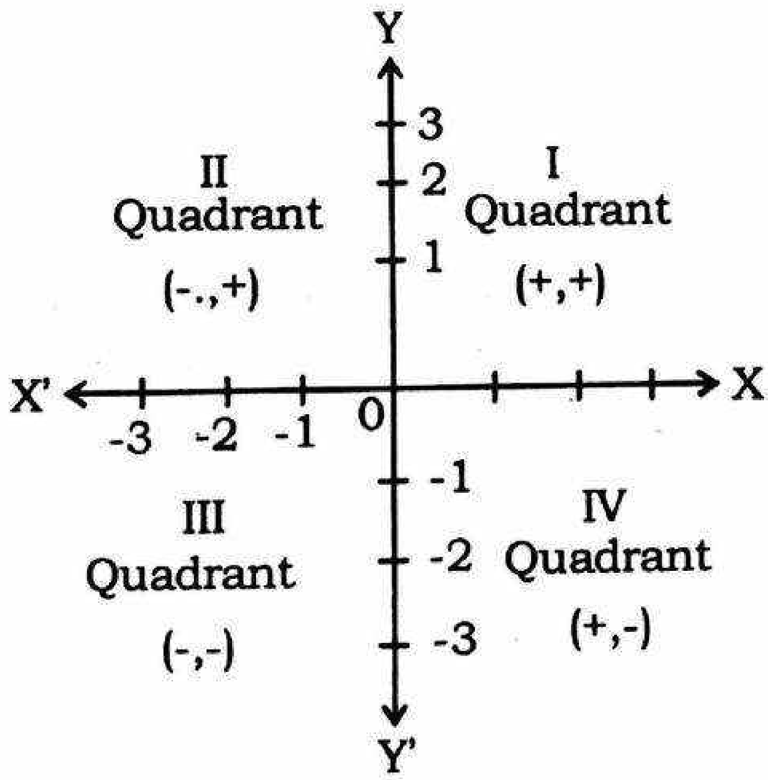
CHAPTER 4

CO-ORDINATE GEOMETRY

CARTESIAN CO-ORDINATE SYSTEM :

- **Rectangular Co-ordinate System :** Let X' OX and Y'OY be two mutually perpendicular lines through any point O in the plane of the paper. Point O is known as the origin. The line X'OX is called the x-axis or axis of x ; the line Y'OY is known as the y-axis or axis of y, and the two lines taken together are called the co-ordinates axes or the axes of co-ordinates.



Region	Quad-rant	Nature of X and Y	Signs of co-ordin-ate
XOY	I	$x > 0, y > 0$	(+, +)
YOX'	II	$x < 0, y > 0$	(- , +)
X'OY'	III	$x < 0, y < 0$	(-, -)
Y'OX	IV	$x > 0, y < 0$	(+, -)

Note- Any point lying on x-axis or y-axis does not lie in any quadrant.
Any point can be represented on the plane described by the co-ordinate axes by specifying its x and y co-ordinates.
The x-co-ordinate of the point is also known as the abscissa while the y-coordinate is also known as the ordinate.

- **Distance Formula :** The distance between two point A (x_1, y_1) and B (x_2, y_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note:

1. Distance is always positive. Therefore, we often write AB instead of |AB|.
2. The distance of a point P (x, y) from the origin $= \sqrt{x^2 + y^2}$
3. The distance between two polar co-ordinates A (r_1, θ_1) and B (r_2, θ_2) is given by

$$AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$

- **Application of Distance Formulae :**

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle.

After finding AB, BC and CA we shall find that the points are :

- **Collinear** - (a) If the sum of any two distances is equal to the third i.e. AB + BC = CA. or AB + CA = BC or BC + CA = AB

(b) If are of Δ ABC is zero

(c) If slope of AB = slope of BC = slope of CA.

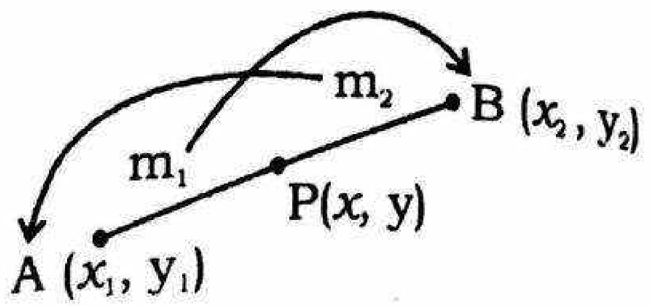
- Vertices of an equilateral triangle if AB = BC = CA
- Vertices of an isosceles triangle if AB = BC or BC = CA or CA = AB.
- Vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ etc.

(ii) For given four points A,B,C,D :

- AB = BC = CD = DA and AC = BD

$\Rightarrow$ ABCD is a square.

$$\frac{AP}{BP} = \frac{m_1}{m_2}$$



2. The co-ordinate of the point $P(x, y)$, dividing the line segment joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m_1:m_2$ are given by

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2}, y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}$$

$$\frac{AP}{BP} = \frac{m_1}{m_2}$$

Division by a Line : A line $ax + by + c = 0$ divides PQ in the ratio

$$= -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$$

- **Area of a triangle :** The area of a triangle ABC whose vertices are (x_1, y_1), $B(x_2, y_2)$ and $C(x_3, y_3)$ is denoted by Δ.

$$\therefore \quad \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

- **Area of Polygon :** The area of the polygon whose vertices are (x_1, y_1), (x_2, y_2),......(x_n, y_n) is -

