CHAPTER 03

# Trusses

## 3.1 Introduction

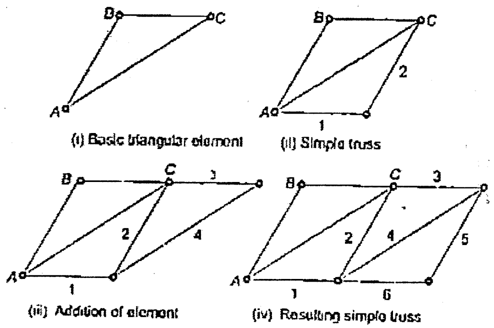
Trusses are most common type of structure used in constructing building roofs, bridges and towers etc. A truss can be constructed by straight slender members joined together at their end by bolting, riveling or welding.

## 3.2 Classification of Trusses

The trusses can be classified into
(a) plane trusses (2-D trusses)
(b) space trusses (3-D trusses)

The plane trusses (2-D) can also be classified into simple truss, compound truss and complex truss.

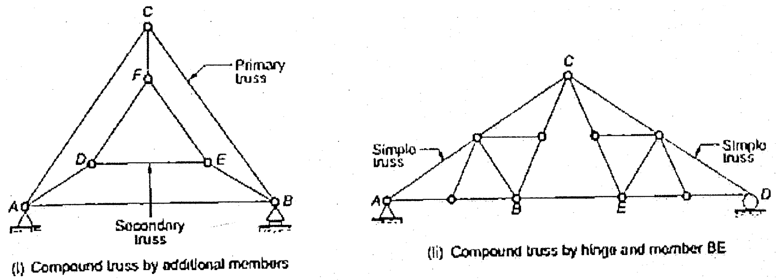
*(i)* **Simple truss:** A simple truss is constructed from the basic triangular element as shown in figure by connecting two members to form an additional element. The other element can be form by connecting two more members to resulting simple truss.



(i) Basic triangular element (ii) Simple truss
(iii) Addition of element (iv) Resulting simple truss

*Fig. 3.1*

*(ii)* **Compound truss:** A compound truss is formed by connecting two or more simple trusses together either by hinge or by additional members.



(i) Compound truss by additional members (ii) Compound truss by hinge and member BE

*Fig. 3.2*

*(iii)* **Complex truss:** If a truss is neither simple nor compound then it is called complex truss. A complex truss has polygonal structure.

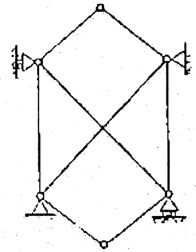


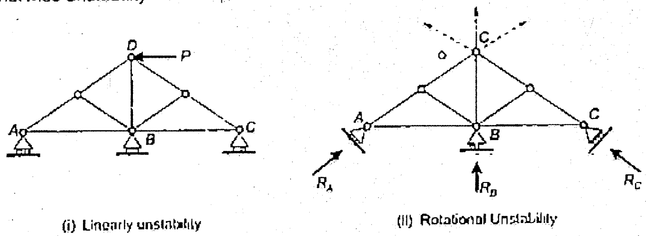
*Fig. 3.3*

## 3.3 Stability of Trusses

(a) **External stability:** For stability of 2-D trusses following three conditions of equilibrium must be satisfied.

(i) $\Sigma F_x = 0$ (ii) $\Sigma F_y = 0$ (iii) $\Sigma M_z = 0$

Apart from above three conditions, all support reactions should not be parallel and concurrent, otherwise unstability will set up.



(i) Linearly unstability (ii) Rotational Unstability

*Fig. 3.4*

(b) **Internal stability:** For the internal stability, no part of the truss can move rigidly relative to the other part to maintain geometry of the structure, however small elastic deformations are permitted. To preserve geometry enough number of members and their adequate arrangement is required.

For plane truss the minimum number of members needed for geometric stability are

$$m = 2j - 3$$

Here, $j$ = Number of joint

$m$ = Members required for geometric stability and all the members should be arranged in such a way that truss can be divided into triangular blocks, i.e. no rectangular or polygonal blocks.

## 3.4 Determinate and Indeterminate Trusses

A truss is said to be determinate when it can by analysed by equations of static equilibrium alone. For determinate truss, the degree of static indeterminacy is zero. If degree of static indeterminacy is greater than zero then truss is called indeterminate truss. To analyse the indeterminate truss, additional compatibility conditions are required.

The degree of static indeterminacy is given by,

$$D_S = m + r_e - 2j \quad \text{(2D-truss)}$$

$$D_S = m + r_e - 3j \quad \text{(3-D truss)}$$

where, $m$ = number of members

$r_e$ = number of external support reactions

$j$ = number of joint

if $D_S = 0$, then truss is stable and determinate, such trusses are called perfect trusses

$D_S > 0$, then truss is stable but indeterminate or over stiff.

$D_S < 0$, then truss is unstable or deficient.

## 3.5 Methods of Analysis of Determinate Truss

The determinate and stable truss can be analysed by using equilibrium methods such as:

(i) Method of joints (ii) Method of sections

(iii) Graphical method–Willot Mohr diagram (iv) Bar chain method

### 3.5.1 Method of Joints

This method is suitable when forces in all members are to be calculated. In this method, equilibrium of each joint is considered. At each joint two equations of equilibrium are available viz.

(i) $\Sigma F_x = 0$ (ii) $\Sigma F_y = 0$

It is also noted that there are only two equations of equilibrium available at each joint. Hence this method can not be applicable when number of unknowns at each joint are more than two.

So in this method the selection of joint is very judicious, proceeding one joint to another that number of unknown forces at a joint are not more than two.

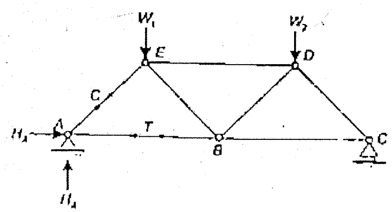
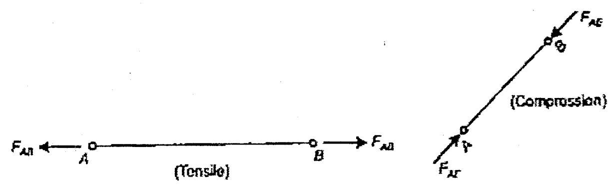
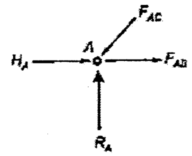



*Fig. 3.5*

**Sign Convention for Member Forces**

The forces in member are shown by double arrows. For tension arrows are pointing away from each other and for compression arrows are pointing towards each other




Consider joint $A$, let the force in $AB$ ($F_{AB}$) is tensile and force in $AC$ ($F_{AC}$) is compressive.

The compressive force in member exerts a reactions pointing towards the joint and a tensile force exerts a reaction pointing away from the joint.

**NOTE**

The unknown forces in members are considered as tensile and tensile forces are taken as positive and vice-versa.

Tensile Force = Positive (+ve)

Compressive force = Negative (–ve)

**Procedure**

**Step-1:** Check for degree of static indeterminacy

if $D_S = 0$

then truss is determinate and stable. Such trusses are called pin-jointed perfect frame. For trusses the external support reactions can be found by using conditions of equilibrium.

