CBSE Board
Class IX Mathematics

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

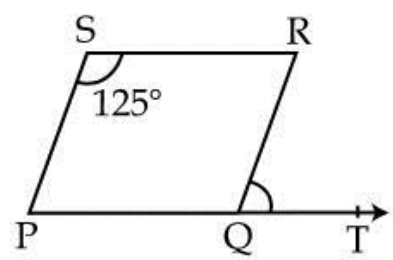
1. Simplify: $\left(6+\sqrt{27}\right)-\left(3+\sqrt{3}\right)+\left(1-2\sqrt{3}\right)$

2. Find the value of the polynomial $x^2 - x - 1$ at $x = -1$.

3. Give the definition of Parallel Lines.

4. Is (2, 0) a solution of $x - 2y = 4$?

5. PQRS is a parallelogram in which m $\angle PSR = 125°$. What is the measurement of $\angle RQT$?



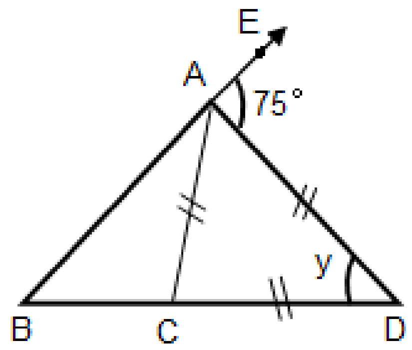
6. Find the Class size, if the Class marks of a frequency distribution are 6, 10, 14, 18, 22, 26 and 30?

Section B
(Questions 7 to 12 carry 2 marks each)

7. The volume of a cuboid is given by the algebraic expression $ky^2 - 6ky + 8k$. Find the possible expressions of the dimensions of the cuboid.

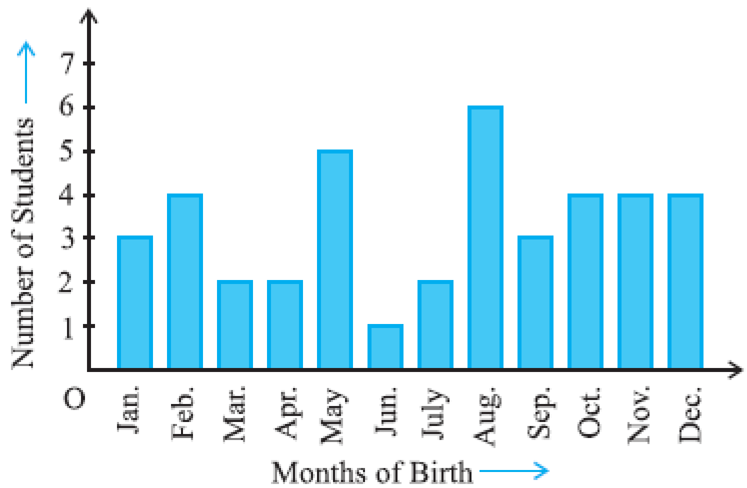
8. If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$ AB. Explain by drawing a figure.

9. In the figure below, BC = AC = AD and $\angle$ DAE = 75°. Find the value of y.



10. The total surface area of a cube is 294 cm^2. Find its volume.

11. In a particular section of Class IX, 40 students were asked about their birth month and the following graph was prepared for the data so obtained:



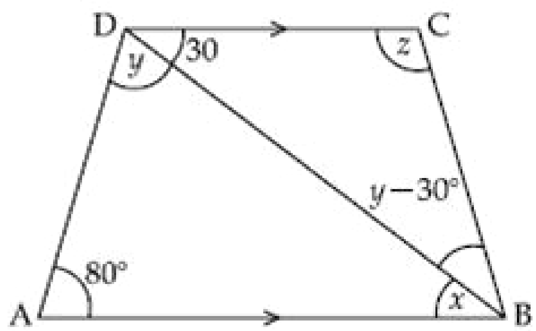
Find the probability that a student of Class IX was born in August.

