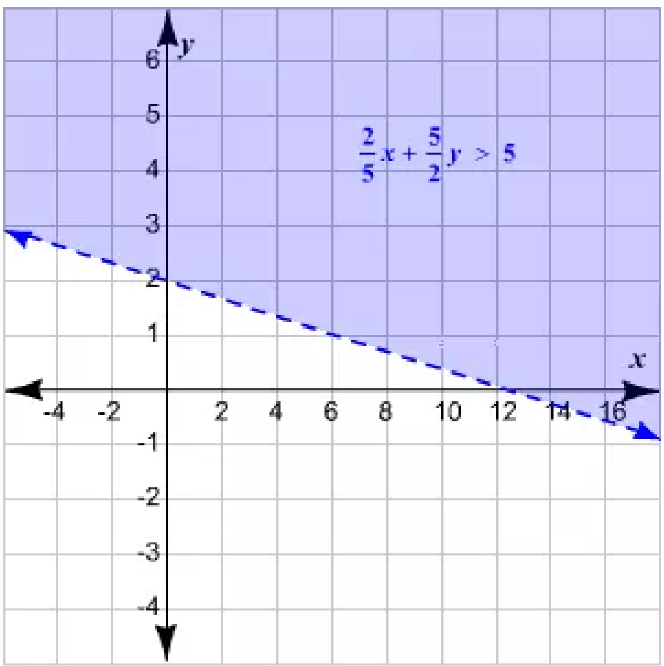
STEP 2 **Test a point.**

Let us take a test point which does not lie on the boundary line, say, (0, 0). Substitute 0 for y, and 0 for x. Check if the test point satisfies the given inequality.

$$\frac{2}{5}(0) + \frac{5}{2}(0) \overset{?}{>} 5$$

$$0 > 5 \qquad \text{FALSE}$$

The test point is not a solution to the inequality. Shade the half-plane that does not contain (0, 0).



Answer 76e.

Consider the polynomial function:

$$f(x)=5x^4-3x^3+4x^2-x+10; x=2 \qquad \ldots\ldots (1)$$

We need to evaluate the polynomial function (1) for the value of $x=2$ by using synthetic substitution.

We write the coefficient of $f(x)$ in order of descending exponents. First we write the value at which $f(x)$ is being evaluated to the left.

$$x\text{-values}\rightarrow 2 \,\big|\, 5 \quad -3 \quad 4 \quad -1 \quad 10 \quad \leftarrow \text{coefficients}$$

Now, we bring down the leading coefficient. Multiply the leading coefficient by the x-value and we write the product under the second coefficient and then add.

$$\begin{array}{r|rrrrr} 2 & 5 & -3 & 4 & -1 & 10 \\ & & 10 & & & \\ \hline & \downarrow & \downarrow & & & \\ & 5 & 7 & & & \end{array}$$

We multiply the previous sum by the x-value. Now we write the product under the third coefficient and then add. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given x-value.

$$\begin{array}{r|rrrrr} 2 & 5 & -3 & 4 & -1 & 10 \\ & & 10 & 14 & 36 & 70 \\ \hline & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 5 & 7 & 18 & 35 & 80 \end{array}$$

Therefore the solution of the polynomial function $f(x)=5x^4-3x^3+4x^2-x+10; x=2$ is: $\boxed{80}$.

Answer 77e.

STEP 1 **Write** the coefficients of $f(x)$ in the order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

In the given function, the x^4 term is missing. Use 0 as the coefficient of the missing term.

$$x\text{-value}\rightarrow -3 \,\big|\, -3 \quad 0 \quad 1 \quad -6 \quad 2 \quad 4 \quad \leftarrow \text{coefficients}$$

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add**.

The leading coefficient is -3. On multiplying -3 by -3, we get 9. Write 9 below the second coefficient 0. Add them.

$$\begin{array}{r|rrrrrr} -3 & -3 & 0 & 1 & -6 & 2 & 4 \\ & & 9 & & & & \\ \hline & -3 & 9 & & & & \end{array}$$

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given x-value.

The previous sum is obtained as 9. Multiply 9 by the x-value -3 and write the result under the third coefficient 1. Add the numbers. Repeat this process.

$$\begin{array}{r|rrrrrr} -3 & -3 & 0 & 1 & -6 & 2 & 4 \\ & & 9 & -27 & 78 & -216 & 642 \\ \hline & -3 & 9 & -26 & 72 & -214 & 646 \end{array}$$

The final sum is 646.

Therefore, the value of $f(x)$ is 646 for the given value of x.

Answer 78e.

Consider the polynomial function:

$$f(x) = 5x^5 - 4x^3 + 12x^2 + 20; x = -2 \qquad \ldots\ldots (1)$$

We need to evaluate the polynomial function (1) for the value of $x = 2$ by using synthetic substitution.

We write the coefficient of $f(x)$ in order of descending exponents. First we write the value at which $f(x)$ is being evaluated to the left.

$$x\text{-values} \rightarrow -2 \left| \begin{array}{cccccc} 5 & 0 & -4 & 12 & 0 & 20 \\ \hline \end{array} \right. \leftarrow \text{coefficients}$$

Now, we bring down the leading coefficient. Multiply the leading coefficient by the x-value and we write the product under the second coefficient and then add.

$$\begin{array}{c|cccccc} -2 & 5 & 0 & -4 & 12 & 0 & 20 \\ & & -10 & & & & \\ \hline & \downarrow & \downarrow & & & & \\ & 5 & -10 & & & & \end{array}$$

We multiply the previous sum by the $x-$value. Now we write the product under the third coefficient and then add. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given $x-$value.

$$\begin{array}{c|cccccc} -2 & 5 & 0 & -4 & 12 & 0 & 20 \\ & & -10 & 20 & -32 & 40 & -80 \\ \hline & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 5 & -10 & 16 & -20 & 40 & -60 \end{array}$$

Therefore the solution of the polynomial function $f(x)=5x^5-4x^3+12x^2+20; x=-2$ is: $\boxed{-60}$.

Answer 79e.

STEP 1 **Write** the coefficients of $f(x)$ in the order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.

In the given function, the x^3 and x^2 terms are missing. Use 0 as the coefficient of the missing term.

$$x\text{-value} \rightarrow 4 \left| \begin{array}{ccccc} -6 & 0 & 0 & 9 & -15 \\ \\ \hline \end{array} \right. \leftarrow \text{coefficients}$$

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add.**

The leading coefficient is -6. On multiplying -6 by 4, we get -24. Write -24 below the second coefficient 0. Add them.

$$\begin{array}{c|ccccc} 4 & -6 & 0 & 0 & 9 & -15 \\ & & -24 & & & \\ \hline & -6 & -24 & & & \end{array}$$

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given x-value.

The previous sum is obtained as −24. Multiply −24 by the x-value 4 and write the result under the third coefficient 0. Add the numbers. Repeat this process.

$$\begin{array}{r|rrrrr} 4 & -6 & 0 & 0 & 9 & -15 \\ & & -24 & -96 & -384 & -1500 \\ \hline & -6 & -24 & -96 & -375 & -1515 \end{array}$$

The final sum is −1515.

Therefore, the value of $f(x)$ is −1515 for the given value of x.