Chapter 20. Probability

Formulae

1. If all the outcomes of an experiment are equally likely and E is an event, then probability of event E, written by P (E), is given by
$$P(E) = \frac{\text{number of outcomes favourable to E}}{\text{total number of possible outcomes of the experiment}}$$
2. $0 \leq P(E) \leq 1$
3. P (not E) = 1 − P (E)
4. P (E) = 1 − P (not E)
5. P (E) + P (not E) = 1
6. The sum of the probabilities of all the elementary events of an experiment = 1
7. The probability of a sure event = 1
8. The probability of an impossible event = 0.

Concept Based Questions

Question 1. An unbiased dice is thrown. What is the probability of getting a number other than 4.

Solution :Sample space = {1, 2, 3, 4, 5, 6}

n (S) = 6

Event = {other than 4}

= {1, 2, 3, 5, 6}

n (E) = 5

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{5}{6}$. Ans.

Question 2. Two dice are thrown simulata- neously. Find the probability of getting six as the product.

Solution :n (S) = 36

Event = {getting 6 as a prodcut}

= {(1, 6), (2, 3), (3, 2), (6, 1)}

n (E) = 4

$\therefore$ $P(E) = \frac{n(E)}{n(S)}$

$= \frac{4}{36} = \frac{1}{9}$ Ans.

Question 3. If the probability of winning a 5 game is 5/11. What is the probability of losing?

Solution : $P(E) = \frac{5}{11}$

$\therefore$ Prob. of lossing $P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{5}{11}$

$= \frac{6}{11}$ Ans.

Question 4. Find the probability of getting a tail in a throw of a coin.

Solution : Sample space (S) $= \{H, T\}$ $n(S) = 2$

Event (E) $= \{T\}$ $n(E) = 1$

$\therefore$ $P(E) = \frac{n(E)}{n(S)}$

$= \frac{1}{2}$ Ans.

Question 5. In a cricket match a batsman hits a boundary 6 times out of 30 balls he play's. Find the probability that he did not hit the boundary?

Solution : $n(S) = 30$

E = {not hitting the boundary}

$n(E) = 30 - 6 = 24$

Prob. of not hitting boundary

$\therefore$ $P(E) = \frac{n(E)}{n(S)}$

$= \frac{24}{30} = \frac{4}{5}$. Ans.

Question 6. It is known that a bax of 600 electric bulbs contain 12 defective bulbs. One bulb is taken out at random from this box. What is the probability that it is a non-defective bulb?

Solution : Number of non-defective bulbs

$= 600 - 12 = 588$

$n(E) = 588$

$n(S) = 600$

$P(E) = ?$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{588}{600} = 0{\cdot}98$

Question 7. 1000 tickets of a lottery were sold and there are 5 prizes on these tickets. If Namita has purchased one lottery ticket, what is the probability of winning a prize?

Solution : $n(S) = 1000$

$n(E) = 5$

$P(E) = ?$

$\therefore \quad P(E) = \frac{n(E)}{n(S)}$

$= \frac{5}{1000} = 0.005$

Question 8. A coin is tossed 100 times with the following frequencies:
Head = 55, Tail = 45
find the probability for each event (i) head (ii) tail.

Solution : (i) Event = {Head}

$n(E) = 55$

$\therefore$ Probability of event $= \frac{n(E)}{n(S)}$

$= \frac{55}{100} = \frac{11}{20}$

(ii) Event = {tail}

$n(E) = 45$

$\therefore$ Probability of event $= \frac{n(E)}{n(S)}$

$= \frac{45}{100} = \frac{9}{20}$

Question 9. Namita tossed a coin once. What is the probability of getting (i) Head (ii) tail?

Solution : Sample space

$S = \{H, T\}$

$n(S) = 2$

Event = {Head}

$n(E) = 1$

$\therefore \quad P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{1}{2}$

(ii) Sample space $S = \{H, T\}$

$n(S) = 2$

Event = {tail}

$n(E) = 1$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}$

Question 10. Two coins are tossed once. Find the probability of getting.

(i)2 heads, (ii) at least 1 tail.

Solution : If two coins are tossed once, then

$$S = \{HH, HT, TH, TT\}$$

$$\Rightarrow \quad N(S) = 4$$

(i) E : getting two heads

$$\therefore \quad N(E) = 1$$

$$\therefore \quad P(E) = \frac{N(E)}{N(S)} = \frac{1}{4} \quad \text{Ans.}$$

(ii) At least one tail

$\therefore$ Favourable out come = 3

Required probability $= \frac{3}{4}$. Ans.

