Algebraic Expressions

Introduction

We have discussed about the addition, subtraction, multiplication and division of the arithmetic expression into previous chapter. In this chapter, we will discuss about the operation on algebraic expression.

Algebraic Expression

It is the combination of constants and variables along with the fundamental operations

Terms: It is the part of an algebraic expression which is separated by the sign of addition and subtraction.

$5x^4y^2, 35x^4y^2 - 13x^2y, 6xy, -3$ is an algebraic expression having $8x^3y^2, -4x^2y, 6xy, -3$ as its term.

Like and Unlike Terms

The terms having similar variable(s) are called like terms otherwise it is unlike.

In an algebraic expression $5x^4y^2, -13x^2y + 6xy - 3 - 35x^4y^2; 5x^4y^2, 35x^4y^2$ are like terms and are unlike terms.

Types of Algebraic Expression

Monomials

An algebraic expression which contains one term is called monomial.

i.e. $3x, 4x, -xy, b^2a$ etc. are monomials.

Binomials

An algebraic expression which contains two terms is called binomial.

i.e. $x + y, \mathrm{a} - \mathrm{b}, \mathrm{b}^2 a + a^2b$ etc. are binomials.

Trinomials

An algebraic expression which contains three terms is called trinomial.

i.e. $(x + y + z)$, $(a + b + c)$, $(a^2 + b^2 + a^2b^2)$ etc. are trinomials.

i.e. $(a - b - c + d), (x + y + z - c)$ etc. are quadrimonials.

Polynomials

An algebraic expression in which the variables have only non-negative integral power is called polynomial.

- $5a^3 - 4a^2 + 6a - 3$ is a polynomial because powers of variable "a" are non-negative.
- $7a^3b^2 - 4a^2b + 6ab - 3$ is a polynomial in two variable.
- $4x^2 + 23x^3 + 37xy + 45$ is a polynomial in two variable.
- $\sqrt{2x} + 3x^2 + 5$ is not a polynomial because the power of first term is $\frac{1}{2}$ which is not a non-negative integer.
- $4e^2 + \frac{1}{6}e + 2\sqrt{3}$ is a polynomial in one variable.

Degree of Polynomial

In case of one variable, the highest power of variable is the degree of polynomial

e.g $5x^4 - 4x^2 + 6x - 3$ is a polynomial of degree 4.

If polynomial is in the more than one variable then the highest sum of degree of the variables in each term is the degree of polynomial.

e.g. $8x^3y^2 - 4x^2y + 6xy - 3$ is a polynomial of degree 5.

Linear Polynomial

A polynomial of one degree is called linear polynomial.

$p(x) = 6x - 3$ is a linear polynomial.

Quadratic Polynomial

A polynomial of two degree is called quadratic polynomial.

$q(x) = 4x2 + 3x + 45$ is a quadratic polynomial.

The value of an Algebraic Expression

Step 1: If possible simplify the given algebraic expression.

Step 2: Replace variable with given numerical value.

Step 3: Simplify it.

Illustrative EXAMPLE

Find the value of $\dfrac{x^3 + y^3 + z^3 - 3xyz}{x^2 + y^2 + z^2 - xy - yz - zx}$, **If** $x = 1, y = 2$ **and** $z = -1$.

(a) 24
(b) 14
(c) 7
(d) 2
(e) None of these

Answer: (d)

Explanation $\dfrac{x^3 + y^3 + z^3 - 3xyz}{x^2 + y^2 + z^2 - xy - yz - zx} = \dfrac{(x + y + z+)(x^2 + y^2 + z^2 - xy - yz - zx)}{(x^2 + y^2 + z^2 - xy - yz - zx)}$

$= (x + y + z)$. Now putting the values of x, y and z we get = 2

Commonly Asked QUESTIONS

Find the value of $4\,xy(x - y) - 6x^2(y - y^2) - 3y^2(2x^2 - x) + 2xy(x - y)$ **for** $x = 5$ **and** $y = 13$.

