Sample Paper-01
Mathematics
Class – XI

Time allowed: 3 hours **M. M: 100**

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

Section – A

1. Compute $(1+2i)i-\frac{3+2i}{1-i}$

2. Write the domain and range of the function $\cos^{-1}x$

3. Find the sign of y if $y=\sin(\cos^{-1}x)$

4. Find $\sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)$

Section B

5. Write the coordinates of the point of intersections of the parabola represented by $y^2=4ax$ and its latus rectum

6. Find x and y if $(x+7,8)=(10,x+y)$

7. Find the inverse of the function $f(x)=x^2-x+1, x>\frac{1}{2}$

8. Find the vertex, axis , Focus , Directrix and latus rectum of the parabola $8y^2+24x-40y+134=0$

9. Express $\frac{7-4i}{3+2i}$ in the form $a+ib$

10. Solve the inequality $(x-2)((x-3)>0$

11. Find the general value of x if $\tan 5x=\frac{1}{\tan 2x}$

12. In a single throw of 2 dies what is the probability of getting a prime number on each die.

Section C

13. If $f(x)=x^3-x;\phi(x)=\sin 2x$ Find the value $f[\phi(\frac{\pi}{12})]$

14. If $\tan A=\frac{m}{m+1}$ *and* $\tan B=\frac{1}{2m+1}$ prove that $\tan A+\tan B+\tan A\tan B=1$

15. If $f:R\to R$ is defined as follows: $f(x)=\begin{cases}1 & if\ x\in Q\\ -1 & if\ x\notin Q\end{cases}$ Find $f(\sqrt{3}, f(3), f(\sqrt{3+1})$

16. Prove that the equation

$sin\theta=x+\frac{1}{x}$ is impossible if x is real

17. Find the domain of the function for which $f(x)=\phi(x);, if\ f(x)=3x^2+1, and \phi(x)=7x-1$

18. Find the limit $\lim_{x\to 0}\frac{1-\cos x}{x}$

19. Solve $2\sin^2 x+14\sin x\cos x+50\cos^2 x=26$

20. Find $\frac{dy}{dx}$ given that $y=(\sin^n x\cos nx)$

21. If $(5a),(a-b),b$ are in GP prove that $\log\left(\frac{1}{3}(a+b)\right)=\frac{1}{2}(\log a+\log b)$

22. If the nth term of a series is denoted by $\frac{7^{n-1}}{10^n}$. Find the sum to infinity of the series.

23. Calculate the variance and standard deviation of the following data $8,12,13,15,22,14$

Section D

24. $f(x)=(1+x)^{\frac{1}{x}}, x\neq 0$. Find $f(1+\frac{a}{y})^{by}$

25. The probability of A hitting a target is $\frac{4}{5}$; the probability of B hitting the target is $\frac{3}{4}$ and the probability of C missing the target is $\frac{1}{3}$.What is the probability of the target being hit at least twice.

26. Find the term independent of x in the expansion $\left(ax^2-\frac{b}{x}\right)^9$

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Answers

Section A

1. **Solution:**

$$\frac{3+2i}{1-i}=\frac{3+2i}{1-i}.\frac{1+i}{1-i}$$
$$=\frac{3+2i+3i+2i^2}{1-i^2}=\frac{1}{2}+\frac{5}{2}i$$
$$(1+2i)i-\frac{3+2i}{1-i}=(-2+i)-(\frac{1}{2}+\frac{5}{2}i)=-\frac{5}{2}-\frac{3}{2}i$$

2. **Solution:**

Domain= $[-1,1]$ Range= $[0,\pi]$

3. **Solution :**

$0\le\cos^{-1}x\le\pi$

sin in this interval is positive and hence y is positive

4. **Solution:**

$$\sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)=\sin^{-1}\left(sin\left(\pi-\frac{\pi}{7}\right)\right)$$
$$=\sin^{-1}\left(sin\left(\frac{\pi}{7}\right)\right)$$
$$=-\frac{\pi}{2}\le\frac{\pi}{7}\le\frac{\pi}{2}$$
$$=\frac{\pi}{7}$$

