Chapter 5

Continuity and Differentiability

Exercise 5.5

Q. 1 Differentiate the functions given in w.r.t. x.

$\cos x.\cos 2x.\cos 3x$

Answer:

Given: $\cos x.\cos 2x.\cos 3x$

Let $y=\cos x.\cos 2x.\cos 3x$

Taking log on both sides, we get

$\log y = \log(\cos x.\cos 2x.\cos 3x)$

$\Rightarrow \log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)$

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log y) = \frac{d}{dx}\log(\cos x) + \frac{d}{dx}\log(\cos 2x) + \frac{d}{dx}(\log\cos 3x)$

$= \frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}.\frac{d}{dx}(\cos x) + \frac{1}{\cos 2x}.\frac{d}{dx}(\cos 2x) + \frac{1}{\cos 3x}\frac{d}{dx}(\cos 3x)$

$= \frac{dy}{dx} = y\left[-\frac{\sin x}{\cos x} - \frac{\sin 2x}{\cos 2x}.\frac{d}{dx}(2x) - \frac{\sin 3x}{\cos 3x}\frac{d}{dx}(3x)\right]$

$= \frac{dy}{dx} = -\cos x.\cos 2x.\cos 3x[\tan x + \tan 2x\ (2) + \tan 3x(3)]$

$= \frac{dy}{dx} = -\cos x.\cos 2x.\cos 3x[\tan x + 2\tan 2x + 3\tan 3x]$

Q. 2 Differentiate the functions given in w.r.t. x.

$(\log x)^{\cos x}$

Answer:

Given: $(\log x)^{\cos x}$

Let y = $(\log x)^{\cos x}$

Taking log on both sides, we get

$\log y = \log (\log x)^{\cos x}$

$\Rightarrow \log y = \cos x . \log (\log x)$

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log y) = \frac{d}{dx}[\cos x . \log(\log x)]$

$= \frac{1}{y}\frac{dy}{dx} = \cos x . \frac{d}{dx}(\log(\log x)) + \log(\log x) . \frac{d}{dx}(\cos x)$

$= \frac{dy}{dx} = y\left[\cos x . \frac{1}{\log x} . \frac{d}{dx}(\log x) + \log(\log x) . (-\sin x)\right]$

$= \frac{dy}{dx} = (\log x)^{\cos x}\left[\cos x . \frac{1}{\log x} . \frac{1}{x} + \log(\log x) . (-\sin x)\right]$

$= \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x . \log x} - (\sin x) . \log(\log x)\right]$

Q. 4 Differentiate the functions given in w.r.t. x.

$x^x - 2^{\sin x}$

Answer:

Given: $x^x - 2^{\sin x}$

Let $y = x^x - 2^{\sin x}$

Let y = u - v

$\Rightarrow$ u = xx and v = $2^{\sin} x$

For, u = x^x

Taking log on both sides, we get

$\log u = \log x^x$

$\Rightarrow \log u = x.\log(x)$

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log u) = \frac{d}{dx}[x.\log(x)]$

$= \frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)$

$= \frac{du}{dx} = u\left[x.\frac{1}{x} + \log x.(1)\right]$

$= \frac{du}{dx} = x^x(1 + \log x)$

For, v = 2sin x

Taking log on both sides, we get

$\log v = \log 2^{\sin x}$

$\Rightarrow \log v = \sin x.\log(2)$

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log v) = \frac{d}{dx}[\sin x.\log(2)]$

$= \frac{1}{v}\frac{dv}{dx} = \log 2.\frac{d}{dx}(\sin x)$

$= \frac{dv}{dx} = v[\log 2.(\cos x)]$

$= \frac{dv}{dx} = 2^{\sin x}.\cos x log\ 2$

Because, y = u - v

$= \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

$dy/dx = x^x(1 + \log x) - 2\sin x.\cos x.\log 2$

Q. 5 Differentiate the functions given in w.r.t. x.

$(x + 3)^2 . (x + 4)^3 . (x + 5)^4$

Answer:

Given: $(x + 3)^2 . (x + 4)^3 . (x + 5)^4$

Let y= $(x + 3)^2 . (x + 4)^3 . (x + 5)^4$

Taking log on both sides, we get

$\log y = \log ((x + 3)^2 . (x + 4)^3 . (x + 5)^4)$

$\Rightarrow \log y = \log (x + 3)^2 + \log (x + 4)^3 + \log (x + 5)^4$

$\Rightarrow \log y = 2.\log (x + 3) + 3.\log (x + 4) + 4.\log (x + 5)^4$

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log y) = \frac{d}{dx}(2.\log(x+3)) + \frac{d}{dx}(3.\log(x+4)) + \frac{d}{dx}(4.\log(x+5))$

$= \frac{1}{y}\frac{dy}{dx} = 2.\frac{1}{x+3}.\frac{d}{dx}(x+3) + 3.\frac{1}{x+4}.\frac{d}{dx}(x+4) + 4.\frac{1}{x+5}.\frac{d}{dx}(x+5)$

$= \frac{dy}{dx} = y\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$

$= \frac{dy}{dx} = (x+3)^2(x+4)^3(x+5)^4\left[\frac{2(x+4)(x+5)+3(x+3)(x+5)+4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$

$= \frac{dy}{dx} = (x+3)^1(x+4)^2(x+5)^3[2(x^2+9x+20) + 3(x^2+8x+15) + 4(x^2+7x+12)]$

$= (x + 3) (x + 4)^2 (x + 5)^3 (9x^2 + 70x + 133)$

Q. 6 Differentiate the functions given in w.r.t. x.

$\left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Answer:

Given: $\left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Let y= $\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$

Also, Let y = u + v

= u $\left(x+\frac{1}{x}\right)^{x}$ and v = $x^{\left(1+\frac{1}{x}\right)}$

for, u = $\left(x+\frac{1}{x}\right)^{x}$

Taking log on both sides, we get

log u = log $\left(x+\frac{1}{x}\right)^{x}$

= log u = x. log $\left(x+\frac{1}{x}\right)$

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log u)=\frac{d}{dx}\left[x.\log\left(x+\frac{1}{x}\right)\right]$

= $\frac{1}{u}-\frac{du}{dx}=x.\frac{d}{dx}\left(\log\left(x+\frac{1}{x}\right)\right)+\log\left(x+\frac{1}{x}\right).\frac{d}{dx}(x)$

= $\frac{du}{dx}=u\left[x.\frac{1}{\left(x+\frac{1}{x}\right)}.\frac{d}{dx}\left(x+\frac{1}{x}\right)+\log\left(x+\frac{1}{x}\right)\right]$

= $\frac{du}{dx}=u\left[x.\frac{1}{\left(x+\frac{1}{x}\right)}.\left(\frac{dx}{dx}+\frac{d}{dx}\left(\frac{1}{x}\right)\right)+\log\left(x+\frac{1}{x}\right)\right]$

= $\frac{du}{dx}=u\left[\frac{x}{\left(x+\frac{1}{x}\right)}.\left(1-\frac{1}{x^{2}}\right)+\log\left(x+\frac{1}{x}\right)\right]$

= $\frac{du}{dx}=u\left[\frac{x}{\left(x+\frac{1}{x}\right)}.\left(\frac{x^{2}-1}{x^{2}}\right)+\log\left(x+\frac{1}{x}\right)\right]$

= $\frac{du}{dx}=\left(x+\frac{1}{x}\right)^{x}\left[\left(\frac{x^{2}-1}{x^{2}+1}\right)+\log\left(x+\frac{1}{x}\right)\right]$

for, v = $x^{\left(1+\frac{1}{x}\right)}$

Taking log on both sides, we get

log v = log $x^{\left(1+\frac{1}{x}\right)}$

= log v = $\left(1+\frac{1}{x}\right).\log x$

Now, differentiate both sides with respect to x

= $\frac{d}{dx}(\log v) = \frac{d}{dx}\left[\left(1+\frac{1}{x}\right).\log x\right]$

= $\frac{1}{v}\frac{dv}{dx} = \log x.\frac{d}{dx}\left(1+\frac{1}{x}\right) + \left(1+\frac{1}{x}\right).\frac{d}{dx}(\log x)$

= $\frac{dv}{dx} = v\left[\log x.\left(0-\frac{1}{x^2}\right) + \left(1+\frac{1}{x}\right).\frac{1}{x}\right]$