12. The angles of a quadrilateral are in the ratio 2 : 5 : 8 : 9. Find all the angles in the quadrilateral.

Section C
(Questions 13 to 22 carry 3 marks each)

13. If $a = 3 + b$, then prove that $a^3 - b^3 - 9ab = 27$.

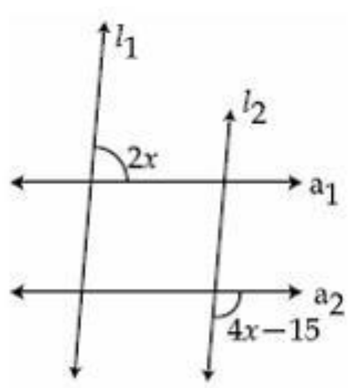
14. In the figure, $AB \parallel DC$, $\angle BDC = 30°$ and $\angle BAD = 80°$, find the values of x, y and z.



15. Use a suitable identity to factorise $27p^3 + 8q^3 + 54p^2q + 36pq^2$.

16. Factorise: $b^2 + c^2 + 2(ab + bc + ca)$.

17. In the figure, $l_1 \parallel l_2$ and $a_1 \parallel a_2$. Find the value of x.



18. A bag contains 12 balls out of which x are white. If one ball is taken out from the bag, find the probability of getting a white ball. If 6 more white balls are added to the bag and the probability now for getting a white ball is twice the previous one, find the value of x.

19. A storehouse measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates, each measuring 1.5 m × 1.25 m × 0.5 m, which can be stored in the storehouse.

20. A hemispherical bowl, made of steel, is 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

21. The length of 40 leaves of a plant are measured correct to one millimeter, and the data obtained is represented in the following table:

Length (in mm)	**Number of leaves**
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Draw a histogram to represent the given data.

i. Is there any other suitable graphical representation for the same data?

ii. Is it correct to conclude that maximum leaves are 153 mm long? Why?

22. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.

Section D
(Questions 23 to 30 carry 4 marks each)

23. If AD is the median of ΔABC, then prove that $AB + AC > 2AD$.

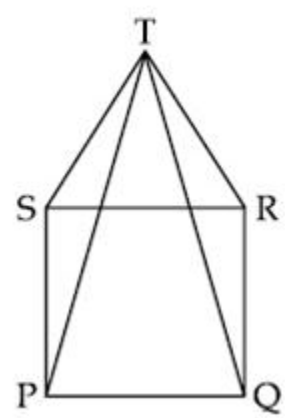
24. Simplify: $\dfrac{16\times 2^{n+1} - 4\times 2^{n}}{16\times 2^{n+2} - 2\times 2^{n+2}}$

25. In the figure, PQRS is a square and SRT is an equilateral triangle. Prove that:

a) $\angle PST = \angle QRT$

b) $PT = QT$

c) $\angle TQR = 15°$



26. (a)Simplify: $\left\{5\left(8^{1/3} + 27^{1/3}\right)^{3}\right\}^{1/4}$

(b) Represent $\sqrt{7}$ on the number line.

27. Find the median of 41, 43, 127, 99, 61, 92, 71, 58, 57.
If 58 is replaced by 85, what will be the new median?

28. The following table gives the distribution of students of two sections according to the marks obtained by them:

Section A		**Section B**	
Marks	**Frequency**	**Marks**	**Frequency**
0 – 10	3	0 – 10	5
10 – 20	9	10 – 20	19
20 – 30	17	20 – 30	15
30 – 40	12	30 – 40	10
40 – 50	9	40 – 50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

29. A circus tent is cylindrical upto a height of 11 m and conical above it. If the diameter of the base is 24 m and the height of the cone is 5 m, find the length of the canvas required to make the tent if the width of the canvas is 5 m.

30. Laxmi purchases some bananas and some oranges. Each banana costs Rs. 2 while each orange costs Rs. 3. If the total amount paid by Laxmi was Rs. 30 and the number of oranges purchased by her was 6, then how many bananas did she purchase?

CBSE Board
Class IX Mathematics
Solution

Time: 3 hrs **Total Marks: 80**

Section A

1.

$$\left(6+\sqrt{27}\right)-\left(3+\sqrt{3}\right)+\left(1-2\sqrt{3}\right)$$
$$=6+3\sqrt{3}-3-\sqrt{3}+1-2\sqrt{3}$$
$$=4$$

2. Given polynomial is $x^2 - x - 1$
Substituting $x = -1$ in $x^2 - x - 1$, we have
$(-1)^2 - (-1) - 1 = 1 + 1 - 1 = 1$

3. Two lines l and m in a plane are said to be parallel lines if they do not have common point, i.e. they do not intersect.

