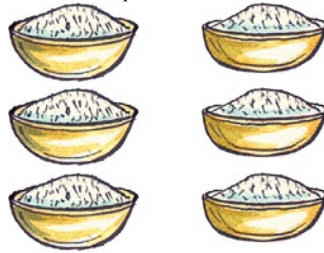


10. Direct and Inverse Proportions

10.0 Introduction

Gopi uses 4 cups of water to cook 2 cups of rice everyday. Oneday when some guests visited his house, he needed to cook 6 cups of rice. How many cups of water will he need to cook 6 cups of rice?



We come across many such situations in our day-to-day life, where we observe change in one quantity brings in change in the other quantity. For example,

- (i) What happens to the quantity of mid day meal needed if more number of students are enrolled in your school? more quantity of mid day meal is required.
- (ii) If we deposit more money in a bank, what can you say about the interest earned? Definitely the interest earned also will be more.
- (iii) What happens to the total cost, if the number of articles purchased decreases? Obviously the total cost also decreases.
- (iv) What is the weight of 20 tea packets, if the weight of 40 tea packets is 1.6 kg? Clearly the weight of 20 tea packets is less.

In the above examples, we observe that variation in one quantity leads to variation in the other quantity.



Do This

Write five more such situations where change in one quantity leads to change in another quantity.

How do we find out the quantity of water needed by Gopi? To answer this question, we now study some types of variations.

10.1 Direct Proportion

On the occasion of Vanamahotsavam, Head of Eco team in a school decided to take up plantation of saplings. Number of Eco club members of each class is furnished here.

Class	VI	VII	VIII	IX	X
Number of Eco student	5	7	10	12	15

Each student has to plant two saplings. Find the number of saplings needed for plantation for each class.



Class	VI	VII	VIII	IX	X
Number of Eco students	5	7	10	12	15
Number of saplings required	10	14	20	24	30

What can you say regarding number of saplings required? What kind of a change do you find in the number of saplings required and the number of students? Either both increase or both decrease.

$$\frac{\text{number of saplings required}}{\text{number of students}} = \frac{10}{5} = \frac{14}{7} = \frac{20}{10} = \dots = \frac{2}{1} = 2 \text{ which is a constant and is called constant of proportion.}$$

As the ratio is the same, we call this variation as direct proportion.

If x and y are any two quantities such that both of them increase or decrease together and $\frac{x}{y}$ remains constant (say k), then we say that x and y are in direct proportion. This is written as $x \propto y$ and read as x is directly proportional to y .

$$\therefore \frac{x}{y} = k \Rightarrow x = ky \text{ where } k \text{ is constant of proportion.}$$

If y_1 and y_2 are the values of y corresponding to the values of x_1 and x_2 of x respectively, then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$



Do These

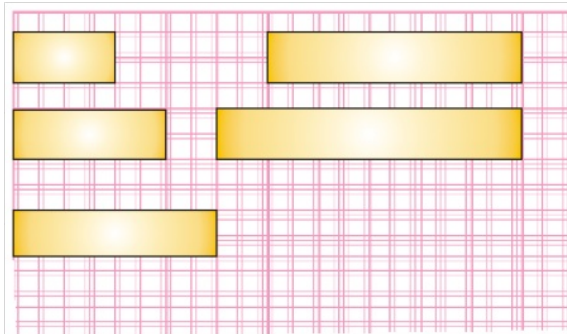
1. Write three situations where you see direct proportion.
2. Let us consider different squares of sides 2, 3, 4 and 5cm, Find the areas of the squares and fill the table.

Side in cm	Area in cm ²
2	
3	
4	
5	

What do you observe? Do you find any change in the area of the square with a change in its side? Further, find the area of a square to the length of its side. Is the ratio same? Obviously not.

∴ This variation is not a direct proportion.

3. The following are rectangles of equal breadth on a graph paper. Find the area for each rectangle and fill in the table.



Rectangle	1	2	3	4	5
Length (cm)					
Area (cm ²)					

Is the area directly proportional to length?

4. Take a graph paper make same rectangles of same length and different width. Find the area for each. What can you conclude about the breadth and area?