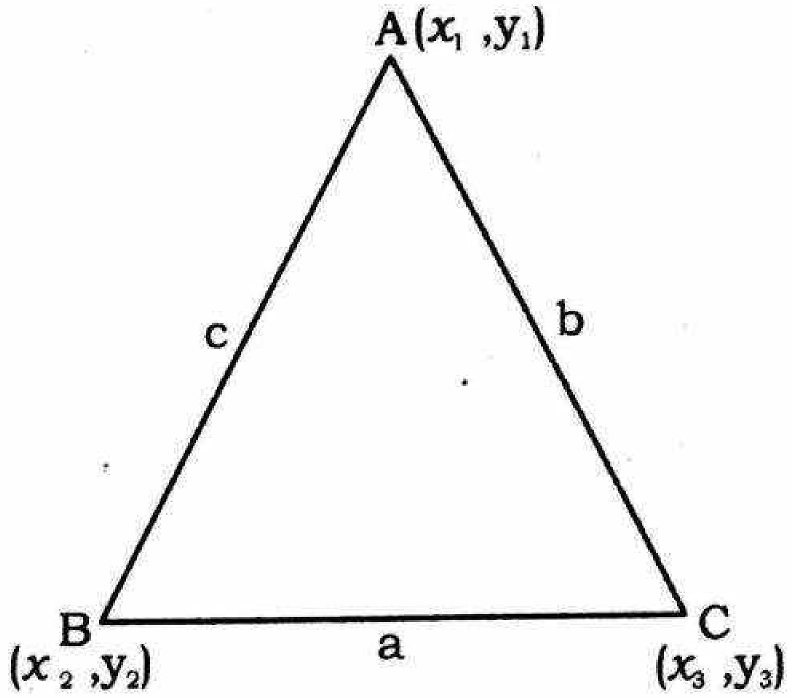
$$= \frac{1}{2}\left[\begin{matrix}(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + \ldots\ldots. \\ \ldots\ldots\ldots\ldots + (x_ny_1 - x_1y_n)\end{matrix}\right]$$

- **Some Important Points in a Triangle :**

- **Centroid :** If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then the co-ordinates of its centroid are -

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

- **Incentre :** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC s.t. $BC = a$, $CA = b$ and $AB = c$, then the co-ordinates of its incentre are



$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$$

- **Circumcentre :** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC, then the co-ordinates of its circumcentre are

$$\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$$

- **Orthocentre :** Co-ordinates of orthocentre are

$$\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C}\right)$$

Note :

- If the traingle is equilateral, then centroid, incentre, orthocentre, circumcentre coincides.
- Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.

- In an isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.
- Incentre divides the angles bisectors in the ratio (b + c) : a, (c + a) : b, (a + b) : c.
- Area of the triangle formed by co-ordinate axes and the line a x + b y + c = 0 is $\frac{c^2}{2ab}$

□ **Straight Line :** A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

□ **Different Forms of the Equations of a Straight Line :**

(a) **General Form :** The general Form of the equation of a straight line is ax + by + c = 0
(First degree equation in x and y). Where a, b and c are real constants and a, b are not simultaneously equal to zero.
In this equation, slope of the line is given

by $-\frac{a}{b}$.

The general form is also given by $y = mx + c$; where m is the slope and c is the intercept on y-axis.

(b) **Line Parallel to the X-axis :** The equation of a straight line to the x-axis and at a distance b from it, is given by $y = b$
Obviously, the equation of the x-axis is $y = 0$

(c) **Line Parallel to Y-axis :** The equation of a straight line parallel to the y-axis and at a distance a from is given by $x = a$
obviously, the equation of y-axis is $x = 0$

(d) **Slope Intercept Form :** The equation of a striaght line passing through the point A (x_1, y_1) and having a slope m is given by

$$(y - y_1) = m(x - x_1)$$

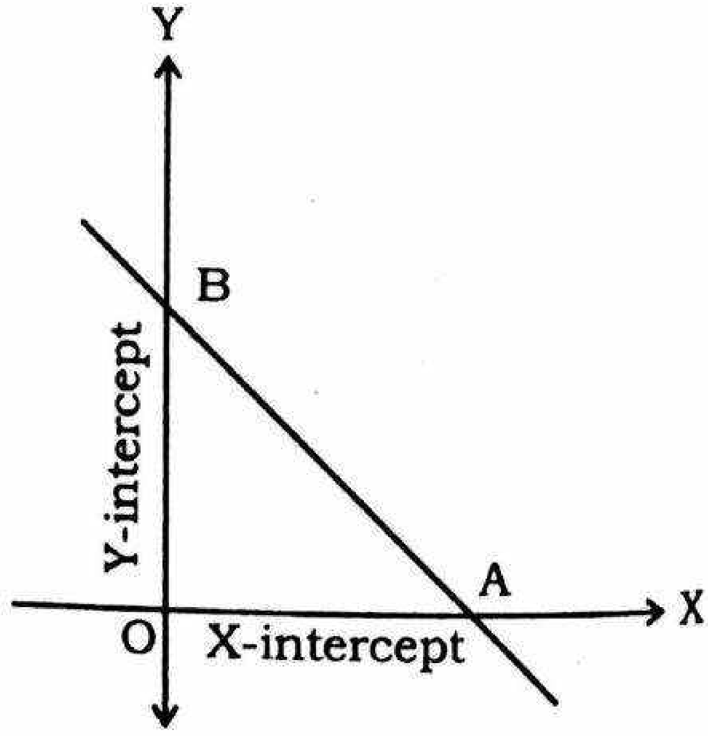
(e) **Two Points Form :** The equation of a straight line passing through two points A(x_1, y_1) and B(x_2, y_2) is given by

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Its slope (m) = $\frac{y_2 - y_1}{x_2 - x_1}$

(e) **Intercept Form :** The equation of a straight line making intercepts a and b on the axes of x and y respectively is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$



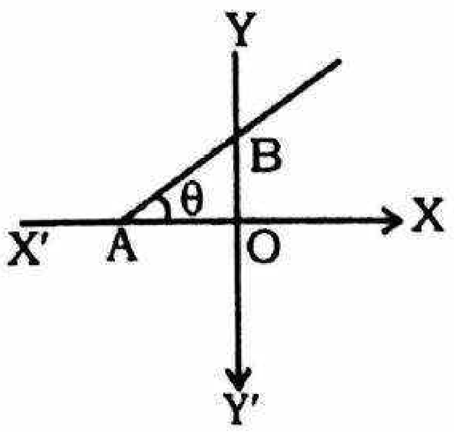
Slope (Gradient) of a Line :