4. Substituting $x = 2$ and $y = 0$ in $x - 2y = 4$, we get
L.H.S $= 2 - 2 \times 0 = 2 \neq 4$
i.e. L.H.S. $\neq$ R.H.S
Therefore, (2, 0) is not a solution of $x - 2y = 4$.

5. $\angle PSR = \angle RQP = 125°$ (opposite angles will be equal since PQRS is a parallelogram)
$\angle PQT = 180°$ (PQT is a straight line)
$\Rightarrow \angle PQR + \angle RQT = 180°$
$\Rightarrow 125° + \angle RQT = 180°$
$\therefore \angle RQT = 55°$

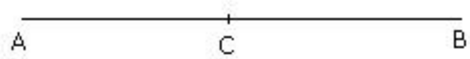
6. Class size is the difference between two successive class marks.
$\therefore$ Class size $= 10 - 6 = 4$

Section B

7. $ky^2 - 6ky + 8k$
$= k(y^2 - 6y + 8)$
$= k(y^2 - 4y - 2y + 8)$
$= k(y - 4)(y - 2)$
Thus, the dimensions of the cuboid are given by the expressions k, $(y - 4)$ and $(y - 2)$.

8.

Given: AC = BC



AC +AC = BC + AC

(If equals are added to equal the wholes are equal)

$\Rightarrow$ 2AC = AB

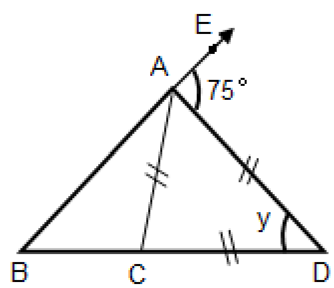
Hence, AC = $\frac{1}{2}$AB

9. Here $\angle ADC = y = \angle ACD$

Ext. $\angle ACD = \angle ABC + \angle BAC$

$\therefore 2\angle BAC = \angle ACD = y$

$\Rightarrow \angle BAC = \frac{y}{2}$



$\therefore \frac{y}{2} + (180° - 2y) = 180° - 75°$

$\Rightarrow \frac{y}{2} + 180° - 2y = 180° - 75$

$\Rightarrow \frac{y}{2} - 2y = -75°$

$\Rightarrow -\frac{3y}{2} = -75°$

$\Rightarrow y = 50°$

10. Let 'l' be the length of the cube.

Now, T.S.A. of the cube = 294 cm^2 ....(given)

$\therefore 6l^2 = 294$

$\therefore l^2 = \frac{294}{6} = 49$

$\therefore$ Side (l) = 7 cm.

Volume of cube = l × l × l = 7 × 7 × 7 = 343 cm^3

11. Number of students born in August = 6

Total number of students = 40

Required probability= $\frac{\text{Number of students born in August}}{\text{Total number of students}} = \frac{6}{40} = \frac{3}{20}$

12. Let the angles of a quadrilateral be 2x, 5x, 8x and 9x respectively.

By the angle sum property of a quadrilateral, we have

$2x + 5x + 8x + 9x = 360°$

$\therefore\ 24x = 360°$

$\therefore\ x = 15°$

Now,

First angle = $2x = 2 \times 15 = 30°$,

Second angle = $5x = 5 \times 15 = 75°$,

Third angle = $8x = 8 \times 15 = 120°$ and

Fourth angle = $9x = 9 \times 15 = 135°$.

Thus, the angles of a quadrilateral are 30°, 75°, 120° and 135°.