Example 1: If the cost of 65 tea-packets of the same size is ₹ 2600, what is the cost of 75 such packets?

Solution: We know if the number of tea packets purchased increases then the cost also increases. Therefore, cost of tea-packets directly varies with number of teapackets.

No. of tea packets(x)	65	75
Cost (y)	2600	?

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} \quad \text{So } \frac{65}{2600} = \frac{75}{y_2} \quad \text{Here } x_1 = 65, y_1 = 2600, x_2 = 75, y_2 = ?$$

$$\text{by substituting, } \frac{65}{2600} = \frac{75}{y_2} \Rightarrow y_2 = \frac{75 \times 2600}{65} = ₹ 3000$$

So cost of 75 such packets is ₹ 3000.

Example 2: Following are the car parking charges near a railway station

Number of Hours	Parking Charge
(x)	(y)
upto 4 hours	₹ 60
8 hours	₹ 100
12 hours	₹ 140
24 hours	₹ 180

Check if the parking charges and parking hours are in direct proportion.

Solution: We can observe that both values are gradually increasing.

Are they in direct proportion? What is the value of $\frac{x}{y}$?

If it is a constant, then they are in direct proportion. Otherwise they are not in direct proportion. Let check in this case.

$$\frac{x}{y} = \frac{4}{60}, \frac{8}{100}, \frac{12}{140}, \frac{24}{180}$$

You can easily observe that all these ratios are not equal. So they are not in direct proportion.

Example 3: A pole of 8 m height casts a 10m long shadow. Find the height of the tree that casts a 40 m long shadow under similar conditions.

Solution: Length of a shadow directly varies to the height of the pole.

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} \quad \text{So } \frac{8}{10} = \frac{x_2}{40} \quad \text{Here } x_1 = 8\text{m}, y_1 = 10\text{m}, x_2 = ? \text{ and } y_2 = 40\text{m}$$

$$\text{Substitute } \frac{8}{10} = \frac{x_2}{40} \Rightarrow x_2 = \frac{8 \times 40}{10} = 32 \text{ m}$$

So height of the tree is 32 m.

Example 4: If a pipe can fill a tank of capacity 50 l in 5 hours. Then how long will it take to fill a tank of capacity 75l.

Solution: Volume of water in a tank \propto time required to fill it.

So here $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ Here $x_1 = 50$ l $y_1 = 5$ hr $x_2 = 75$ l and $y_2 = ?$

$$\frac{50}{5} = \frac{75}{y_2} \Rightarrow y_2 = \frac{75 \times 5}{50} = \frac{375}{50} = 7\frac{1}{2} \text{ hr}$$

Time required to fill a tank of capacity 75 l is $7\frac{1}{2}$ hr

Example 5: If the cost of 20m of a cloth is ₹ 1600, then what will be the cost of 24.5 m of that cloth.

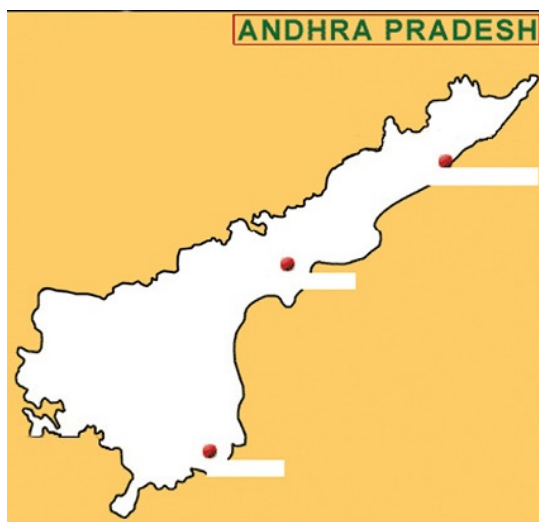
Solution: Cost directly varies with the length of cloth. So $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ where $x_1 = 20$ m

$y_1 = ₹ 1600$, $x_2 = 24.5$ m and $y_2 = ?$

$$\text{Substitute } \frac{20}{1600} = \frac{24.5}{y_2} \Rightarrow x_2 = \frac{1600 \times 24.5}{20} = ₹ 1960$$

Cost of 24.5 m of cloth is ₹ 1960.

