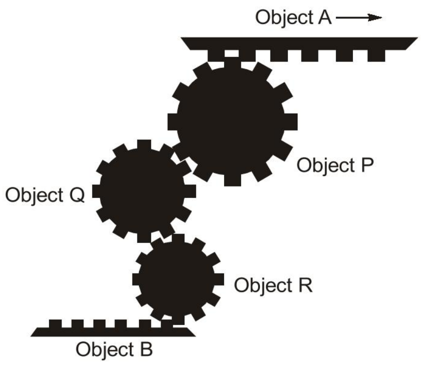
Electrical Engineering
(Afternoon Session)
Exam Date- 11-02-2024

SECTION - A GENERAL APTITUDE

Q.1 The asembly shown below has three teethed circular objects (Pinions) and two teethed flat objects (Racks), which are perfectly mating with each other. Pinions can only rotate clockwise or anti-clockwise staying at its own center. Racks can translate towards the left (←) and the right (→) direction.



If the object A (Rack) is translating towards the right (→) direction, the correct statement among the following is

(a) Object B translates towards the right direction.
(b) Object Q rotates in the clockwise direction.
(c) Object B translates towards the left direction.
(d) Object R rotates in the anti-clockwise direction.

Ans. (c)

Object B translates towards the left direction.

End of Solution

Q.2 If, for non-zero real variables x, y and real parameter $a > 1$,

$$x : y = (a + 1) : (a - 1),$$

then, the ratio $(x^2 - y^2) : (x^2 + y^2)$ is

(a) $a : (a^2 - 1)$
(b) $a : (a^2 + 1)$
(c) $2a : (a^2 + 1)$
(d) $2a : (a^2 - 1)$

Ans. (c)

$$x : y = (a + 1) : (a - 1)$$

$$\frac{x}{y} = \frac{(a+1)}{(a-1)}$$

$$\frac{x^2 - y^2}{x^2 + y^2} = \frac{\left(\frac{x}{y}\right)^2 - 1}{\left(\frac{x}{y}\right)^2 + 1}$$

$$= \frac{\left(\frac{(a+1)}{(a-1)}\right)^2 - 1}{\left(\frac{(a+1)}{(a-1)}\right)^2 - 1}$$

$$= \frac{(a+1)^2 - (a-1)^2}{(a+1)^2 + (a-1)^2}$$

$$= \frac{4a}{2a^2 + 2} = \frac{2a}{a^2 + 1}$$

End of Solution

Q.3 If '$\rightarrow$' denotes increasing order of intensity, then the meaning of the words [talk $\rightarrow$ shout $\rightarrow$ scream] is analogous to [please $\rightarrow$ __________ $\rightarrow$ pander].

Which one of the given options is appropriate to fill the blank?

(a) flutter
(b) fritter
(c) frizzle
(d) flatter

Ans. **(d)**

End of Solution

Q.4 The decimal number system uses the characters 0, 1, 2, ..., 8, 9, and the octal number system uses the characters 0, 1, 2, ..., 6, 7.

For example, the decimal number 12 ($= 1 \times 10^1 + 2 \times 10^0$) is expressed as 14 ($= 1 \times 8^1 + 4 \times 8^0$) in the octal number system.

The decimal number 108 in the octal number system is

(a) 150
(b) 168
(c) 108
(d) 154

Ans. **(d)**

$$(108)_{10} = (154)_8$$

$$\begin{aligned} 154 &= 1 \times 8^2 + 5 \times 8^1 + 4 \times 8^0 \\ &= 64 + 40 + 4 \times 1 \\ &= 108 \end{aligned}$$

End of Solution

Q.5 In the given text, the blanks are numbered (i)-(iv). Select the best match for all the blanks.

Following a row __(i)__ the shopkeeper __(ii)__ the price of a frying pan, the cook stood __(iii)__ a row to withdraw cash __(iv)__ the ATM booth.

(a) (i) with; (ii) over; (iii) in; (iv) at
(b) (i) with; (ii) over; (iii) at; (iv) with
(c) (i) over; (ii) with; (iii) over; (iv) at
(d) (i) at; (ii) over; (iii) over; (iv) in

Ans. **(a)**

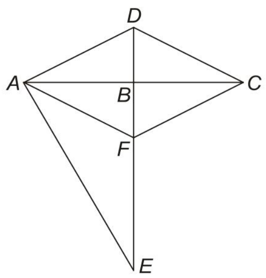
End of Solution

Q.6 In the following figure,

$$CD = 5 \text{ cm}, BE = 10 \text{ cm}, AE = 12 \text{ cm}$$

$$\angle DAB = \angle DCB \text{ and } \angle DAE = \angle DBC = 90°$$

Points AFCD create a rhombus.



The length of BF (in cm) is

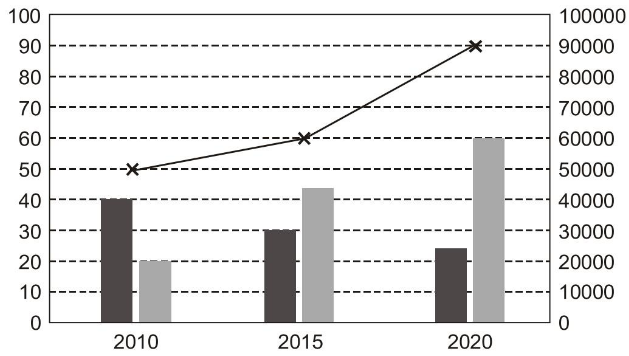
(a) 2 (b) 6

(c) 4 (d) 3

Ans. **(*)**

End of Solution

Q.7 The chart below shows the data of the number of cars bought by Millennials and Gen X people in a country from the year 2010 to 2020 as well as the yearly fuel consumption of the country (in Million liters).



Considering the data presented in the chart, which one of the following options is true?

(a) The increase in the number of Millennial car buyers from 2015 to 2020 is less than the decrease in the number of Gen X car buyers from 2010 to 2015.

(b) The decrease in the number of Gen X car buyers from 2015 to 2020 is more than the increase in the number of Millennial car buyers from 2010 to 2015.

(c) The increase in the number of Millennial car buyers from 2010 to 2015 is more than the decrease in the number of Gen X car buyers from 2010 to 2015.

(d) The percentage increase in fuel consumption from 2010 to 2015 is more than the percentage increase in fuel consumption from 2015 to 2020.

Ans. (c)

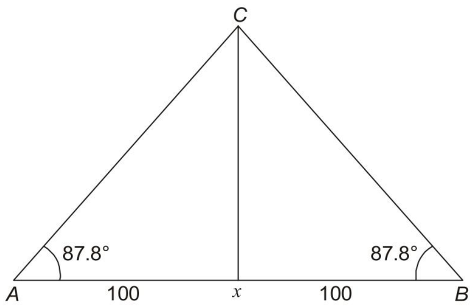
End of Solution

Q.8 A surveyor has to measure the horizontal distance from her position to a distant reference point C. Using her position as the center, a 200 m horizontal line segment is drawn with the two endpoints A and B. Points A, B and C are not collinear. Each of the angles ∠CAB and ∠CBA are measured as 87.8°. The distance (in m) of the reference point C from her position is nearest to

(a) 2606 (b) 2306

(c) 2603 (d) 2063

Ans. (c)



$$\tan\theta = \frac{Cx}{Ax}$$

$$\tan 87.8 = \frac{Cx}{100}$$

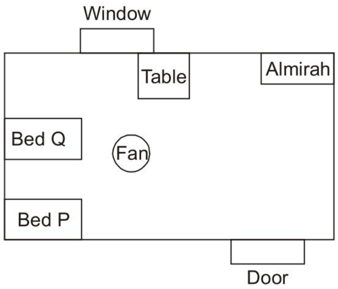
$$Cx = 100 \times \tan 87.8$$

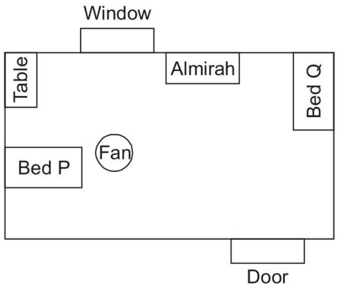
$$= 2603$$

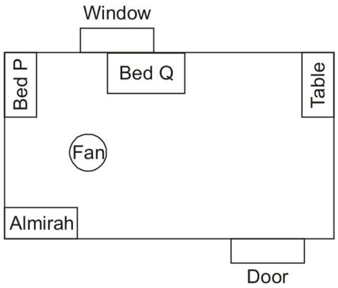
End of Solution

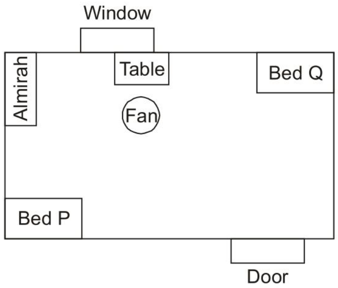
Q.9 P and Q have been allotted a hostel room with two beds, a study table, and an almirah. P is an avid bird-watcher and wants to sit at the table and watch birds outside the window. Q does not mind that as long as his bed is close to the ceiling fan.

Which one of the following arrangements suits them the most?

(a) 

(b) 

(c) 

(d) 

Ans. (a)

End of Solution

Q.10 A shopkeeper buys shirts from a producer and sells them at 20% profit. A customer has to pay ₹ 3,186.00 including 18% taxes, per shirt. At what price did the shopkeeper buy each shirt from the producer?