Section C

13. Given: $a = 3 + b$

$\Rightarrow a - b = 3$

Applying the cubic identity on both the sides, we have

$(a - b)^3 = 3^3$

$\Rightarrow a^3 - b^3 - 3(a)(b)(a - b) = 27$

$\Rightarrow a^3 - b^3 - 3ab(3) = 27$ $\qquad (\because a - b = 3)$

$\Rightarrow a^3 - b^3 - 9ab = 27$

14. Since AB ∥ DC,

$\angle x = 30°$ [Alternate angles]

In ΔABD,

$80° + 30° + \angle y = 180°$

$\Rightarrow \angle y = 180° - 110° = 70°$

In ΔBDC,

$30° + (70° - 30°) + \angle z = 180°$

$\Rightarrow \angle z = 110°$

15. $27p^3 + 8q^3 + 54p^2q + 36pq^2$

$= (3p)^3 + (2q)^3 + 18pq(3p+2q)$

$= (3p)^3 + (2q)^3 + 3 \times 3p \times 2q\ (3p + 2q)$

$= (3p + 2q)^3$ $[(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$ [where a = 3p and b = 2q]

$= (3p + 2q)\ (3p + 2q)\ (3p + 2q)$

16. $b^2 + c^2 + 2(ab + bc + ca)$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2$ [Adding and subtracting a^2]

$= [a^2 + b^2 + c^2 + 2ab + 2bc + 2ca] - a^2$

$= (a + b + c)^2 - (a)^2$ [Using $x^2 + y^2 + 2xy + 2yz + 2zx = (x + y + z)^2$]

$= (a + b + c + a)(a + b + c - a)$ [Because $a^2 - b^2 = (a+ b)\ (a - b)$]

$= (2a + b + c)(b + c)$

17. $2x = z$ (Alternate angles, as $l_1 \parallel l_2$)

$y = z$ (Alternate angles, as $a_1 \parallel a_2$)

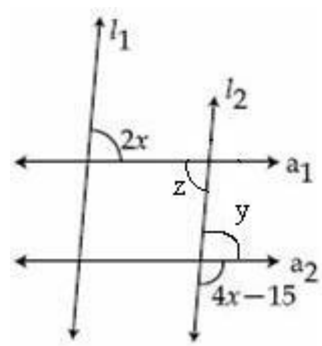
So, $2x = y$

Now, $y + 4x - 15 = 180°$ (linear pair)

$2x + 4x - 15 = 180°$

$\Rightarrow 6x = 195°$

$\Rightarrow x = 32.5$



18. Number of white balls = x

Total number of balls = 12

$\therefore P(\text{white ball}) = \dfrac{x}{12}$

If 6 white balls are added, we have

Total number of balls = 18

Number of white balls = x + 6

Now, $P(\text{getting a white ball}) = \dfrac{x+6}{18}$

According to the given information,

$\dfrac{x+6}{18} = 2\left(\dfrac{x}{12}\right)$

$\therefore \dfrac{x+6}{18} = \dfrac{x}{6}$

$\therefore 6x + 36 = 18x$

$\therefore 12x = 36$

$\therefore x = 3$

19. Length (l_1) of the storehouse = 40 m

Breadth (b_1) of the storehouse = 25 m

Height (h_1) of the storehouse = 10 m

Volume of storehouse = $l_1 \times b_1 \times h_1 = (40 \times 25 \times 10)\ m^3 = 10000\ m^3$

Length (l_2) of a wooden crate = 1.5 m

Breadth (b_2) of a wooden crate = 1.25 m

Height (h_2) of a wooden crate = 0.5 m

Volume of a wooden crate = $l_2 \times b_2 \times h_2 = (1.5 \times 1.25 \times 0.5)\ m^3 = 0.9375\ m^3$

Let the number of wooden crates stored in the storehouse be 'n'.

Hence, volume of 'n' wooden crates = Volume of storehouse

$0.9375 \times n = 10000$

$$\therefore n = \frac{10000}{0.9375} = 10666.66$$

Thus, 10666 wooden crates can be stored in the storehouse.