Do This



Scale : 1 cm = 300 km

Measure the distance in the given map and using that calculate actual distance between (i) Vijayawada and Vishakhapatnam, (ii) Tirupati and Vijayawada. (Scale is given)

Scale shows how lengths between two cities are reduced in drawing

Scale : 1 cm = 300 km

Convert scale to ratio or 1 cm : 30000000 cm using centimetres as common unit.

Exercise - 10.1

- The cost of 5 meters of a particular quality of cloth is ₹ 210. Find the cost of (i) 2 (ii) 4 (iii) 10 (iv) 13 meters of cloth of the same quality .
- Fill the table.

No. of Apples	1	4	7	12	20
Cost of Apples (in ₹)	8
- 48 bags of paddy costs ₹ 16, 800 then find the cost of 36 bags of paddy.
- The monthly average expenditure of a family with 4 members is ₹ 2,800. Find the monthly average expenditure of a family with only 3 members.
- In a ship of length 28 m, height of its mast is 12 m. If the height of the mast in its model is 9 cm what is the length of the model ship?
- A vertical pole of 5.6 m height casts a shadow 3.2 m long. At the same time find (i) the length of the shadow cast by another pole 10.5 m high (ii) the height of a pole which casts a shadow 5m long.
- A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?
- If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weigh $16\frac{2}{3}$ kilograms?
- A train moves at a constant speed of 75 km/hr.
 (i) How far will it travel in 20 minutes?
 (ii) Find the time required to cover a distance of 250 km.
- The design of a microchip has the scale 40:1. The length of the design is 18cm, find the actual length of the micro chip?
- The average age of consisting doctors and lawyers is 40. If the doctors average age is 35 and the lawyers average age is 50, find the ratio of the number of doctors to the number of lawyers.

Project work

- Take a map of India. Note the scale used there. Measure the map distance between any two cities using a scale. Calculate the actual distance between them.
- The following ingredients are required to make halwa for 5 persons: Suji /Rawa = 250 g, Sugar = 300 g, Ghee = 200 g, Water = 500 ml. Using the concept of proportion, estimate the changes in the quantity of ingredients, to prepare halwa for your class.

10.2 Inverse Proportion

A parcel company has certain number of parcels to deliver. If the company engages 36 persons, it takes 12 days. If there are only 18 person, it will take 24 days to finish the task. You see as the number of persons are halved time taken is doubled, if company engages 72 person, will time taken be half?

Yes of course . Let's have a look at the table.

No. of persons	36	18	9	72	108
Time taken	12	24	48	6	4

How many persons shall a company engage if it want to deliver the parcels with in a day?

Two quantities change in such a manner that, if one quantity increases, the other quantity decreases in same proportion and vice versa, is called inverse proportion. In above example ,the number of persons engaged and number of days are inversely proportional to each other.

Symbolically, this is expressed as

number of days required $\propto \frac{1}{\text{number of persons engaged}}$

If x and y are in inverse proportion then $x \propto \frac{1}{y}$

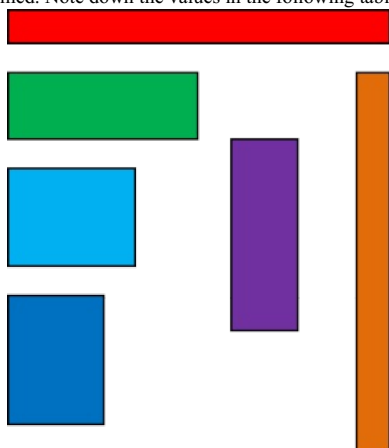
$x = \frac{k}{y}$ where k is constant of proportionality.

$xy = k$.

If y_1 and y_2 are the values of y corresponding to the values x_1 and x_2 of x respectively then $x_1 y_1 = x_2 y_2$ (= k), or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

Do These

- Write three situations where you see inverse proportion.
- To make rectangles of different dimensions on a squared paper using 12 adjacent squares. Calculate length and breadth of each of the rectangles so formed. Note down the values in the following table.