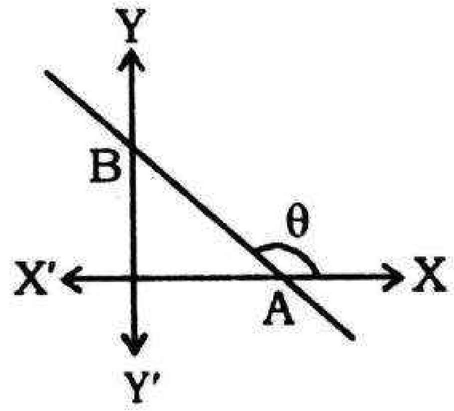
$$m = \tan\theta = -\frac{a}{b}$$

$\{\because ax + by + c = 0 \Rightarrow y = -\frac{a}{b}x - \frac{c}{b}$

$\Rightarrow y = mx + c$, where $m = -\frac{a}{b}$ and c is a constant }

Here m is called the slope or gradient of a line and c is the intercept on y-axis. The slope of a line is always measured in anticlockwise.



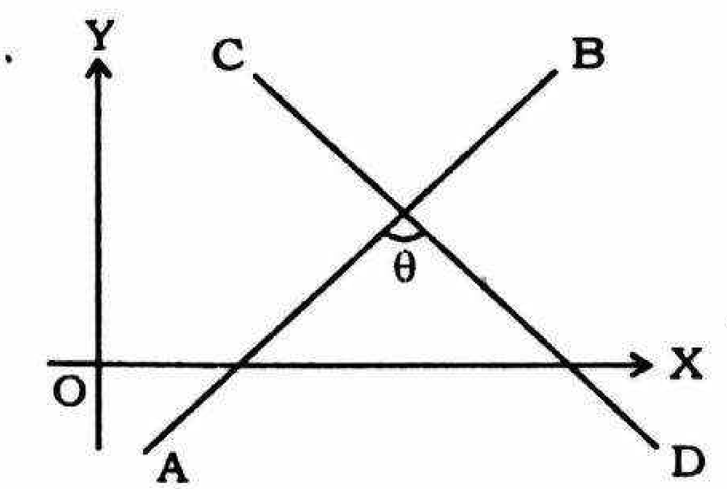


Slope of a line in terms of co-ordinates any two points on it :-
If (x_1, y_1) and (x_2, y_2) are co-ordinates of any two points on a line, then its slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Difference of ordinates}}{\text{Difference of abscissa}}$$

Angle between two lines :

$$\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$



- **Condition of Parallellism of lines :** If the slopes of two lines is m_1 and m_2 and if they are parallel, then,

$$\boxed{m_1 = m_2}$$

- **Length of Perpendicular it y or Distance of a Point from a Line :** The length of perpendicular from a given point $(x1, y1)$ to a line $ax + by + c = 0$ is :

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

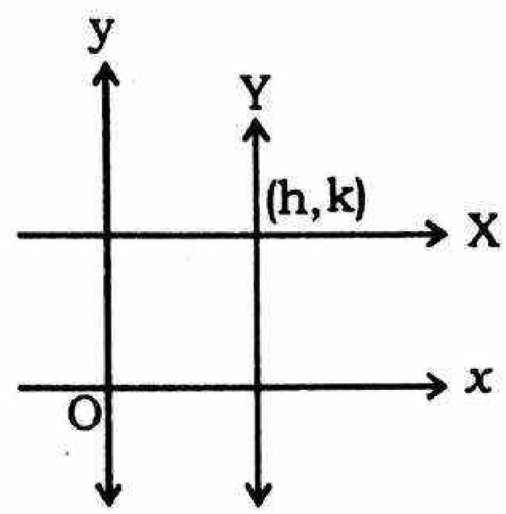
Note: The length of Perpendicular from the t origin to the line $ax + by + c = 0$ is given by

$$\frac{|c|}{\sqrt{a^2 + b^2}}$$

- **Distance between two Parallel Lines :** If two lines are parallel, the distance between them will always be the same. When two straight lines are parallel whose equations are $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$, then the distance between them is given by

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}.$$

- **Changes of Axes :** If origin (0, 0) is shifted to (h, k) then the coordinates of the point (x, y) referred to the old axes and (X, Y) referred to the new axes can be related with the relation $x = X + h$ and $y = Y + k$



- **Point of Intersection of Two Lines:** Point of intersection of two lines can be obtained by solving the equations as simultaneous equations.
- If the given equations of straight line are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then

(i) The angle between the lines 'θ' is given by $\tan\theta = \dfrac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}$

(ii) If the lines are parallel, then

$$a_2b_1 - a_1b_2 = 0 \text{ or } \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

(iii) If the lines are perpendicular, then

$$a_1a_2 + b_1b_2 = 0$$

(iv) Coincident : $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

- Angle between lines $x\cos\alpha + y\sin\alpha = P_1$ and $x\cos\beta + y\sin\beta = P_2$ is $|\alpha - \beta|$

Exercise
LEVEL - 1

1. The point (–5, 7) lies in the quadrant:
(a) First (b) Second
(c) Third (d) Fourth

2. The point (7, –5) lies in the quadrant:
(a) First (b) Second
(c) Third (d) Fourth

3. Find the distance between the points (–6,2) and (2 , 4) :
(a) $2\sqrt{17}$ (b) $4\sqrt{17}$
(c) $2\sqrt{5}$ (d) 10

4. The distance between the points A (b,o) and B (0, a) is :
(a) $\sqrt{a^2 - b^2}$ (b) $\sqrt{a^2 + b^2}$
(c) $\sqrt{a + b}$ (d) a + b

5. The distance between the points A (7, 4) and B(3, 1) is :
(a) 6 units (b) 3 units
(c) 4 units (d) 5 units

6. The co-ordinates of point situated on x-axis at a distance of 5 units from y-axis is :
(a) (0, 5) (b) (5, 0)
(c) (5, 5) (d) (–5, 5)

7. The co-ordinates of a point situated on y-axis at a distance of 7 units from x-axis is :
(a) (0, 7) (b) (7, 0)
(c) (7, 7) (d) (–7, 7)