20. Inner radius of hemispherical bowl = 5 cm

Thickness of the bowl = 0.25 cm

$\therefore$ Outer radius (r) of hemispherical bowl = (5 + 0.25) cm = 5.25 cm

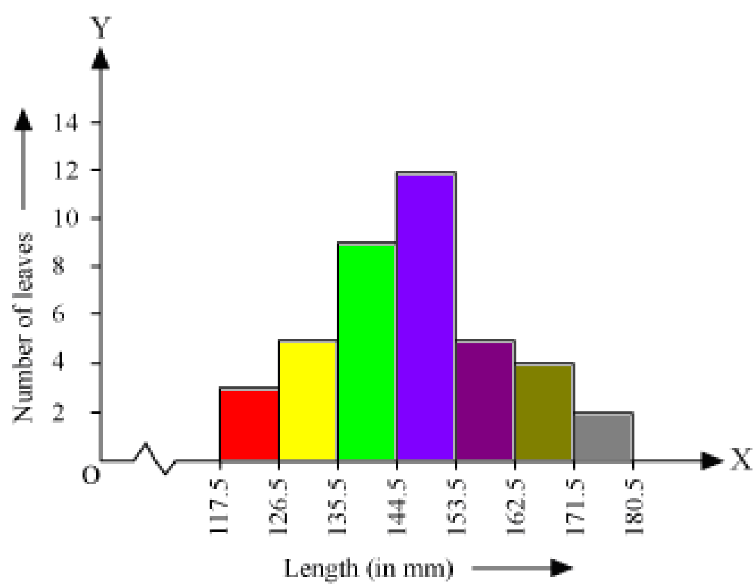
Outer C.S.A. of hemispherical bowl = $2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2 = 173.25\ cm^2$

Thus, the outer curved surface area of the bowl is $173.25\ cm^2$.

21. Lengths of the leaves are represented in discontinuous class intervals. Hence we have to add 0.5 mm to each upper class limit and also have to subtract 0.5 mm from the lower class limits so as to make our class intervals continuous.

Length (in mm)	Number of leaves
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2

Now taking the length of leaves on the x-axis and number of leaves on the y-axis we can draw the histogram of this information as below:

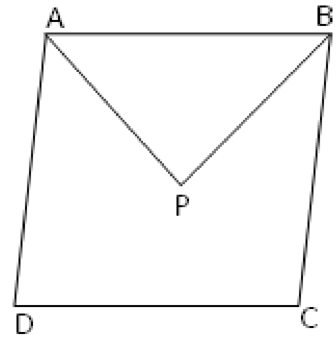


Here 1 unit on the y-axis represents 2 leaves.

i. Other suitable graphical representation of this data could be a frequency polygon.

ii. No, as maximum numbers of leaves (i.e. 12) have their length in between of 144.5 mm and 153.5 mm. It is not necessary that all have a length of 153 mm.

22. Given: ABCD is a parallelogram such that angle bisector of adjacent angles A and B intersect at point P.



To prove: $\angle APB = 90°$

AD || BC

$\therefore \angle A + \angle B = 180°$ [Consecutive interior angles]

$\therefore \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$

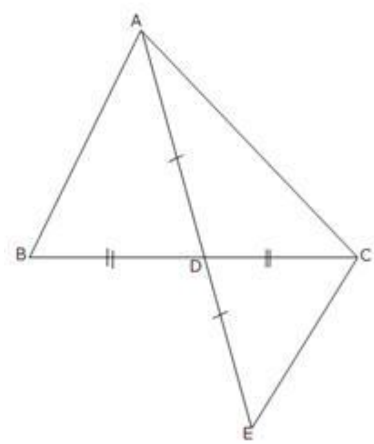
But, $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle APB = 180°$ ...(Angle sum property of a Δ)

$\therefore 90° + \angle APB = 180° \Rightarrow \angle APB = 90°$

Thus, the angle bisectors of two adjacent angles intersect at right angles.

Section D

23. Given: AD is median of triangle ABC



To Prove: AB + AC > 2AD

Proof: Produce AD so that AD = DE

Now, in triangles ADB and EDC,

AD = DE

BD = DC

$\angle$ADB = $\angle$EDC

Thus, ΔADB $\cong$ ΔED (By SAS congruence criterion)

Hence, AB = EC (CPCT)

Now, in ΔAEC,

AC + CE > AE

AC + CE > 2AD

AC + AB > 2AD (since, AB = EC, proved above)

24.

$$\frac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}=\frac{2^{4}\times 2^{n+1}-2^{2}\times 2^{n}}{2^{4}\times 2^{n+2}-2\times 2^{n+2}}$$

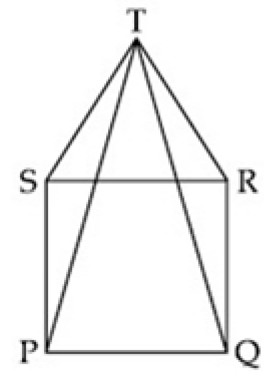
$$=\frac{2^{n+5}-2^{n+2}}{2^{n+6}-2^{n+3}}$$

$$=\frac{2^{n+5}-2^{n+2}}{2.2^{n+5}-2.2^{n+2}}$$

$$=\frac{\left(2^{n+5}-2^{n+2}\right)}{2\left(2^{n+5}-2^{n+2}\right)}$$

$$=\frac{1}{2}$$

25. □PQRS is a square.



