Algebra

Self-Evaluation Test

1. The ratio of the volume of three buckets is 3 : 4 : 5. Buckets contains the mixture of water and alcohol. If the mixture contains water and alcohol in the ratio 1 : 4, 1: 3 and 2 : 5, respectively then find the ratio of water and alcohol when the mixture in all containers are poured in fourth container.

(a) 35 : 57 (b) 53:157
(c) 157 : 53 (d) 35 : 157
(e) None of these

2. Compounded ratio of (x + y):(x - y), $(x^2 + y^2)$: $(x+ y)^2$ and $(x^2 - y^2)$: $(x^4 - y^4)$ is:

(a) (x + y) : 1
(b) (x - y) : (x + y)
(c) $(1 : (x^2 - y^2)$
(d) $(x^2+y^2):(x^2-y^2)$
(e) None of these

3. The monthly income of X and Y are in the ratio of 4 : 5 and their monthly expenditure are in the ratio 7 : 9. If both save $ 100 per month then monthly expenditure of X will be:

(a) $ 800
(b) $ 700
(c) $ 900
(d) $ 200
(e) None of these

4. A person buys some pen and pencil from the market at a rate such that a pencil is twice as costly as pen. He sells them such that the cost of pencil is thrice the cost of pen. By selling a pen at twice of its cost price, he makes 150 % profit on the whole. Find the ratio of pencils to pen.

(a) 1 : 2 (b) 4 : 5
(c) 3 : 4 (d) 7 : 4
(e) None of these

5. In a container, water and milk are mixed in the ratio 5 : 3. If 16 litres of mixture are removed and same quantity of milk be added then the ratio becomes 3 : 5. What is the volume of the container?

(a) 40 (b) 50
(c) 60 (d) 70
(e) None of these

6. The ratio of ages of Marry and Mariya is 4 : 5. After 12 years their ratio becomes 5 : 6. What will be the age of Marry after 2 years?

(a) 49 (b) 50
(c) 60 (d) 70
(e) None of these

7. In a certain year the population of London is 200000. If it increases at the rate of 6.5 % per annum then what will be its population after 2 years?

(a) 226845 (b) 228645
(c) 224685 (d) 228465
(e) None of these

8. Stephen's mathematics test had 85 problems, which contains 20 algebra, 30 statistics and 35 geometry problems. He answered 70% of algebra, 40% of the statistics and 60% of geometry problems correctly. He did not pass the test because he got less than 60% of the problem correct. How many more questions he would have needed to answer correctly to earn 60% passing grade?

(a) 5 (b) 4
(c) 1 (d) 3
(e) None of these

9. If the duty on an article be reduced by 40% of its present amount, then by how much percent must the consumption be increased in order that the revenue may remain unaltered?

(a) 50 %
(b) $166\frac{2}{3}$ %
(c) 40%
(d) 20 %
(e) None of these

10. What is the total number of candidates appear in an examination, if 31% is fail and the number of passed candidates are 247 more than the number of fail candidates?

(a) 650
(b) 750
(c) 800
(d) 900
(e) None of these

11. Find the value of $(3a + 5b)^3$

(a) $27a^3+125b^3+135a^2b+2225ab^2$
(b) $27a^3-125b^3-135a^2b+2225ab^2$
(c) $27a^2+155b^2-135a^2b-225a^2b$
(d) $29a^2-156b^2-156a^2b-225a^4c$
(e) None of these

12. Find the cube of x - 2y.

(a) $x^3+8y^2+6x^2y-12xy^3$
(b) $x^3-8y^3-6x^2y+12xy^2$
(c) $x^2+87y+7xy-7xy$
(d) $7xy^3-6x-74x^2y+2xy$
(e) None of these

13. The factor of $4p^2 + q^2 + 16 - 4pq + 8q - 16p$ is:

(a) (-2p – q + 4) (-2p + q + 4)
(b) (2p + q + 4) (2p +q + 4)
(c) (-2p + q + 4) (-2p + q + 4)
(d) (-2p + q - 4) (-2p + q - 4)
(e) None of these

14. If $x - \frac{1}{x} = 2$, then find the value of $x^3 - \frac{1}{x^3}$.

(a) 14 (b) 17
(c) 16 (d) 15
(e) None of these

15. Factories: $27x^3+64y^3+108x^2+144xy^2$.

(a) $(3x+4y)^3$
(b) $(2x+4y)^2$
(c) $(3x-4y)^2$
(d) $(9x+2y)^6$
(e) None of these

16. What should be added to the sum of (x^2+y^2+xy) and $(4x^2+4xy)$ to get $(2x^2+3xy)$?

(a) $(3x^2+y^2+2xy)$ (b) $(3x^2-y^2-2xy)$
(c) $-(3x^2+y^2+2xy)$ (d) $(-3x^2+y^2+2xy)$
(e) None of these

17. A steamer goes downstream from one port to another in 6 hours. It covers the same distance up stream in 7 hours. If the speed of the stream is 2km/hours then find the speed of the steamer in still water.