= $\frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left[-\frac{\log x}{x^2} + \left(\frac{1}{x}+\frac{1}{x^2}\right)\right]$

= $\frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left[\frac{-\log x+x+1}{x^2}\right]$

= $\frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1-\log x}{x^2}\right]$

Because, y = u + v

= $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

= $\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\left(\frac{x^2-1}{x^2+1}\right) + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1-\log x}{x^2}\right]$

Q. 7 Differentiate the functions given in w.r.t. x.

$(\log x)^{x} + x^{\log x}$

Answer:

Given: $(\log x)^{x} + x^{\log x}$

Let y= $(\log x)\, x + x^{\log x}$

Let y = u + v

$\Rightarrow$ u = $(\log x)^{x}$ and v = $x^{\log x}$

For, u = $(\log x)^{x}$

Taking log on both sides, we get

log u =log $(\log x)^{x}$

$\Rightarrow$log u = x.log (log(x))

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log u) = \frac{d}{dx}[x.\log(\log x)]$$

$$= \frac{1}{u} - \frac{du}{dx} = x.\frac{d}{dx}\log(\log x)) + \log(\log x).\frac{d}{dx}(x)$$

$$= \frac{du}{dx} = u\left[x.\frac{1}{\log x}\frac{d}{dx}(\log x) + \log(\log x).(1)\right]$$

$$= \frac{du}{dx} = (\log x)^{x}\left[\frac{x}{\log x}.\frac{1}{x} + \log(\log x).(1\_\right]$$

$$= \frac{du}{dx} = (\log x)^{x}\left[\frac{1+\log(\log x).(\log x)}{\log x}\right]$$

$$= \frac{du}{dx} = (\log x)^{x-1}[1 + \log x.\log(\log x)]$$

For, v = $x^{\log x}$

Taking log on both sides, we get

log v =log (x log x)

$\Rightarrow$log v = log x. log x

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log v) = \frac{d}{dx}[(\log x)^{2}]$$

$$= \frac{1}{v}\frac{dv}{dx} = 2.\log x\frac{d}{dx}(\log x)$$

$$= \frac{dv}{dx} = v\left[2.\frac{\log x}{x}\right]$$

$= \frac{dv}{dx} = x^{\log x}\left[2.\frac{\log x}{x}\right]$

$= \frac{dv}{dx} = 2.x^{\log x - 1}.\log x$

Because, y = u + v

$= \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= \frac{dy}{dx} = (\log x)^{x-1}\,[1 + \log x.\log(\log x)] + 2.x^{\log x - 1}.\log x$

Q. 8 Differentiate the functions given in w.r.t. x.

$(\sin x)^{x} + \sin^{-1}\sqrt{x}$

Answer:

Given: $(\sin x)^{x} + sin^{-1}\sqrt{x}$

Let y= $(\sin x)^{x} + sin^{-1}\sqrt{x}$

Let y = u + v

= u = $(\sin x)^{x}$ and v = $sin^{-1}\sqrt{x}$

for, u = $(\sin x)^{x}$

Taking log on both sides, we get

log u = log $(\sin x)^{x}$

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log u) = \frac{d}{dx}[x.\log(\sin x)$

$= \frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}\log(\sin x)) + \log(\sin x).\frac{d}{dx}(x)$

$= \frac{du}{dx} = u\left[x.\frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log(\sin x).(1)\right]$

$= \frac{dy}{dx} = (\sin x)^{x}\left[\frac{x}{\sin x}.\cos x + \log(\sin x).(1)\right]$

$= \frac{dy}{dx} = (\sin x)^x\ [x.\cot x + \log \sin x]$

for, v = $sin^{-1}\sqrt{x}$

Now, differentiate both sides with respect to x

$= \frac{dv}{dx} = \frac{d}{dx}\left[sin^{-1}\sqrt{x}\right]$

$= \frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{d}{dx}(\sqrt{x})$

$= \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2(\sqrt{x})}$

$= \frac{dv}{dx} =$

$\frac{1}{2\sqrt{x-x^2}}$Because, y = u + v

$= \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= \frac{dy}{dx} = (\sin x)^x\ [x.\cot x + \log \sin x] + \frac{1}{2\sqrt{x-x^2}}$