(i) $\Sigma F_x = 0$ (ii) $\Sigma F_y = 0$ (iii) $\Sigma M_z = 0$

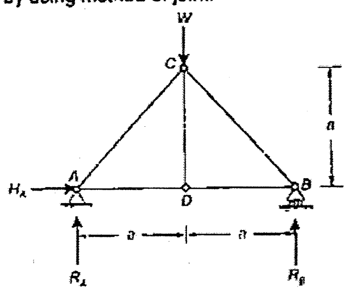
The above conditions are applied for truss as a whole.

**Step-2:** Draw free body diagram of that joint at which only two unknown forces are present. Now consider equilibrium of joint and apply following conditions of equilibrium to find unknowns.

(i) $\Sigma F_x = 0$ (ii) $\Sigma F_y = 0$

Now proceed to another joints one by one at which only two unknown forces are present and use above conditions of equilibrium to find unknowns.

**Example 3.1** Find forces in all members of truss by using method of joint.




***Solution:***

Reaction:

$\Sigma F_x = 0;$ $H_A = 0$

$\Sigma F_y = 0;$ $R_A + R_B = W$ ...(i)

$\Sigma M_A = 0;$ $R_B \times 2a - W \times a = 0$

$\therefore$ $R_B = \frac{W}{2}$

and $R_A = \frac{W}{2}$

Method of Joint:

Joint $A$:

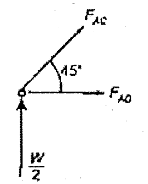
Consider equilibrium of joint '$A$'

$\Sigma F_x = 0;$ $F_{AD} + F_{AC}\cos 45 = 0$

$F_{AD} = \frac{W}{2}$ (Tension)

$\Sigma F_y = 0;$ $\frac{W}{2} + F_{AC}\sin 45 = 0$

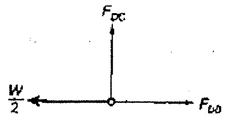
$F_{AC} = -\frac{W}{\sqrt{2}}$ (Compression)




Joint *D*:
Consider equilibrium of joint *D*

$\Sigma F_x = 0;$ $F_{DB} = \frac{W}{2}$ (Tension)

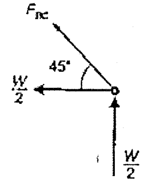
$\Sigma F_y = 0;$ $F_{DC} = 0$

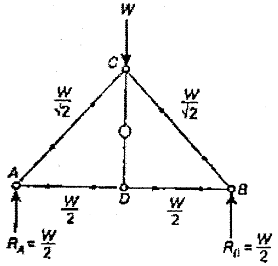



Joint *B*:
Consider equilibrium of joint *B*.

$\Sigma F_x = 0;$ $\frac{W}{2} + F_{BC}\sin 45 = 0$

$F_{BC} = -\frac{W}{\sqrt{2}}$ (Compression)

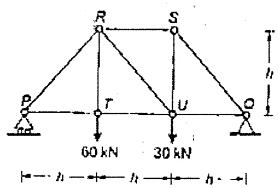



**Example 3.2** Consider a loaded truss shown in the given figure.

Match List-I (Member) with List-II (Force) and select the correct answer using the codes below the lists:

| List-I | List-II |
|---|---|
| A. *PR* | 1. 40 kN (Tension) |
| B. *RS* | 2. 40 kN (Compression) |
| C. *SU* | 3. 60 kN (Tension) |
| D. *RT* | 4. $50\sqrt{2}$ kN (Compression) |




Codes:

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 3 | 2 | 1 | 4 |
| (b) | 3 | 1 | 2 | 4 |
| (c) | 4 | 1 | 2 | 3 |
| (d) | 4 | 2 | 1 | 3 |

[IES : 2001]

**Ans.** (d)

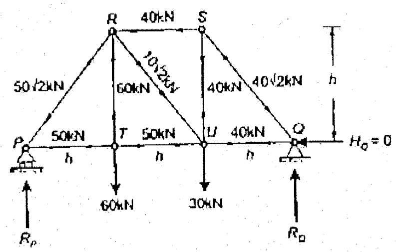
Reactions:

$\Sigma F_y = 0;$ $R_P + R_Q = 60 + 30 = 90$ ...(i)

$\Sigma M_Q = 0;$ $3h \times R_P = 60 \times 2h + 30 \times h$

$\therefore$ $R_P = 50$ kN

and $R_Q = 40$ kN




Method of joints:

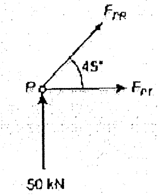
Joint *P*:
Consider equilibrium of joint *P*.

$\Sigma F_x = 0;$ $F_{PT} + F_{PR}\cos 45° = 0$

$F_{PT} = -\frac{F_{PR}}{\sqrt{2}}$ ...(ii)

$\Sigma F_y = 0;$ $50 + F_{PR}\sin 45° = 0$

$\frac{F_{PR}}{\sqrt{2}} = -50$ kN



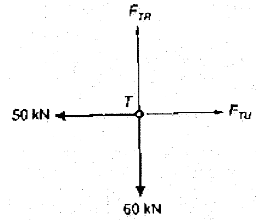

$\Rightarrow$ $F_{PR} = -50\sqrt{2}$ kN (Compression)

From equation (i), $F_{PT} = +50$ (Tension)

Joint *T*:
Consider equilibrium of joint *T*.

$\Sigma F_x = 0;$ $F_{TU} = 50$ kN (Tension)

$\Sigma F_y = 0;$ $F_{TR} = 60$ kN (Tension)




Joint *Q*:
Consider equilibrium of joint *Q*.

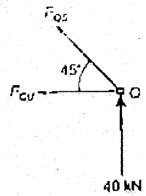
$\Sigma F_x = 0;$ $F_{QU} + F_{QS}\cos 45° = 0$

$F_{QU} = -\frac{FQs}{\sqrt{2}}$ ...(iii)

$\Sigma F_y = 0;$ $F_{QS}\sin 45° + 40 = 0$

$F_{QS} = -40\sqrt{2}$ kN (Compression)

From equation (ii), $F_{QU} = +40$ kN (Tension)



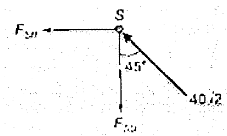

Joint *S*:
Consider equilibrium of joints *S*.

$\Sigma F_x = 0;$ $F_{SR} + 40\sqrt{2}\sin 45° = 0$ ...(iv)

$F_{SR} = -40$ kN (Compression)

$\Sigma F_y = 0;$ $F_{SU} = 40\sqrt{2}\cos 45°$

$= 40$ kN (Tension)




Hence option (d) is correct.

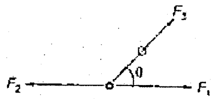
**Identifying Zero Force Members**

There are following two thumb rules to identify the members having zero forces.

1. If at a joint three members meet, out of three two members are collinear and there is no external force or support reaction at that joint. Then non collinear member will carry zero force.

$\Sigma F_y = 0;$ $\qquad F_3 \sin\theta = 0$

$\therefore$ $\qquad F_3 = 0$




Also, note that,

$\Sigma F_x = 0$

$$F_1 - F_2 + F_3 \cos\theta = 0$$

$$F_1 = F_2 \qquad [\because F_3 = 0]$$

It mean if at a joint two collinear member meet and there is no external load or support reaction at that joint, then both the collinear members will have equal and like forces. Eg.