$\therefore PQ = QR = RS = SP$ ....(i)

Also $\angle RSP = \angle SRQ = \angle RQP = \angle SPQ = 90^\circ$ ....(ii)

Also ΔTSR is equilateral.
$TS = TR = SR$......(iii)

Also $\angle STR = \angle TSR = \angle TRS = 60^\circ$
$TR = QR$.....from (i) and (ii)

Also $\angle TSP = \angle RSP + \angle TSR$
$\angle TSP = 90^\circ + 60^\circ = 150^\circ$

Similarly $\angle TRQ = 150^\circ$

In ΔTSP and ΔTRQ,
$PS = QR$.....$(\because$ by (i)$)$
$\angle TSP = \angle TRQ$......$(\because$ Both $150^\circ)$
$TS = TR$......$(\because$ by (iii)$)$
$\therefore \Delta TSP \cong \Delta TRQ$ ....(by SAS criterion)
$\therefore PT = QT$ .....(c.p.c.t)

Now, in ΔTRQ
$TR = RQ$ Given
$\therefore \angle TQR = \angle RTQ$
$\therefore \angle RTQ + \angle RQT + \angle TRQ = 180^\circ$ (angle sum property)
$\therefore 2\angle TQR + 150^\circ = 180^\circ$
$\therefore \angle TQR = 15^\circ$

26.

a)

$$\left\{5\left(2^{3\times\frac{1}{3}}+3^{3\times\frac{1}{3}}\right)^{3}\right\}^{\frac{1}{4}}$$

$$=\left[5(2+3)^{3}\right]^{\frac{1}{4}}$$

$$=\left(5\times5^{3}\right)^{\frac{1}{4}}$$

$$=5^{4\times\frac{1}{4}}$$

$$=5$$

b) In order to represent $\sqrt{7}$ on number line, we follow the steps given below:

Step 1: Draw a line and mark a point A on it.

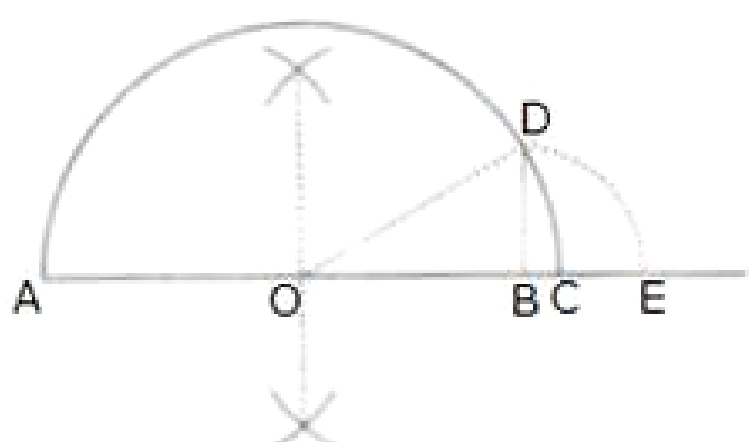
Step 2: Mark a point B on the line drawn in step 1 such that AB = 7 cm.

Step 3: Mark a point C on AB produced such that BC = 1 unit.

Step 4: Find mid-point of AC. Let the mid-point be O.

Step 5: Taking O as the centre and OC = OA as radius draw a semicircle. Then, draw a line passing through B perpendicular to OB. Let the perpendicular cut the semicircle at D.

Step6: Taking B as the centre and radius BD draw an arc cutting OC produced at E. Point E so obtained represents $\sqrt{7}$.