8. The co-ordinates of a point below x-axis at a distance of 6 units from x-axis but lying on y-axis is :
(a) (0, 6) (b) (–6, 0)
(c) (0, –6) (d) (6, – 6)

9. The distance of the point (6, –8) from the origin is :
(a) 2 units (b) 14 units
(c) 7 units (d) 10 units

10. The point of intersection of the lines $2x + 7y = 1$ and $4x + 5y = 11$ is :
(a) (4, –1) (b) (2, 3)
(c) (–1, 4) (d) (4, –2)

11. The line $4x + 7y = 12$ meets x-axis at the point :
(a) (3, 1) (b) (0, 3)
(c) (3, 0) (d) (4, 0)

12. The line $4x - 9y = 11$ meets y-axis at the point :
(a) $\left(-\frac{11}{9}, 0\right)$ (b) $\left(0, -\frac{11}{9}\right)$
(c) $\left(0, \frac{11}{4}\right)$ (d) $\left(0, -\frac{11}{4}\right)$

13. The slope of the line $3x + 7y + 8 = 0$ is :
(a) 3 (b) 7
(c) $-\frac{3}{7}$ (d) $\frac{3}{7}$

14. The slope of the line joining P(–4, 7) and Q(2, 3) is :
(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$
(c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

15. The equation of a line parallel to x-axis at a distance of 6 units and above x-axis is :
(a) $x = 6$ (b) $y = 6x$
(c) $x = 6y$ (d) $y = 6$

16. The equation of a line parallel to y-axis at a distance of 5 units to the left of y-axis, is :
(a) $y = -5$ (b) $x = -5$
(c) $x + 5y = 0$ (d) $y + 5x = 0$

17. The equation of a line parallel to x-axis and at a distance of 7 units below x-axis is :
(a) $y = -7$ (b) $x = 7$
(c) $x = -7$ (d) $y = -7x$

18. The area of the triangle whose vertices are P (4, 5), Q(–3, 8) and R (3, –4), (in square units) is :

(a) 66 (b) $16\frac{1}{2}$

(c) 33 (d) 35

19. The points A(0, 0), B(0, 3) and C(4, 0) are the vertices of a triangle which is :

(a) Isosceles

(b) Right angled

(c) Equilateral

(d) None of these

20. The co-ordinates of the centroid of ΔPQR with vertices P(–2, 0), Q(9, –3) and R (8, 3) is :

21. The equation of a line passing through the points A (0, –3) and B (–5, 2) is :

(a) $x + y + 3 = 0$ (b) $x + y - 3 = 0$

(c) $x - y + 3 = 0$ (d) $x - y - 3 = 0$

22. The length of perpendicular from the origin to the line $12x + 5y + 7 = 0$ is:

(a) 2 units (b) 1 unit

(c) $\frac{7}{13}$ units (d) $\frac{7}{11}$ units

23. The angle which the line joining the points $(\sqrt{3}, 1)$ and $(\sqrt{15}, \sqrt{5})$ makes with x-axis is :

(a) 30° (b) 45°

(c) 60° (d) 90°

24. The lines whose equations are 2x-5y + 7 = 0 and 8x - 20y + 28 = 0 ...

Exercise
LEVEL - 2

1. If the distance of the point $P(x, y)$ from $A(a, 0)$ is $a + x$, then $y^2 = ?$
   (a) $2ax$ (b) $4ax$
   (c) $6ax$ (d) $8ax$
2. If the point (x, y) is equidistant from the points $(a + b, b - a)$ and $(a-b, a + b)$ then $bx = ?$
   (a) a^2y (b) ay^2
   (c) ay (d) a^2y^2
3. If the sum of the square of the distance of the point (x, y) from the point $(a, 0)$ and $(-a, 0)$ is $2b^2$, then :
   (a) $x^2 + a^2 = b^2 + y^2$
   (b) $x^2 + a^2 = 2b^2 - y^2$
   (c) $x^2 - a^2 = b^2 + y^2$
   (d) $x^2 + a^2 = b^2 - y^2$
4. $P(-4, a)$ and $Q(2, a + 4)$ are two points and the co-ordinates of the middle point of PQ are $(-1, 4)$. The value of a is :
   (a) 0 (b) 2
   (c) −2 (d) 3
5. If the points $P(2, 3)$, $Q(5, a)$ and $R(6, 7)$ are collinear, the value of a is :
   (a) 5/2 (b) −4/3
   (c) 6 (d) 5
6. The equation of a line parallel to x-axis and passing through $(-6, -5)$ is :
   (a) $y = -5$ (b) $x = -6$
   (c) $y = -5x$ (d) $y = -6x - 5$
7. The equation of a line parallel to y-axis and passing through $(2, -5)$ is :
   (a) $x = 2$ (b) $y = -5$
   (c) $y = 2x$ (d) $x = -5y$
8. Two vertices of a triangle PQR are $P(-1, 0)$ and $Q(5, -2)$ and its centroid is $(4, 0)$. The co-ordinates of R are :
   (a) $(8, -2)$ (b) $(8, 2)$
   (c) $(-8, 2)$ (d) $(-8, -2)$
9. The co-ordinates of the point of intersection of the medians of a triangle with vertices $P(0, 6)$, $Q(5, 3)$ and $R(7, 3)$ are :
   (a) $(4, 5)$ (b) $(3, 4)$
   (c) $(4, 4)$ (d) $(5, 4)$
10. The ratio in which the line segment joining $A(3, -5)$ and $B(5, 4)$ is divided by x-axis is :
    (a) 4 : 5 (b) 5 : 4
    (c) 5 : 7 (d) 6 : 5
11. The ratio in which the line segment joining $P(-3, 7)$ and $Q(7, 5)$ is divided by y-axis is :
    (a) 3 : 7 (b) 4 : 7
    (c) 3 : 5 (d) 4 : 5
12. The ratio in which the point $P\left(1, \frac{10}{3}\right)$ divides the join of the point $A(-3, 2)$ and $B(3, 4)$ is :
    (a) 2 : 3 (b) 1 : 2
    (c) 2 : 1 (d) 3 : 1
13. The equation of a line with slope 5 and passing through the point $(-4, 1)$ is :
    (a) $y = 5x + 21$ (b) $y = 5x - 21$
    (c) $5y = x + 21$ (d) $5y = x - 21$
14. The value of a so that the lines $x + 3y - 8 = 0$ and $ax + 12y + 5 = 0$ are parallel is :
    (a) 0 (b) 1
    (c) 4 (d) −4