Q. 9 Differentiate the functions given in w.r.t. x.

$x^{\sin x} + (\sin x)^{\cos x}$

Answer:

Given: $x^{\sin x} + (\sin x)^{\cos x}$

Let y= $x^{\sin x} + (\sin x)^{\cos x}$

Let y = u + v

⇒ u = $x^{\sin x}$ and v = $(\sin x)^{\cos x}$

For, u = $x^{\sin x}$

Taking log on both sides, we get

log u =log (x sin x)

$\Rightarrow \log u = \sin x . \log(x)$

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log u) = \frac{d}{dx}[\sin x . \log x$

$= \frac{1}{u}\frac{du}{dx} = \sin x . \frac{d}{dx}(\log x) + \log x . \frac{d}{dx}(\sin x)$

$= \frac{du}{dx} = u\left[\sin x . \frac{1}{x} + \log x . \cos x\right]$

$= \frac{du}{dx} = (x)^{\sin x}\left[\frac{\sin x}{x} + \log x . \cos x\right]$

For, v = $(\sin x)^{\cos x}$

Taking log on both sides, we get

log v =log $(\sin x)^{\cos x}$

$\Rightarrow \log v = \cos x . \log(\sin x)$

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log v) = \frac{d}{dx}[\cos x . \log(\sin x)]$

$= \frac{1}{v}\frac{dv}{dx} = \cos x . \frac{d}{dx}\log(\sin x) + \log \sin x . \frac{d}{dx}(\cos x)$

$= \frac{dv}{dx} = v\left[\cos x . \frac{1}{\sin x} . \frac{d}{dx}(\sin x) + \log(\sin x) . (-\sin x)\right]$

$= \frac{dv}{dx} = (\sin x)^{\cos x}\left[\frac{\cos x}{\sin x} . \cos x + \log \sin x . (-\sin x)\right]$

$= \frac{dy}{dx} = (\sin x)^{\cos x}[\cot x . \cos x - \sin x . \log \sin x]$Because, y = u + v

$= \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= \frac{dy}{dx} = (x)^{\sin x}\left[\frac{\sin x}{x} + \log x . \cos x\right] + (\sin x)^{\cos x}[\cot x . \cos x -$
$\sin x . \log \sin x]$

Q. 10 Differentiate the functions given in w.r.t. x.

$x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Answer:

Given: $x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Let y= $x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Let y = u + v

= u = $x^{x\cos x}$ and v = $\frac{x^2+1}{x^2-1}$

for, u = $x^{xcos\, x}$

Taking log on both sides, we get

log u = log $x^{x\cos x}$

⇒log u = x. cos x.log

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log u) = \frac{d}{dx}[x.\cos x.\log x]$

= $\frac{1}{u}\frac{du}{dx} = \cos x log\ x.\frac{d}{dx}(x) + x.\log x.\frac{d}{dx}(\cos x) + x.\cos x.\frac{d}{dx}(\log x)$

= $\frac{du}{dx} = u\left[\cos x.\log x + x.\log x(-\sin x) + x.\cos x.\left(\frac{1}{x}\right)\right]$

= $\frac{du}{dx} = x^{x\cos x}[\cos x.\log x - x.\log x.\sin x + \cos x]$

= $\frac{dy}{dx} = x^{x\cos x}[\cos x(1 + \log x) - x.\log x.\sin x]$

for, v = $\frac{x^2+1}{x^2-1}$

Taking log on both sides, we get

log v = log $\left(\frac{x^2+1}{x^2-1}\right)$

⇒ log v = log (x^2 + 1) – log (x^2 – 1)

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log v) = \frac{d}{dx}[\log(x^2+1) - \log(x^2-1)]$

$= \frac{1}{v}\frac{dy}{dx} = \frac{1}{x^2+1}.\frac{d}{dx}(x^2) - \frac{1}{x^2-1}.\frac{d}{dx}(x^2)$

$= \frac{dy}{dx} = v.\left[\frac{1}{x^2+1}.(2x) - \frac{1}{x^2-1}.(2x)\right]$

$= \frac{dy}{dx} = \left(\frac{x^2+1}{x^2-1}\right).\left[\frac{2x(x^2-1)-2x(x^2+1)}{(x^2+1)(x^2-1)}\right]$