Section B

5. Solution: $(a,2a)$,$(a,-2a)$
6. **Solution:**

$x+7=10$

$x=3$

$x+y=8$

$y=5$

7. **Solution:**

$y=x^2-x+1$

$y=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$

$y-\frac{3}{4}=\left(x-\frac{1}{2}\right)^2$

$x=\frac{1}{2}+\sqrt{y-\frac{3}{4}}$

$f^{-1}(x)=\frac{1}{2}+\sqrt{x-\frac{3}{4}}$

8. **Solution:**

$Equation\ is\ 8y^2+24x-40y+134=0$

$=4y^2+12x-20y+67=0$

This can be written as

$y^2-5y=-3x-\frac{67}{4}$

$(y-\frac{5}{2})^2=-3x-\frac{67}{4}+\frac{25}{4}-3(x+\frac{7}{2})$

$Let\ Y=y-\frac{5}{2}$

$X=x+\frac{7}{2}$

$Y^2=-3X$

$This\ is\ of\ the\ form\ y^2=-4ax$

$Latus\ rectum\ is=3$

$Vertex(-\frac{7}{2},\frac{5}{2})$

$Axis\ \ y=\frac{5}{2}$

$Focus\ (-\frac{7}{2}-\frac{3}{4},\frac{5}{2})$

$Directrix: referred\ to\ New\ axis: X=a=\frac{3}{4}$

$Directrixreferred\ to\ Old\ axis: \frac{3}{4}=x+\frac{7}{2}$

$x=\frac{3}{4}-\frac{7}{2}$

$x=-\frac{11}{4}$

9. **Solution:**

$\frac{7-4i}{3+2i}=\frac{7-4i}{3+2i}\times\frac{3-2i}{3+21}$

$\frac{13-26i}{13}=1-2i$

10. **Solution**

Either both factors are negative or both factors are positive to have this in equality. if $x<2$ both factors are negative and if $x>3$ both factors are positive. Hence the Solution is $x\in\{(-\infty,2)\cup(3,\infty)\}$

11. **Solution**

tan5x = cot2x

$\tan 5x=\tan(\frac{\pi}{2}-2x)$

$$5x = (\frac{\pi}{2} - 2x)$$

$$5x = n\pi + (\frac{\pi}{2} - 2x)$$

$$7x = n\pi + \frac{\pi}{2}$$

$$x = \frac{1}{7}(n\pi + \frac{\pi}{2})$$

12. **Solution**

Total number of occurrence = $6 \times 6 = 36$

On each die there are 3 prime numbers $\{2,3,5\}$

Hence total number of favorable cases $3 \times 3 = 9$

Probability of getting a prime in each die = $\frac{9}{36} = \frac{1}{4}$

Section C

13. **Solution:**

$$\phi(\frac{\pi}{12}) = \sin 2.(\frac{\pi}{12})$$

$$= \sin\frac{\pi}{6} = \frac{1}{2}$$

$$f(x) = (\frac{1}{2})^3 - \frac{1}{2}$$

$$= \frac{1}{8} - \frac{1}{2} = -\frac{3}{8}$$

14. **Solution:**

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1} \cdot \frac{1}{2m+1}} = 1$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B + \tan A \tan B = 1$$

15. **Solution:**

$$f(\sqrt{3}) = -1$$

$$f(3) = 1$$

$$f(\sqrt{3+1}) = 1$$

16. **Solution:**

Use the inequality $AM \geq GM$

$$AM \ between \ x, \frac{1}{x} = \frac{x + \frac{1}{x}}{2}$$

$GM\ between\ x, \frac{1}{x} = \sqrt{x.\frac{1}{x}} = 1$

$\frac{x + \frac{1}{x}}{2} \geq 1$

$x + \frac{1}{x} \geq 2$

$Since\ -1 \leq \sin\theta \leq 1$

$\sin\theta = x + \frac{1}{x}\ is\ impossible$

17. **Solution:**

$f(x) = \phi(x)$

$f(x) = 3x^2 + 1$

$\phi(x) = 7x - 1$

$3x^2 + 1 = 7x - 1$

$3x^2 - 7x + 2 = 0$

$(x-2)(3x-1) = 0$

$x = 2\ , x = \frac{1}{3}$

$Hence\ f(x)\ and\ \phi(x)\ are\ equal\ when\ the\ domain\ is\ in\ the\ set\ \left\{\frac{1}{3}, 2\right\}$