(a) ₹ 2,250.00 (b) ₹ 1,975.40
(c) ₹ 2,500.00 (d) ₹ 2,548.00

Ans. (a)

Let Cost price $= x$

$$x \times \frac{120}{100} \times \frac{118}{100} = 3186$$

$$x = \frac{100 \times 100 \times 3186}{120 \times 118}$$

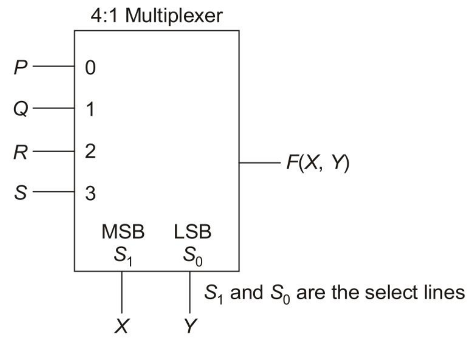
$$x = 2250$$

End of Solution

■■■■

SECTION - B TECHNICAL

Q.1 To obtain the Boolean function $F(X, Y) = X\overline{Y} + \overline{X}$, the inputs *PQRS* in the figure should be



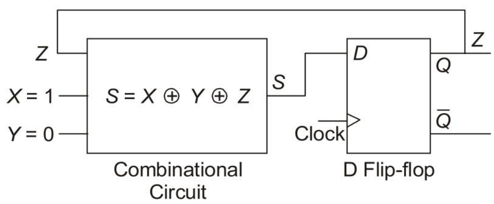
(a) 1110 (b) 1010

(c) 0110 (d) 0001

Ans. (a)

End of Solution

Q.2 In the circuit, the present value of Z is 1. Neglecting the delay in the combinatorial circuit, the values of S and Z, respectively, after the application of the clock will be



(a) $S = 0, Z = 0$ (b) $S = 1, Z = 1$

(c) $S = 0, Z = 1$ (d) $S = 1, Z = 0$

Ans. (d)

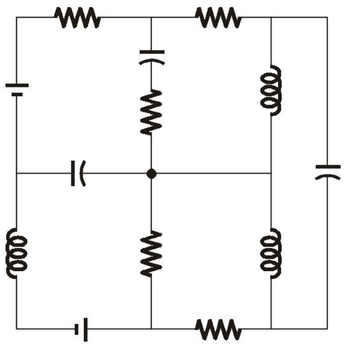
Based on this trick

$$x \oplus x = 0$$
$$x \oplus \overline{x} = 1$$
$$x \oplus 1 = \overline{x}$$
$$x \oplus 0 = x$$

$$\overline{x \oplus y} = x \odot y = 0 \odot 1 = 0$$

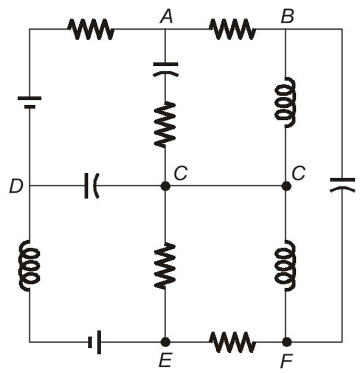
End of Solution

Q.3 The number of junctions in the circuit is



(a) 8 (b) 6
(c) 7 (d) 9

Ans. **(b)**



A point at which more than two elements are joints together is called Junction.
A, B, C, D, E and F are junction.

End of Solution

Q.4 If $u(t)$ is the unit step function, then the region of convergence (ROC) of the Laplace transform of the signal

$$x(t) = e^{t^2}[u(t-1) - u(t-10)]$$

is

(a) $\text{Re}(s) \geq 10$ (b) $-\infty < \text{Re}(s) < \infty$
(c) $\text{Re}(s) \leq 1$ (b) $1 \leq \text{Re}(s) \leq 10$

Ans. **(b)**

Since $x(t)$ is finite duration signal. So ROC will be $-\infty < \sigma < \infty$.

End of Solution

Q.5 Which one of the following matrices has an inverse?

(a) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 2 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 0.5 & 2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 3 & 12 & 24 \end{bmatrix}$

Ans. (a)

Option (b) :

$$\det = 0$$

Cannot have inverse.

Option (c) :

$$\det = 0$$

Cannot have inverse.

Option (d) :

$$\det = 0$$

It cannot have inverse.

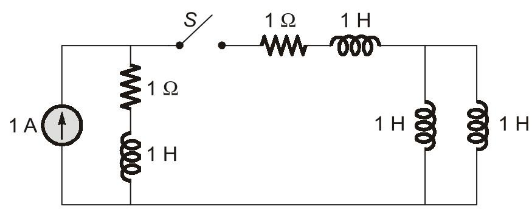
Option (a) :

$$|A| \neq 0$$

$\therefore$ It has inverse.

End of Solution

Q.6 The circuit shown in the figure with the switch S open, is in steady state. After the switch S is closed, the time constant of the circuit in seconds is



(a) 1.5 (b) 1.25

(c) 0 (d) 1

Ans. (b)

$$L_{eq} = 1 + 1 + (1 \| 1) = 2.5 \text{ H}$$

$$R_{eq} = 2\ \Omega$$

$$\text{Time constant, } \tau = \frac{L_{eq}}{R_{eq}} = \frac{2.5}{2} = 1.25$$

End of Solution

Q.7 If the following switching devices have similar power ratings, which one of them is the fastest?

(a) SCR
(b) GTO
(c) Power MOSFET
(d) IGBT

Ans. (c)

End of Solution

Q.8 Suppose signal $y(t)$ is obtained by the time-reversal of signal $x(t)$, i.e., $y(t) = x(-t)$, $-\infty < t < \infty$. Which one of the following options is always true for the convolution of $x(t)$ and $y(t)$?

(a) It is an odd signal
(b) It is an even signal
(c) It is an anti-causal signal
(d) It is a causal signal

Ans. (b)

$$z(t) = x[t] * y[t]$$
$$z(t) = x[t] * x[-t]$$

Replace t with $-t$

$$z[-t] = x[-t] * x[t]$$
$$z[-t] = x[t] * x[-t]$$
$$z[-t] = z[t]$$

even function.

End of Solution

Q.9 A single-phase triac based AC voltage controller feeds a series RL load. The input AC supply is 230 V, 50 Hz. The values of R and L are 10 Ω and 18.37 mH, respectively. The minimum triggering angle of the triac to obtain controllable output voltage is

(a) 30°
(b) 15°
(c) 45°
(d) 60°

Ans. (a)

Output voltage is controlled if

$$\alpha > \theta$$

Given: $f = 50$ Hz, $R = 10\,\Omega$, $L = 18.37$ mH

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

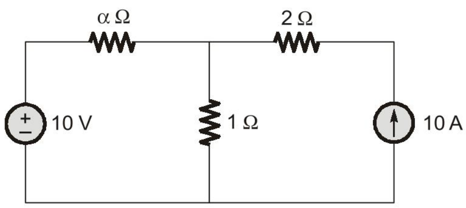
$$\theta = \tan^{-1}\left(\frac{2\pi \times 50 \times 18.37 \times 10^{-3}}{10}\right)$$

$$\theta = 29.989°$$

So, α_{min} can be 30°. Hence answer is (a).

End of Solution

Q.10 All the elements in the circuit are ideal. The power delivered by the 10 V source in watts is

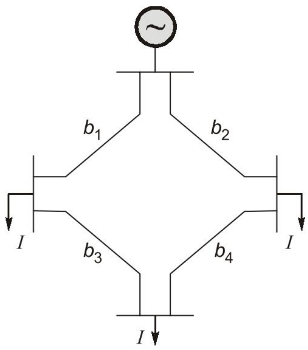


(a) 100 (b) 0
(c) dependent on the value of α (d) 50

Ans. (b)

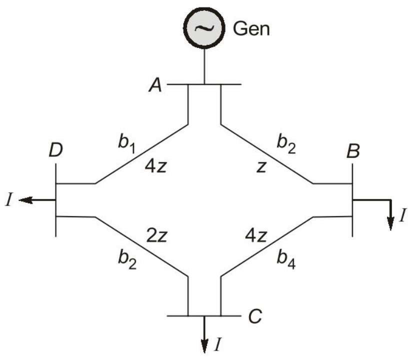
End of Solution

Q.11 The figure shows the single line diagram of a 4-bus power network, Branches b_1, b_2, b_3 and b_4 have impedances $4z$, z, $2z$ and $4z$ per-unit (pu), respectively, where $z = r + jx$, with $r > 0$ and $x > 0$. The current drawn from each load bus (marked as arrows) is equal to I pu, where $I \neq 0$. If the network is to operate with minimum loss, the branch that should be opened is



(a) b_3 (b) b_1
(c) b_4 (d) b_2

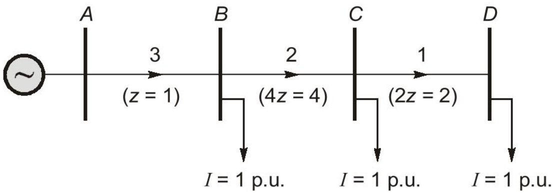
Ans. (a)



Assume, $|z| = |r + jX| = 1$ p.u.,

$I = 1$ p.u.

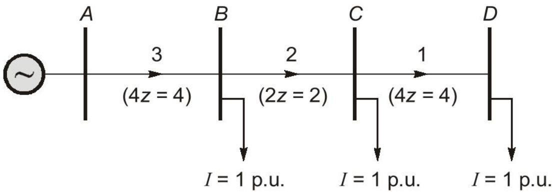
By opening branches ring becomes radial open b_1



Total power loss,

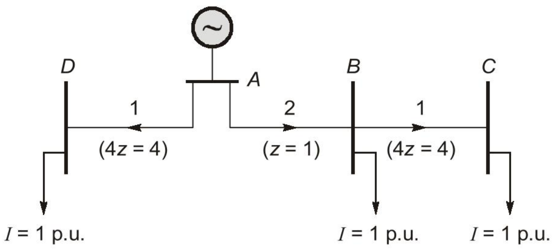
$$P_{L1} = 1^2 \times 2 + 2^2 \times 4 + 3^2 \times 1$$
$$= 27 \text{ p.u.}$$

Open b_2 :



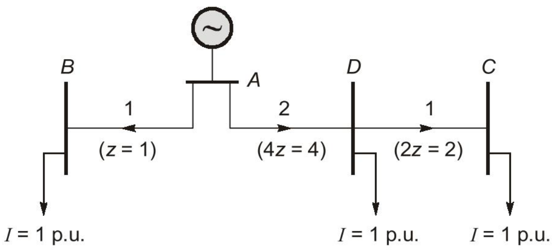
$$P_{L2} = (3^2 \times 4 + 2^2 \times 2 + 1^2 \times 4)$$
$$= 48 \text{ p.u.}$$

Open b_3 :



$$P_{L3} = (2^2 \times 1 + 1^2 \times 4 + 1^2 \times 4)$$
$$= 12 \text{ p.u.} \leftarrow \text{mininum power loss}$$

Open b_4 :



$$P_{L4} = (1^2 \times 1 + 2^2 \times 4 + 1^2 \times 2)$$
$$= 19 \text{ p.u.}$$

By opening branch b_3 Power loss is minimum.

End of Solution

Q.12 The sum of the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$ is ______ (rounded off to the nearest integer).

Ans. **(29)**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

$$\lambda_1 + \lambda_2 = \text{Trace (A)}$$
$$= 22 + 7$$
$$= 29$$

End of Solution

Q.13 A three phase, 50 Hz, 6 pole induction motor runs at 960 rpm. The stator copper loss, core loss, and the rotational loss of the motor can be neglected. the percentage efficiency of the motor is

(a) 96 (b) 98

(c) 92 (d) 94

Ans. **(a)**

$$s = \frac{1000 - 960}{1000} = 0.04$$

$$\frac{\text{Rotor output}}{\text{Rotor input}} = 1 - s$$

$$\eta = 1 - 0.04 = 0.96 \text{ or } 96\%$$

End of Solution

Q.14 The table lists two instrument transformers and their features :

Instrument Transformers	Features
	(P) Primary is connected in parallel to the grid
(X) Current Transformer (CT)	(Q) Open circuited secondary is not desirable
(Y) Potential Transformer (PT)	(R) Primary current is the line current
	(S) Secondary burden affects the primary current

The correct matching of the two column is

(a) X matches with P and Q; Y matches with R and S.