15. The value of P for which the lines $3x + 8y + 9 = 0$ and $24x + py + 19 = 0$ are perpendicualar is :

(a) −12 (b) − 9

(c) − 11 (d) 9

16. The value of a so that line joining P(−2, 5) and Q (0, −7) and the line joining A (−4, −2) and B(8, a) are perpendicular to each other is :

(a) −1 (b) 5

(c) 1 (d) 0

17. The angle between the lines represented by the equations $2y - \sqrt{12}x - 9 = 0$ and $\sqrt{3}y - x + 7 = 0$, is:

(a) 30° (b) 45°

(c) 60° (d) $22\frac{1}{2}°$

18. If P(3, 5), Q (4, 5) and R(4, 6) be any three points, the angle between PQ and PR is :

(a) 30° (b) 45°

(c) 60° (d) 90°

19. Given a ΔPQR with vertices P (2, 3), Q (− 3, 7) and R (− 1, −3). The equation of median PM is :

(a) $x - y + 10 = 0$

(b) $x - 4y - 10 = 0$

(c) $x - 4y + 10 = 0$

(d) None of these

20. The co-ordinates of the point P which divides the join of A(3, −2) and $B\left(\frac{11}{2}, \frac{21}{2}\right)$ in the ratio 2 : 3 are :

(a) (4, 3) (b) (4, 5)

Exercise
LEVEL - 3

1. The length of the portion of the straight line $8x + 15y = 120$ intercepted between the axes is :
   (a) 14 units (b) 15 units
   (c) 16 units (d) 17 units

2. The equation of the line passing through the point (1, 1) and perpendicular to the line $3x + 4y - 5 = 0$, is :
   (a) $3x + 4y - 7 = 0$
   (b) $3x + 4y + k = 0$
   (c) $3x - 4y - 1 = 0$
   (d) $4x - 3y + 1 = 0$

3. The equation of a line passing through the point (5, 3) and parallel to the line $2x - 5y + 3 = 0$, is :
   (a) $2x - 5y - 7 = 0$
   (b) $2x - 5y + 5 = 0$
   (c) $2x - 2y + 5 = 0$
   (d) $2x - 5y = 0$

4. The sides PQ, QR, RS and SP of a quadrilateral have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively, then the angle between the diagonals PR and QS is:
   (a) 30° (b) 45°
   (c) 60° (d) 90°

5. The equations of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, –10). The equation of the third side is :
   (a) $x - 3y - 31 = 0$ but not $x - 3y - 31 = 0$
   (b) neither $3x + y + 7 = 0$ nor $x - 3y - 31 = 0$
   (c) $3x + y + 7 = 0$ or $x - 3y - 31 = 0$
   (d) $3x + y + 7 = 0$ but not $x - 3y - 31 = 0$

6. If P_1 and P_2 be perpendicular from the origin upon the straight lines $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x \cos\theta - y \sin\theta = a \cos 2\theta$ respectively, then the value of $4P_1^2 + P_2^2$ is :
   (a) a^2 (b) $2a^2$
   (c) $\sqrt{2}\,a^2$ (d) $3a^2$

7. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9 ?
   (a) $x + 2y - 6 = 0$ but not $2x + y - 6 = 0$
   (b) neither $x + 2y - 6 = 0$ nor $2x + y - 6 = 0$
   (c) $2x + y - 6 = 0$ but not $x + 2y - 6 = 0$
   (d) $x + 2y - 6 = 0$ or $2x + y - 6 = 0$

Hints and Solutions

LEVEL - 1

1.(b) The point (–5, 7) lies in the second quadrant.

2.(d) The point (7, –5) lies in the fourth quadrant.

3.(a) Distance between two points

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

here $(x_1, y_1) = (-6, 2)$ and $(x_2, y_2) \equiv (2, 4)$

$\therefore$ Required distance

$= \sqrt{(-6-2)^2 + (2-4)^2}$

$= \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}$unit

4.(b)

$AB = \sqrt{(b-0)^2 + (0-a)^2} = \sqrt{b^2 + a^2}$

$= \sqrt{a^2 + b^2}$

5.(d) $AB^2 = (7-3)^2 + (4-1)^2$

$= 4^2 + 3^2$

$= 16 + 9$

$= 25$

$\left[x_1(y_2 - y_3) + x_2(y_1 - y_2)\right] + x_3$

$\Rightarrow AB = \sqrt{25} = 5$ units

6.(b) Clearly, the point of x -axis has ordinate 0 and abscissa 5.
So, the point is (5, 0)

7.(a) Clearly, the point on y-axis has abscissa 0.
So, the point is (0, 7)

8.(c) Clearly, the point is (0, 6)

9.(d) Required distance

$= \sqrt{(6-0)^2 + (-8-0)^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$ units

10.(a) $2x + 7y = 1$ .... (i)

$4x + 5y = 11$ ......(ii)

on solving (i) and (ii), we get $x = 4$ and $y = -1$

$\therefore$ Required point of intersection = (4, –1)

11.(c) Equation of x-axis is $y = 0$

put $y = 0$ in $4x + 7y = 12$ we get $x = 3$

$\therefore$ Required point = (3, 0)

12.(b) Equation of y-axis is $x = 0$

put $x = 0$ in $4x - 9y = 11$ we get $y = -\frac{11}{9}$

$\therefore$ Required point $= \left(0, -\frac{11}{9}\right)$

13.(c) $3x + 7y + 8 = 0 \Rightarrow 7y = -3x - 8$

$\Rightarrow y = \left(-\frac{3}{7}\right)x - \left(\frac{8}{7}\right)$

$\therefore$ Slope of the line is $= -\frac{3}{7}$

14.(a) Slope of PQ $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-7}{2-(-4)} = \frac{-4}{6} = -\frac{2}{3}$