(a) – 195
(b) 2535
(c) – 2535
(d) 215
(e) None of these

Answer: (c)

Explanation

$4xy(x - y) - 6x^2(y - y^2) - 3y^2(2x^2 - x) + 2xy(x - y)$

$4x^2y - 4xy^2 - 6x^2y + 6x^2y^2 - 6y^2x^2 + 3xy^2 + 2x^2y - 2xy^2$ After simplification, we get

$-3xy^2 = -3 \times 5 \times 13 \times 13 = -2535$

The value of A, B, C, D, E, F, G, H and I for which $(ABC)\times(DEF)=GHI$

(a) (6, 5, 7, 8, 2, 6, 1, 8, 4, 0)
(b) (7, 9, 5, 2, 6, 4, 8, 4, 0)
(c) (4, 6, 7, 8, 5, 7, 0, 8, 9)
(d) (1, 0, 3, 7, 8, 4, 2, 1, 0)
(e) None of these

Answer: (d)

Explanation

If we put the value of A, B, C, D, E, F, G, H and I from option D in the left hand side and right hand side of given expression, then it becomes zero.

Find the value of $3x^3y+4x^y+2y^x$, if $x=2$ $x=2$ and $y=-2.$

(a) – 39
(b) 39
(c) 13
(d) Cannot be determined
(e) None of these

Answer: (a)

Find the value of $a^2+b^2+c^2-ab-bc-ca$, if $a=2b=-2$ and $c=1.$

(a) 12
(b) 1
(c) 13
(d) 7
(e) None of these

Answer: (d)

Which one of the following options is the correct value or the expression $\frac{axy+byz+cxy}{ax+by+cz+1}$, if

$a=1, b=2, c=-1, x=-1, y=2, z=3?$

(a) 12
(b) 10
(c) 13
(d) 11
(e) None of these

Answer: (a)

Operations on Algebraic Expression

Addition and subtraction of an algebraic expressions mean addition and subtraction of like terms.

Addition of an Algebraic Expression

For addition of algebraic expression we may follow any one of the following methods:

Row Method

In this method, write all expression in a single row then arrange the terms to collect all like terms together and add it.

Column Method

In this method, arrange each expression in such a way that each like term is placed one below to other in a column.

Illustrative EXAMPLE

Add $3x+2y+3z$ and $2x-3y+4z$

Solution:

$$(3x+2y+3z)+(2x-3y+4z)$$

$=(3x+2x)+(2y-3y)+(3z+4z)=5x-y+7z$ Column method

$$\begin{array}{r} 3x+2y+3z \\ \underline{+2x-3y+4z} \\ 5x-y+7z \end{array}$$

Subtraction of an Algebraic Expression

For subtraction also you may follow any one of the following method

Row Method

We arrange algebraic expression in a row and change the sign (from + to –, – or – to +) of all terms which is to be subtracted. The two expression then added as above.

Column Method

Arrange two expression in such a way that like terms are placed one below the other and change the sign (from + to –,or – to +) of algebraic expression which is to be subtracted.

Illustrative EXAMPLE

Subtract $5a^2b^2+6a^2b^2+4$ from $7a^2b-6a^2b^2+5$

Solution:

By row method,

$(7a^2b-6a^2b^2+5)-(5a^2b-6a^2b^2+4)$

$=7a^2b-6a^2b^2+5-5a^2b-6a^2b^2-4$

Arrange like terms and add, we get

$=2a^2b-12a^2b^2+1$

By column method,

$$\begin{array}{l} 7a^2b-6a^2b^2+5 \\ \underline{5a^2b+6a^2b^2+4} \\ 2a^2b-12a^2b^2+1 \end{array}$$

Algebraic identities

Abatement of equality which holds, for all values of the variable is called algebraic identities.

Now we recall some important algebraic identities:

- $(\mathrm{a}+\mathrm{b})^2=a^2+2ab+b^2$
- $(\mathrm{a}-\mathrm{b})^2=a^2-2ab+b^2$
- $a^2-b^2=(a-b)(\mathrm{a}+\mathrm{b})$

- $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$
- $a^3+b^3=(a+b)^3-3ab(a+b)$ or $(a+b)^3 = a^3+b^3+3ab(a+b)$
- $(a-b)^3 = a^3-b^3-3ab(a-b)$
- $a^3+b^3=(a+b)^3-3ab(a+b)$ or $(a+b)(a^2-ab+b^2)$
- $a^3-b^3=(a-b)^3+3ab(a-b)$ or $(a-b)(a^2+ab+b^2)$
- $a^3+b^3+c^3-3abc=(a+b+c)\ (a^2+b^2+c^2-ab-bc-ca)$

  If $(a+b+c)=0$ $a^3+b^3+c^3=3abc$

Illustrative EXAMPLE

Simplify: $(2p+3q-+4r)^2+(2p-3q-4r)^2$

(a) $2(4p^2+9q^2+16r^2-16rp)$

(b) $-2(4p^2+9q^2+16r^2-16rp)$

(c) $2(-4p^2+9q^2-16r^2+16rp)$

(d) $2(5p^2+9q^2-16r^2-98rp)$

(e) None of these

Answer: (a)