(b) X matches with Q and S; Y matches with P and R.

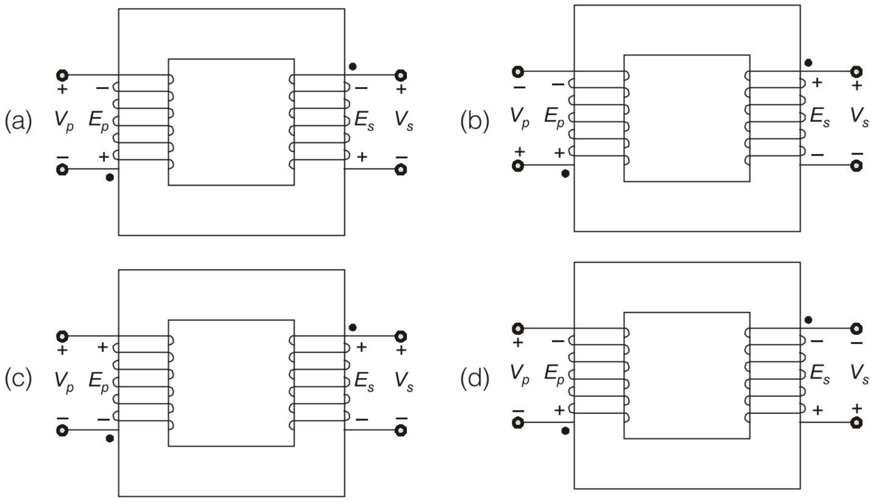
(c) X matches with Q and R; Y matches with P and S.

(d) X matches with P and R; Y matches with Q and S.

Ans. **(c)**

End of Solution

Q.15 Which one of the following options represents possible voltage polarities in a single phase two winding transformer? Here, V_p is the applied primary voltage, E_p is the induced primary voltage, V_s is the open circuit secondary voltage, and E_s is the induced secondary voltage.



Ans. **(b)**

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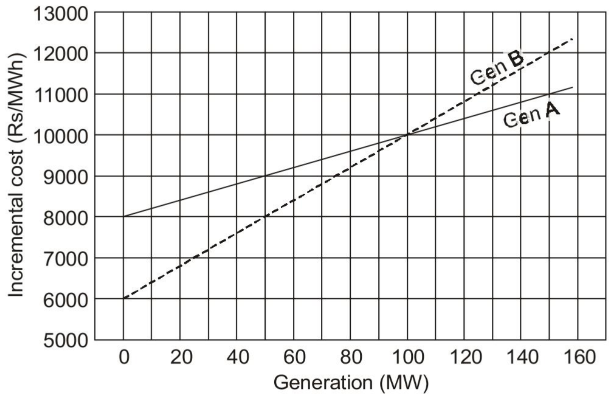
End of Solution

Q.16 The incremental cost curves of two generators (Gen A and Gen B) in a plant supplying a common load are shown in the figure. If the incremental cost of supplying the common load is Rs. 7400 per MWh, then the common load in MW is ______ (rounded off to the nearest ingeger).



Ans. **(35)**

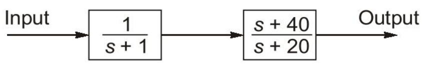
Given, $\lambda = 7400$ Rs/MWhr

$$I_{CA}(P_{GA}) = \frac{2000}{100}P_{GA} + 8000$$

$$I_{CB}(P_{GB}) = 40P_{GB} + 6000$$
$$\lambda = I_{CA} = I_{CB} = 7400$$
$$200P_{GA} + 8000 = 7400$$
$$P_{GA} = -600$$
$$P_{GA} < 0$$
$$P_{GB} = \frac{7400-6000}{40} = 35$$
$$P_{GA} + P_{GB} = 35 \text{ MW}$$

End of Solution

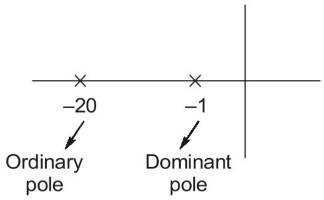
Q.17 Consider the cascaded system as shown in the figure. Neglecting the faster component of the trasient response, which one of the following options is a first-order pole-only approximation such that the steady-state values of the unit step responses of the original and the approximated system are same?



(a) $\frac{2}{s+20}$ (b) $\frac{2}{s+1}$

(c) $\frac{1}{s+20}$ (d) $\frac{1}{s+1}$

Ans. (b)

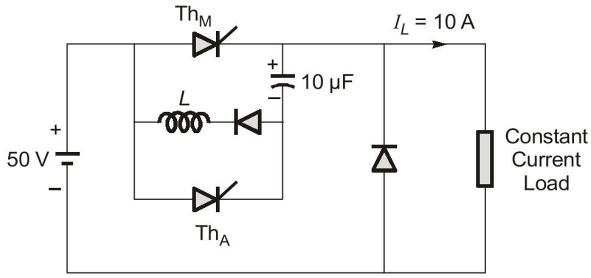


Transfer function $\rightarrow \dfrac{40\left(1+\frac{s}{40}\right)}{20\left(1+\frac{s}{40}\right)(1+s)}$

Answer $= \dfrac{2}{s+1}$

End of Solution

Q.18 A forced commutated thyristorized step-down chopper is shown in the figure. Neglect the ON-state drop across the power devices. Assume that the capacitor is initially charged to 50 V with the polarity shown in the figure. The load current (I_L) can be assumed to be constant at 10 A. Initially, Th_M is ON and Th_A is OFF. The turn-off time available to Th_M in microseconds, when Th_A is triggered, is ______ (rounded off to the nearest integer).



Ans. **(50)**

$$t_{cm} = \frac{C}{I_o} V_s$$

$$= \frac{10 \cdot 10^{-6}}{10} \cdot 50$$

$$= 50\ \mu s$$

End of Solution

Q.19 Which of the following complex functions is/are analytic on the complex plane?

(a) $f(z) = \text{Im}\ (z)$
(b) $f(z) = z^2 - z$
(c) $f(z) = e^{|z|}$
(d) $f(z) = j\ \text{Re}\ (z)$

Ans. **(b)**

Let us take (a), $f(z) = z^2 - z$

$$u + iv = x^2 - y^2 + i(2xy) - x - iy$$

$\Rightarrow$ $u = x^2 - y^2 - x,\ V = 2xy - y$

Now by C–R equation,

$$u_x = V_y \text{ and } u_y = -V_x$$

$\Rightarrow$ $(2x - 1) = (2x - 1)$ and $-2y = -(2y)$

i.e. C-R equation are satisfied.

End of Solution

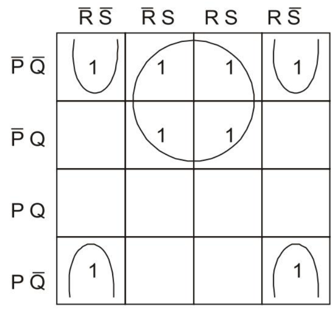
Q.20 Simplified form of the Boolean function :

$$F(P, Q, R, S) = \bar{P}\bar{Q} + \bar{P}QS + P\bar{Q}\bar{R}\bar{S} + P\bar{Q}R\bar{S}$$

is

(a) $\bar{P}\bar{Q} + \bar{Q}\bar{S}$
(b) $P\bar{S} + Q\bar{R}$
(c) $\bar{P}Q + R\bar{S}$
(d) $\bar{P}S + \bar{Q}\bar{S}$

Ans. **(d)**



Minimize function = $\bar{Q}\bar{S} + \bar{P}S$

End of Solution

Q.21 Consider the complex function $f(z) = \cos z + e^{z^2}$. The coefficient of z^5 in the Taylor series expansion of $f(z)$ about the origin is ______ (rounded off to 1 decimal place).

Ans. **(0)**

$$f(z) = \left(1 + z^2 + \frac{z^4}{\underline{|2}} + \frac{z^6}{\underline{|6}} + ...\right) + \left(1 - \frac{z^2}{\underline{|2}} + \frac{z^4}{\underline{|4}} + ...\right)$$

It is series is of even powers.

$\therefore$ Coefficient of z^5 = 0.

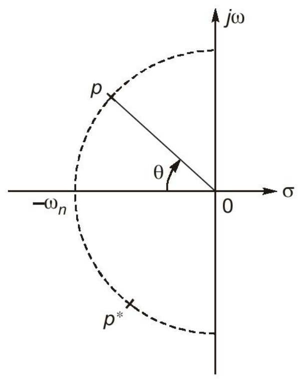
End of Solution

Q.22 Consider the standard second-order system of the form $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ with the poles p and p* having negative real parts. The pole locations are also shown in the figure. Now consider two such second-order systems as defined below :

System 1 : ω_n = 3 rad/sec and θ = 60°

System 2 : ω_n = 1 rad/sec and θ = 70°

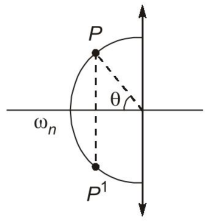
Which one of the following statements is correct?



(a) Settling time cannot be computed from the given information.
(b) Settling time of System 1 is more than that of System 2.
(c) Settling times of both the systems are the same.
(d) Settling time of System 2 is more than that of System 1.

Ans. (d)

In system -1



$$\xi_1 = \cos\theta = \cos 60° = 0.5$$
$$\omega_{n1} = 3 \text{ rad/sec}$$
$$\text{Settling time} - t_{s1} = \frac{4}{\xi\omega_n} = \frac{4}{0.5 \times 3} = \frac{8}{3} = 2.67 \text{ sec}$$

In system -2

$$\xi_2 = \cos\theta_2 = \cos 70°$$
$$\omega_{n2} = 1 \text{ rad/sec}$$
$$\text{Settling time} - t_{s2} = \frac{4}{\xi\omega_n} = \frac{4}{\cos 70° \times 1} = 11.69 \text{ sec}$$

So, $t_{s2} > t_{s1}$

End of Solution

Q.23 Let X be a discrete random variable that is uniformly distributed over the set $\{-10, -9, ..., 0, ..., 9, 10\}$. Which of the following random variables is/are uniformly distributed?

