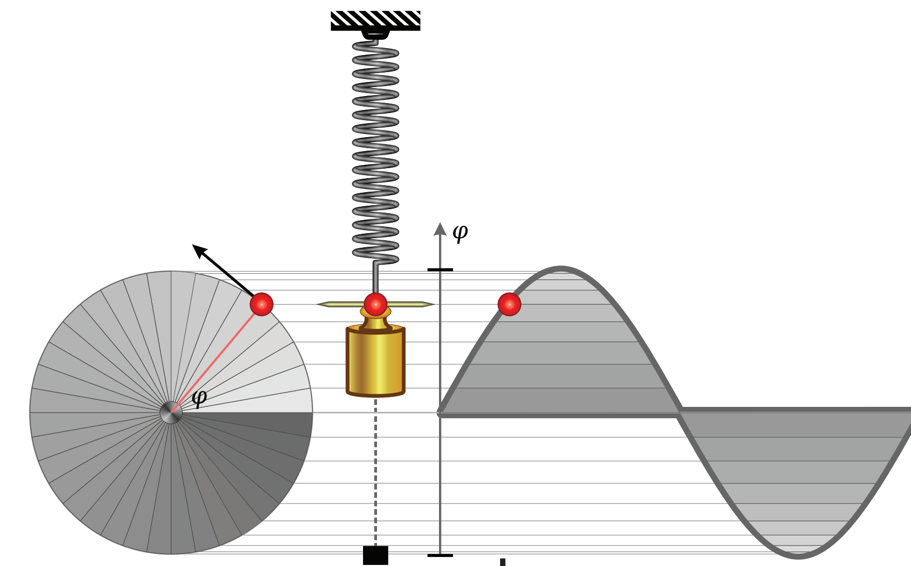
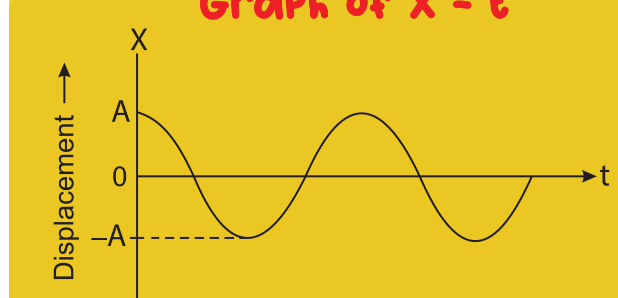
SIMPLE HARMONIC MOTION



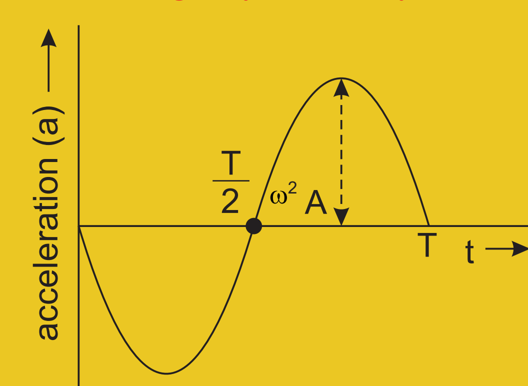
CHARACTERISTICS OF LINEAR SHM

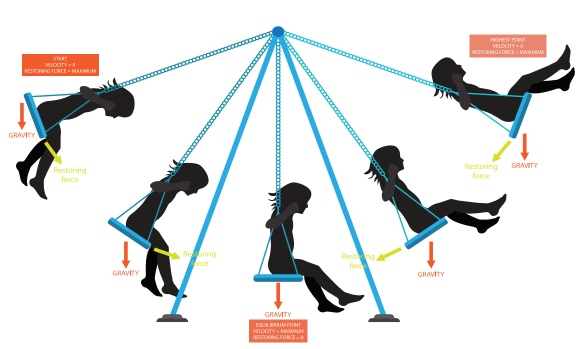
- Differential Equation of S.H.M $\frac{d^2x}{dt^2} + \omega_x^2 = 0$
- Displacement - x = A sin (ωt + ϕ)
- Velocity - V = $\frac{dx}{dt}$ = ω A Cos(ωt + ϕ)
- Acceleration - a = A Sin(wt + ϕ) = $-\omega^2 x$