Question 11. A die has 6 faces marked by the given numbers as shown below:
The die is thrown once. What is the probability of getting
(i) a positive integer.
(ii) an integer greater than – 3.
(iii) the smallest integer.

1	2	3	–1	–2	–3

The die is thrown once. What is the probability of getting

(i) a positive integer.

(ii) an integer greater than – 3.

(iii) the smallest integer.

Solution :

1	2	3	–1	–2	–3

Total number of out comes = 6

(i) P (a positive integer) $= \frac{3}{6} = \frac{1}{2}$ Ans.

(ii) P (an integer greater than – 3) $= \frac{5}{6}$ Ans.

(iii) P (the smallest integer) $= \frac{1}{6}$ Ans.

Question 12. 1800 families with 2 children were selected randomly and the following data were recorded:

No. of girls in a family	2	1	0
No. of families	700	850	250

Compute the probability of a family chosen at random having :

(i) 2 girls (ii) 1 girl (iii) No girl

Solution : Total no. of families n (S) = 1800

Event = {2 girls}

n (E) = 700

P (E) = ?

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{700}{1800} = \frac{7}{18}$. Ans.

(ii) Event = {1 girl}

n (E) = 850

P (E) = ?

$\therefore$ $P(E) = \frac{n(E)}{n(S)}$

$= \frac{850}{1800} = \frac{17}{36}$. Ans.

(iii) Event = {No girl}

n (E) = 250

P (E) = ?

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{250}{1800} = \frac{5}{36}$ Ans.

Question 13. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that at the card drawn is neither a red card nor a qeen.

Solution : n (S) = 52

Event = {getting neither a red card nor a queen}

$\therefore$ There are 26 red cards and 2 more queens are there.

Number of cards each one of which is either a red card or a queen = 28.

The event that the card drawn is neither a red card nor a queen = 52 – 28 = 24.

n (E) = 24

n (S) = 52

P (E) = ?

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{24}{52} = \frac{6}{13}$. Ans.

Question 14. A dice is thrown once. What is the probability that the
(i) number is even
(ii) number is greater than 2?

Solution :

(i) $n(S) = 6$

Event = {Even number}

$= \{2, 4, 6\}$

$n(E) = 3$

$P(E) = ?$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$ Ans.

(ii) Event = {number is greater than 2}

$= \{3, 4, 5, 6\}$

$n(E) = 4$

$P(E) = ?$

$\therefore \quad P(E) = \frac{n(E)}{n(S)}$

$= \frac{4}{6} = \frac{2}{3}.$ Ans.

Question 15. A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box.

Solution : Let the number of Black Balls $= x$

White Balls $= 30$

Total Balls $= x + 30$

$P\text{ (Black Ball)} = \frac{x}{x+30}$

$P\text{ (White Ball)} = \frac{30}{x+30}$

According to the question

$P\text{ (Black Ball)} = \frac{2}{5} P.\text{ (White Ball)}$

$\frac{x}{x+30} = \frac{2}{5} \times \frac{30}{x+30}$

or $x = \frac{2}{5} \times 30$

$x = 12$

$\therefore$ Number of Black Balls $= 12$ Ans.

Question 16. From a pack of 52 playing cards all cards whose numbers are multiples, of 3 are removed. A card is now drawn at random.
What is the probability that the card drawn is:

(i) a face card (King, Jack or Queen)
(ii) an even numbered red card?

Solution : The numbers which are multiple of 3 in 52 playing cards are 3, 6 & 9 *i.e.* 3 cards of each denomination

$\therefore$ All cards whose numbers are multiples of 3 are $= 4 \times 3 = 12$ cards

$\left[\begin{array}{l}\text{Jack, Queen \& King} \\ \text{of each denomination}\end{array}\right]$

Remaining cards $= 52 - 12 = 40$

Now, No. of face cards $= 12$

(i) $P(\text{face card}) = \frac{12}{40} = \frac{3}{10}$ Ans.

Again, even numbered cards are 2, 4, 6, 8 and 10 each of heart (red) & Diamond (red)

$\therefore$ Total even numbered red card

$= 5 \times 2 = 10$

(ii) P (even numbered red card)

$= \frac{10}{40} = \frac{1}{4}$ Ans.

Question 17. One card is randomly drawn from a pack of 52 cards. Find the probability that:
(i) the drawn card is red.
(ii) the drawn card is an ace.
(iii) the drawn card is red and a king.
(iv) the drawn card is red or king.