15.(d) Clearly; the equation of the line is, $y = 6$

16.(b) Clearly, the equation of the line is, $x = -5$

17.(a) Clearly, the equation of the line is $y = -7$

18.(c) $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3 (y_1 - y_2)\right]$

$\Delta = \frac{1}{2}\left|4(4+8) - 3(-4-5) + 3(5-8)\right|$

$= \frac{1}{2}|66| = 33$ sq. units

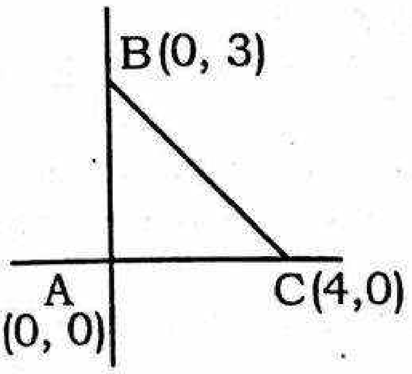
19.(b) $AB = \sqrt{(0+0)^2 + (0-3)^2} = 3$

$AC = \sqrt{(4-0)^2 + (0+0)^2} = 4$

and BC $= \sqrt{(4-0)^2+(0-3)^2} = 5$

$\because$ $AB^2 + AC^2 = BC^2$

$\therefore$ ΔABC is a right angled triangle.



20.(d) The co-ordinates of the centroid of ΔPQR are -

$$\left(\frac{-2+9+8}{3}, \frac{0-3+3}{3}\right) = (5, 0)$$

21.(a) The required equation is

$$(y+3) = \frac{2+3}{-5-0}(x-0)$$

$$\Rightarrow y + 3 = -x \Rightarrow x + y + 3 = 0$$

22.(c) Length of perpendicular =

$$= \frac{12\times0+5\times0+7}{\sqrt{12^2+5^2}} = \frac{7}{13} \text{ units}$$

23.(a) The slope of the line is

$$\frac{\sqrt{5}-1}{\sqrt{15}-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \tan\theta = \frac{1}{\sqrt{3}} \text{ or } \theta = 30°$$

24.(c) Here, $\frac{a_1}{a_2} = \frac{2}{8} = \frac{1}{4}$, $\frac{b_1}{b_2} = \frac{-5}{-20} = \frac{1}{4}$

and $\frac{c_1}{c_2} = \frac{7}{28} = \frac{1}{4}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, So the given lines are coincident.

LEVEL - 2

1.(b) $\sqrt{(x-a)^2+(y-0)^2} = a+x$

$\Rightarrow (x-a)^2+y^2=(a+x)^2$

$\Rightarrow y^2=(x+a)^2-(x-a)^2$

$\Rightarrow y^2=4ax$

2.(c) Let (x, y), Q(a + b, b – a) and R(a – b, a + b) are given points.

$\therefore$ PQ = PR.

$\Rightarrow \sqrt{\{x-(a+b)\}^2+\{y-(b-a)\}^2}$

$= \sqrt{\{x-(a-b)\}^2+\{y-(a+b)\}^2}$

$\Rightarrow x^2-2x(a+b)+(a+b)^2+y^2-2y(b-a)+(b+a)^2 = x^2+(a-b)^2-2x(a-b)+y^2+(a+b)^2-2y(a+b)$

$\Rightarrow ax+bx+by-ay=ax-bx+ay+by$

$\Rightarrow 2bx=2ay$

$\Rightarrow bx=ay$

3.(d) Let A (x, y), P(a, 0) and Q(–a, 0), Then,

$\Rightarrow AP^2+AQ^2=2b^2$

$\Rightarrow [(x-a)^2+(y-0)^2]+[(x+a)^2+(y-0)^2]=2b^2$

$\Rightarrow x^2+a^2-2ax+y^2+x^2+a^2+2ax+y^2=2b^2$

$\Rightarrow 2(x^2+a^2+y^2)=2b^2$

$\Rightarrow x^2+a^2+y^2=b^2$

$\Rightarrow x^2+a^2=b^2-y^2$

4.(d) co-ordinates of middle point $\equiv$ (–1, 4)

$\therefore \frac{a+a+4}{2}=4$

$\Rightarrow 2a+4=8$

$\Rightarrow 2a=4$

$a=2$

5.(c) Since, P,Q and R collinear

$\therefore$ slope of PQ = slope of PR

$\Rightarrow \frac{a-3}{5-2}=\frac{7-3}{6-2} \Rightarrow \frac{a-3}{3}=\frac{4}{4}$

$\Rightarrow a-3=3 \Rightarrow a=6$

6.(a) The equation of a line parallel to x-axis is y = b.
Since, it passes through (–6, –5), so b = –5

$\therefore$ The required equation is, $y = -5$

7.(a) The equation of a line parallel to y-axis is, $x = a$.
Since, it passes through (2, –5), so a = 2

$\therefore$ The required equation is, $x = 2$

8.(b) Let the co-ordinates of R be (x, y). Then,

$\frac{-1+5+x}{3}=4$ and $\frac{0-2+y}{3}=0$

or $4+x=12$ and $-2+y=0$

or $x=8$ and $y=2$

$\therefore$ R = (x, y) = (8, 2)

9.(c) Since, point of intersection of median is "centroid".