27. Arranging the given data in ascending order, we have

41, 43, 57, 58, 61, 71, 92, 99, 127

Here, n = 9 (odd)

$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ value} = \left(\frac{9+1}{2}\right)^{th} \text{ value} = 5^{th} \text{ value} = 61$

If 58 is replaced by 85, we get the following data:

41, 43, 57, 61, 71, 85, 92, 99, 127

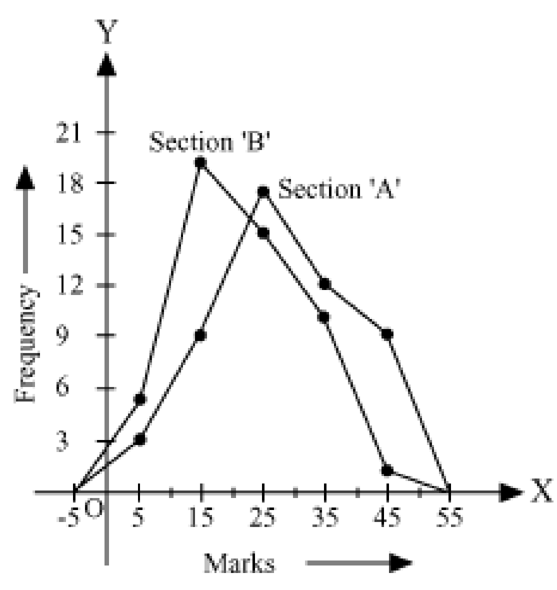
$\therefore \text{New median} = \left(\frac{n+1}{2}\right)^{th} \text{ value} = \left(\frac{9+1}{2}\right)^{th} \text{ value} = 5^{th} \text{ value} = 71$

28. We can find class marks of the given class intervals by using the formula –

$\text{Class mark} = \frac{\text{upper class limit} + \text{lower class limit}}{2}$

Section A			**Section B**		
Marks	Class marks	Frequency	Marks	Class marks	Frequency
0 – 10	5	3	0 – 10	5	5
10 – 20	15	9	10 – 20	15	19
20 – 30	25	17	20 – 30	25	15
30 – 40	35	12	30 – 40	35	10
40 – 50	45	9	40 – 50	45	1

Now taking the class marks on the x-axis and frequency on the y-axis and choosing an appropriate scale (1 cm = 3 units on the y-axis) we can draw a frequency polygon as below:



From the graph we can see that the performance of students of section ‘A’ is better than the students of section ‘B’.

29. Diameter = 24 m ⇒ radius = 12 m

Radius of the conical part = Radius of the cylindrical part (r) = 12 m

Height of cylindrical part (h) = 11 m, height of the cone (h) = 5 m

For the conical part of the circus tent,

$l^2 = r^2 + h^2$

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ m}$

Surface area of the tent = Curved surface area of the conical part + Curved surface area of the cylindrical part

$$\begin{aligned}\therefore \text{Surface area of the tent} &= \pi rl + 2\pi rh \\ &= \pi r(l + 2h) \\ &= \frac{22}{7} \times 12(13 + 22) \\ &= \frac{22}{7} \times 12 \times 35 \\ &= 1320 \text{ m}^2\end{aligned}$$

Breadth of the canvas (B) = 5 m

Let the length of the canvas = L

Now, area of canvas required = surface area of the tent

$\therefore$ L × B = 1320 ⇒ L × 5 = 1320 ⇒ L = 264 m

Thus, 264 m long canvas is required to make the tent.

30. Let us assume that Laxmi purchased x bananas and y oranges.

Since each banana costs Rs. 2, x bananas cost Rs. 2 × x = Rs. 2x

Similarly, each orange costs Rs. 3.

Thus, y oranges cost Rs. 3 × y = Rs. 3y

Thus, the total amount paid by Laxmi is Rs. (2x + 3y), which equals Rs. 30

Thus, we can express the given information in the form of a linear equation as 2x + 3y = 30

Now, we know that Laxmi purchased 6 oranges, i.e., the value of y is 6.

Substitute this value of y in the equation 2x + 3y = 30, thereby reducing it to a linear equation in one variable.

We can then solve the equation to obtain the value of x.

2x + 3 × 6 = 30 ⇒ 2x + 18 = 30

This is a linear equation in one variable.

⇒ 2x = 30 – 18

⇒ 2x = 12

⇒ x = 6

Thus, we see that the value of x is 6, i.e., Laxmi purchased 6 bananas.