(a) 20 km/h (b) 30 km/h
(c) 26 km/h (d) 48 km/h
(e) None of these

18. The sum of the digits of a two digit number is 10. The number obtained by interchanging the digits exceeds the original number by 54, find the original number.

(a) 29 (b) 28
(c) 55 (d) 95
(e) None of these

19. A monkey climbing up a pole ascends 10 metres and slips down 2 metres in alternative minutes. If the pole is 57 metres high, how long will take him to reach the top of pole?

(a) 14 minutes, 6 seconds
(b) 16 minutes, 4 seconds
(c) 20 minutes, 30 seconds
(d) 10 minutes, 18 seconds
(e) None of these

20. Two trams of equal length are running on parallel tracks in the same direction at 46 km per hour. The faster train passes the slower train in 36 seconds, the length of each train is:

(a) 46 m
(b) 33 m
(c) 53 m
(d) cannot be determined
(e) None of these

21. The denominator of a number is greater than its numerator by 8. If the numerator increased by one, the number obtained is $\frac{2}{3}$. The number is:

(a) $\frac{3}{11}$ (b) $\frac{13}{21}$

(c) $\frac{11}{19}$ (d) $\frac{14}{22}$

(e) None of these

22. The perimetre of a rectangle is 100 m. If the length is decreased by 2 m and the breadth is increased by 3 m then area increased by 44 m^2. Find the length and breadth of the rectangle.

(a) 30 m, 20 m (b) 40 m, 30 m

(c) 50 m, 40 m (d) 100 m, 90 m

(e) None of these

23. A number is 7 less than the other and its square is 77 less than the square of the greater number. The smaller number is:

(a) 9 (b) 1

(c) 4 (d) 5

(e) None of these

24. The ratio of two smaller sides of a right-angled triangle is 4 : 3, A rectangle is on the largest side of the triangle in such a way that largest side will be the length of the rectangle. The breadth of rectangle is four fifth of its length. Find the length of shortest side of triangle if the perimetre of rectangle is 1.8 m.

(a) 60 cm (b) 40 cm

(c) 15 cm (d) 30 cm

(e) None of these

25. Find the dimension of rectangle when its length is 20 m more than its width. If its width is reduced by 20 m and length is increased by 100 m then the perimetre will be twice the perimetre of original one.

(a) 30 m, 50 m

(b) 40 m, 60 m

(c) 20 m, 40 m

(d) 10 m, 30 m

(e) None of these

26. Evaluate

$$\left[\left(\frac{36}{25}\right)^{\frac{3}{2}}\right]^{\frac{5}{3}}$$

(a) $\frac{7776}{3125}$

(b) 1

(c) $\frac{75}{31}$

(d) 2

(e) None of these

27. Evaluate:

$$\left[\left\{\left(\frac{1}{x}\right)^{-12}\right\}^{\frac{1}{4}}\right]^{-\frac{2}{3}}$$

(a) $\frac{1}{x}$

(b) $\frac{1}{x^2}$

(c) $\frac{1}{x^3}$

(d) $\frac{1}{x^4}$

(e) None of these

28. Find the value of x, if $(\sqrt{6})^{x-2} = 1$.

(a) 1 (b) 2

(c) 3 (d) 4

(e) None of these

29. The value of $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^{x}}$ is:

(a) 1 (b) 0

(c) -1 (d) 10

(e) None of these

30. If $9^{p-1} + 18 = 3^{2P-1}$ then which one of the following options is incorrect?

(a) p is a natural number

(b) $p = 2$

(c) $1 < p < 4$

(d) Can't determine for natural numbers

(e) None of these

Answer – Key

1. (B)	**2.** (C)	**3.** (B)	**4.** (A)	**5.** (A)
6. (B)	**7.** (A)	**8.** (B)	**9.** (B)	**10.** (A)
11. (A)	**12.** (B)	**13.** (C)	**14.** (A)	**15.** (A)
16. (C)	**17.** (C)	**18.** (B)	**19.** (A)	**20.** (D)
21. (B)	**22.** (A)	**23.** (B)	**24.** (D)	**25.** (C)
26. (A)	**27.** (B)	**28.** (B)	**29.** (A)	**30.** (D)

Explanation for Selected Questions

2. **Explanation**

$$\frac{x+y}{x-y}\times\frac{x^2+y^2}{(x+y)^2}\times\frac{x^2-y^2}{x^4-y^4}=1:(x^3-y^2)$$

x^2-y^2

3. **Explanation**

Income of $X=4x$ and income of $Y=5x$

Expenditure of $X=7y$ and expenditure of $Y=9y$

$\Rightarrow 4x-7y=100$ ..... (i)

$\Rightarrow 5x-9y=100$ ..... (ii)

On solving (i) and (ii), we get $x=200$ and $y=100$.