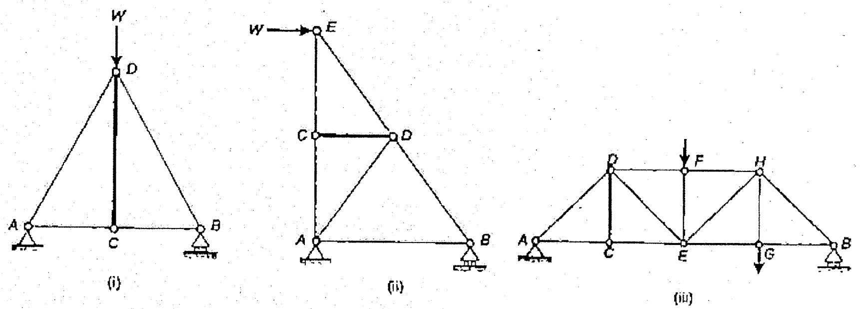
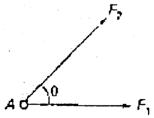



*Fig. 3.6*

In above figures member *CD* will carry zero force.

2. If at a joint only two members meet which are non-collinear and there is no external load or support reaction at that joint then both the members will carry zero forces.




Consider equilibrium of joint *A*.

$\Sigma F_x = 0;$ $\qquad F_1 + F_2 \cos\theta = 0$ $\qquad \ldots(i)$

$\Sigma F_y = 0;$ $\qquad F_2 \sin\theta = 0$

$\therefore$ $\qquad F_2 = 0$

From equation (*i*), we get

$$F_1 + 0 \cdot \cos\theta = 0$$

$\therefore$ $\qquad F_1 = 0$

Hence, $\qquad F_1 = F_2 = 0$

For eg.

In above truss member *CE*, *CA*, *BD* and *FD* will carry zero force.

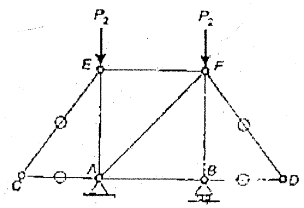
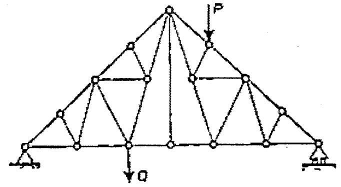



*Fig. 3.7*

**Example 3.3** For the plane truss shown in the figure, the number of zero force members for the given loading is

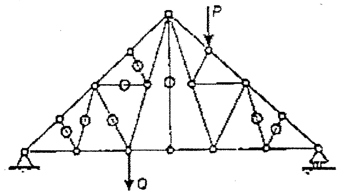



(a) 4 (b) 8
(c) 11 (d) 13

[IES : 2004]

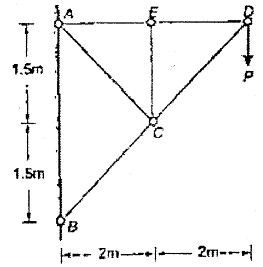
*Ans. (b)*

If three members meet at a joint and two of them are collinear, then the third member will carry zero force if there is no load on that joint. Thus, using above condition we identify 8 members which carry zero forces.




Hence option (b) is correct.

**Example 3.4** What is the force in the member CE of a cantilever truss shown in the adjoining figure?



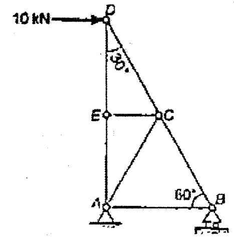

(a) P (tensile) (b) P (compression)
(c) 2P (tensile) (d) Zero

[IES : 2003]

*Ans. (d)*

There is no force at joint *E*, so the force in member *EC* is zero. Hence option (d) is correct.

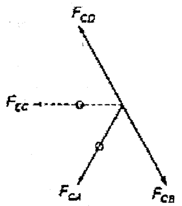
**Example 3.5** Member(s) of the frame shown below carries/carry zero force is/are




(a) *EC* only
(b) *EC* and *AB*
(c) *EC* and *AC*
(d) *EC*, *AC* and *AB*

[IES : 2003]

***Ans. (c)***



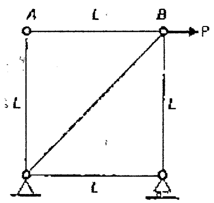

Since member *DE* and *EA* are collinear and no force on joint *E*. Hence member *EC* will carry zero force. Now at joint *C* two members *DC* and *CB* are collinear and no external force on joint *C*. Hence member *CA* will carry zero force. $[F_{EC} = 0]$

Hence option (c) is correct.

**Example 3.6** What is the force in member AB of the pin-jointed frame as shown below?

(a) *P* (Tension)
(b) *P* (Compression)
(c) $\frac{P}{\sqrt{2}}$ (Compression)
(d) Zero

[IES : 2006]



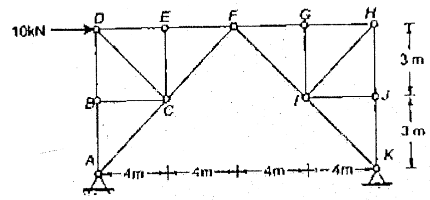

***Ans. (d)***

At joint *A*, two members meet and there is no external load on joint *A*. Hence both member meeting at *A* will carry zero force.

$\therefore \quad F_{AB} = 0$

Hence option (d) is correct.

**Example 3.7** All members of the truss shown in the below figure are pin jointed. Calculate the reactions and forces in all the members.




[IES : 2002]

***Solution:***

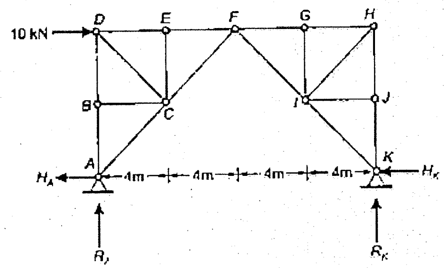
The degree of static indeterminacy of the given truss may be given as

$$D_S = m + r_a - 2j$$

Here, $m = 18$, $r_a = 4$, $j = 11$

$$D_S = 18 + 4 - 2 \times 11$$
$$= 0$$

Thus, the given truss is statically determinate.




$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$ & $\cos\theta = \frac{3}{5}$

Reactions:

$\Sigma F_x = 0 \qquad H_A + H_K = 10 \qquad \ldots(i)$

$\Sigma F_y = 0: \qquad R_A + R_K = 0 \qquad \ldots(ii)$

$\Sigma M_A = 0: \qquad R_K \times 16 - 10 \times 6 = 0$

$\therefore \qquad R_K = 3.75 \text{ kN} (\uparrow)$

From equation (*ii*), we get

$$R_A = -R_K = -3.75 \text{ kN} (\downarrow)$$

Also, $\Sigma M_F = 0$ (From right)

$$R_K \times 8 - H_K \times 6 = 0$$

$$H_K = \frac{3.75 \times 8}{6} = 5 \text{ kN} (\leftarrow)$$

From equation (*i*), we get

$$H_A = 10 - 5 = 5 \text{ kN} (\leftarrow)$$

**Joint A:** Consider equilibrium of joint A.

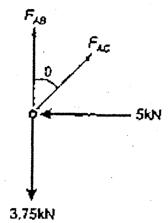
$\Sigma F_x = 0$: $F_{AC}\sin\theta - 5 = 0$

$$F_{AC} \times \frac{4}{5} = 5$$

∴ $F_{AC} = 6.25$ kN (Tension)

$\Sigma F_y = 0$: $F_{AC}\cos\theta + F_{AB} - 3.75 = 0$

$$F_{AB} = 3.75 - 6.25 \times \frac{3}{5} = 0$$




Since $F_{AB} = 0$, therefore $BC$ and $BD$ will carry zero forces.