(a) X^2
(b) $(X + 10)^2$
(c) X^3
(d) $(X - 5)^2$

Ans. (b, c)

$$X^2 = \{100, 81, 64, ..., 4, 1, 0, 1, 4, 9, ..., 81, 100\}$$

$$P(\text{Choosing } 0) = \frac{1}{21}, \text{ P (choosing 1)} = \frac{2}{21}$$

So, X^2 is not uniformly distributed.

$$(X - 5)^2 = \{225, 199, ..., 4, 1, 0, 1, 4, 9, 16, 25\}$$

Again probability of choosing any number is not equal so $(X - 5)^2$ is also not uniformly - distributed.

$$(X + 10)^2 = \{0, 1, 4, 9, 16, ... 81, 100, 121, ..., 400\}$$

$P(\text{Choosing any number}) = \frac{1}{21}$ so is it uniformly-distributed

Now, $X^3 = \{-1000, -729, -512, ..., -8, -1, 0, 1, 8, ... 729, 1000\}$

Again P(Choosing any number) $= \frac{1}{21}$ = Constant

So, it is also uniformly-distributed.

End of Solution

Q.24 Let $X(\omega)$ be the Fourier transform of the signal, $x(t) = e^{-t^4} \cos t$, $-\infty < t < \infty$. The value of the derivative of $X(\omega)$ at $\omega = 0$ at is ______ (rounded off to 1 decimal place)

Ans. **(0)**

$$tx(t) \rightleftarrows j\frac{dX(\omega)}{d\omega}$$

$$f(t) = \frac{tx(t)}{j} \rightleftarrows \frac{dX(\omega)}{d\omega} = F(\omega)$$

$$\left.\frac{dX(\omega)}{d\omega}\right|_{\omega=0} = F(\omega)\big|_{\omega=0} = \text{area of } f(t)$$

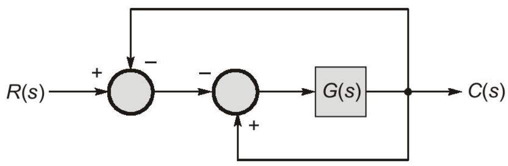
$$= \int_{-\infty}^{\infty} \frac{tx(t)}{j} dt = \text{area of odd-function}$$

$$= 0$$

$$\left[\begin{array}{l} \because x(t) = \text{even-function} \\ \therefore \frac{t}{j}x(t) = \text{odd-function} \end{array}\right]$$

End of Solution

Q.25 For the block-diagram shown in the figure, the transfer function $\frac{C(s)}{R(s)}$ is



(a) $\frac{G(s)}{1-2G(s)}$

(b) $-\frac{G(s)}{1+2G(s)}$

(c) $-\frac{G(s)}{1-2G(s)}$

(d) $\frac{G(s)}{1+2G(s)}$

Ans. **(c)**

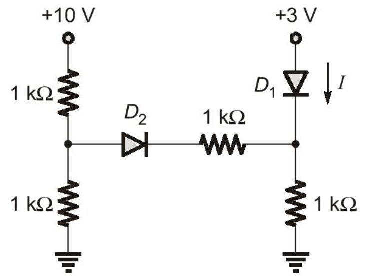
Transfer function

$$\frac{C(s)}{-R(s)} = \frac{G(s)}{1-G(s)-G(s)} = \frac{G(s)}{1-2G(s)}$$

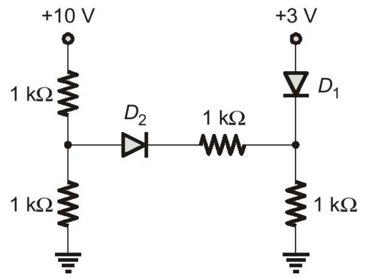
$$\frac{C(s)}{R(s)} = \frac{-G(s)}{1-2G(s)}$$

End of Solution

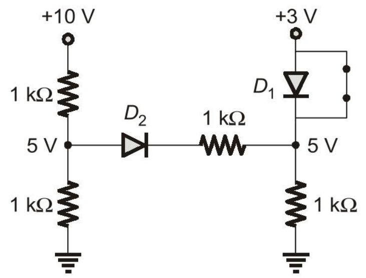
Q.26 In the given circuit, the diodes are ideal. The current I through the diode D_1 in milliamperes is _______ (rounded off to two decimal places).



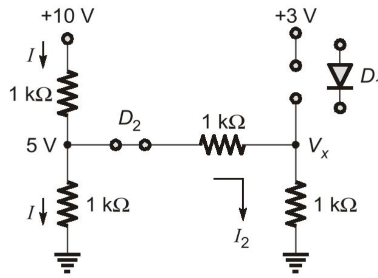
Ans. **1.67 (1.65 to 1.69)**

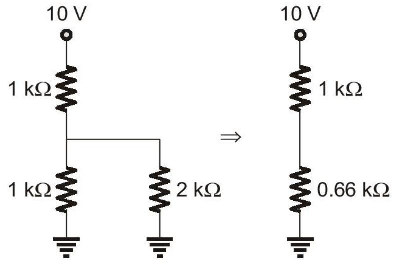


Assume D_2 OFF



Tested : D_2 ON
Assume D_1 OFF





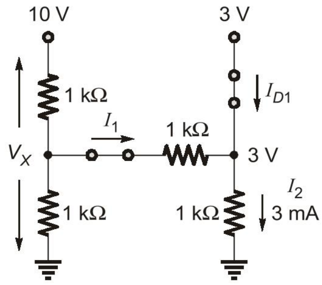
$$I = \frac{10}{1.66\text{ K}} = 6.02\text{ mA}$$

$$I_2 = I \times \frac{1\text{k}\Omega}{3\text{k}\Omega} = 6.02\text{ mA} \times \frac{1}{3} = 2\text{ mA}$$

Tested : D_1 ON

$$V_X = 2\text{ mA} \times 1\text{ k}\Omega$$
$$= 2\text{ V}$$

States: D_1 and D_2 ON



$$\frac{V_X - 10}{1\text{k}\Omega} + \frac{V_X - 3}{1\text{k}\Omega} + \frac{V_X}{1\text{k}\Omega} = 0$$

$$3V_X = 13$$

$$V_X = \frac{13}{3} = 4.33\text{ V}$$

$$I_1 + I_{D1} = I_2$$

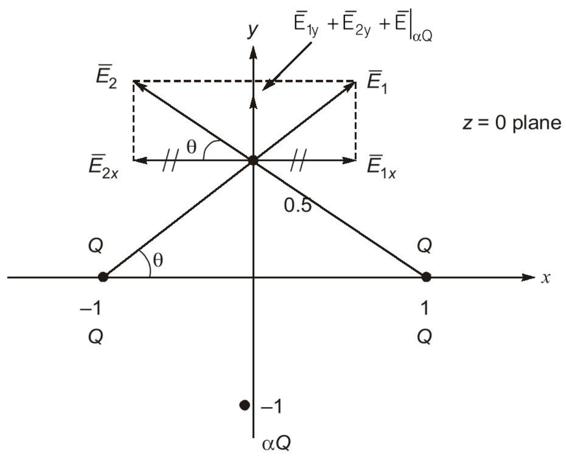
$$\frac{4.33 - 3}{1\text{k}\Omega} + I_{D1} = 3\text{ mA}$$

$$I_{D1} = 3\text{ mA} - 1.33\text{ mA}$$
$$= 1.67\text{ mA}$$

End of Solution

Q.27 In the (x, y, z) coordinate system, three point-charges Q,Q. and αQ are located in free space at (–1, 0, 0), (1, 0, 0) and (0, –1, 0) respectively. The value of α for the electric field to be zero at (0, 0, 5, 0) is _______ (rounded off to 1 decimal places).

Ans. **–1.6 (–1.7 to –1.5)**



From the figure,

$$\bar{E}_{net} = \bar{E}_{1y}+\bar{E}_{2y}+\bar{E}\big|_{\alpha Q} = 0 \text{ [as per question]}$$

Here, $\bar{E}_{1y} = \bar{E}_{2y} = \bar{E}\big|_{Q}$ (say)

$\therefore$ $\bar{E}_{net} = 2E\big|_Q + 2E\big|_{\alpha Q} = 0$

Now, $$\bar{E}\big|_Q = \frac{kQ}{\left[\sqrt{1+0.5^2}\right]^2}\sin\theta\hat{a}_y; \qquad k=\frac{1}{4\pi \in_o}$$

$\Rightarrow$ $$\bar{E}\big|_Q = \frac{kQ}{1.25}\times\frac{0.5}{\sqrt{1+0.5^2}}\hat{a}_y = 0.357\, kQ\hat{a}_y$$

Also, $$\bar{E}\big|_{\alpha Q} = \frac{k(\alpha Q)}{(1.5)^2}\hat{a}_y = 0.444k(\alpha Q)\hat{a}_y$$

Hence, $$\bar{E}_{net} = 2\times 0.357kQ\hat{a}_y + 0.444k(\alpha Q)\hat{a}_y = 0$$

$\Rightarrow$ $0.715kQ = –0.444\text{ k}(\alpha Q)$

$\Rightarrow$ $$\alpha = \frac{-0.715}{0.444} = -1.61$$

$\therefore$ $\alpha = –1.61$

End of Solution

Q.28 Consider the discrete-time system T_1 and T_2 defined as follows:

$$\{T_1x\}[n] = x[0] + x[1] + \ldots + x[n]$$

$$\{T_2x\}[n] = x[0] + \frac{1}{2}x[1] + \cdots + \frac{1}{2^n}x[n]$$

Which one of the following statement is true?

(a) T_1 and T_2 are not BIBO stable.

(b) T_1 is BIBO stable but T_2 is not BIBO stable.

(c) T_1 is not BIBO stable but T_2 is BIBO stable.

(d) T_1 and T_2 are BIBO stable.

Ans. (c)

$$\{T_1x\}[n] = x[0] + x[1] + \ldots + x[n]$$

$$\{T_1x\}[n]^- = \sum_{K=0}^{n} x[K]$$

Let, $x[K] = u[K]$

$$|x[K]| = |u[K]|$$

$$|\{T_1x\}[n]| = \left|\sum_{K=0}^{n} u[K]\right| = \sum_{K=0}^{n} 1 = n$$

$$|[T_1x(n)|_{n=\infty} = \infty \text{ system is not BIBO stable}$$

$$\{T_2x\}[n] = x[0] + \frac{1}{2}x[1] + \cdots + \frac{1}{2^n}x[n]$$

$$= \sum_{K=0}^{n} \left(\frac{1}{2}\right)^K x(K)$$

Let, $x(K) = u[K]$

$$|x[K]| = |u[K]|$$

$$\{T_2x\}[n] = \sum_{K=0}^{n} \left|\left(\frac{1}{2}\right)^K u(K)\right|$$

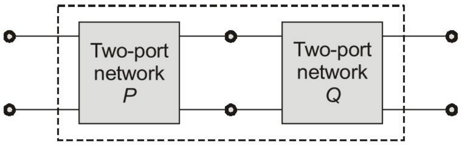
$$\{T_2x\}[n] = \sum_{K=0}^{n} \left(\frac{1}{2}\right)^K$$

$$|[T_2x(n)|_{n=\infty} = \sum_{K=0}^{\infty} \left(\frac{1}{2}\right)^K = \frac{1}{1-\frac{1}{2}} = 2 \text{ stable}$$

T_1 is not BIBO stable but T_2 is BIBO stable.