Explanation Let us first solve,

$[2p+3q+(-4r)]^2=(2p)^2+(3q)^2+(-4r)^2+2(2p)(3q)+2(3q)(-4r)$

$+2(-4r)(2p)=4p^2+4p^2+9q^2+16r^2+12pq-24qr-16rp.........(i)$

Now solve,

$(2p-3q-4r)^2=[2p+(-3q)+(-4r)]^2$

$=(2p)^2+(-3q)^2+(-4r)^2+2(2p)(-3q)+2(-3q)\ (-4r)+2(-4r)(2p)$

$=4p^2+9q^2+16r2-12pq+24qr-16rp$ ....(ii) Adding, (i) & (ii) we get,

$(2p+3a-4r)^2+(2p-3q-4r)^2$

$=4p^2+9q^2+16r^2+12pq-24qr-16rp+\ (4p^2+9q^2+16r^2-12pq+24qr-16rp)$

$=4p^2+9q^2+16r^2+12pq-24qr-16rp+4p^2+9q^2 \quad +16r^2-12pq+24qr-16rp$

$=8p^2+18q^2+32r^2-32rp$

$=2(4p^2+9q^{2+}16r2-16rp)$

Commonly Asked QUESTIONS

The expanded form of $(2x+3y-5z)^2$ **is:**

(a) $4x^2+9y^2+25z^2+12xy-30yz-20zx$

(b) $5x^2-6y^3+15z^3+12xy-36y^6+21xy^2$

(c) $8a^2+a^3-c^2+7x^2+2c^2+9c^2y$

(d) $1z^2-z^4-8^c-75^2+2c^2-98c^2$

(e) None of these

Answer: (a)
Explanation
$(2x+3y-5z)^2 = (2x)^2 + (3y)^2 + (-5z)^2 + 2(2x)(3y) + 2(3y)(-5z) + 2(5z)(2x)$
$= 4x^2 + 9y^2 + 25z^2 + 12xy - 30yz - 20zx$

Expand: $(3a-b+4c)^2$
(a) $5x^2 - 6y^3 + 15z^3 + 12xy - 36y + 21xy$
(b) $9a^2 + b^2 + 16c^2 - 6ab - 8bc + 24ac$
(c) $2x^2 + 7y^2 + 2z^2 + xy + 3yz + 2zx$
(d) $15a2 + 22b^2 + 2c^4 + ab + 4bc + 102ca$
(e) $5x^2 - 6y^3 + 15z^3 + 12xy - 36y + 21xy$

Answer: (b)
Explanation
$(3a-b+4c)^2 = (3a(-\mathrm{b}) + 4\mathrm{c})^2 = (3a)^2 + (-b)^2 + (4c)^2 + 2(3a)(-b) + 2(-b)$
$(4c) + 2(4c)(3a) = 9a^2 + b^2 + 16c^2 - 6ab - 8bc + 24ac$

Find the value of $(3a+5b)^3$.
(a) $27a^3 + 125b^3 + 135a^2b + 225ab^2$
(b) $27a^3 - 125b^3 - 135a^2b + 2225ab^2$
(c) $27a^2 + 155b^2 - 135a^2b - 225a^2b$
(d) $29a^2 - 156b^2 - 156a^2b - 225a^4c$
(e) None of these

Answer: (a)

Find the Cube of $x-2y$.
(a) $x^3 + 8y^2 + 6x^2y - 12xy^3$
(b) $x^3 - 8y^3 - 6x^2y + 12xy^2$ v
(c) $x^2 + 87y + 7xy - 7xy$
(d) $7xy^3 - 6x - 74x^2y + 2xy$
(e) None of these

Answer: (b)

Evaluate: $\left(-6p + \frac{1}{3}q - r\right)^2$
(a) $36p^2 \frac{9}{1} p^2 + 5^2 - 4pq - \frac{2}{3} gr + 12rp$
(b) $36p^3 \frac{9}{1} q^2 + p^2 + 4qp - \frac{3}{2} qr + 13rp$
(c) $36p^2 + \frac{1}{9} q^2 + r^2 - 4qp - \frac{2}{3} qr + 12rp$
(d) $36x^2 - \frac{99}{8} s^2 - s^2 - 4pq - \frac{2}{3} qr + 14rp$
(e) None of these

Answer: (c)

Factorization of the Polynomials

Let us recall that an algebraic expression that is expressed as the product of two or more expressions & each of these is a factor of the given algebraic expression.