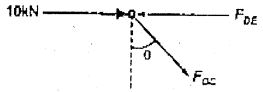
**Joint D:** Consider equilibrium of joint D.

$\Sigma F_y = 0$: $F_{DC}\cos\theta = 0$

∴ $F_{DC} = 0$

$\Sigma F_x = 0$: $F_{DC}\sin\theta + F_{DE} + 10 = 0$

$F_{DE} = -10$ kN (Compression)



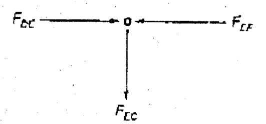

**Joint E:** Consider equilibrium of joint E.

$\Sigma F_x = 0$: $F_{EF} + F_{DE} = 0$

$F_{EF} + 10 = 0$

∴ $F_{EF} = -10$ kN (Compression)

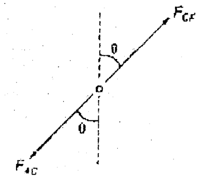
$\Sigma F_y = 0$: $F_{EC} = 0$




**Joint C:** Consider equilibrium of joint C.

$F_{CF} = F_{AC}$

∴ $F_{CF} = 6.25$ kN (Tension)




**Joint K:** Consider equilibrium of joint K.

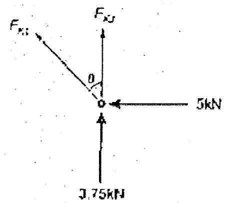
$\Sigma F_x = 0$: $F_{KI}\sin\theta + 5 = 0$

$$F_{KI} \times \frac{4}{5} = -5$$

$F_{KI} = -6.25$ kN (Compression)

$\Sigma F_y = 0$: $F_{KI}\cos\theta + F_{KJ} + 3.75 = 0$

$$F_{KJ} = -3.75 - (-6.25) \times \frac{3}{5} = 0$$




Since $F_{KJ} = 0$, therefore $JI$ and $JH$ will carry zero forces.
As a result $HI$ and $HG$ will also carry zero force.

**Joint I:** Consider equilibrium of joint I.

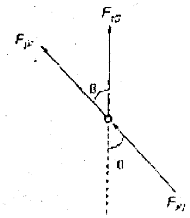
$\Sigma F_x = 0$: $-F_{KI}\sin\theta - F_{IF}\sin\theta = 0$

$F_{IF} + F_{KI} = 0$

$F_{IF} = -6.25$ kN (Compression)

$\Sigma F_y = 0$: $F_{IG} + F_{KI}\cos\theta + F_{IF}\cos\theta = 0$

$$F_{IG} = -\frac{3}{5}(6.25 - 6.25) = 0$$

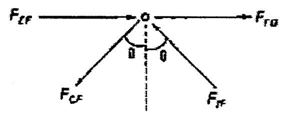



**Joint F:** Consider equilibrium of joint F.

$\Sigma F_x = 0$: $F_{FG} - F_{IF}\sin\theta + F_{EF} - F_{CF}\sin\theta = 0$

$$F_{FG} - 6.25 \times \frac{4}{5} + 10 - 6.25 \times \frac{4}{5} = 0$$

∴ $F_{FG} = 0$




| Member | Force (kN) | Nature |
|---|---|---|
| AB | 0 | – |
| AC | 6.25 | Tension |
| BC | 0 | – |
| BD | 0 | – |
| DC | 0 | – |
| DE | –10 | Compressive |
| EC | 0 | – |
| EF | –10 | Compressive |

| Member | Force (kN) | Nature |
|---|---|---|
| CF | 6.25 | Tension |
| KJ | 0 | – |
| KI | –6.25 | Compressive |
| JI | 0 | – |
| JH | 0 | – |
| HI | 0 | – |
| HG | 0 | – |
| IF | –6.25 | Compressive |
| IG | 0 | – |
| FG | 0 | – |

### 3.5.2 Method of Sections

This method is suitable when forces are required only in few members. In this method whole truss is cut into two portions and equilibrium of each portion is considered. For two portions of truss, the following equations of equilibrium should be satisfied.

(i) $\Sigma F_x = 0$ (ii) $\Sigma F_y = 0$ (iii) $\Sigma M_z = 0$

To determine the unknown member forces, an imaginary section is cut in such a way that unknown force can be determined using above equations.

The imaginary section may be vertical, horizontal, zig-zag or any shape.

Consider a truss show below and forces in members HG, CD and HD are required, cut the truss along section x-x and consider equilibrium of either left free body or right free body.

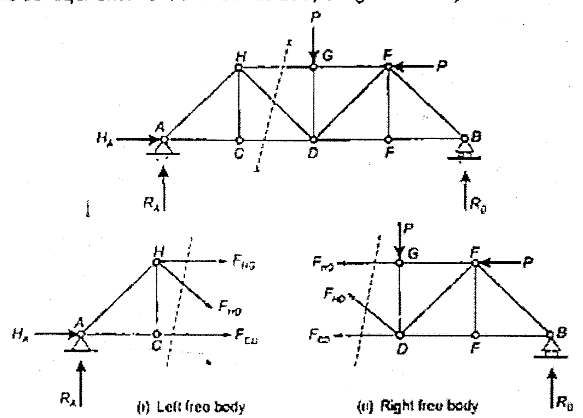



*Fig. 3.8*

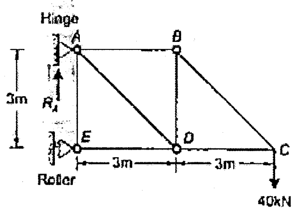
The unknown member forces $F_{HG}$, $F_{HD}$ and $F_{CD}$ can be found by the following conditions of equilibrium either for left free body or right free body.

(i) $\Sigma F_x = 0$ (ii) $\Sigma F_y = 0$ (iii) $\Sigma M_z = 0$

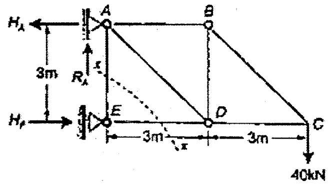
**Example 3.8** The pin-jointed cantilever truss is loaded as shown in the given figure. The force in member *ED* is

(a) 40 kN (Compressive)
(b) 80 kN (Tensile)
(c) 80 kN (Compressive)
(d) 120 kN (Compressive)

[IES : 2001]

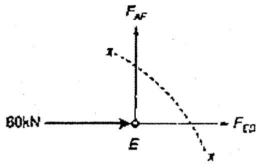



*Ans.* (c)



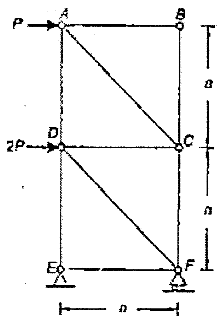

$\Sigma F_x = 0$; $H_A = H_E$ ...(i)
$\Sigma F_y = 0$; $R_A = 40$ kN
$\Sigma M_A = 0$; $40 \times 6 - H_E \times 3 = 0$
$\Rightarrow$ $H_E = 80$ kN
Hence, $H_A = 80$ kN

Using method of section,
$\Sigma F_x = 0$; $F_{ED} + 80 = 0$
$F_{ED} = -80$ kN (Compressive)




Hence option (c) is correct.