Rectangle Number	Length (in cm)	Breadth (in cm)	Area (sq.cm)
1	l_1, \dots	b_1, \dots	\dots
2	l_2, \dots	b_2, \dots	\dots
3	l_3, \dots	b_3, \dots	\dots
4	l_4, \dots	b_4, \dots	\dots
5	l_5, \dots	b_5, \dots	\dots
6	l_6, \dots	b_6, \dots	\dots

What do you observe? As length increases, breadth decreases and vice-versa (for constant area).

Are length and breadth inversely proportional to each other?

Example 6: If 36 workers can build a wall in 12 days, how many days will 16 workers take to build the same wall ? (assuming the number of working hours per day is constant)

Solution: If the number of workers decreases, the time to taken built the wall increases in the same proportion. Clearly, number of workers varies inversely to the number of days.

So here $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ where

$x_1 = 36$ workers $y_1 = 12$ days

$x_2 = 16$ workers and $y_2 = (?)$ days

No. of workers No. of days

36 12

16 y_2

Substitute, $\frac{36}{16} = \frac{y_2}{12} \Rightarrow y_2 = \frac{12 \times 36}{16} = 27$ days.

Therefore 16 workers will build the same wall in 27 days.

Think Discuss and Write

Can we say that every variation is a proportion.

A book consists of 100 pages. How do the number of pages read and the number of pages left over in the book vary?

No. of pages read (x)	10	20	30	50	70
Left over pages (y)	90	80	70	50	30

What happened to the number of left over pages, when completed pages are gradually increasing? Are they vary inversely? Explain.

Exercise - 10.2

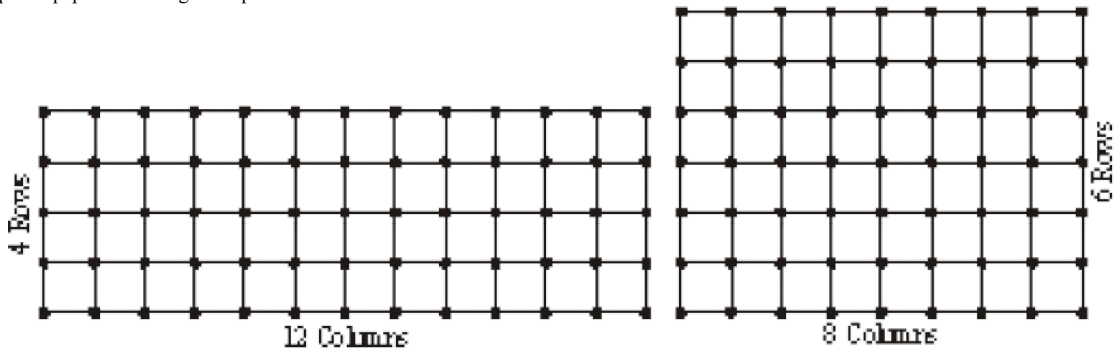
Observe the following tables and find which pair of variables (x and y) are in inverse proportion

(i)	x	50	40	30	20		(ii)	x	100	200	300	400
	y	5	6	7	8			y	60	30	20	15
(iii)	x	90	60	45	30	20	5					
	y	10	15	20	25	30	25					

2. A school wants to spend ₹ 6000 to purchase books. Using this data, fill the following table.

Price of each book (in ₹)	40	50	75
Number of books that can be purchased	150	100	75

3. Take a squared paper and arrange 48 squares in different number of rows as shown below:



Number of Rows (R)	2	3	4	6	8
Number of Columns (C)	---	---	12	8	---

What do you observe? As R increases, C decreases

(i) Is $R_1 : R_2 = C_2 : C_1$?

(ii) Is $R_3 : R_4 = C_4 : C_3$?

(iii) Is R and C inversely proportional to each other?

(iv) Do this activity with 36 squares.

Class Project

Prepare a table with number of students present and number of students absent in your class for a week.

Discuss with your friends and write your observations in your note book.

Day of the week	Number of students present (x)	Number of students absent (y)	x : y
Monday			
Tuesday			
Wednesday			
Thursday			
Friday			
Saturday			

Now let us solve some examples.