18. **Solution**

$\lim_{x \to 0} \frac{1 - \cos x}{x}$

$= \lim_{x \to 0} \frac{1 - (1 - 2\sin^2 \frac{x}{2})}{x}$

$= \lim_{x \to 0} \frac{\sin^2 \frac{x}{2}}{x}$

$= \lim_{x \to 0} \frac{\sin \frac{x}{2}}{2\frac{x}{2}} \sin \frac{x}{2}$

$= \frac{1}{2}.1.0$

$= 0$

19. **Solution:**

$2\sin^2 x + 14\sin x\cos x + 50\cos^2 x = 26$

$= 2\sin^2 x + 14\sin x\cos x + 50\cos^2 x = 26(\sin^2 x + \cos^2 x)$

$= -24\sin^2 x + 14\sin x\cos x + 24\cos^2 x = 0$

$= 24\sin^2 x - 14\sin x\cos x - 24\cos^2 x = 0$

$= 24\tan^2 x - 14\tan x - 24 = 0$

$$\tan x = \frac{14 \pm \sqrt{196+2304}}{48}$$

$$\tan x = \frac{14 \pm \sqrt{2500}}{48}$$

$$\tan x = \frac{14 \pm 50}{48}$$

$$\tan x = \frac{64}{48}\ ;or\ ;-\frac{36}{48}$$

$$\tan x = \frac{4}{3}\ or -\frac{3}{4}$$

20. **Solution**

$$\frac{dy}{dx} = \sin^n x.\{-\sin nx).(n)\} + \cos nx.\{n.\sin^{n-1} x.\cos x\}$$

$$\frac{dy}{dx} = n\sin^{n-1} x(\cos nx.\cos x - \sin x.\sin nx)$$

$$\frac{dy}{dx} = n\sin^{n-1}[\cos(n+1)x]$$

21. **Solution**

$$(a-b)^2 = 5ab$$

$$a^2 + b^2 - 2ab = 5ab$$

$$a^2 + b^2 = 7ab$$

$$(a+b)^2 = 9ab$$

$$a + b = 3\sqrt{ab}$$

$$\frac{1}{3}(a+b) = \sqrt{ab}$$

$$\log\left(\frac{1}{3}(a+b)\right) = \frac{1}{2}(\log a + \log b)$$

22. **Solution**

First term = $\frac{1}{10}$

Second term= $\frac{7}{10^2}$

Third term= $\frac{7^2}{10^3}$

r = $\frac{7}{10}$

This is a GP

Sum to infinity= $\dfrac{\frac{1}{10}}{1-\frac{7}{10}}$ = $\frac{1}{3}$

23. **Solution**

Mean= 14= $\frac{8+12+13+15+22+14}{6}$

x_i	$x_i - Mean$	$(x_i - Mean)^2$
8	-6	36
12	-2	4
13	-1	1
15	1	1
22	8	64
14	0	0
		$\Sigma(x_i - Mean)^2 = 106$

Variance= $\frac{1}{n}\Sigma(x_i - Mean)^2 = \frac{106}{6} = 17.66$

SD= $\sqrt{Variance} = \sqrt{17.66} = 4.2$

Section D

24. **Solution**

$Let\ \frac{a}{y} = x$

$by = \frac{ab}{x}$

$f(1+\frac{a}{y})^{by} = f\left[(1+x)^{\frac{1}{x}}\right]^{ab}$

25. **Solution:**

Probability of all the three hitting the target= $\frac{4}{5}.\frac{3}{4}.\frac{2}{3} = \frac{2}{5}$

Probability of A alone missing the target = $\frac{1}{5}.\frac{3}{4}.\frac{2}{3} = \frac{1}{10}$

Probability of B alone missing the target = $\frac{4}{5}.\frac{1}{4}.\frac{2}{3} = \frac{2}{15}$

Probability of C alone missing the target = $\frac{4}{5}.\frac{3}{4}.\frac{1}{3} = \frac{1}{5}$

The probability that the target being hit at least two= $\frac{2}{5} + \frac{1}{10} + \frac{2}{15} + \frac{1}{5} = \frac{5}{6}$

26. **Solution**

Let T_{r+1} be the term that is independent of x

Then,

$T_{r+1} = {}^9C_r(ax^2)^r(-\frac{b}{x})^{9-r}$

$2r + (r-9) = 0$

$r = 3$

4th term is independent of x

$T_4 = {}^9C_3(a)^3(-b)^6$

$= {}^9C_3(a)^3(b)^6$