$= \frac{dy}{dx} = \left(\frac{x^2+1}{x^2-1}\right).\left[\frac{-4x}{(x^2+1)(x^2-1)}\right]$

$= \frac{dy}{dx} = \left[\frac{-4x}{(x^2-1)^2}\right]$

Because, y = u + v

$= \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= \frac{dy}{dx} = x^{x\cos x}[\cos x(1+\log x) - x.\log x.\sin x] - \left[\frac{4x}{(x^2-1)^2}\right]$

Q. 11 Differentiate the functions given in w.r.t. x.

$(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Answer:

Given: $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Let y= $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Let y = u + v

= u = $(x\cos x)^x$ and v = $(x\sin x)^{\frac{1}{x}}$

for, u = $(x\cos x)^x$

Taking log on both sides, we get

log u = log $(x \cos x)^{x}$

⇒log u = x. log $(x \cos x)$

⇒ log u = x (log x + log (cos x))

⇒ log u = x (log x) +x (log (cos x))

Now, differentiate both sides with respect to x

$= \frac{dy}{dx}(\log x) = \frac{d}{dx}[x.\log(x)] + \frac{d}{dx}[x.\log(\cos x)]$

$= \frac{1}{u}\frac{du}{dx} = \left\{x.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)\right\} + \left\{x.\frac{d}{dx}(\log\cos x) + \log\cos x.\frac{d}{dx}(x)\right\}$

$= \frac{du}{dx} = u\ \left[\left\{x.\frac{1}{x} + \log x.(1)\right\} + \left\{x.\frac{1}{\cos x}.\frac{d}{dx}(\cos x) + \log\cos x.(1)\right\}\right]$

Taking log on both sides, we get

log v = log $(x \sin x)^{\frac{1}{x}}$

Now, differentiate both sides with respect to x

$\frac{d}{dx}(\log v) = \frac{d}{dx}\left[\frac{1}{x}.(\log x)\right] + \frac{d}{dx}\left[\frac{1}{x}.\log(\sin x)\right]$

$= \frac{1}{v}\frac{dy}{dx} = \left\{\frac{1}{x}.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}\left(\frac{1}{x}\right)\right\} + \left\{\frac{1}{x}.\frac{d}{dx}(\log\sin x) + \log\sin x.\frac{d}{dx}\left(\frac{1}{x}\right)\right\}$

$= \frac{dy}{dx} = v\left[\left\{\frac{1}{x}.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}\left(\frac{1}{x}\right)\right\} + \left\{\frac{1}{x}.\frac{d}{dx}(\log\sin x) + \log\sin x.\frac{d}{dx}\left(\frac{1}{x}\right)\right\}\right]$

$= \frac{dy}{dx} = (x\sin x)^{\frac{1}{x}}\left[\left\{\frac{1}{x^2}(1-\log x)\right\} + \left\{\frac{\cos x}{x.\sin x}.-\frac{\log\sin x}{x^2}\right\}\right]$

$= \frac{dy}{dx} = (x \sin x)^{\frac{1}{x}} \left[\frac{1-\log x}{x^2} + \frac{\cot x}{x} - \frac{\log \sin x}{x^2}\right]$

$= \frac{dy}{dx} = (x \sin x)^{\frac{1}{x}} \left[\frac{1-\log x + x \cot x - \log \sin x}{x^2}\right]$

$= \frac{dy}{dx} = (x \sin x)^{\frac{1}{x}} \left[\frac{1+x \cot x - \log(x.\sin x)}{x^2}\right]$

$= \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= \frac{dy}{dx} = (x \cos x)^x\, [1 - x.\tan x + \log(x.\cos x)] +$

$(x \sin x)^{\frac{1}{x}} \left[\frac{1+x \cot x - \log(x.\sin x)}{x^2}\right]$

Q. 12 Find dy/dx of the functions.

$x^y + y^x = 1$

Answer:

Given: $x^y + y^x = 1$

Let y= $x^y + y^x = 1$

Let u = x^y and v = y^x

Then, ⇒ u + v = 1

$= \frac{du}{dx} + \frac{dv}{dx} = 0$

For, u = xy

Taking log on both sides, we get

Log u =log xy

⇒log u = y.log(x)