Solution : In randomly drawing a card from 52 cards.

$n(S) = 52$

(i) Let A denote the event that the drawn card is red.

$n(A) = 26$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$ Ans.

(ii) Let B denote the event that drawn card is an ace

$n(B) = 4$

$n(S) = 52$

$P(B) = ?$

$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$ Ans.

(iii) Let C denote the event that the drawn card is red and a king.

$n(C) = 2$

$P(C) = ?$

$n(S) = 52$

$\therefore \quad P(C) = \frac{n(C)}{n(S)}$

$= \frac{2}{52} = \frac{1}{26}$. Ans.

(iv) Let D denote the event that the drawn card is red or a king

$$n(D) = 26 \text{ (red cards)} + 2 \text{ (kings)}$$
$$= 28$$
$$P(D) = ?$$
$$n(S) = 52$$
$$\therefore \quad P(D) = \frac{n(D)}{n(S)} = \frac{28}{52} = \frac{7}{13}. \quad \text{Ans.}$$

Question 18. One card is drawn from a pack of 52 cards, each of the 52 cards being equall likely to be drawn. Find the probability that the card drawn is (i) An ace, (ii) red, (iii) either red or king, (iv) red and a king, (v) a face card, (vi) a red face card, (vii) '2' of spade, (viii) '10' of a blacksuit.

Solution : $n(S) = 52$

(i) Event = {an ace}

$$n(E) = 4$$
$$P(E) = ?$$
$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}. \quad \text{Ans.}$$

(ii) Event = {red cards}

$$n(E) = 26$$
$$P(E) = ?$$
$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}. \quad \text{Ans.}$$

(iii) Event = {either red or king}

= {26 red cards + 2 kings}

= 28

$$n(E) = 28$$
$$P(E) = ?$$
$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}. \quad \text{Ans.}$$

(iv) Event = {red and a king}

= (there are 2 red kings in a pack of 52 cards}.

$$n(E) = 2$$
$$P(E) = ?$$
$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}. \quad \text{Ans.}$$

(v) Event = {face cards}

$$n(E) = 12$$
$$P(E) = ?$$
$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}. \quad \text{Ans.}$$

(vi) Event = {red face cards}

= { 3 diamonds + 3 hearts}

n (E) = 6

P (E) = ?

$\therefore$ P (E) = $\frac{n(E)}{n(S)}$

$= \frac{6}{52} = \frac{3}{26}$. Ans.

(vii) Event = {'2' of spade}

n (E) = 1

P (E) = ?

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{1}{52}$. Ans.

(viii) Event = {'10' of blacksuit}

n (E) = 2

P (E) = ?

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$. Ans.

Question 19. Two dice are thrown simulta-neously. Find the probability of getting:
(i) an even number as the stun, (ii) the sum as a prime number, (iii) a total of at least 10, (iv) a doublet of even number, (v) a multiple of 3 as the sum.

Solution : n (S) = 36

such as

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1) (, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2) (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5, (6, 6)

(i) Event = {even number as the sum}

i.e., = {(1, 1), (1, 3), (3, 1), (2, 2), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (2, 6), (6, 2), (4, 4), (5, 3), (3, 5), (5, 5), (6, 4), (4, 6) and (6, 6)}

n (E) = 18

P (E) = ?

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$. Ans.

(ii) Event = {the sum as a prime number}

i.e., = {(1, 1), (1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (6, 5) and (5, 6)}

n (E) = 15

P (E) = ?

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$. Ans.

(iii) Event = {getting a total of at least 10}

= {(6, 4), (4, 6), (5, 5), (6, 5), (6, 6)}

n (E) = 6

P (E) = ?

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$. Ans.

(iv) Event = {getting a doublet of even number}

= {(2, 2), (4, 4), (6, 6)}

n (E) = 3

P (E) = ?

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$. Ans.

(v) Event = {multiple of 3 as a sum}

= {(1, 2), (2, 1), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (3, 6), (6, 3), (5, 4), (4, 5), (6, 6)}

n (E) = 12

P (E) = ?

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$. Ans.

Question 20. Find the probability that leap year selected at random, will contain 53 Sundays.

Solution : In a leap year there are 366 days. In 366 days, we have 52 weeks and 2 days, Thus we can say that leap year has always 52 sundays.

The remaining two days can be

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday.

From above it is clear that there are 7 elementary events associated with this random experiment.

Clearly the event A will happen if the last two days of the leap year are either Sunday and Monday or Saturday and Sunday.

n (E) = 2

P (E) = 2

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$. Ans.