End of Solution

Q.29 Two passive two-port network P and Q are connected as shown in the figure. The impedance matrix of network P is $Z_P = \begin{bmatrix} 40\,\Omega & 60\,\Omega \\ 80\,\Omega & 100\,\Omega \end{bmatrix}$. The admittance matrix of network Q is $Y_Q = \begin{bmatrix} 5S & -2.5S \\ -2.5S & 1S \end{bmatrix}$. Let the $ABCD$ matrix of the two-port network R in the figure be $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. The value of β in Ω is ________ . (rounded off to 2 decimal places)



Two-port network R

Ans. **−19.80 (−19.90 to −19.70)**

$$Z_P = \begin{bmatrix} 40 & 60 \\ 80 & 100 \end{bmatrix}; \quad Y_Q = \begin{bmatrix} 5S & -2.5S \\ -2.5S & 1S \end{bmatrix}$$

From Z_P matrix

$$V_1 = 40I_1 + 60I_2 \qquad \text{...(i)}$$

$$V_2 = 80I_1 + 100I_2 \qquad \text{...(ii)}$$

Convert Z-parameters in to ABCD parameters

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

From equation (ii),

$$I_1 = \frac{V_2}{80} - \frac{100}{80}I_2$$

$$I_1 = \frac{V_2}{80} - \frac{5}{4}I_2 \qquad \text{...(iii)}$$

Sub equation (iii) in equation (i),

$$V_1 = 40\left(\frac{V_2}{80} - \frac{5}{4}I_2\right) + 60I_2$$

$$V_1 = \frac{V_2}{2} + 10I_2 \qquad \text{...(iv)}$$

From equation (iii) and (iv),

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 0.5 & -10 \\ \frac{1}{80} & \frac{5}{4} \end{bmatrix}$$

$$[Y]_Q = \begin{bmatrix} 5 & -2.5 \\ -2.5 & 1 \end{bmatrix}$$

$$I_1 = 5V_1 - 2.5V_2 \quad \text{...(v)}$$
$$I_2 = -2.5V_1 + V_2 \quad \text{...(vi)}$$

Convert Y-parameters to ABCD parameters

From equation (vi),

$$V_1 = \frac{V_2}{2.5} - \frac{I_2}{2.5}$$
$$V_1 = 0.4V_2 - 0.4I_2 \quad \text{...(vii)}$$

Sub equation (vii) in equation (v),

$$I_1 = 5(0.4V_2 - 0.4I_2) - 2.5V_2$$
$$= 2V_2 - 2I_2 - 2.5V_2$$
$$I_1 = -0.5V_2 - 2I_2 \quad \text{...(viii)}$$

From equation (vii) and equation (viii),

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.4 \\ -0.5 & 2 \end{bmatrix}$$

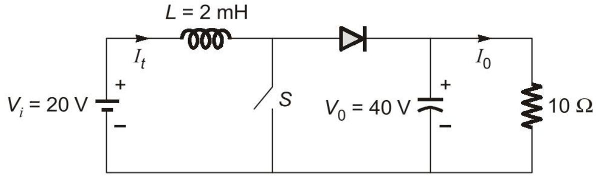
$$T = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} * \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & -10 \\ \frac{1}{80} & \frac{5}{4} \end{bmatrix} * \begin{bmatrix} 0.4 & 0.4 \\ -0.5 & 2 \end{bmatrix}$$

$$\beta = 0.5 \times 0.4 + (-10) \times 2$$
$$= -19.8\ \Omega$$

End of Solution

Q.30 In the DC-DC converter shown in the figure, the current through the inductor is continuous, The switching frequency is 500 Hz. The voltage (V_0) across the load is assumed to be constant and ripple free. The peak inductor current in amperes is ________ (rounded off to the nearest integer).



Ans. (10)

$$V_0 = \frac{V_S}{1-\alpha}$$

$$40 = \frac{20}{1-\alpha}$$

$$\alpha = 0.5$$

$$I_0 = \frac{V_0}{R} = \frac{40}{10} = 4\text{ A}$$

Boost :

$$I_L = \frac{I_0}{1-\alpha} = \frac{4}{1-0.5} = 8 \text{ A}$$

$$\Delta I_L = \frac{\alpha V_s}{fC} = \frac{0.5\times 20}{500\times 2.10^{-3}} = 10$$

$$I_{mx} = (iL)_{\text{peak}}$$

$$= I_L + \frac{\Delta I_L}{2} = 8 + \frac{10}{2} = 13 \text{ A}$$

End of Solution

Q.31 The given equation represents a magnetic field strength $\bar{H}(r,\theta,\phi)$ in the spherical coordinate system, in free space. Here, $\hat{r}$ and $\hat{\theta}$ represent the unit vectors along r and θ, respectively. The value of P in the equation should be _______ (rounded off to the nearest integer).

$$\bar{H}(r,\theta,\phi) = \frac{1}{r^3}(\hat{r}P\cos\theta + \hat{\theta}\sin\theta)$$

Ans. **(2)**

$$\nabla\cdot\bar{B} = 0$$

$$\nabla\cdot\bar{H} = 0$$

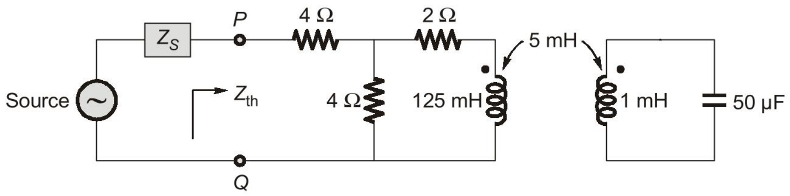
Spherical coordinate system,

$$h_1\ h_2\ h_3 = 1\ r\ r\sin\theta$$

On substituting $P = 2$.

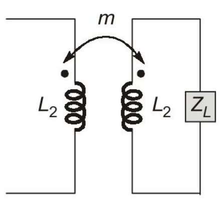
End of Solution

Q.32 For the circuit shown in the figure, the source frequency is 5000 rad/sec. The mutual inductance between the magnetically coupled inductors is 5 mH with their self inductances being 125 mH and 1 mH. The Thevenin's impedance. Z_{th}, between the terminals P and Q in Ω is ________ (rounded off to 2 decimal places).



Ans. **5.33 (5.30 to 5.40)**

Given: $\omega = 5000$ rad/sec

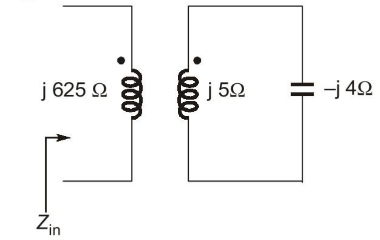


$$z_{in} = jX_{L1} + \frac{(\omega m)^2}{jX_{L2} + Z_L}$$

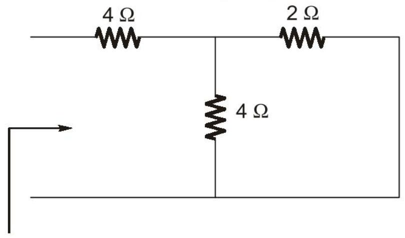
$$jX_{L1} = 125 \text{ mH} \times 5000 = j625\ \Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{5000 \times 50 \times 10^{-6}} = -j4\Omega$$

$$jX_{L2} = 1 \text{ mH} \times 5000 = +j5 \ \Omega$$



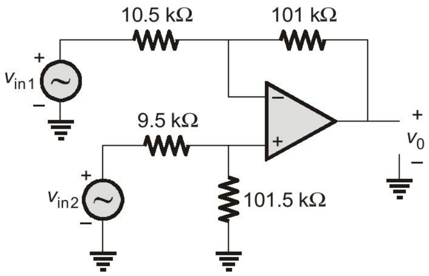
$$z_{in} = j625 + \frac{(5000 \times 5 \times 10^{-3})^2}{j5 - j4} = j625 - j625 = 0 \ \Omega$$



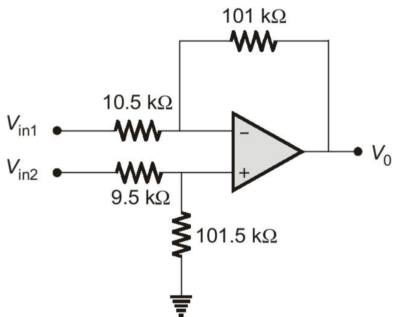
$$z_m = \frac{4 \times 2}{4+2} + 4 = \frac{16}{3}\Omega = 5.33 \ \Omega$$

End of Solution

Q.33 A difference amplifier is shown in the figure. Assume the op-amp to be ideal. The CMRR (in dB) of the difference amplifier is _______ (rounded off to 2 decimal places).



Ans. 40.52 (40.49 to 40.53)



$$V_o = \left(1 + \frac{101}{10.5}\right)V^+ + \frac{-101}{10.5}V_{in1}$$

$$= 10.62 \times V_{in2} \times \frac{101.5}{101.5 + 9.5} + \frac{-101}{10.5}V_{in1}$$

$$= \underbrace{9.71}_{A_1}V_{in2} + \underbrace{-9.619}_{A_2}V_{in1}$$

$$\text{CMRR} = \frac{A_d}{A_c}$$

$$A_d = \frac{A_1 - A_2}{2}$$

$$= \frac{9.71 + 9.619}{2} = 9.6645$$

$$A_c = A_1 + A_2$$

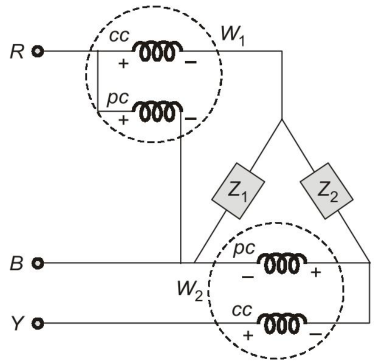
$$= 9.71 - 9.619 = 0.091$$

$$\text{CMRR} = \frac{9.6645}{0.091} = 106.20$$

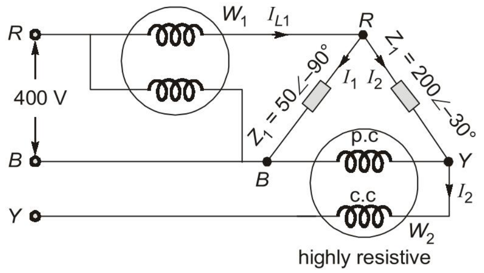
$$\text{CMRR} = 20\log(106.20) = 40.52 \text{ dB}$$

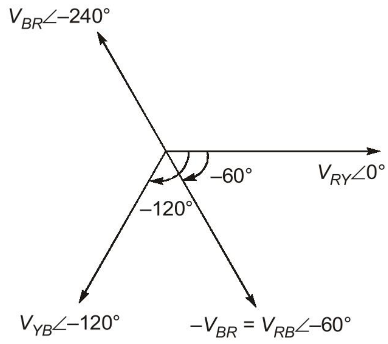
End of Solution

Q.34 In the circuit shown, $Z_1 = 50\angle{-90°}$ and $Z_2 = 200\angle{-30°}\ \Omega$. It is supplied by a three phase 400 V source with the phase sequence being R-Y-B. Assume the watt meters W_1 and W_2 to be ideal. The magnitude of the difference between the readings of W_1 and W_2 in watts is ________ (rounded off to 2 decimal places).