The process of writing a given algebraic expression as the product of two or more factors is called factorization.

Some of the common methods of factorization of the algebraic expressions are as follows:

- Factorization by taking out a common factor.
- Factorization by using identities
- Factorization by regrouping the terms.

Factorization of the Algebraic Expression $ax^2 + bx + c$

Step 1: Find the product of constant term and coefficient of x2 i.e. ac.

Step 2: Find factors of "ac".

Step 3: Select the factors of 'ac' in such a way that addition or subtraction of factors must be equal to the coefficient of x i. e "b".

Step 4: If the product 'ac' is positive then both the factors are either positive or negative.

Step 5: If the product 'ac' is negative then the two factors of 'ac' will have different sign.

Illustrative EXAMPLE

Find the factors of $x^2 + 9x + 18$

Solution:

Step 1: The product of constant term and coefficient of x^2 is 18

Step 2: The product is positive. Therefore, both the factors of 18 will be either be positive or negative.

Step 3: But the coefficient of x is positive therefore, both the factors of 18 will be positive.

Step 4: Factors of 18 are $1\times18, 2\times9$ and 3×6. One pair whose sum is 9 should betaken.

Step 5: The required number are 3 & 6.

$\therefore x^2 + 9x + 18 = x^2 + (6+3)x + 18$

$= (x+6)(x+3)$ [By using identity

$(x+a)(x+b) = x^2 + (\mathrm{a}+\mathrm{b})\mathrm{x} + ab$]

or $x^2 + 9x + 18 = x^2 + 6x + 3x + 18$

$= x(x+6) + 3(x+6) = (x+6)(x+3)$.

Taking out x from first two terms and 3 common from last two terms.

Commonly Asked QUESTIONS

Factorize: $x^2 - 11x + 30$.

(a) $(x-6)(x-5)$
(b) $(x-6)(x+5)$
(c) $(7x^2-6)(8x-9)$
(d) $\left(9x^2-62\right)\left(2x+3\right)$
(e) None of these

Answer: (a)
Explanation
Let us find two numbers whose product is 30 & sum is (-11)
Since, the product is positive, therefore, either both the numbers will be positive or negative.
But the sum is negative hence, both the numbers will also be negative.
Required factors of 30 are (-6) and (-5).

$\therefore x^2 - 11x + 30 = x^2 + \{(-6)+(-5)\}\ x + 30$

$= x^2 - 6x - 5x + 30 = x(x-6) - 5(x-6) = (x-6)(x-5)$

Factorize: $x^2 + x - 12$

(a) $(x+5)(x-3)$
(b) $(x-9)(x-52)$
(c) $(x+4)(x-3)$
(d) $(7x+5^2)(6x-7)$
(e) None of these

Answer: (c)
Explanation
To find two numbers whose product is (- 12) & sum is 1.
Since the product is negative, one number will be positive & the other number will be negative.
Since the sum is positive, the numerically greater of two numbers will be positive.
So, the required factors are 4 & (-3)

$x^2 + x - 12 = x^2 + \{4+(-3)\}\, x - 12$

$= x^2 + 4x - 3x - 12 = x(x+4) - 3(x+4) = (x+4)(x-3)$

If $x - \frac{1}{x} = 7$ then the value of $x^4 - \frac{1}{x^4}$ is:

(a) 2600
(b) 2599
(c) 2601
(d) 2401
(e) None of these

Answer: (b)

Factorize: $(5a+4b)^2 - (3a-2b)^2$

(a) $4(a+3b)(4a+b)$
(b) $2(a+3b)(4a+b)$

(c) $3(a+3b)(4a+b)$ (d) $(a+3b)(4a+b)$

(e) None of these

Answer: (a)

The factor of $4p^2+q^2+16-4pq+8q-16p$ is:

(a) $(-2p-q+4)(-2p+q+4)$

(b) $(2p+q+4)(2p+q+4)$

(c) $(-2p+q+4)(-2p+q+4)$

(d) $(-2p+q-4)(-2p+q-4)$

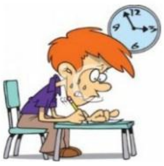
(e) None of these

Answer: (c)