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log u) = \frac{d}{dx}[y.\log(x)]$

$= \frac{1}{u}\frac{du}{dx} = \left\{y.\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(y)\right\}$

$= \frac{du}{dx} = u\left[y.\frac{1}{x} + \log x.\left(\frac{dy}{dx}\right)\right]$

$= \frac{dy}{dx} = x^y\left[\frac{y}{x} + \log x.\left(\frac{dy}{dx}\right)\right]$

For, v = y^x

Taking log on both sides, we get

Log v =log y^x

⇒log v = x.log(y)

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log v) = \frac{d}{dx}[x.\log(y)]$

$= \frac{1}{v}\frac{dv}{dx} = \left\{x.\frac{d}{dx}(\log y) + \log y.\frac{d}{dx}x\right\}$

$= \frac{dv}{dx} = v\left[x.\frac{1}{y}.\frac{dy}{dx} + \log y.\left(\frac{dy}{dx}\right)\right]$

$= \frac{dy}{dx} = y^x\left[\frac{x}{y}.\frac{dy}{dx} + \log y\right]$

because, $\frac{du}{dx} + \frac{dv}{dx} = 0$

so, $x^y\left[\frac{y}{x} + \log x.\left(\frac{dy}{dx}\right)\right] + y^x\left[\frac{x}{y}.\frac{dy}{dx} + \log y\right] = 0$

$= (x^y \log x + xy^{x-1}).\frac{dy}{dx} + (yx^{y-1} + y^x \log y) = 0$

$= (x^y \log x + xy^{x-1}).\frac{dy}{dx} = -(yx^{y-1} + y^x \log y)$

$= \frac{dy}{dx} = -\frac{(yx^{y-1}+y^x \log y)}{(x^y \log x+xy^{x-1})}$

Q. 13 Find dy/dx of the functions.

$y^x = x^y$

Answer:

Given: $y^x = x^y$

Taking log on both sides, we get

log yx =log x^y

⇒x log y = y log x

Now, differentiate both sides with respect to x

$$x.\frac{d}{dx}\log y + \log y.\frac{d}{dx}x = y.\frac{d}{dx}\log x + \log x.\frac{d}{dx}y$$

$$x.\frac{1}{y}.\frac{dy}{dx} + \log y.(1) = y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

$$\frac{x}{y}.\frac{dy}{dx} - \log x.\frac{dy}{dx} = y.\frac{1}{x} - \log y$$

$$= \frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \frac{y - x\log y}{x}$$

$$= \frac{dy}{dx}\left(\frac{x - y\log x}{y}\right) = \frac{y - x\log y}{x}$$

$$= \frac{dy}{dx} = \frac{y}{x}\left(\frac{y - x\log y}{x - y\log x}\right)$$

Q. 14 Find dy/dx of the functions.

$(\cos x)^{y} = (\cos y)^{x}$

Answer:

Given: $(\cos x)^{y} = (\cos y)\, x$

Taking log on both sides, we get

$\log (\cos x)^{y} = \log (\cos y)^{x}$

⇒y log (cos x) = x log (cos y)

Now, differentiate both sides with respect to x

$y.\frac{d}{dx}\log(\cos x) + \log(\cos x).\frac{d}{dx}y = x.\frac{d}{dx}\log(\cos y) + \log\cos y.\frac{d}{dx}x$

$= y.\frac{1}{\cos x}.\frac{d}{dx}(\cos x) + \log(\cos x).\frac{dy}{dx} = x.\frac{1}{\cos y}.\frac{d}{dx}(\cos y) +$
$\log(\cos y).\frac{dy}{dx}$

$= \frac{y}{\cos x}.(-\sin x) + \log(\cos x).\frac{dy}{dx} = \frac{x}{\cos y}.(-\sin y).\frac{dy}{dx} +$
$\log(\cos y).(1)$

$= \frac{dy}{dx}\left(\frac{x.\sin y}{\cos y} + \log(\cos x)\right) = y.\frac{\sin x}{\cos x} + \log(\cos y)$

$= \frac{dy}{dx}(x\tan x + \log(\cos x)) = y.\tan x + \log(\cos y)$

$= \frac{dy}{dx} = \left(\frac{y.\tan x+\log(\cos y)}{x.\tan x+\log(\cos x)}\right)$