Ans. 692.80 (691.00 to 694.00)





$$V_{RY} = 400\angle 0^\circ$$
$$V_{YB} = 400\angle{-120^\circ}$$
$$V_{BR} = 400\angle{-240^\circ}$$

$W(1) \Rightarrow$
$$V_{PC} = V_{RB} = 400\angle{-60^\circ}$$

$$I_1 = \frac{V_{RB}}{Z_1} = \frac{400\angle -60^\circ}{50\angle -90^\circ} = 8\angle 30^\circ$$

$$I_2 = \frac{V_{RY}}{Z_2} = \frac{400\angle 0^\circ}{200\angle -30^\circ} = 2\angle 30^\circ \Rightarrow I_{CC\,(2)}$$

$$I_{L1} = I_{CC\,(1)} = I_1 + I_2 = 8\angle 30^\circ + 2\angle 30^\circ$$
$$= 10\angle 30^\circ \Rightarrow I_{CC\,(1)}$$
$$W = V_{RB} \times I_{2\theta} \cos(\angle V_{RB} \text{ and } I_{LO})$$
$$W_1 = 400 \times 10 \times \cos(-60^\circ - 30^\circ) = 0 \text{ Watt}$$
$$W_2 = V_{YB} \times I_2 \cos(\angle V_{YB} \text{ and } I_2)$$
$$= 400 \times 2 \times \cos(-120^\circ - 30^\circ) = -692.8 \text{ Watt}$$
$$W_1 - W_2 = 0 - (-692.8) = 692.8 \text{ Watt}$$

End of Solution

Q.35 Which of the following option is/are correct for the Automatic Generation Control (AGC) and Automatic Voltage Regulator (AVR) installed with synchronous generators?

(a) AGC regulates the generator's mechanical power input while AVR regulates the field current of the synchronous generator.

(b) AGC response has a local effect on frequency while AVR response lias a global effect on voltage.

(c) AGC response has a global effect on frequency while AVR response has a local effect on voltage.

(d) AGC regulates the field current of the synchronous generator while AVR regulates the generator's mechanical power input.

Ans. (a, c)

End of Solution

Q.36 Which of the following differential equations is/are nonlinear?

(a) $x(t)+e^{\left(\frac{dx(t)}{dt}\right)}=1$, $x(0) = 0$

(b) $t\,x(t)+\left(\frac{dx(t)}{dt}\right)=t^2e^t$, $x(0) = 0$

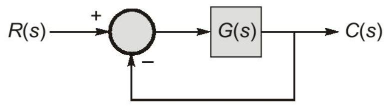
(c) $\frac{1}{2}e^t+x(t)\left(\frac{dx(t)}{dt}\right)=0$, $x(0) = 0$

(d) $x(t)\cos t-\frac{dx(t)}{dt}\sin t=1$, $x(0) = 0$

Ans. **(a, c)**

End of Solution

Q.37 Consider the stable closed-loop system shown in the figure. The asymptotic Bode magnitude plot of $G(s)$ has a constant slope of –20 dB/decade at least till 100 rad/sec with the gain crossover frequency being 10 rad/sec. Hie asymptotic Bode phase plot remains constant at –90° at least till ω = 10 rad/sec. The steady-state error of the closed-loop system for a unit ramp input is _______ (rounded off to 2 decimal places).

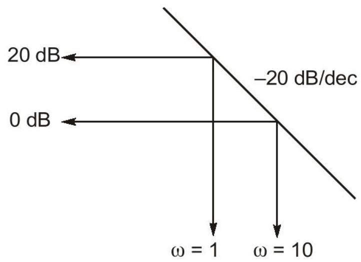


Ans. **0.10 (0.09 to 0.11)**

Initial slppe = –20 dB/sec

Open loop transfer function = $\frac{k}{s}$

$$20\log_{10}k = 20$$
$$k = 10$$



e_{ss} for type -1 system for ramp input

$$e_{ss} = \frac{1}{k_v}=\frac{1}{10}=0.1$$

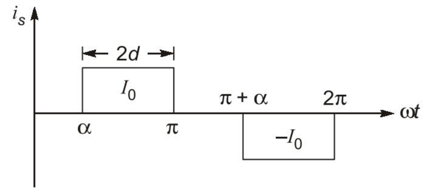
End of Solution

Q.38 A single-phase half-controlled bridge converter supplies an inductive load with ripple free load current. The triggering angle of the converter is 60°. The ratio of the rms value of the fundamental component of the input current to the rms value of the total input current of the bridge is __________(rounded off to 3 decimal places).

Ans. 0.955 (0.950 to 0.960)

1-ϕ Half controlled bridge rectifier

$$\alpha = 60°$$



$$g = \frac{I_{s1}}{I_{sr}} = \frac{\frac{2\sqrt{2}}{\pi} I_0 \cdot \cos\frac{\alpha}{2}}{I_0\left(\frac{\pi-\alpha}{\pi}\right)^{1/2}} = \frac{2\sqrt{2}\cdot\cos\frac{\alpha}{2}}{\sqrt{\pi\cdot(\pi-\alpha)}}$$

$$= \frac{2\sqrt{2}\cdot\cos\left(\frac{60}{2}\right)}{\sqrt{\pi\cdot\left(\pi-\frac{\pi}{3}\right)}} = \frac{3}{\pi} = 0.955$$

Alternate solution:

$$2d = \pi - \alpha$$

$$= \pi - \frac{3}{\pi}$$

$$2d = \frac{2\pi}{\pi} = 120°$$

$$g = \frac{3}{\pi} = 0.955$$

End of Solution

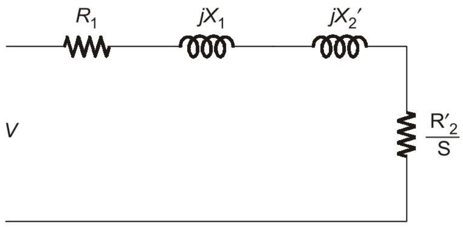
Q.39 A 3-phase star connected slip ring induction motor has the following parameters referred to the stator:

$$R_s = 3\ \Omega,\quad X_s = 2\ \Omega,$$
$$X_r' = 2\ \Omega,\quad R_r' = 2.5\ \Omega,$$

The per phase stator to rotor effective turns ratio is 3 :1. The rotor winding is also star connected. The magnetizing reactance and core loss of the motor can be neglected. To have maximum torque at starting, the value of the extra resistance in ohms (referred to the rotor side) to be connected in series with each phase of the rotor winding is ______ (rounded off to 2 decimal places).

Ans. **0.28 (0.26 to 0.30)**

Condition for the maximum torque



$$\frac{R'_2}{s} = \sqrt{3^2 + 4^2} = 5$$

Substituting, $s = 1$

As maximum torque at starting

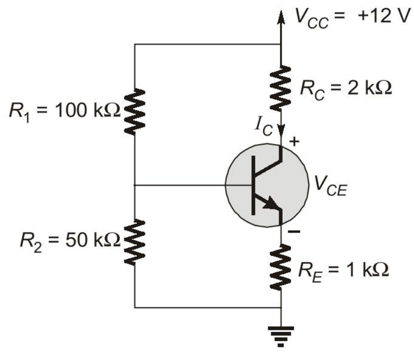
$$R'_2 + R'_{ext} = 5$$

$$R'_{ext} = 5 - 2.5 = 2.5\ \Omega$$

$$\frac{2.5}{3^2} = 0.277\ \Omega$$

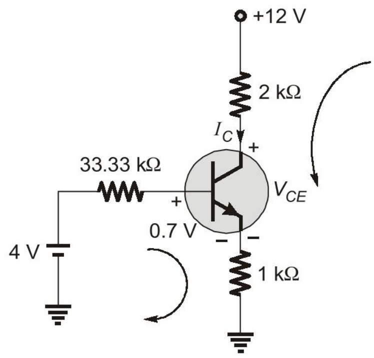
End of Solution

Q.40 A BJT biasing circuit is shown in the figure, where $V_{BE} = 0.7$ V and $\beta = 100$. The Quiescent Point values of V_{CE} and I_C are respectively



(a) 4.6 V and 2.46 mA
(b) 2.61 V and 3.13 mA
(c) 4.6 V and 3.13 mA
(d) 3.5 V and 2.46 mA

Ans. **(a)**



$$V_{Th} = \frac{12 \times 50k}{150k} = 4\text{ V}$$

$$R_{Th} = 50k \parallel 100k = \frac{50k \times 100k}{150k} = 33.33\text{ k}\Omega$$

KVL in input loop,

$4 - 33.33k \times I_B - 0.7 - I_E R_E = 0$

$3.3 - 33.33k \times I_B - (1 + \beta)I_B \times 1k = 0$

$$\therefore \quad I_B = \frac{3.3}{33.33k + 101k} = 24.56\,\mu A$$

$$I_C = \beta I_B = 100 \times 24.56 \times 10^{-6} = 2.46 \text{ mA}$$

$$I_E = (1 + \beta)I_B = 2.47 \text{ mA}$$

KVL in outer loop :

$12 - 2k \times I_C - V_{CE} - I_E R_E = 0$

$$12 - V_{CE} = 2k \times I_C + I_E \times 1k$$

$$12 - V_{CE} = 7.39 \quad \Rightarrow \quad V_{CE} = 4.61 \text{ V}$$

End of Solution

Q.41 For a two-phase network, the phase voltages V_p and V_q are to be expressed in terms of sequence voltages V_α and V_β as $\begin{bmatrix} V_p \\ V_q \end{bmatrix} = S \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$. The possible option(s) for matrix S is/are

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

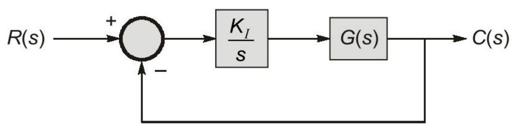
(d) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

Ans. (b, d)

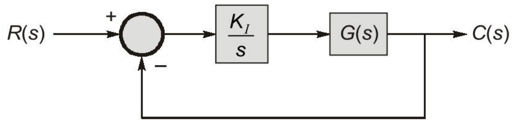
End of Solution

Q.42 Consider the stable closed-loop system shown in the figure. The magnitude and phase values of the frequency response of $G(s)$ are given in the table. The value of the gain $K_1(> 0)$ for a 50° phase margin is _______ (rounded off to 2 decimal places).