Thus, expenditure of x will be \$ 700.

4. **Explanation**

Let there are a pens and b pencils.

Also let C.P of one pen $=x\Rightarrow$ C.P of a pens = ax

then C.P of one pencil $2x\Rightarrow$C.P of b pencils = 2bx

S.P. of one pen $=2x\Rightarrow$ S.P. of a pens = 2ax

S.P of one pencil $=6x\Rightarrow$ S.P. of b pencils = 6bx

Now, $\frac{S.P-C.P}{C.P}\times 100=Profit\%$

$\Rightarrow (2ax+6bx)\frac{-(ax+2bx)}{ax+2bx}=\frac{150}{100}$

$\Rightarrow \frac{a+4b}{a+2b}=\frac{3}{2}$

$\Rightarrow 2a+8b=3a+6b$

$\Rightarrow 8b-6b=3a-2a$

$\Rightarrow 2b=a\Rightarrow\frac{b}{a}=\frac{1}{2}$

5. **Explanation**

Let the quantity of water be 5x litre and quantity of milk be 3x litre in the container.

Removed quantity contains,

$\frac{16}{8}\times 5=10$ litre water and $(16-10)$ litre = 6 litre of milk.

Now, $\frac{5x-10}{(3x-6)+16}=\frac{3}{5}$

$\Rightarrow 5(5x-10)=3(3x+10)\Rightarrow 25x-50=9x+30$

$\Rightarrow 25x-9x=30+50\Rightarrow 16x=80\Rightarrow x=5$

Thus, the volume of container is $8\times 5=40$ litre.

6. **Explanation**

Let the age of Mary be 4x and age of Mariya be 5x

Age of Mary after 12 years $=(4x+12)$ years.

Age of Mariya after 12 years $=(5x+12)$ years.

Now, $\frac{4x+12}{5x+12}=\frac{5}{6}$

$\Rightarrow 6(4x+12)=5(5x+12)$

$\Rightarrow 24x+72=25x+60\Rightarrow 25x-24x=72-60$

$\Rightarrow x=12$ years.

$\therefore$ The age of Mary after 2 years $=(4\times 12+2)$ years $=50$ years.

7. **Explanation**

Population after 2 years

$=200000\left(1+\frac{6.5}{100}\right)^2=226845.$

8. **Explanation**

Number of questions attempted correctly $=(70\%\text{ of }20+40\%\text{ of }30+60\%\text{ of }35)$

$=(14+12+21)=47$

Questions to be answered correctly to obtained $60\%=60\%$ of $85=51$

Required number of question $=(51-47)=4.$

14. **Explanation**

$$x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\times(x)\times\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

$=2^3+3\times 2=8+6=14.$

15. Explanation

$(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

16. Explanation

$(x^2+y^2+xy)+(4x^2+4xy)+A = 2x^2+3xy$

$\Rightarrow (5x^2+5xy+y^2)+A = 2x^2+3xy$

$\Rightarrow A = 2x^2+3xy-(5x^2+5xy+y^2)$

$= -(3x^2+2xy+y^2).$

17. Explanation

Let the speed of the steamer in still water be x km/h.

It is given that while going down stream the steamer takes 6 hours to cover the distance between two ports.

$\therefore$ Speed of the steamer down-stream $= (x+2)$ km/ h.

Distance covered in $1\,h = (x+2)$ km

Distance covered in $6\,h = 6(x+2)$ km

$\therefore$ Distance between 2 ports $= 6(x+2)$ km ..... (i)

It is given that while going up stream, the steamer takes 7 hours to cover the distance.

Speed of the steamer up stream $= (x-2)$ km/ h

Distance covered in $1\,h = (x-2)$ km

Distance covered in $7\,h = 7(x-2)$ km

$\therefore$ Distance covered in this case $= 7(x-2)$ km ..... (ii)

$\therefore$ The distance between two ports is same

$\therefore$ From (i) and (ii) we get

$6(x+2) = 7(x-2)$

$6x+12 = 7x-14$

$\Rightarrow 6x-7x = -14-12$ [Transposing 7x to L.H.S. and 12 to R.H.S.]