Example 7: Ration is available for 100 students in a hostel for 40 days.

How long will it be last, if 20 more students join in the hostel after 4 days?

Solution: As the number of students increase, ration will last for less number of days in the same proportions are in inverse proportion.

	No. of days ration available	No. of students
	40	100
After 4 days	36	100
	x	120

Now the question is if there is availability of rice for 36 days for 100 students. How long will it last for 120 students.

$$\frac{36}{x} = \frac{120}{100}$$

$$x = \frac{36 \times 100}{120} = 30 \text{ days}$$

Since the number of students are increasing

$$100 \times x = 120 \Rightarrow x = \frac{120}{100}$$

So the number of days will decrease in same proportion.

i.e. $36 \div x$

$$= 36 \div \frac{120}{100}$$

$$\Rightarrow 36 \times \frac{100}{120} = 30 \text{ days}$$

Example 8: A car takes 4 hours to reach the destination by travelling at a speed of 60 km/h. How long will it take if the car travels at a speed of 80 km/h?

Solution: As speed increases, time taken decreases in same proportion. So the time taken and varies inversely to the speed of the vehicle, for the same distance.

method I		method II	
Speed	Time	Speed	Time
60	4	60	4
80	x	80	y

or

$$\frac{60}{80} = \frac{x}{4}$$

$$60 \times 4 = 80 \times x$$

$$x = \frac{60 \times 4}{80} = 3 \text{ hr.}$$

$$60 \times x = 80 \text{ and } 4 \div x = y$$

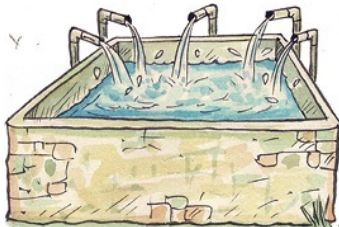
$$x = \frac{80}{60}$$

$$4 \div \frac{80}{60} = y$$

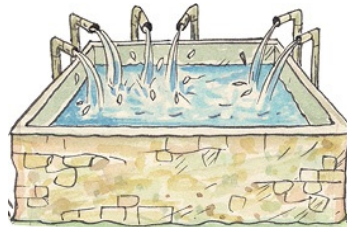
$$y = \frac{4 \times 60}{80} = 3 \text{ hr.}$$

Example 9: 6 pumps are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pumps of the same type are used?

Solution:



5 pipes in a tank



6 pipes in a tank

Let the desired time to fill the tank be x minutes. Thus, we have the following table.

Number of pumps	6	5
Time (in minutes)	80	x

Lesser the number of pumps, more will be the time required by it to fill the tank. So, this is a case of inverse proportion.

Hence, $80 \times 6 = x \times 5$ [$x_1 y_1 = x_2 y_2$]

$$\frac{80 \times 6}{5} = x$$

$$\text{or } x = 96 \text{ minutes.}$$

Thus, time taken to fill the tank by 5 pumps is 96 minutes or 1 hour 36 minutes.

Exercise - 10.3

- Siri has enough money to buy 5 kg of potatoes at the price of ` 8 per kg. How much can she buy for the same amount if the price is increased to ` 10 per kg?
- A camp has food stock for 500 people for 70 days. If 200 more people join the camp, how long will the stock last?
- 36 men can do a piece of work in 12 days. In how many days 9 men can do the same work?
- A cyclist covers a distance of 28 km in 2 hours. Find the time taken by him to cover a distance of 56 km with the same speed.
- A ship can cover a certain distance in 10 hours at a speed of 16 nautical miles per hour. By how much should its speed be increased so that it takes only 8 hours to cover the same distance? (A nautical mile is a unit of measurement used at sea distance or sea water i.e. 1852 metres).
- 5 pumps are required to fill a tank in $1\frac{1}{2}$ hours. How many pumps of the same type are used to fill the tank in half an hour.
- If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?
- A School has 8 periods a day each of 45 minutes duration. How long would each period become, if the school has 6 periods a day? (assuming the number of school hours to be the same)
- If z varies directly as x and inversely as y . Find the percentage increase in z due to an increase of 12% in x and a decrease of 20% in y .
- If $x + 1$ men will do the work in $x + 1$ days, find the number of days that $(x + 2)$ men can finish the same work.
- Given a rectangle with a fixed perimeter of 24 meters, if we increase the length by 1m the width and area will vary accordingly. Use the following table of values to look at how the width and area vary as the length varies.