$\therefore$ co-ordinates of centroid

$=\left(\frac{0+5+7}{3},\frac{6+3+3}{3}\right)$

$=\left(\frac{12}{3},\frac{12}{3}\right)=(4,4)$

10.(b) Let the ratio be k : 1
The ordinate of a point lying on x-axis must be zero

$\therefore \frac{4k-5\times1}{k+1}=0 \Rightarrow 4k=5 \Rightarrow k=\frac{5}{4}$

$\therefore$ Required ratio is $\frac{5}{4}:1=5:4$

11.(a) Let the ratio be k : 1
The abcissa of a point lying on y-axis must be zero

$\therefore \frac{7k-3\times1}{k+1}=0 \Rightarrow 7k-3=0 \Rightarrow k=\frac{3}{7}$

$\therefore$ Required ratio is $\frac{3}{7}:1=3:7$

12.(c) Let the ratio be k : 1

$\therefore \frac{3k - 3\times 1}{k+1} = 1$

$\Rightarrow 3k - 3 = k + 1 \Rightarrow 2k = 4 \Rightarrow k = 2$

$\therefore$ Required ratio is 2 : 1

13.(a) Let the equation be $y = 5x + c$
Since it passes through (–4, 1), we have $1 = 5(-4) + c$

$\therefore c = 21$ so, its equation is,
$y = 5x + 21$

14.(c) Condition of parallelism $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\therefore \frac{1}{a} = \frac{3}{12} \Rightarrow a = 4$

Alternatively,

$x + 3y - 8 + 0$

$\Rightarrow y = \left(-\frac{1}{3}\right)x + \left(\frac{8}{3}\right)$

$\therefore m_1 = -\frac{1}{3}$

$ax + 12y + 5 + 0 = 0$

$\Rightarrow y = \left(-\frac{a}{12}\right)x - \frac{5}{12}$

$\therefore m_2 = -\frac{a}{12}$

for parallelism; $m_1 = m_2$

$\therefore -\frac{1}{3} = -\frac{a}{12} \Rightarrow a = 4$

15.(b) Condition of perpendicularism,
$a_1a_2 + b_1b_2 = 0$

$\therefore 3\times 24 + 8\times p = 0$

$\Rightarrow 8p = -3\times 24 \Rightarrow p = -9$

Alternatively-

$3x + 8y + 9 = 0 \Rightarrow$

$y = \left(-\frac{3}{8}\right)x - \frac{9}{8} \quad \therefore m_1 = -\frac{3}{8}$

$24x + py + 19 = 0 \Rightarrow y = \left(-\frac{24}{p}\right)x - \frac{19}{p}$

$\therefore m_2 = -\frac{24}{p}$

for perpendicularism, $m_1.m_2 = -1$

$\therefore = \left(-\frac{3}{8}\right)\left(-\frac{24}{p}\right) = -1$

$\Rightarrow P = -9$

16.(d) m_1 = Slope of PQ

$= \frac{-7-5}{0+2} = \frac{-12}{2} = -6$

m_2 = Slope of AB $= \frac{a+2}{8+4} = \frac{a+2}{12}$

$\therefore m_1m_2 = -1 \Rightarrow -6\times\frac{a+2}{12} = -1$

$\Rightarrow a + 2 = 2 \Rightarrow a = 0$

17.(a) $2y - \sqrt{12}.x - 9 = 0 \Rightarrow y = \frac{\sqrt{12}}{2}x + \frac{9}{2}$

$\Rightarrow m_1 = \frac{\sqrt{12}}{2} = \sqrt{3}$

$\sqrt{3}y - x + 7 = 0 \Rightarrow y = \left(\frac{1}{\sqrt{3}}\right)x - \frac{7}{\sqrt{3}}$

$\Rightarrow m_2 = \frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \frac{m_1 - m_2}{1 + m_1.m_2} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}.\frac{1}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$

So, $\theta = 30°$

18.(b) Slope of PQ, $m_1 = \frac{5-5}{4-3} = 0$

Slope of PR, $m_2 = \frac{6-5}{4-3} = 1$

$\therefore\ \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{0-1}{1+0}\right| = 1$

So, $\theta = 45°$

19.(c) Clearly, M is the mid-point of QR.

$\therefore$ Co-ordinates of M are $\left(\frac{-3-1}{2}, \frac{7-3}{2}\right)$

i.e. (–2, 2)

Now, find the equation of the line joining P(2, 3) and M(–2, 2)

Required equation is , $(y-3)$

$= \frac{2-3}{-2-2}(x-2)$

$\Rightarrow y - 3 = \frac{1}{4}(x-2) \Rightarrow 4y - 12 = x - 2$

$\Rightarrow x - 4y + 10 = 0$

20.(a) Required point is :

$$\left(\frac{3\times 3 + 2\times\frac{11}{2}}{3+2}, \frac{3(-2)+2\frac{21}{2}}{3+2}\right)$$

$$= \left(\frac{20}{5}, \frac{15}{5}\right) = (4, 3)$$

LEVEL - 3

1.(d) Point of intersection at x-axis $= (x, 0)$

$\therefore\ 8x + 15y = 120$

$\Rightarrow\ 8x + 15 \times 0 = 120 \Rightarrow x = 15$

$\Rightarrow$ point ot intersection = (15, 0)

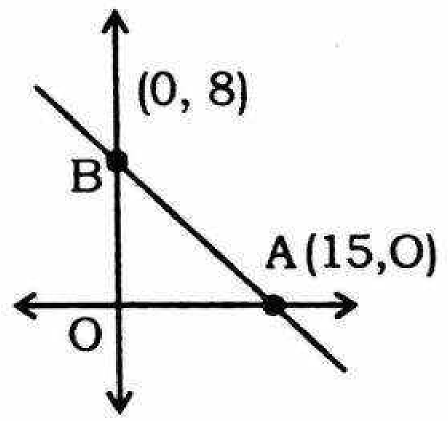
Point of intersection at y-axis $= (0, y)$

$\therefore\ 8x + 15y = 120$

$\Rightarrow\ 0 + 15y = 120 \Rightarrow y = 8$

$\Rightarrow$ Point of intersection = (0, 8)

$\therefore$ Required length = AB



$= \sqrt{(15-0)^2 + (0-8)^2}$

$= \sqrt{225+64} = \sqrt{289}$

$= 17$ units

2.(c) Given line - $3x + 4y - 5 = 0$

$$\Rightarrow y = \left(-\frac{3}{4}\right)x + \frac{5}{4}$$

$\therefore$ its slope, $m_1 = -\frac{3}{4}$

Let m_2 be the slope of required line.