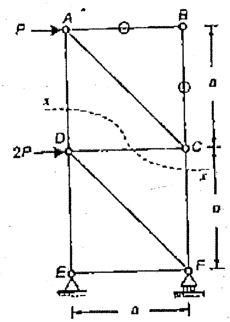
**Example 3.9** The force in the member CD is



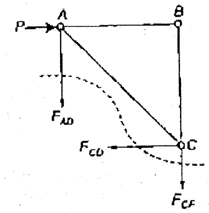

(a) *P* Tensile (b) *P* Compressive
(c) 2*P* Tensile (d) 2*P* Compressive

[IES : 2011]

*Ans.* (a)




Using method of section



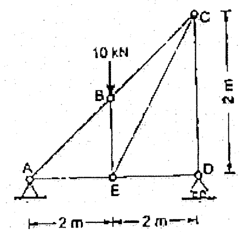

Considering equilibrium of free body shown above.
$\Sigma F_x = 0$; $F_{CD} = P$ (Tensile)
Hence option (a) is correct.

**Example 3.10** The figure below shows a pin-jointed frame.

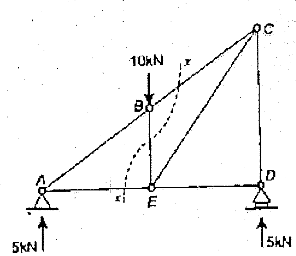
What are the forces in members *BE*, *CD* and *ED*?

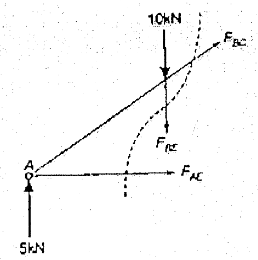
(a) 10 kN, 5 kN and 5 kN
(b) 10 kN, 5 kN and Zero
(c) 5 kN, 10 kN and Zero
(d) 5 kN, 5 kN and Zero

[IES : 2009]



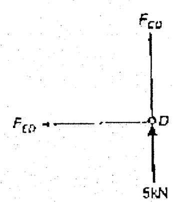

*Ans.* (b)
Cutting a section through *BC*, *BE* and *AE* and consider left free body of given pin-jointed frame.



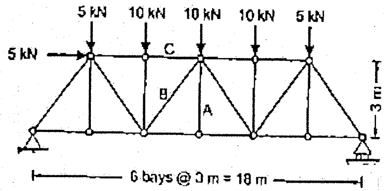

$\Sigma M_A = 0$; $10 \times 2 + F_{BE} \times 2 = 0$
$\therefore$ $F_{BE} = -10$ kN (Compression)

Consider equilibrium of joint *D*



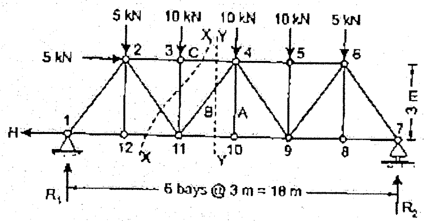

$\Sigma F_x = 0$; $F_{ED} = 0$
$\Sigma F_y = 0$; $F_{CD} + 5 = 0$
$\therefore$ $F_{CD} = -5$ kN (Compression)

**Example 3.11** Find the member forces in the member marked *A*, *B* and *C* for the truss as show.




[IES : 2013]

***Solution:***




$$H = 5\text{ kN}$$

$$R_1 + R_2 = 40\text{ kN}$$

Taking moment about joint 1

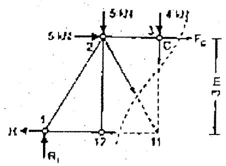
$$R_2(18) = 5(3) + 5(3) + 10(6) + 10(9) + 10(12) + 5(15)$$

$\Rightarrow$ $$R_2 = 20.833\text{ kN}$$

$\therefore$ $$R_1 = 19.1667\text{ kN}$$

Section X-X is as shown above.

Considering the equilibrium of LHS of section x-x and taking moment about joint 11,




$$F_c(3) + 5(3) + R_1(6) = 5(3)$$

$\Rightarrow$ $$F_c(3) + 15 + 115 = 15$$

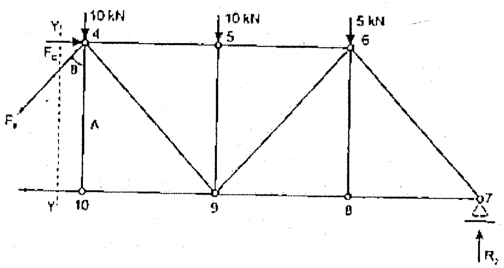
$\Rightarrow$ $$F_c = -\frac{115}{3} = -38.333\text{kN}$$

$\therefore$ $$F_c = 38.333\text{ kN (compression)}$$

$$F_A = 0$$

(due to eq. of joint 10)

Consider section-y as shown in figure.




$$\theta = 45°$$

Considering the vertical equilibrium of RHS of section y-y

$$F_B\cos 45° + 10 + 10 + 5 = R_2$$

$\Rightarrow$ $$\frac{F_B}{\sqrt{2}} + 25 = 20.8333$$

$\Rightarrow$ $$F_B = -5.8926\text{ kN}$$

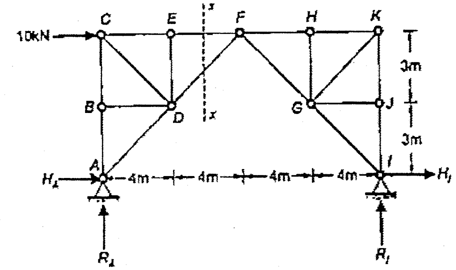
$\therefore$ $$F_B = 5.8926\text{ kN (Compression)}$$

$\therefore$ Force in member $A = 0$

Force in member $B = 5.8926$ kN (Compression)

Force in member $C = 38.333$ kN (Compression)

**Example 3.12** Find forces in member DF and EF for the truss shown below.




***Solution:***

$$D_S = m + r_e - 2j$$

$$= 18 + 4 - 2 \times 11$$

$$= 0$$

Thus given truss is determinate.

**Reaction:**

$\Sigma F_x = 0$: $$H_A + H_I + 10 = 0$$

$$H_A + H_I = -10 \quad ...(i)$$

$\Sigma F_y = 0$: $$R_A + R_I = 0 \quad ...(ii)$$

$\Sigma M_I = 0$: $$R_A \times 16 + 10 \times 6 = 0$$

$$R_A = -3.75\text{ kN} (\downarrow)$$

and $$R_I = +3.75\text{ kN} (\uparrow)$$

Also, $\Sigma M_F = 0$ (From left)

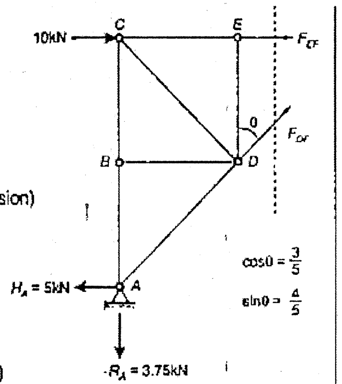
$\therefore$ $$R_A \times 8 - H_A \times 6 = 0$$

$$H_A = -5\text{ kN} (\leftarrow)$$

Hence, $$H_I = -5\text{ kN} (\leftarrow)$$

Consider a section x-x as shown in figure.