What do you observe? Write your observations in your note books

Length(in cm)	1	2	3	4	5	6	7	8	9
Width(in cm)	11	10
Area (in cm ²)	11	20

10.3 Compound Proportion

Some times change in one quantity depends upon the change in two or more quantities in some proportion. Then we equate the ratio of the first quantity to the compound ratio of the other two quantities.

- (i) One quantity may be in direct proportion with the other two quantities.
- (ii) One quantity may be in inverse proportion with the other two quantities
- (iii) One quantity may be in direct proportion with the one of the two quantities and in inverse proportion with the remaining quantity.

Example 10: Consider the mess charges for 35 students for 24 days is ₹ 6300. How much will be the mess charges for 25 students for 18 days.

Solution: Here, we have three quantities i.e mess charges, number of students and number of days.

Mess charges in ₹	Number of students	Number of days
6300	35	24
? (x)	25	18
$6300 : x$	$35 : 25 = 7 : 5$	$24 : 18 = 4 : 3$

Mess charges is directly proportional to number of students.

Mess charges \propto number of students.

$$6300 : x = 7 : 5$$

Again mess charges are directly proportional to number of days.

Mess charges \propto number of days.

$$6300 : x = 4 : 3$$

Since, mess charges depends upon both the value i.e number of students and number of days so we will take a compound ratio of these two variables.

Mess charges \propto compound ratio of ratio of number of students and ratio of number of days.

$$6300 : x = \text{compound ratio of } 7 : 5 \text{ and } 4 : 3$$

$$6300 : x = 7 \times 4 : 5 \times 3$$

$$6300 : x = 28 : 15$$

Product of means = product of extremes.

$$28 \times x = 15 \times 6300$$

$$x = \frac{15 \times 6300}{28}$$

$$x = ₹ 3375.$$

Hence, the required mess charges is ₹ 3375.

Example 11: 24 workers working 6 hours a day can finish a piece of work in 14 days. If each worker works 7 hours a day, find the number of workers to finish the same piece of work in 8 days.

Solution: Here we have three quantities i.e number of workers, number of hours per day and number of days.

No. of workers	No. of hours per day	No. of days
24	6	14
? (x)	7	8
$24 : x$	$6 : 7$	$14 : 8 = 7 : 4$

Number of workers inversely proportional to number of hours per day.

$$\text{Number of workers} \propto \frac{1}{\text{number of hours per day}}$$

$$24 : x = \text{inverse ratio of } 6 : 7 \text{ i.e. } 7 : 6$$

$$\Rightarrow 24 : x \text{ is directly proportional to } 7 : 6.$$

Again, number of days is inversely proportional to number of workers.

$$\text{Number of workers} \propto \frac{1}{\text{number of days}}$$

$$24 : x = \text{inverse ratio of } 7 : 4 \text{ i.e. } 4 : 7$$

As, number of workers depends upon two variables i.e number of days and number of hours per day. Therefore,

Number of workers \propto compound ratio of inverse ratio of number of hours per day and inverse ratio of number of days.

$$24 : x = \text{compound ratio of } 7 : 6 \text{ and } 4 : 7$$

$$24 : x = 7 \times 4 : 6 \times 7$$

$$24 : x = 4 : 6$$

$$24 : x = 2 : 3$$

Product of means = product of extremes .

$$2 \times x = 24 \times 3$$

$$x = 36$$

Hence the required number of workers = 36.

Alternate method

$$\frac{24}{x} = \frac{7}{6} \times \frac{4}{7}$$

$$\frac{24}{x} = \frac{2}{3}$$

$$2 \times x = 24 \times 3$$

$$x = \frac{72}{2} = 36$$

Example 12: 12 painters can paint a wall of 180 m long in 3 days. How many painters are required to paint 200 m long wall in 5 days?