SUMMARY

- $(a+b)^2=a^2+2ab+b^2$
- $(a-b)^2=a^2-2ab+b^2$
- $a^2-b^2=(a-b)(a+b)$
- $(a+b)^3=a^3+b^3+3a^2b+3ab^2$ or $(a+b)^3=a^3+b^3+3ab(a+b)$
- $(a-b)^3=a^3-b^3-3a^2b+3ab^2$ or
  $(a-b)^3=a^3-b^3-3ab(a-b)$
- $a^3+b^3=(a+b)^3-3ab(a+b)$ or $(a+b)(a^2-ab+b^2)$
- $a^3-b^3=(a-b)^3+3ab(a-b)$ or
  $(a-b)(a^2+ab+b^2)$
- $a^3+b^3+c^3-3abc(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
- If $(a+b+c)=0$, then $a^3+b^3+c^3=3abc$

Self Evaluation TEST



Duration 10 Minutes

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1. **Simplify:** $(y-5)(y+2)$

   (a) $y^3-3y+10$ (b) $y^2-3y+10$

   (c) $x^2-2y-10$ (d) $x^3+3y+15$

   (e) None of these

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2. **Simplify:** $(z-1)(z-2)$

   (a) z^2-3z+2 (b) z^2-4z-2

   (c) x^2+5x+2 (d) $x+5y-3$

   (e) None of these

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3. **If** $a+b+c=12$ & $a^2+b^2+c^2=66$, **then find the value of** $ab+be+ca$.

   (a) 50 (b) 89

   (c) 99 (d) 39

   (e) None of these

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4. **If** $a+b=7$, $ab=15$, **then find** a^3+b^3.

   (a) 29 (b) 28

   (c) 27 (d) 26

   (e) None of these

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5. **If** $x-\frac{1}{x}=2$, **then find the value of** $x^3-\frac{1}{x^3}$.

   (a) 14 (b) 17

   (c) 16 (d) 15

   (e) None of these

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6. **Factorize:** $x^2-4x-45$

   (a) $(x-9)(x+5)$

   (b) $(x-9)(x-5)$

   (c) $(9-x)(2+x)$

   (d) $(x-9)(x-5)$

   (e) None of these

7. **Factorize:** $27x^3 + 64y^3 + 108x^2y + 144xy^2$

(a) $(3x + 4y)^3$
(b) $(2x + 4y)^2$
(c) $(3x - 4y)^2$
(d) $(9x + 2y)^6$
(e) None of these

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8. **Factorize:** $x^3 - 8y^3 - 6x^2y + 12xy^2$ **.**

(a) $(x - 2y)^3$
(b) $(x + 2y)^3$
(c) $(-x - 2y)^3$
(d) $(x + 2x)^3$
(e) None of these

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9. **What should be added to the sum of** $(x^2 + y2 + xy)$ **and** $(4x^2 + 4xy)$ **to get** $(2x^2 + 3xy)$?

(a) $(3x^2 + y^2 + 2xy)$
(b) $(3x^2 - y^2 - 2xy)$
(c) $-(3x^2 + y^2 + 2xy)$
(d) $(-3x^2 + y^2 + 2xy)$
(e) None of these

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10. **What should be subtracted from** $(2a + 8b + 10)$ **to get** $(-3a + 7b + 16)$?

(a) $5a - b - 6$
(b) $-5a - b + 6$
(c) $5a + b - 6$
(d) All of these
(e) None of these

Answers – Self Evaluation Test

1.	B	**2.**	A	**3.**	D	**4.**	B	**5.**	A	**6.**	A	**7.**	A	**8.**	A	**9.**	C	**10.**	C

Self Evaluation Test
SOLUTIONS

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1. $(y-5)(y+2)=y^2-5y+2y-10=y^2-3y-10$

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2. $(z-1)(z-2)=z^2-2z-z+2=z^2-3z+2$

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3. $(a+b+c)^2\ a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow 12^2=66+2(ab+bc+ca)$, solve this equation and get result 39

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4. $a^3+b^3=(a+b)^3-3ab(a+b)=7^3-3\times7\times15=28$

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5. $x^3-\dfrac{1}{x^3}=\left(x-\dfrac{1}{x}\right)^3+3\bullet\cancel{x}\dfrac{1}{\cancel{x}}\left(x-\dfrac{1}{x}\right)=2^3+3\times2=8+6=14$

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6. $x^2-4x-45=x^2-9x+5x-45=(x-9)(x+5)$

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7. $(a+b)^3=a^3+b^3+3a^2b+3ab^2$

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8. $a^3-b^3-3a^2b+3ab^2=(a-b)^3$

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9. $\left(x^2+y^2+xy\right)+\left(4x^2+4xy\right)=\left(5x^2+5xy+y^2\right)$