ω in rad/sec	Magnitude in dB	Phase in degrees
0.5	–7	–40
1.0	–10	–80
2.0	–18	–130
10.0	–40	–200



Ans. (1.12)



$$GH(s) = \frac{K_I}{s} G(s)$$

$$\text{PM} = 180° + \angle GH(j\omega)_{\omega_{gc}} = 50°$$

$$\Rightarrow \quad 180° + [-90° + \angle G(j\omega)]_{\omega_{gc}} = 50°$$

$$\therefore \quad \angle G(j\omega) = -40°$$

at $\omega_{gc} \Rightarrow$ $\quad |GH(j\omega)| = 0 \text{ dB}$

But at $\omega = 0.5$;

$$|G(j\omega)| = -7 \text{ dB and } \angle G(j\omega) = -40°$$

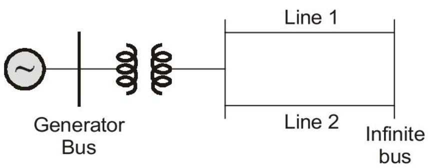
$$\therefore \quad \left|\frac{K_I}{s}\right|_{\omega=0.5} = +7 \text{ dB}$$

$$\therefore \quad \frac{K_I}{0.5} = 2.24$$

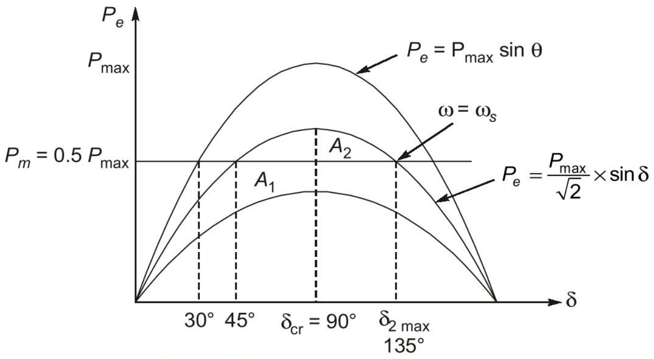
$$\therefore \quad K_I = 1.12$$

End of Solution

Q.43 The single line diagram of a lossless system is shown in the figure. The system is operating in steady-state at a stable equilibrium point with the power output of the generator being P_{max} sin δ. where δ is the load angle and the mechanical power input is 0.5 P_{max}. A fault occurs on line 2 such that the power output of the generator is less than 0.5.P_{max} during the fault. After the fault is cleared by opening line 2. The power output of the generator is $\{P_{max} / \sqrt{2}\}\sin\delta$. If the critical fault clearing angle is π/2 radians, the accelerating area on the power angle carve is _______ times P_{max} (rounded off to 2 decimal places).



Ans. (0.1)



$$A_1 = A_2$$

$$\int_{90°}^{135°} \left(\frac{P_{max}}{\sqrt{2}} \sin\delta - 0.5 P_{max} \right) d\delta = 0.1\,P_{max}$$

End of Solution

Q.44 Let $f(t)$ be a real-valued function whose second derivative is positive for $-\infty < t < \infty$. Which of the following statements is/are always true?

(a) $f(t)$ cannot have two distinct local minima.
(b) The minimum value of $f(t)$ cannot be negative.
(c) $f(t)$ has at least one local maximum.
(d) $f(t)$ has at least one local minimum.

Ans. (a)

Since $f''(t) > 0$

If $t \in (-\infty, \infty)$

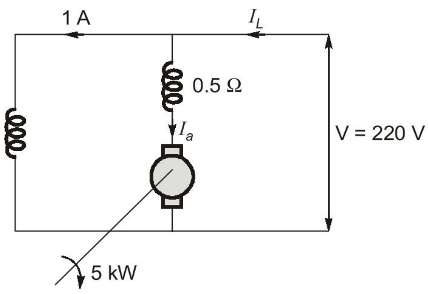
$\Rightarrow$ $f(t)$ is a parabola open upwards. So, it has only one minima at its stationary point.

$\therefore$ $f(t)$ cannot have two distinct local minima.

End of Solution

Q.45 A 5 kW. 220 V DC shunt motor lias 0.5 Ω armature resistance including brushes. The motor draws a no-load current of 3 A. The field current is constant at 1 A. Assuming that the core and rotational losses are constant and independent of the load, the current (in amperes) drawn by the motor while delivering the rated load, for the best possible efficiency, is _________ (rounded off to 2 decimal places).

Ans. (27.28)



At no load, $E_b = V - I_a R_a$

$= 220 - (3 - 1)0.5$

$E_b = 219$ V

No load power developed

$=$ Mechanical loss $= E_b . I_a$

$= 219 \times 2 = 438$ W

At rated load :

Given : $P_{out} = 500$ W

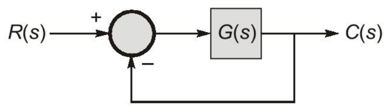
$$E'_b I'_a = 5000 + \text{Mechanical Loss}$$
$$(V - I'_a R_a)(I'_a) = 5438 \text{ W}$$
$$-I'^2_a + 0.5 + V.I'_a = 5438$$
$$0.5I'^2_a - 220I'_a + 5438 = 0$$
$$I'_a = 26.28 \text{ A}$$

Current drawn by motor

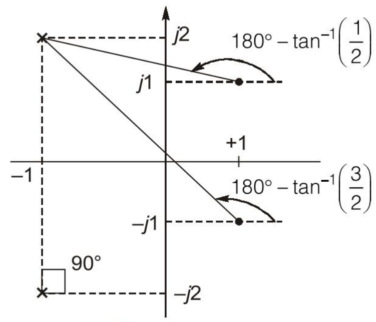
$$I_L = I'_a + 1 = 27.28 \text{ A}$$

End of Solution

Q.46 Consider the closed-loop system shown in the figure with $G(s) = \dfrac{K(s^2 - 2s + 2)}{(s^2 + 2s + 5)}$. The root locus for the closed-loop system is to be drawn for $0 \le K < \infty$. The angle of departure (between 0° and 360°) of the root locus branch drawn from the pole $(-1 + j2)$, in degree, is ______ (rounded off to the nearest integer).



Ans. **(7)**



$$\phi_D = \pm(180 + \phi)$$
$$\phi = \Sigma\phi_z - \Sigma\phi_p$$
$$= \left\{\left[180 - \tan^{-1}\left(\frac{1}{2}\right)\right] + \left[180 - \tan^{-1}\left(\frac{3}{2}\right)\right] - 90°\right\}$$
$$\phi_D = \pm 7°$$

End of Solution

Q.47 The input $x(t)$ and the output $y(t)$ of a system are related as

$$y(t) = e^{-t}\int_{-\infty}^{t} e^{\tau} x(\tau) d\tau \quad , \quad -\infty < t < \infty$$

The system is

(a) nonlinear

(b) linear but not time-invariant

(c) linear and time-invariant

(d) noncasual

Ans. **(c)**

- System will follow law of superposition. Therefore, system will be linear.
- Time-invariance check:

$$y(t-t_o) = e^{-(t-t_o)}\int_{-\infty}^{t-t_o} e^{\tau}\cdot x(\tau)d\tau \quad \text{... (i)}$$

For input $x(t - t_o)$, output will be

$$y'(t) = e^{-t}\int_{-\infty}^{t} e^{\tau}\cdot x(\tau - t_o)d\tau$$

Put $\tau - t_o = k$, then $d\tau = dk$ and for $\tau = t$, $k = t - t_o$

$$= e^{-t}\int_{-\infty}^{t-t_o} e^{k+t_o}\cdot x(k)dk$$

$$= e^{-t}\times e^{to}\times\int_{-\infty}^{t-t_o} e^{k}\cdot x(k)dk = e^{-(t-t_o)}\times\int_{-\infty}^{t-t_o} e^{k}\cdot x(k)dk$$

$$= e^{-(t-t_0)}\times\int_{-\infty}^{t-t_0} e^{\tau}\cdot x(\tau)d\tau \quad \text{... (ii)}$$

Since, $y'(t) = y(t - t_o)$

So, the system is time-invariant.

End of Solution

Q.48 If the energy of a continuous-time signal $x(t)$ is E and the energy of the signal $2x(2t - 1)$ is cE, then c is _____ (rounded off to 1 decimal place).

Ans. **(2)**

$$x(t) \rightarrow E \text{ (energy)}$$

$$x(t-1) \rightarrow E$$

$$x(2t-1) \rightarrow \frac{E}{2}$$

$$2x(2t-1) \rightarrow (2)^2(E/2) = 2E = CE$$

$$C = 2$$

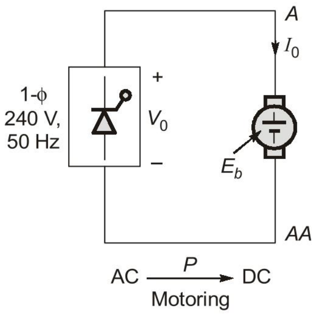
End of Solution

Q.49 A single-phase full-controlled thyristor converter bridge is used for regenerative braking of a separately excited DC motor with the following specifications:

Rated armature voltage	210 V
Rated armature current	10 A
Rated speed	1200 rpm
Armature resistance	1 Ω
Input to the converter bridge	240 V at 50 Hz
The armature of the DC motor is fed from the full-controlled bridge and the field current is kept constant.	

Assume that the motor is running at 600 rpm and the armature terminals of the motor are suitably reversed for regenerative braking. If the armature current of the motor is to be maintained at the rated value, the triggering angle of the converter bridge in degrees should be ______ (rounded off to 2 decimal places).

Ans. 114.58 (114.55 to 114.65)



Rating of DC machine

210 V, 10 A, 1200 rpm,

$$R_a = 1\ \Omega$$

$$I_F \rightarrow \text{constant}$$

$$\therefore \quad \phi \rightarrow \text{constant}$$

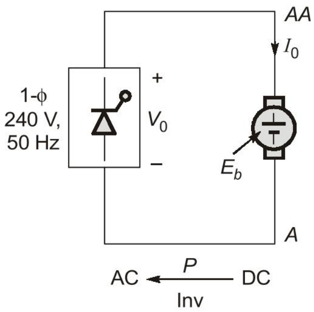
$$\therefore \quad E_b = K_m W = K_m \cdot \frac{2\pi \cdot N}{60}$$

$$V_0 = E_b + I_0 R_a$$

$$210 = K_m \cdot \frac{2\pi N}{60} + I_0 R_a$$

$$210 = K_m \cdot \frac{2\pi \cdot 1200}{60} + (10 \times 1)$$

$$K_m = 1.59 \text{ (V-s)/rad}$$



Armature terminals reversed

$$\therefore \quad E_b \rightarrow \text{Negative}$$

Inv for regenerative braking

$$V_0 = -E_b + I_0 R_a$$

$$\frac{2V_m}{\pi} \cos\alpha = -E_b + I_0 R_a$$

$$\frac{2.240\sqrt{2}}{\pi}\cos\alpha = -K_m \cdot \frac{2\pi N}{60} + I_0 R_a$$

$$\frac{2 \cdot 240\sqrt{2}}{\pi} \cdot \cos\alpha = -1.59\left(\frac{2\pi \cdot 600}{60}\right) + 10$$

$$\alpha = 114.58°$$

End of Solution

Q.50 Consider the function $f(t) = (\max(0, t))^2$ for $-\infty < t < \infty$, where $\max(a, b)$ denotes the maximum of a and b. Which of the following statements is/are true?