Solution: Here number of painters are in direct proportion to length of the wall and inversely proportional to number of days.

No. of painter	Length of the wall (m)	No. of days
12	180	3
x	200	5
$12 : x$	$180 : 200 = 9 : 10$	$3 : 5$

Number of painters \propto length of the wall

$$12 : x = 9 : 10 \text{ ---- (1) and}$$

Number of painters $\propto \frac{1}{\text{number of days}}$

$$12 : x = \text{inverse ratio of } 3 : 5$$

$$12 : x = 5 : 3 \text{ ---- (2)}$$

from (1) and (2)

$$12 : x = \text{compound ratio of } 9 : 10 \text{ and } 5 : 3$$

$$12 : x = (9 : 10) \times (5 : 3)$$

$$12 : x = 9 \times 5 : 10 \times 3$$

$$12 : x = 45 : 30 = 3 : 2$$

$$12 : x = 3 : 2 \text{ (product of extremes = product of means)}$$

$$12 \times 2 = 3 \times x$$

$$3 \times x = 12 \times 2$$

$$x = \frac{24}{3} = 8$$

Number of painters required = 8

Alternate method

$$\frac{12}{x} = \frac{9}{10} \times \frac{5}{3}$$

$$\frac{12}{x} = \frac{3}{2}$$

$$12 \times 2 = 3 \times x$$

$$x = \frac{12 \times 2}{3} = 8$$

Exercise - 10.4

1. Rice costing ` 480 is needed for 8 members for 20 days. What is the cost of rice required for 12 members for 15 days ?
2. 10 men can lay a road 75 km. long in 5 days. In how many days can 15 men lay a road 45 km. long ?
3. 24 men working at 8 hours per day can do a piece of work in 15 days. In how many days can 20 men working at 9 hours per day do the same work ?
4. 175 men can dig a canal 3150 m long in 36 days. How many men are required to dig a canal 3900 m. long in 24 days?
5. If 14 typists typing 6 hours a day can take 12 days to complete the manuscript of a book, then how many days will 4 typists, working 7 hours a day, can take to do the same job?



What we have discussed

- If x and y are in direct proportion, the two quantities vary in the same ratio i.e. if $\frac{x}{y} = k$ or $x = ky$. We can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ or $x_1 : y_1 = x_2 : y_2$ [y_1, y_2 are values of y corresponding to the values x_1, x_2 of x respectively]
- Two quantities x and y are said to vary in inverse proportion, if there exists a relation of the type $xy = k$ between them, k being a constant. If y_1, y_2 are the values of y corresponding to the values x_1 and x_2 of x respectively, then $x_1 y_1 = x_2 y_2$ ($= k$), or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.
- If one quantity increases (decreases) as the other quantity decreases (increases) in same proportion, then we say it varies in the inverse ratio of the other quantity. The ratio of the first quantity ($x_1 : x_2$) is equal to the inverse ratio of the second quantity ($y_1 : y_2$). As both the ratios are the same, we can express this inverse variation as proportion and it is called inverse proportion.
- Sometimes change in one quantity depends upon the change in two or more other quantities in same proportion. Then we equate the ratio of the first quantity to the compound ratio of the other two quantities.

Diffy with fractions

The process in this activity is called Diffy. The name comes from the process of taking successive differences of numbers and the activity provides practicing skills in subtraction.

Directions:

Step1 : Make an array of circles as shown and choose four fractions in the top four circles

Step2 : In the first three circles of the second row write the difference of the fractions above and to the right and left of the circle in the question, always being careful to subtract the smaller of these two fractions from the larger. In the fourth circle of second row place the difference of fractions in the first and fourth circles in the preceding row, again always subtracting the smaller fraction from the larger.

Step3 : Repeat step 2 to fill the successive rows of the circles. You may stop if you obtain a row of zeros.

Step4 : Repeat steps 1, 2 and 3 several times and each time start with different fractions.

Try Fraction Diffy with the fractions in first row $\frac{2}{7}, \frac{4}{5}, \frac{3}{2}, \frac{5}{6}$