Now consider equilibrium of left portion of truss.




$\Sigma F_y = 0;$ $\quad F_{DF}\cos\theta - 3.75 = 0$

$$F_{DF} = \frac{3.75}{\cos\theta}$$

$$= \frac{3.75}{(3/5)} = 6.25 \text{ kN} \quad \text{(Tension)}$$

$\Sigma F_x = 0;$

$$F_{EF} + 10 + H_A + F_{DF}\sin\theta = 0$$

$$F_{EF} + 10 - 5 + 6.25 \times \frac{4}{5} = 0$$

$$F_{EF} = -10 \text{ kN} \quad \text{(Compression)}$$

## 3.6 Methods of Analysis of Indeterminate Truss

If the degree of static indeterminacy is greater than zero then truss is called indeterminate or redundant. The indeterminate truss can be analyse by following methods:

(a) Force methods – examples are unit load method, strain energy method and Maxwell's method.

(b) Displacement method/Stiffness method/Equilibrium method.

(c) Graphical method.

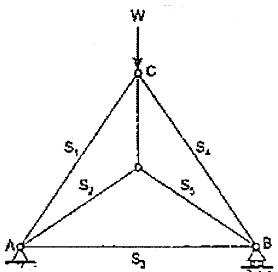
In trusses, generally $D_S$ is much lower than $D_K$. Hence force methods are preferred.

### 3.6.1 Unit Load Method

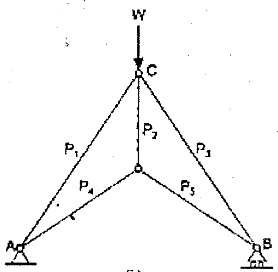
Consider a truss shown below.

$$D_S = m + r_e - 2j$$
$$= 6 + 3 - 2 \times 4$$
$$= 6 + 3 - 8 = +1$$
$$D_{Se} = r_e - 3$$
$$D_{Se} = 3 - 3 = 0$$

and
$$D_{Si} = m - (2j - 3)$$
$$= 6 - (2 \times 4 - 3) = 1$$




(i)




(ii)

Thus the above truss is indeterminate to 1st degree and indeterminacy is internal i.e. internal force is redundant force.

Let $S_1, S_2, S_3 \ldots\ldots S_n$ are the final forces in the members due to given external loading ($W$).

Since the truss is indeterminate to 1st degree. Hence remove one of the member say $AB$ in order to make the remaining truss determinate.

The above determinate truss may be analysed by using method of joint or method of sections. Let due to the given loading, force in truss are $P_1, P_2, P_3 \ldots\ldots P_n$ (Say $P$-system of forces).

Remove given loading, consider there is $X$ force in $AB$. Since member is removed, hence apply force equal to member force $X$ at $A$ and $B$.

Let $Q_1, Q_2, Q_3 \ldots\ldots Q_n$ (Say $Q$-system of forces) developed in members due to $X$ force in member $AB$.

Now a unit load is applied at $A$ and $B$ such that forces in members are $K_1, K_2, K_3 \ldots\ldots K_n$ (Say $K$-system of forces).

The final forces in members will be

$$S_1 = P_1 + Q_1 = P_1 + XK_1$$
$$S_2 = P_2 + Q_2 = P_2 + XK_2$$
$$\vdots \quad\quad \vdots \quad\quad \vdots$$
$$S_n = P_n + Q_n = P_n + XK_n$$

where, $\quad X$ = force in redundant member.

The true value of $X$ will be such that the total strain energy stored in the system is minimum.

The strain energy stored in all members is

$$U = \sum \frac{S^2 L}{2AE} = \sum \frac{(P + XK)^2 L}{2AE}$$

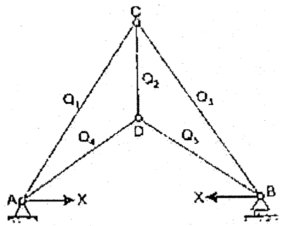
For minimum strain energy,

$$\frac{\partial U}{\partial X} = 0$$

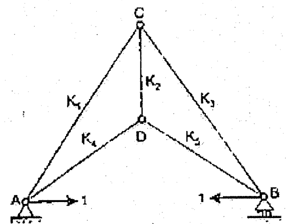
$$\frac{\partial U}{\partial X} = \sum \frac{2(P + XK)KL}{2AE} = 0$$

$$\sum \frac{PKL}{AE} + X\sum \frac{K^2 L}{AE} = 0$$

$$X = -\frac{\sum \frac{PKL}{AE}}{\sum \frac{K^2 L}{AE}}$$




(iii)




(iv)

*Fig. 3.9*

**Procedure**

**Step-1:** Check the degree of static indeterminacy and assure that $D_S$ = 1 and then identify the redundant.

**Step-2:** If truss is redundant to first degree then remove the redundant judiciously so that rest of the truss will be determinate. Find $P$-system of forces $P_1, P_2, P_3 \ldots\ldots P_n$ due to given loading.

**Step-3:** Remove all external loads and apply unit load at between the joints where member is removed in the direction of member removed. Due to unit load at joints find $K$-system of forces $K_1, K_2, K_3 \ldots\ldots K_n$.

**Step-4:** Calculate value of redundant force $X = -\dfrac{\sum \frac{PKL}{AE}}{\sum \frac{K^2 L}{AE}}$.

**Step-5:** Find final forces in all members (s-system of forces)

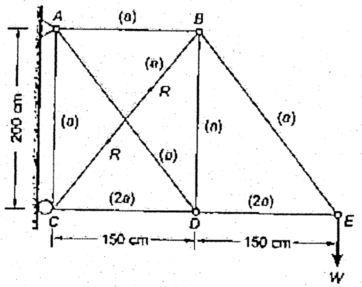
$$S_1 = P_1 + XK_1$$
$$S_2 = P_2 + XK_2$$
$$\vdots \quad \vdots \quad \vdots$$
$$S_n = P_n + XK_n$$

Note: Calculation of this method should be done in tabular manner.

| Member | P | k | L | PkL | $k^2L$ | S = P + Xk |
|---|---|---|---|---|---|---|
| | | | | | | |
| | | | | ΣPkL | $\Sigma k^2L$ | |

**Case-I: Internally Indeterminate Trusses**

**Example 3.13** Find the axial force in the member *BC* of the frame shown in figure. The figure brackets indicate the cross-sectional area in cm². The members are all of the same material.




[UPPSC: 2007(II)]

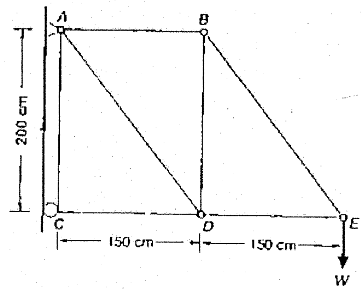
***Solution:***

$$D_s = m + r_e - 2j$$

Here, $m = 8$, $r_e = 3$ and $j = 5$

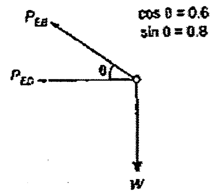
$\therefore$
$$D_s = 8 + 3 - 2 \times 5 = 11 - 10 = 1$$

Let member *BC* is redundant. After removal of member *BC* rest of the truss is determinate.