Q. 15 Find dy/dx of the functions.

$xy = e^{(x-y)}$

Answer:

Given: $xy = e^{(x-y)}$

Taking log on both sides, we get

$\log(xy) = \log(e\,(x-y))$

$\Rightarrow \log x + \log y = (x - y)\log e$

$\Rightarrow \log x + \log y = (x - y)\ .1$

$\Rightarrow \log x + \log y = (x - y)$

Now, differentiate both sides with respect to x

$\frac{d}{dx}\log x + \frac{d}{dx}\log y = \frac{d}{dx}x - \frac{d}{dx}y$

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\left(1+\frac{1}{y}\right)\frac{dy}{dx}=1-\frac{1}{x}$

$\frac{1+y}{y}\frac{dy}{dx}=\frac{x-1}{x}$

$\frac{dy}{dx}=\frac{y(x-1)}{x(1+y)}$

Q. 16 Find the derivative of the function given by f (x) = $(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$ and hence find f ′ (1).

Answer:

Given: $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$

Taking log on both sides, we get

$\log f(x) = \log(1 + x) + \log(1 + x^2) + \log(1 + x^4) + \log(1 + x^8)$

Now, differentiate both sides with respect to x

$\frac{d}{dx}\log f(x)=\frac{d}{dx}\log(1+x)+\frac{d}{dx}\log(1+x^2)+\frac{d}{dx}\log(1+x^4)+\frac{d}{dx}\log(1+x^8)$

$=\frac{1}{f(x)}.\frac{d}{dx}[f(x)]$

$=\frac{1}{1+x}.\frac{d}{dx}(1+x)+\frac{1}{1+x^2}\frac{d}{dx}(1+x^2)+\frac{1}{1+x^4}\frac{d}{dx}(1+x^4)+\frac{1}{1+x^8}\frac{d}{dx}(1+x^8)$

$= f'(x) = f(x) = \left[\frac{1}{1+x}+\frac{1}{1+x^2}.(2x)+\frac{1}{1+x^4}.(4x^3)+\frac{1}{1+x^8}(8x^7)\right]$

$=f'(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}\right]$

$=f'(x)=(1+1)(1+1^2)(1+1^4)(1+1^8)\left[\frac{1}{1+1}+\frac{2(1)}{1+1}+\frac{4(1)^3}{1+(1)^4}+\frac{8(1)^7}{1+(1)^8}\right]$

$= f'(1) = (2)(2)(2)(2)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$

$= f'(1) = 16\left[\frac{1+2+4+8}{2}\right]$

$= f'(1) = 16\left(\frac{15}{2}\right)$

$= f'(1) = 120$

Q. 17 Differentiate (x^2 – 5x + 8) (x^3 + 7x + 9) in three ways mentioned below:

(i) by using product rule

(ii) by expanding the product to obtain a single polynomial.

(iii) by logarithmic differentiation.

Do they all give the same answer?

Answer:

Given: (x^2 – 5x + 8) (x^3 + 7x + 9)

Let y = (x^2 – 5x + 8) (x^3 + 7x + 9)

(i) By applying product rule differentiate both sides with respect to x

$\frac{dy}{dx} = \frac{dy}{dx}(x^2 - 5x + 8)(x^3 + 7x + 9)$

$= \frac{dy}{dx} = (x^3 + 7x + 9).\frac{d}{dx}(x^2 - 5x + 8) + (x^2 - 5x + 8).\frac{d}{dx}(x^3 + 7x + 9)$

$= \frac{dy}{dx} = (x^3 + 7x + 9).(2x - 5) + (x^2 - 5x + 8).(3x^2 + 7)$

$= \frac{dy}{dx} = 2x^4 + 14x^2 + 18x - 5x^3 - 35x - 45 + 3x^4 + 7x^2 - 15x^3 - 35x + 24x^2 + 56$

$= \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11 \ldots..(1)$

(ii) by expanding the product to obtain a single polynomial

y = (x^2 – 5x + 8) (x^3 + 7x + 9)

y = x^5 + 7x^3 + 9x^2 - 5x^4 – 35x^2 - 45x + 8x^3 + 56x + 72

y = x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72

Now, differentiate both sides with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^5) - \frac{d}{dx}(5x^4) + \frac{d}{dx}(15x^3) - \frac{d}{dx}(26x^2) + \frac{d}{dx}(11x) + \frac{d}{dx}(72)$$

$$\frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11 \ldots (2)$$

(iii) by logarithmic differentiation

y = (x^2 – 5x + 8) (x^3 + 7x + 9)