Graph of X - t

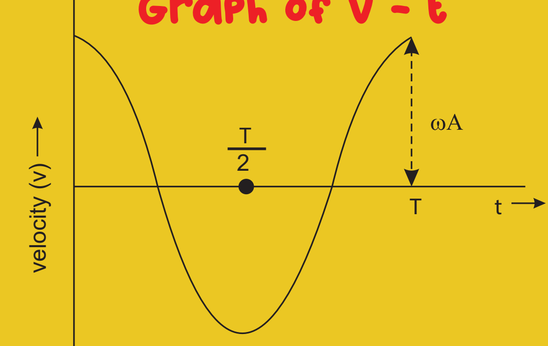


Graph of a - t





Graph of v - t



TIME PERIOD CALCULATION

(1) Force → $\vec{F} = -m\omega_x^2$ or $\vec{F} = -k\vec{x}$; $\left(\omega = \sqrt{\frac{k}{m}}\right)$ Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

K → spring Constant

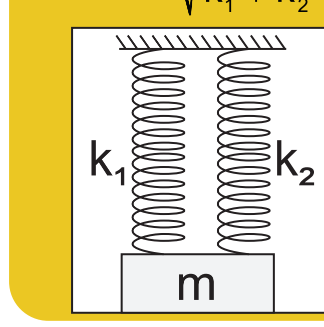
Spring Block System

Time Period → $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

(i) $k_{eq} = K_1 + K_2$

$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$;

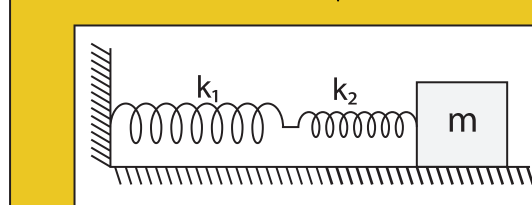
$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$



(ii) $K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$;

$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{K_1 K_2}}$



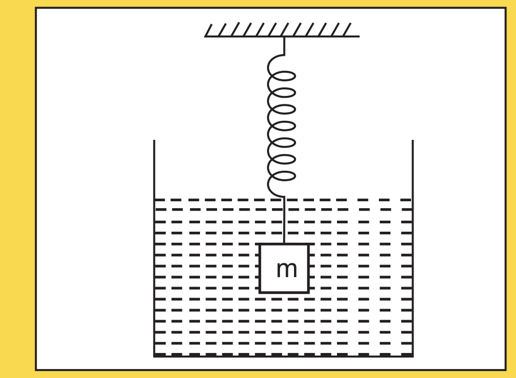
DAMPED AND FORCE OSCILLATIONS

DAMPED OSCILLATION

(1) Amplitude → $A^1 = Ae^{-bt/2m}$

(2) Angular Frequency → $w^1 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$,

where – b = damping constant



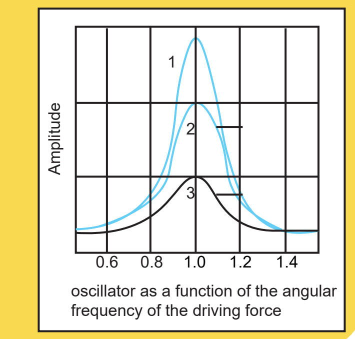
FORCED OSCILLATION

(1) Amplitude (For → $w_d >> \omega$) → $A^1 = \frac{F_o}{m(w^2 - w_d^2)}$

(2) Amplitude → $A^1 = F_o/w_d b$

ω_d → Driving Frequency

ω → Natural Frequency



ANGULAR S.H.M

(i) Different Equation → $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

⇒ Displacement → θ = θ_o sin (ωt + S)

⇒ Torque → T = Kθ

⇒ Angular Velocity → $W = \sqrt{\frac{K}{I}}$; Angular accelartion → $\propto = \frac{-K\theta}{I}$