**P-system of force**

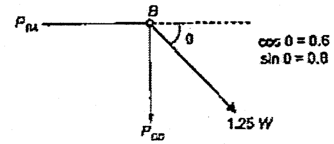
Joint *E*: Consider equilibrium of joint *E*.




$\Sigma F_y = 0;$
$$P_{EB}\sin\theta = W$$
$$0.8\,P_{EB} = W$$
$\therefore$
$$P_{EB} = 1.25\,W \text{(Tension)}$$

$\Sigma F_x = 0;$
$$P_{ED} + P_{EB}\cos\theta = 0$$
$$P_{ED} + 1.25\,W \times 0.6 = 0$$
$$P_{ED} = -0.75\,W \text{(Compression)}$$

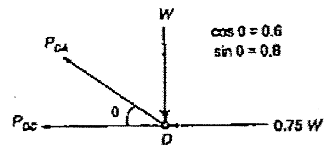
Joint B: Consider equilibrium of joint *B*.




$\Sigma F_x = 0;$
$$P_{BA} - 1.25\,W\cos\theta = 0$$
$$P_{BA} = 1.25 \times 0.6\,W$$
$$P_{BA} = 0.75\,W \text{(Tension)}$$

$\Sigma F_y = 0;$
$$P_{BD} + 1.25\,W\sin\theta = 0$$
$$P_{BD} = -1.25\,W\sin\theta$$
$$P_{BD} = -1.25\,W \times 0.8 = -W \text{(Compression)}$$

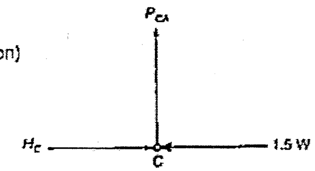
Joint *D*: Consider equilibrium of joint *D*.




$\Sigma F_y = 0;$
$\therefore$
$$P_{DA}\sin\theta = W$$
$$P_{DA} \times 0.8 = W$$
$$P_{DA} = 1.25\,W \text{(Tension)}$$

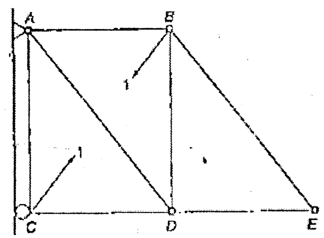
$\Sigma F_x = 0;$
$$P_{DC} + P_{DA}\cos\theta + 0.75\,W = 0$$
$$P_{DC} + 1.25\,W \times 0.6 + 0.75\,W = 0$$
$$P_{DC} = -1.5\,W \text{(Compression)}$$

Joint C: Consider equilibrium of joint *C*.




$\Sigma F_y = 0;$
$$P_{CA} = 0$$

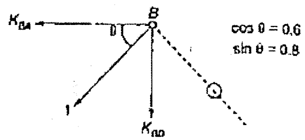
*k*-system of force: Remove given loading and apply unit load at *B* and *C* in the direction of member *BC* and find k-system of force.



**Joint E:** Consider equilibrium of joint $E$.

At joint $E$ there is no load.

$$\therefore \quad k_{EB} = k_{DE} = 0$$




**Joint B:** Consider equilibrium of joint $B$.

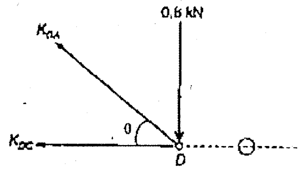
$\Sigma F_x = 0;$ $\quad k_{BA} + 1\cos\theta = 0$

$k_{BA} = -\cos\theta$

$= -0.6$ kN (Compression)

$\Sigma F_y = 0;$ $\quad k_{BD} + 1\sin\theta = 0$

$k_{BD} = -0.8$ kN (Compression)




**Joint D:** Consider equilibrium of joint $D$.

$\Sigma F_y = 0;$

$k_{DA}\sin\theta = 0.8$

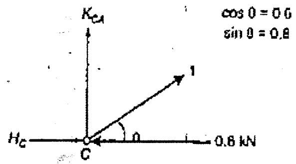
$0.8\,k_{DA} = 0.8$

$k_{DA} = 1$ kN (Tension)

$\Sigma F_x = 0;$ $\quad k_{DC} + k_{DA}\cos\theta = 0$

$k_{DC} = -k_{CA} \times 0.6$

$= -0.6$ kN (Compression)




**Joint C:** Consider equilibrium of joint $C$.

$\Sigma F_y = 0;$

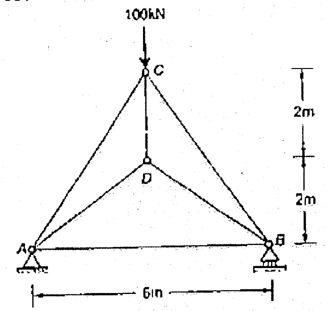
$k_{CA} + 1\sin\theta = 0$

$k_{CA} = -0.8$ kN (Compression)

| Member | Area | $P$ | $k$ | $L$ (cm) | $\frac{PkL}{AE}$ | $\frac{k^2L}{AE}$ |
|---|---|---|---|---|---|---|
| AB | $a$ | 0.75 W | −0.6 | 150 | −67.5 W/a | 54/a |
| AC | $a$ | 0 | −0.8 | 200 | 0 | 128/a |
| AD | $a$ | 1.25 W | 1 | 250 | 312.5 W/a | 250/a |
| BD | $a$ | −W | −0.8 | 200 | 160 W/a | 128/a |
| BE | $a$ | 1.25 W | 0 | 250 | 0 | 0 |
| DC | 2a | −1.5 W | −0.6 | 150 | 67.5 W/a | 27/a |
| DE | 2a | −0.75 W | 0 | 150 | 0 | 0 |
| BC | $a$ | 0 | 1 | 250 | 0 | 250/a |
| | | | | $\Sigma$ | $\frac{472.5W}{a}$ | $\frac{837}{a}$ |

Force in redundant member $BC$, $\quad R = \dfrac{-\Sigma \frac{PkL}{AE}}{\Sigma \frac{k^2L}{AE}} = \dfrac{-\frac{472.5W}{aE}}{\frac{837}{aE}} = -0.564\,W$ (Compression)

**Example 3.14** Find the forces in the member of truss shown in figure. All the members have same material and area cross-section.




**Solution:**

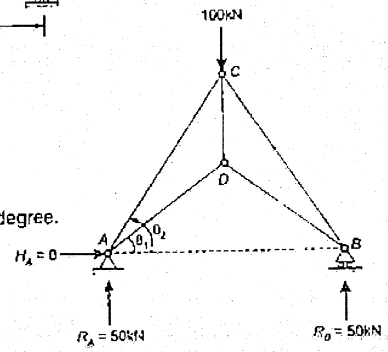
$D_{Se} = r_e - 3$

$= 3 - 3 = 0$

and $\quad D_{Si} = m - (2j - 3)$

$= 6 - (2 \times 4 - 3)$

$= 1$




Thus the given truss is internally indeterminate to 1st degree.

Let us consider the member $AB$ is redundant.

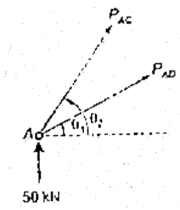
Now remove $AB$ and remaining truss is determinate.