Taking log on both sides, we get

log y = log ((x^2 – 5x + 8) (x^3 + 7x + 9))

log y = log (x^2 – 5x + 8) + log (x^3 + 7x + 9)

Now, differentiate both sides with respect to x

$$\frac{dy}{dx}(\log y) = \frac{d}{dx}\log(x^2 - 5x + 8) + \frac{d}{dx}\log(x^3 + 7x + 9)$$

$$= \frac{1}{y}\frac{d}{dx}(y) = \left[\frac{1}{(x^2-5x+8)}.\frac{d}{dx}(x^2 - 5x + 8) + \frac{1}{(x^3+7x+9)}.\frac{d}{dx}(x^3 + 7x + 9)\right]$$

$$= \frac{1}{y}\frac{d}{dx}(y) = \left[\frac{1}{(x^2-5x+8)}.(2x - 5) + \frac{1}{(x^3+7x+9)}.(3x^2 + 7)\right]$$

$$= \frac{d}{dx}(y) = y.\left[\frac{(2x-5)}{(x^2-5x+8)} + \frac{(3x^2+7)}{(x^3+7x+9)}\right]$$

$$= \frac{d}{dx}(y) = y.\left[\frac{(2x-5)(x^3+7x+9)+(3x^2+7)(x^2-5x+9)}{(x^2-5x+8)(x^3+7x+9)}\right]$$

$= \frac{d}{dx}(y) = y.\left[\frac{2x^4+14x^2+18x-5x^3-35x-45+3x^4-15x^3+24x^2+7x^2-35x+56}{(x\quad -5x+8)(x\quad +7x+9)}\right]$

$= \frac{d}{dx}(y) = (x^2-5x+8)(x^3+7x+9).\left[\frac{5x^4-20x^3-45x^2-52x+11}{(x^2-5x+8)(x^3+7x+9)}\right]$

$= \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11 \ldots\ldots (3)$

From equation (i), (ii) and (iii), we can say that value of given function after differentiating by all the three methods is same.

Q. 18 If u, v and w are functions of x, then show that

$\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$

in two ways – first by repeated application of product rule, second by logarithmic differentiation.

Answer:

To prove: $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$

Let y=u. v. w=u. (v. w)

(a) by applying product rule differentiate both sides with respect to x

$\frac{dy}{dx} = (v.w).\frac{du}{dx} + u.\frac{d}{dx}(v.w)$

$= \frac{dy}{dx} = (v.w).\frac{du}{dx} + u.\left[v.\frac{d}{dx}(w) + w.\frac{d}{dx}(v)\right]$

$= \frac{dy}{dx} = (v.w).\frac{du}{dx} + (u.v).\frac{dw}{dx} + (u.w.).\frac{dv}{dx}$

(b) Taking log on both sides, we get

as, y=u. v. w

log y = log (u. v. w)

log y = log u + log v + log w

Now, differentiate both sides with respect to x

$= \frac{d}{dx}(\log y) = \frac{d}{dx}\log u + \frac{d}{dx}\log v + \frac{d}{dx}\log w$

$= \frac{1}{y}.\frac{d}{dx}(y) = \frac{1}{u}.\frac{d}{dx}(u) + \frac{1}{v}.\frac{d}{dx}(v) + \frac{1}{w}.\frac{d}{dx}(w)$

$= \frac{dy}{dx}(y) = y\left[\frac{1}{u}.\frac{du}{dx} + \frac{1}{v}.\frac{dv}{dx} + \frac{1}{w}.\frac{dw}{dx}\right]$

$= \frac{dy}{dx} = u.v.w\left[\frac{1}{u}.\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}.\frac{dw}{dx}\right]$

$= \frac{dy}{dx} = v.w.\frac{du}{dx} + u.w.\frac{dv}{dx} + u.v.\frac{dw}{dx}$

From equation (i), (ii) and (iii), we can say that value of given function after differentiating by all the three methods is same.