$\Rightarrow -x = -26$

$\Rightarrow x = 26$

$\therefore$ The speed of the streamer in still water $= 26$ km/ hr.

18. Explanation

Since the required number is a two digit number so, we have to find its units digit and tens digit.

Let the digit at ones place be x.

It is given that the sum of the digit of the number is 10.

$\therefore$ The digit at the tens place $= 10-x$

Thus the Original number $= 10x(10-x)+x$

$= 100-10x+x = 100-9x$

On interchanging the digits of the given number the digit at the ones place becomes $(10-x)$ and the digit at the tens place becomes x.

$\therefore$ New number $= 10x+(10-x) = 9x+10$

It is given that the new number exceeds the original number by 54.

i.e.. New number-original number = 54

$(9x+10)-(100-9x) = 54$

$\therefore 9x+10-100+9x = 54$

Or, $18x-90 = 54$

$\therefore 18x = 54+90$

Or, $18x = 144$

Or, $x = \frac{144}{18} = 8$

$\therefore$ The digit at the ones place = 8

The digit at the tens place $= (10-8) = 2$

$\therefore$ Original number $= 28$.

22. Explanation

Let the length be x and breadth be y, then according to question

$2(x+y) = 100$

$\Rightarrow x+y = 50$

$\Rightarrow y = 50-x$

$(x-2)(53-x) = x(50-x)+44$

$\Rightarrow -X^2+53x+2x-106 = 50x-X^2+44$

$\Rightarrow 5x = 106+44 \Rightarrow 5x = 150 \Rightarrow x = 30\,m$

$\Rightarrow y = 50-x = 20$ m.

23. Explanation

Let the smaller number be x.

Then the larger number $= x+7$

$(x+7)^2 = x^2+77$

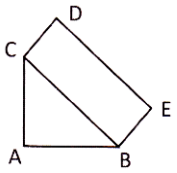
$\Rightarrow x^2+14x+49 = x^2+77$

$\Rightarrow 14x = 77-49$

$\Rightarrow x = \frac{28^2}{14}$

$\Rightarrow x = 2$

24. Explanation



Let the small sides are 4x and 3x.

From the figure

$BC^2 = AB^2 = AC^2$ (Pythagoras theorem)

$\Rightarrow BC^2 = (3x)^2 + 4x^2$

$\Rightarrow BC^2 = 9x^2 + 16x^2$

$\Rightarrow BC^2 = 25x^2 BC = 5x$

According to question,

Length of rectangle $= 5x$

Breadth of rectangle $\Rightarrow \frac{4}{5} \times 5x = 4x$

Perimeter of rectangle $= 2(5x + 4x) = 18x$

$\Rightarrow 18x = 180$ cm

$\Rightarrow x = 10$ cm

The length of shortest side $= 3 \times 10 = 30$ cm.

25. Explanation

Let the breadth of rectangle be x.

Then length will be $x + 20$

According to question

$2(x + 100 + x - 20) = 2[2(x + x + 20)]$

$\Rightarrow 2(2x + 80) = 4(2x + 20)$

$4x + 160 = 8x + 80$

$4x = 80$

$\Rightarrow x = 20$

breadth $= 20$ m

length $= 20 + 20 = 40$ m.

26. Explanation

$$\left(\frac{36}{25}\right)^{\frac{\cancel{2}}{\cancel{2}} \times \frac{5}{\cancel{2}}} = \left(\frac{36}{25}\right)^{\frac{5}{2}} = \left(\frac{6}{2}\right)^{\cancel{2} \times \frac{5}{\cancel{2}}} = \left(\frac{6}{5}\right)^5 = \frac{7776}{3125}$$

27. Explanation

$$\left(\frac{1}{x}\right)^{-12x\left(\frac{1}{4}\right) \times \frac{-2}{3}} = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$$

28. Explanation

We can write the given expression as,

$\left(\sqrt{6}\right)^{x-2} = \left(\sqrt{6}\right)^0$ [$\because a^0 = 1$ where a is any number except zero]

$\Rightarrow x - 2 = 0$

$\Rightarrow x = 2$

29. Explanation

$$= \frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^x}$$

$$= \frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times (2 \times 3)^{y+1}}{(2 \times 3)^{x+1} \times (5 \times 2)^{y+3} \times (5 \times 3)^x}$$

$$= \frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 2^{y+1} \times 3^{y+1}}{2^{x+1} \times 3^{x+1} \times 5^{y+3} \times 2^{y+3} \times 5^x \times 3^x}$$

$$= \frac{2^{x+y+4} \times 3^{2x+1} \times 5^{x+y+3}}{2^{x+y+4} \times 3^{2x+1} \times 5^{x+y+3}} = 1$$