Chapter 15

Statistics

Exercise 15.3

Q. 1 From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Answer:

The group having higher coefficient of variation will be more variable

So, we will calculate

Where σ is standard deviation is mean

For group A

Marks	Group A f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	$(y_i)^2$	$f_i y_i$	$f_i (y_i)^2$
10-20	9	15	$\frac{15-45}{10}$ = -3	$(-3)^2$ =9	-27	81
20-30	17	25	$\frac{25-45}{10}$ = -2	$(-2)^2$ =4	-34	68
30-40	32	35	$\frac{35-45}{10}$ = -1	$(-1)^2$ =1	-32	32
40-50	33	45	$\frac{45-45}{10}$ = 0	$0^2 = 0$	0	0
50-60	40	55	$\frac{55-45}{10}$ = 1	$(1)^2$ =1	40	40

60-70	10	65	$\frac{65-45}{10} = 2$	$(2)^2 = 4$	20	40
70-80	9	75	$\frac{75-45}{10} = 3$	$(3)^2 = 9$	27	81
total	150				-6	342

Mean

Where A is assumed mean = 45

h = class size= 20- 10 = 10

Mean = $A + \frac{\sum_{i=1}^{7} x_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} = 45 - 0.4 = 44.6$

Variance $\sigma_1^2 = \frac{h^2}{N^2}\left(N\sum_{i=1}^{7} f_i {x_i}^2 - \left(\sum_{i=1}^{7} f_i x_i\right)^2\right)$

$= \frac{100}{22500}[150 \times 342 - (-6)^2]$

$= \frac{1}{225} \times 51264$

= 227.84

Standard deviation $\sigma 1 = \sqrt{227.84} = 15.09$

The standard deviation of group B is calculated as follows.

Marks	Group B f_i	Mid-point x_i	$y_i = \frac{x_i - A}{h}$	$(y_i)^2$	$f_i y_i$	$f_i(y_i)^2$
10-20	10	15	$\frac{15-45}{10}$ = -3	9	-30	90
20-30	20	25	$\frac{25-45}{10}$ = -2	4	-40	80
30-40	30	35	$\frac{35-45}{10}$ = -1	1	-30	30
40-50	25	45	$\frac{45-45}{10}$ = 0	0	0	0
50-60	43	55	$\frac{55-45}{10}$ = 1	1	43	43

60-70	15	65	$\frac{65-45}{10}=2$	4	30	160
70-80	7	75	$\frac{75-45}{10}=3$	9	21	189
total	150				-6	592

Variance $\sigma_2{}^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i x_i{}^2 - \left(\sum_{i=1}^{7} f_i x_i\right)^2\right]$

$= \frac{100}{22500}[150 \times 366 - (-6)^2]$

$= \frac{1}{225}[54864] = 243.84$

Standard deviation = $\sqrt{243}.84 = 15.61$

Since the mean of both the group is same, the group with greater standard deviation will be more variable.

Thus, group B has more variability in the marks.

Q. 2 From the prices of shares x and y below, find out which is more stable in value:

X	35	54	53	56	58	52	50	51	49
Y	108	107	105	106	107	104	103	104	101

Answer:

The prices of the shares x are

35, 54, 52, 53, 56, 58, 52, 50, 51, 49

Here. The number of observation, N = 10

Mean $\overline{x} = \frac{1}{N}\sum_{i=1}^{10} x_i = \frac{1}{10} \times 510 = 51$

X (x_i)	Y(y_i)	$x_i{}^2$	$y_i{}^2$
35	108	1225	11664
54	107	2916	11449
52	105	2704	11025

53	105	2809	11025
56	106	8136	11236
58	107	3364	11449
52	104	2704	10816
50	103	2500	10609
51	104	2601	10816
49	101	2401	10201
Total = 510	1050	26360	110290

Variance $(\sigma_1{}^2) = \frac{1}{N}\sum_{i=1}^{10}(x_i - \overline{x})^2 = \frac{1}{10} \times 350 = 35$

Standard deviation (σ1) = $\sqrt{35} = 5.91$

C.V. (shares x) = $\frac{\sigma_1}{x} \times 100 = \frac{5.91}{51} \times 100 = 11.58$

The prices of shares Y are

108, 107, 105, 105, 106, 107, 104, 103, 104, 101

Mean $\overline{x} = \frac{1}{N}\sum_{i=1}^{10} y_i = \frac{1010}{11} \times 1050 = 105$

The following table is obtained corresponding to shares y.

y_i	$(y_i - \overline{y})$	$(y_i - \overline{y})^2$
108	3	9
107	2	4
105	0	0
105	0	0
106	1	1
107	2	4
104	-1	1
103	-2	4
104	-1	1
101	-4	16
		40

Variance $(\sigma_1{}^2) = \frac{1}{N}\sum_{i=1}^{10}(y_i - \overline{y})^2 = \frac{1}{10} \times 40 = 4$

Standard deviation $(\sigma_2) = \sqrt{4} = 2$

C.V. (shares Y) = $\frac{\sigma^2}{y} \times 100 = \frac{2}{105} \times 100 = 1.9$

C.V. of prices of shares X is greater than the C.V. of prices of shares y.