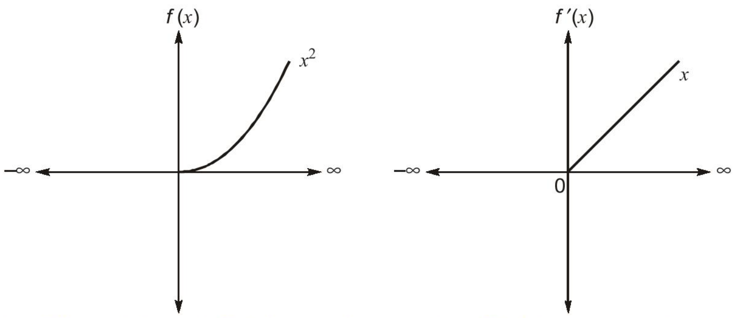
(a) $f(t)$ and its derivative are differentiable.

(b) $f(t)$ is differentiable but its derivative is not continuous.

(c) $f(t)$ is not differentiable.

(d) $f(t)$ is differentiable and its derivative is continuous.

Ans. (d)



$\therefore$ $f(x)$ is differential and $f'(x)$ is continuous but $f''(x)$ is not continuous.

End of Solution

Q.51 If the Z-transform of a finite-duration discrete-time signal $x[n]$ is $X(z)$, then the Z-transform of the signal $y[n] = x[2n]$ is

(a) $Y(z) = X(z^2)$

(b) $Y(z) = \frac{1}{2}\left[X\left(z^2\right) + X\left(-z^2\right)\right]$

(c) $Y(z) = \frac{1}{2}\left[X\left(z^{-1/2}\right) + X\left(-z^{-1/2}\right)\right]$

(d) $Y(z) = \frac{1}{2}\left[X\left(z^{1/2}\right) + X\left(-z^{1/2}\right)\right]$

Ans. (d)

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} = \sum_{n=-\infty}^{\infty} x(2n) z^{-n}$$

Let $2n = m$

then $n = \frac{m}{2} = \sum_{m=\text{even-int.}} x(m) z^{-m/2}$

$$= \sum_{m=-\infty}^{\infty} \left[\frac{x(m) + (-1)^m x(m)}{2}\right] z^{-m/2}$$

$$= \frac{1}{2}\left[\sum_{m=-\infty}^{\infty} x(m) z^{-m/2} + \sum_{m=-\infty}^{\infty} (-1)^m x(m) z^{-m/2}\right]$$

$$= \frac{1}{2}\left[\sum_{m=-\infty}^{\infty} x(m)(z^{1/2})^{-m} + \sum_{m=-\infty}^{\infty} x(m)(-z^{1/2})^{m}\right]$$

$$= \frac{1}{2}\left[x(z^{1/2}) + x(-z^{1/2})\right]$$

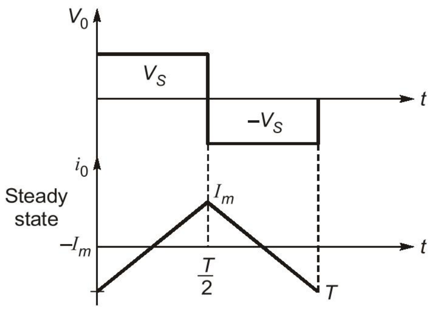
End of Solution

Q.52 A single-phase full bridge voltage source inverter (VSI) feeds a purely inductive load. The inverter output voltage is a square wave in 180° conduction mode. The fundamental frequency of the output voltage is 50 Hz. If the DC input voltage of the inverter is 100 V and the value of the load inductance is 20 mH, the peak-to-peak load current in amperes is _______ (rounded off to the nearest integer).

Ans. (50)

1-ϕ VSI, $f_1 = f_0 = 50$ Hz

$V_S = 100$ V



$$V_0 = V_S \quad \left[0 \le t \le \frac{T}{2}\right]$$

$$L \cdot \frac{di}{dt} = V_S$$

$$\frac{di}{dt} = \frac{V_S}{L}$$

$$\int di_0 = \frac{V_S}{L}\int dt$$

$$i_0 = \frac{V_S}{L} t + K$$

$$-I_m = \frac{V_S}{L}(0) + K$$

$$i_0 = \frac{V_S}{L} \cdot t - I_m \qquad \text{At } t = 0, i_0 = -I_m$$

$$\therefore \qquad K = -I_m$$

At $t = \frac{T}{2}$, $$i_0 = I_M$$

$$i_0 = \frac{V_S}{L} \cdot t - I_m$$

$$I_M = \frac{V_S}{L} \cdot \frac{I}{2} - I_m$$

$$2I_m = \frac{V_S}{2fL}$$

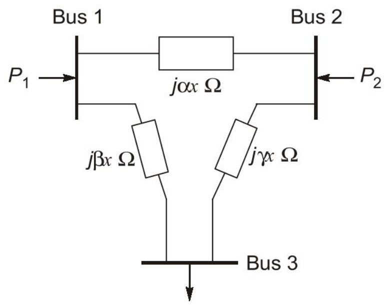
$$I_m = \frac{V_S}{4fL}$$

$\therefore$ Peak to peak load current,

$$2I_m = \frac{V_S}{2fL} = \frac{100}{2.50(20 \cdot 10^{-3})} = 50 \text{ A}$$

End of Solution

Q.53 For the three-bus lossless power network shown in the figure, the voltage magnitudes at all the buses are equal to 1 per unit (pu), and the differences of the voltage phase angles are very small. The line reactances are marked in the figure, where α, β, γ and x are strictly positive. The bus injections P_1 and P_2 are in pu. If $P_1 = mP_2$, where $m > 0$, and the real power flow from bus 1 to bus 2 is 0 pu. then which one of the following options is correct?



(a) $\alpha = m\beta$
(b) $\beta = m\gamma$
(c) $\gamma = m\beta$
(d) $\alpha = m\gamma$

Ans. (c)

$$P_1 = mP_2$$

$$P_1 = \frac{1 \times 1}{\beta x} \sin\theta$$

$$P_2 = \frac{1 \times 1}{\gamma x} \sin\theta$$

$$\frac{1 \times 1}{\beta x} \sin\theta = m \frac{1 \times 1}{\gamma x} \sin\theta$$

$$\frac{1}{\beta} = \frac{m}{\gamma} \Rightarrow \gamma = m\beta$$

End of Solution

Q.54 Consider a vector $\bar{u} = 2\hat{x} + \hat{y} + 2\hat{z}$, where $\hat{x}, \hat{y}, \hat{z}$ represent unit vector along the coordinate axes x, y, z respectively. The directional derivative of the function $f(x, y, z) = 2\ln(xy) + \ln(yz) + 3\ln(xz)$ at the point $(x, y, z) = (1, 1, 1)$ in the direction of $\bar{u}$ is

(a) 21
(b) 7
(c) $\dfrac{7}{5\sqrt{2}}$
(d) 0

Ans. **(b)**

We know that: directional derivative of 'f' at P(1, 1, 1) in the direction of $\vec{a}$ is given by

$$\text{Directional derivative} = \nabla f \cdot \frac{\hat{a}}{|\bar{a}|}$$

$$\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\nabla f = i\frac{5}{x} + j\frac{3}{y} + k\frac{4}{z}$$

$$(\nabla f)_{(1,1,1)} = 5i + 3j + 4k$$

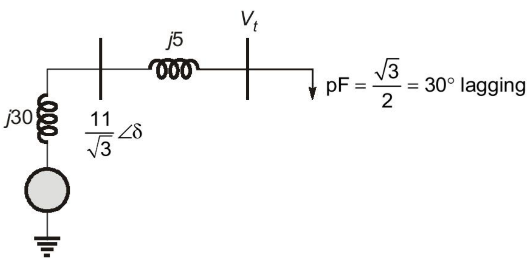
$$\therefore \text{ Directional derivative} = \overline{\nabla f} \cdot \frac{\bar{a}}{|\bar{a}|} = (5i + 3j + 4k) \cdot \frac{(2i + j + 2k)}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{21}{3} = 7$$

End of Solution

Q.55 A 3-phase. 11 kV. 10 MVA synchronous generator is connected to an inductive load of power factor $(\sqrt{3}/2)$ via a lossless line with a per-phase inductive reactance of 5 Ω. The per-phase synchronous reactance of the generator is 30 Ω with negligible armature resistance. If the generator is producing the rated current at the rated voltage, then the power factor at the terminal of the generator is

(a) 0.63 lagging
(b) 0.87 lagging
(c) 0.63 leading
(d) 0.87 leading

Ans. **(a)**



$$\text{pF} = \frac{\sqrt{3}}{2} = 30^\circ \text{ lagging}$$

$$E = \sqrt{(V\cos 30 + IR)^2 + (V\sin 30^\circ + IX)^2}$$

$$E = \sqrt{(V\cos + IR)^2 + (V\sin + IX)}$$

$$\frac{11K}{\sqrt{3}} = \sqrt{\left(V\cdot\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{V}{2} + 524.86(5)\right)^2}$$

$$\left(\frac{11K}{\sqrt{3}}\right)^2 = V^2\cdot\frac{3}{4} + \left(\frac{V}{2} + 2624.319\right)^2$$

$$\left(\frac{11K}{\sqrt{3}}\right)^2 = \frac{3}{4}V^2 + \frac{V^2}{4} + 2624.319V + 2624.319^2$$

$$V_t = 4618.105 \text{ V}$$

$$(IZ)^2 = E^2 + V^2 - 2EV\cos\delta$$

$$(2624.319)^2 = \left(\frac{11K}{\sqrt{3}}\right)^2 + (4618.105)^2 - 2\left(\frac{11K}{\sqrt{3}}\right)(4618.105)\cos\delta$$

$\therefore$ $\delta = 20.96$

$\therefore$ pF = cos (20.96 + 30) = 0.629 lagging

End of Solution

■■■■