Then, $m_1 m_2 = -1$ or $\left(-\frac{3}{4}\right)$ m2 = −1

$\Rightarrow m_2 = \frac{4}{3}$

Let the required equation be, $y = m_2 x + c$

$\Rightarrow y = \frac{4}{3}x + c$

Since, it passes through (1, 1)

$\therefore\ 1 = \frac{4}{3} \times 1 + c \Rightarrow c = 1 - \frac{4}{3} = -\frac{1}{3}$

$\therefore$ the required equation is, $y = \frac{4}{3}x - \frac{1}{3}$

or $4x - 3y - 1 = 0$

3.(b) $2x - 5y + 3 = 0$

$\Rightarrow\ y = \left(\frac{2}{5}\right)x + \left(\frac{3}{5}\right)$

$\therefore$ its slope $m_1 = \frac{2}{5}$

Let the slope of line which is parall;el to the given line is m_2

$\therefore\ m_2 = m_1 = \frac{2}{5}$

Let the required equation be,

$y = \frac{2}{5}x + c$

Since, it passes through (5, 3)

$\therefore\ 3 = \frac{2}{5} \times 5 + c \Rightarrow c = 1$

$\therefore$ Required equation is, $y = \frac{2}{5}x + 1$

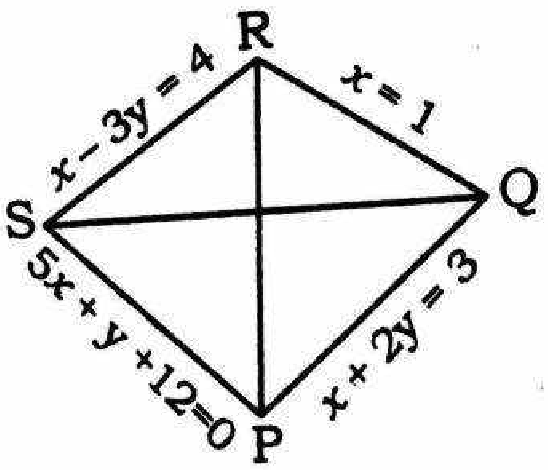
or $2x - 5y + 5 = 0$

4.(d) $x + 2y = 3$ .... (i)

$5x + y = -12$ ....(ii)

On solving (i) and (ii), we get $x = -3$, $y = 3$

$\therefore$ co-ordinates of P(− 3, 3)



Similarly, Q(1, 1), R(1, −1) and S(−2, 2)

Now, m_1 = slope of PR = $\frac{-1-3}{1+3} = -1$

m_2 = slope of QS = $\frac{-2-1}{-2-1} = 1$

$\because$ $m_1 m_2 = -1$

$\therefore$ the required angle is 90°

5.(c) $\because$ Third side passes through (1, −10) so its equation $y + 10 = m(x - 1)$ ....(i)

This side makes equal angle with the given two sides.

let this angle be θ.

Now, slope of line $7x - y + 3 = 0$ is m_1,

$\therefore m_1 = 1$

and slope of line $x + y - 3 = 0$ is m_2,

$\therefore m_2 = -1$

angle between (i) and $7x - y + 3 = 0$ = angle between (i) and $x + y - 3 = 0$

$$\therefore \tan\theta = \frac{m-7}{1+7m} = \frac{m-(-1)}{1+m(-1)}$$

$\Rightarrow m = -3$ or $1/3$

Hence possible equations of third side are $y + 10 = -3(x - 1)$

and $y + 10 = \frac{1}{3}(x-1)$

or $3x + y + 7 = 0$ and $x - 3y - 31 = 0$

6.(a) P_1 = lenght of perpendicular from (0, 0) on $x\sec\theta + y\,\text{cosec}\,\theta = a$

$$\therefore P_1 = \frac{a}{\sqrt{\sec^2\theta + \text{cosec}^2\theta}} = \frac{a}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}$$

$= a\sin\theta . \cos\theta$

or $2P_1 = a(2\sin\theta . \cos\theta) \Rightarrow 2P_1 = a \sin 2\theta$

Similarly,

$$P_2 = \frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} = a\cos 2\theta$$

$$\therefore \quad 4P_1^2 + P_2^2 = a^2\left(\sin^2 2\theta + \cos^2 2\theta\right) = a^2$$

7.(d) Let a and b are the intercepts on x and y-axes respectively.

$\therefore$ $a + b = 9 \Rightarrow b = 9 - a$ ....(i)

and the equation of the line is

$\frac{x}{a} + \frac{y}{b} = 1$ ....(ii)

From (i) and (ii)

$\frac{x}{a} + \frac{y}{9-a} = 1$ ......(iii)

this line also passes through the point (2,2)

$\therefore$ from (iii) $\frac{2}{a} + \frac{2}{9-a} = 1$

On solving we get a = 6 or a = 3

If a = 6 then b = 9 − 6 = 3

$\therefore$ equation of the line is $\frac{x}{6} + \frac{y}{3} = 1$

or $x + 2y - 6 = 0$

If a = 3 then b = 9 − 3 = 6

$\therefore$ equation of the line is $\frac{x}{3} + \frac{y}{6} = 1$

or $2x + y - 6 = 0$

Hence, required equation is $x + 2y - 6 = 0$ or $2x + y - 6 = 0$

Note : Solve this type of question with the help of given options.

Answer Key

LEVEL - 1

1. (b)	2. (d)	3. (a)
4. (b)	5. (d)	6. (b)
7. (a)	8. (c)	9. (d)
10. (a)	11. (c)	12. (b)
13. (c)	14. (a)	15. (d)
16. (b)	17. (a)	18. (c)
19. (b)	20. (d)	21. (a)
22. (c)	23. (a)	24. (c)

LEVEL - 2

1. (b)	2. (c)	3. (d)
4. (b)	5. (c)	6. (a)
7. (a)	8. (b)	9. (c)
10. (b)	11. (a)	12. (c)
13. (a)	14. (c)	15. (b)
16. (d)	17. (a)	18. (b)
19. (c)	20. (a)	

LEVEL - 3

1. (d)	2. (c)	3. (b)
4. (d)	5. (c)	6. (a)
7. (d)		