Thus, the prices of shares Y are more stable than the prices of shares X.

Q. 3 An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No of wage earners	586	648
Mean of monthly wages	$5253	$5253
Variance of the distribution of wages	100	121

(i) Which firm A or B pays larger amount as monthly wages?

(ii) Which firm, A or B, shows greater variability in individual wages?

Answer:

Here

(i) monthly wages of firm A = 5253

No. of wage earners = 586

Total amount paid = 586 × 5253 = 3078258

Mean monthly wages of firm B = 5253

No. of wage earners = 648

Total amount paid = 648 × 5253 = 340 3944

Thus, firm B pays the larger amount as monthly wages as the number of wage earners in firm B are more than the number of wage earners in firm A.

(ii) Variance of the distribution of wages in firm A (${\sigma_1}^2$) = 100

Standard deviation of the distribution of wages in firm

A $(\sigma_1) = \sqrt{100} = 10$

Variance of the distribution of wages in firm B (${\sigma_2}^2$) = 121

Standard deviation of the distribution of wages in firm B (${\sigma_2}^2$) = $\sqrt{121} = 11$

The mean of monthly wages of both the firms is same i.e., 5253.

Therefore, the firm with greater

Thus, firm B has greater variability in the individual wages.

Q. 4 The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goal. Find which team may be considered more consistent?

Answer:

No. of goals scored x_i	No. of matches f_i	$f_i x_i$	x_i^2	fix_i^2
0	1	0	0	0

1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
Total	$\sum f_i = 25$	$\sum f_i x_i = 50$		$\sum f_i x_i^2 = 130$

Mean = $\frac{\sum_{i=1}^{3} f_i x_i}{\sum_{i=1}^{3} f_i} = \frac{50}{25} = 2$

Thus, the mean of both the teams is same.

$\sigma = \frac{1}{N}\sqrt{N \sum f_i {x_i}^2 - (\sum f_i x_i)^2}$

$= \frac{1}{25}\sqrt{25 \times 130 - (50)^2}$

$= \frac{1}{25}\sqrt{750}$

$= \frac{1}{25} \times 27.38$

= 1.09

The standard deviation of team B is 1.25 goals.

The average number of goals scored by both the teams is same i.e., 2.

Therefore, the team with lower standard deviation will be more consistent.

Thus, team A is more consistent than team B.

Q. 5 The sum and sum of squares corresponding to length x (in cm) and weight y (in gm) of 50 plant products are given below:

Which is more varying, the length or weight?

Answer:

$$\sum_{i=1}^{50} x_1 = 212, \sum_{i=1}^{50} {x_1}^2 = 902.8$$

Here, N = 50

Mean, $\overline{x} = \frac{\sum_{i=1}^{50} y_i}{N} = \frac{212}{50} = 4.24$

Variance $({\sigma_1}^2) = \frac{1}{N}\sum_{i=1}^{50}(x_i - \overline{x})^2$

$= \frac{1}{50}\sum_{i=1}^{50}(x_i - 4.24)^2$

$= \frac{1}{50}\sum_{i=1}^{50}[{x_i}^2 - 8.48x_i + 17.97]$

$= \frac{1}{50}\left[\sum_{i=1}^{50} {x_1}^2 - 8.48\sum_{i=1}^{50} x_i + 17.97 \times 50\right]$

$= \frac{1}{50}[902.8 - 8.48 \times (212) + 898.5]$

$= \frac{1}{50}[1801.3 - 1797.76]$

$= \frac{1}{50} \times 3.58$

$= 0.07$

Standard deviation, σ_1(Length) = $\sqrt{0.07} = 0.26$

C.V. (Length) = $\frac{Standard\ deviation}{Mean} \times 100 = \frac{0.26}{4.24} \times 100 = 6.13$

$\sum_{i=1}^{50} y_i = 261, \sum_{i=1}^{50} {y_1}^2 = 1457.6$

Mean, $\overline{y} = \frac{1}{N}\sum_{i=1}^{50} y_i = \frac{1}{50} \times 261 = 5.22$

Variance $({\sigma_2}^2) = \frac{1}{N}\sum_{i=1}^{50}(y_i - \overline{y})^2$

$= \frac{1}{N}\sum_{i=1}^{50}(y_i - 5.22)^2$

$= \frac{1}{50}\sum_{i=1}^{50}[{y_i}^2 - 10.44y_i + 27.24]$

$$= \frac{1}{50}\left[\sum_{i=1}^{50} {y_i}^2 - 10.44 \sum_{i=1}^{50} y_i + 27.24 \times 50\right]$$

$$= \frac{1}{50}[1457.6 - 10.44 \times (212) + 1362]$$

$$= \frac{1}{50}[2819.6 - 2724.84]$$

$$= \frac{1}{50} \times 94.76$$

= 1.89

Standard deviation, σ_2(weight) = 1.37

C.V.(weight) = $\frac{Standard\ deviation}{Mean} \times 100 = \frac{1.37}{5.22} \times 100 = 26.24$

Thus, C.V. of weight is greater than the C.V. of lengths.

Therefore, weights very more than the lengths.