⇒ Time period – $T = 2\pi\sqrt{\frac{I}{K}}$

PENDULUM

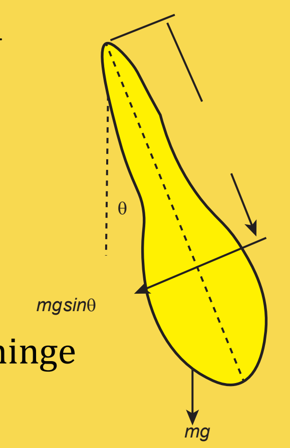
Physical Pendulum

:- Time period → $T = 2\pi\sqrt{\frac{I}{mgd}}$

I : MoI of system

M : Mass of System

d: distance between com and hinge

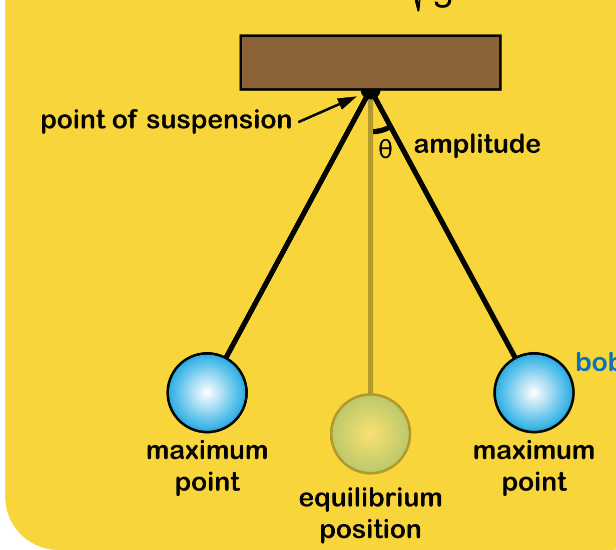


Simple Pendulum

F ∝ -θ;

F = -Kθ;

Time period → = $2\pi\sqrt{\frac{\ell}{g}}$



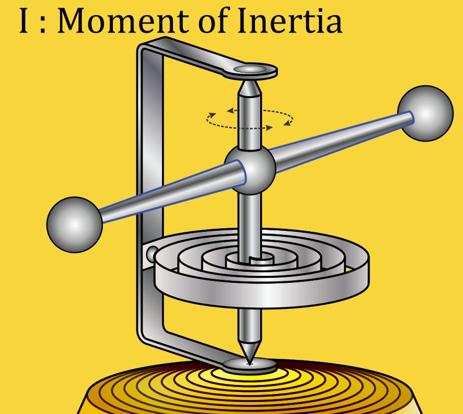
Torsional Pendulum

T ∝ θ

T = -Cθ [C = Torsional Constant]

Time period – $T = 2\pi\sqrt{\frac{I}{C}}$

I : Moment of Inertia



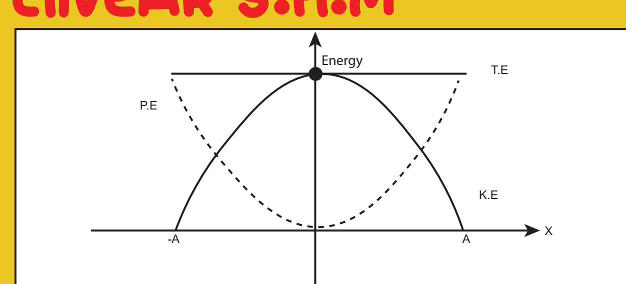
ENERGY OF LINEAR S.H.M

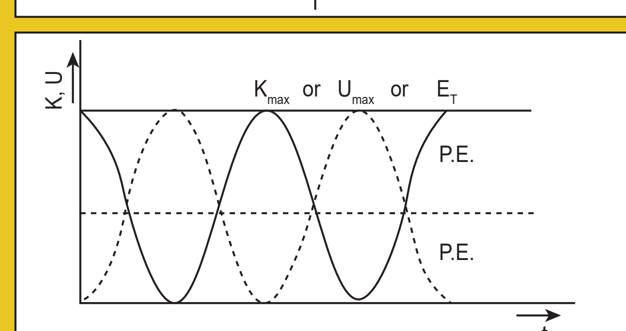
→ P.E → $U = \frac{1}{2}Kx^2$

K.E → $K = \frac{1}{2}K(A^2 - x^2)$

→ P.E → $U = \frac{1}{2}KA^2\sin^2(\omega t + \phi)$

K.E → $K = \frac{1}{2}KA^2\cos^2(\omega t + \phi)$





TWO BLOCKS SPRING SYSTEM

Reduced Mass: $\mu = \frac{m_1 m_2}{m_1 + m_2}$

$T = 2\pi\sqrt{\frac{m_1 m_2}{K(m_1 + m_2)}} = 2\pi\sqrt{\frac{\mu}{k}}$