**P-system of Forces:**

Joint $A$:

$\cos\theta_1 = \dfrac{3}{\sqrt{13}}$ and $\sin\theta_1 = \dfrac{2}{\sqrt{13}}$

$\cos\theta_2 = \dfrac{3}{5}$ and $\sin\theta_2 = \dfrac{4}{5}$




$\Sigma F_x = 0;$ $\quad P_{AD}\cos\theta_1 + P_{AC}\cos\theta_2 = 0$

$$\frac{3 \times P_{AD}}{\sqrt{13}} + \frac{3}{5}P_{AC} = 0 \quad \ldots(i)$$

$\Sigma F_y = 0;$ $\quad P_{AC}\sin\theta_2 + P_{AD}\sin\theta_1 = 50$

$$\frac{4}{5}P_{AC} + \frac{2}{\sqrt{3}}P_{AD} = -50 \quad \ldots(ii)$$

Solving $(i)$ and $(ii)$, we get

$P_{AD} = +25\sqrt{13}$ kN $= 90.13$ kN (Tension)

$P_{AC} = -125$ kN (Compression)

**Joint B:**

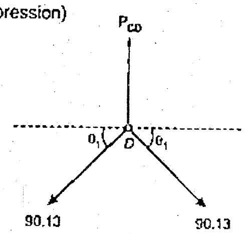
Similarly,

$$P_{BD} = P_{AD} = +25\sqrt{13}\text{ kN} = 90.13\text{ kN} \quad \text{(Tension)}$$

$$P_{BC} = P_{AC} = -125\text{ kN} \quad \text{(Compression)}$$

**Joint D:**

$\Sigma F_y = 0;$



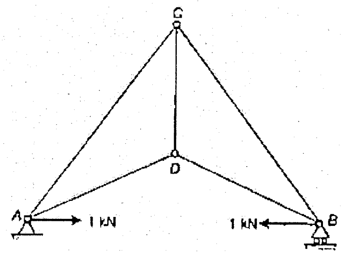

$$P_{CD} - 90.13\sin\theta_1 - 90.13\sin\theta_1 = 0$$

$$P_{CD} = 2\times 90.13\times\frac{2}{\sqrt{13}}$$

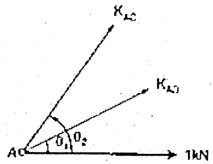
$$= +100\text{ kN} \quad \text{(Tension)}$$

**K-system of Forces:**

Now remove external loading and apply unit load at *A* and *B* in the direction of member removed.




**Joint A:**




$$\cos\theta_1 = \frac{3}{\sqrt{13}} \text{ and } \sin\theta_1 = \frac{2}{\sqrt{13}}$$

$$\cos\theta_2 = \frac{3}{5} \text{ and } \sin\theta_2 = \frac{4}{5}$$

$\Sigma F_x = 0;$ $\quad 1 + K_{AD}\cos\theta_1 + K_{AC}\cos\theta_2 = 0$

$$1+\frac{3}{\sqrt{13}}K_{AD}+\frac{3}{5}K_{AC} = 0 \quad \ldots(i)$$

$\Sigma F_y = 0;$ $\quad K_{AC}\sin\theta_2 + K_{AD}\sin\theta_1 = 0$

$$\frac{2}{\sqrt{13}}K_{AD}+\frac{4}{5}K_{AC} = 0 \quad \ldots(ii)$$

Solving (*i*) and (*ii*), we get

$$K_{AD} = -\frac{2}{3}\sqrt{13}\text{ kN} \quad \text{(Compression)}$$

$$K_{AC} = +\frac{5}{3}\text{ kN} \quad \text{(Tension)}$$

**Joint B:**

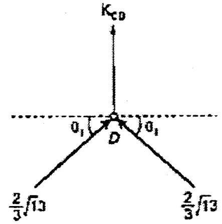
Similarly,

$$K_{BD} = K_{AD} = -\frac{2}{3}\sqrt{13}\text{ kN} \quad \text{(Compression)}$$

$$K_{BC} = K_{AC} = +\frac{5}{3}\text{ kN} \quad \text{(Tension)}$$

**Joint D:**

$\Sigma F_y = 0;$




$$K_{CD}+2\times\frac{2\sqrt{13}}{3}\sin\theta_1 = 0$$

$$\therefore \quad K_{CD} = -\frac{2\times 2\sqrt{13}}{3}\times\frac{2}{\sqrt{13}}$$

$$= -\frac{8}{3}\text{ kN} \quad \text{(Compression)}$$

| Member | P | K | L | PKL | $K^2L$ | $S = P + XK$ |
|---|---|---|---|---|---|---|
| AC | −125 | $+\frac{5}{3}$ | 5 | −1041.67 | 13.89 | −47.31 |
| CB | −125 | $+\frac{5}{3}$ | 5 | −1041.67 | 13.89 | −47.31 |
| AD | $+25\sqrt{13}$ | $\frac{-2\sqrt{13}}{3}$ | $\sqrt{13}$ | −781.20 | 20.83 | −21.90 |
| BD | $+25\sqrt{13}$ | $\frac{-2\sqrt{13}}{3}$ | $\sqrt{13}$ | −781.20 | 20.83 | −21.90 |
| CD | 100 | $-\frac{8}{3}$ | 2 | −533.33 | 14.22 | −24.30 |
| AB | 0 | 1 | 6 | 0 | 6 | 46.61 |
| | | | | ΣPKL = −4179.07 | $\Sigma K^2L$ = 89.66 | |

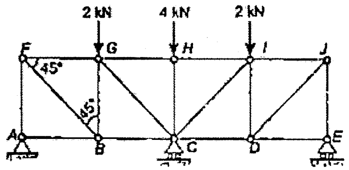
The value of redundant force $X = -\dfrac{\Sigma PKL}{\Sigma K^2L}$

$$= -\frac{(-4179.07)}{89.66} = +46.61\text{ kN} \quad \text{(Tension)}$$

The final forces (s-system of forces) are tabulated above.

**Case-II: Externally Indeterminate Trusses**

**Example 3.15** Find the forces in the members of the truss shown in figure. The truss is hinged at *A* and simply supported at *C* and *E*. It carries 2 kN, 4 kN and 2 kN at nodes *G*, *H* and *I* respectively. The diagonals make 45° with the horizontal and vertical members.




[IES : 2004]

***Solution:***

The degree of indeterminacy of the given truss may be given as

$$D_S = m + r_e - 2j$$

Here, $m = 17$, $r_e = 4$, $j = 10$

$\therefore$ $D_S = 17 + 4 - 2 \times 10$

$\Rightarrow$ $D_S = 1$

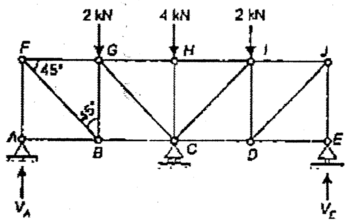
However, this truss is internally determinate. It is externally indeterminate because total number of external reactions are more than the available equilibrium equations.

**P-system of Forces:**

Removing the support at $C$, the truss will become statically determinate.

$\therefore$ $D_S = 17 + 3 - 2 \times 10$

$\Rightarrow$ $D_S = 0$




Since the truss is loaded symmetrically, the reactions at the supports will be equal

$\therefore$ $V_A = V_E = 4$ kN

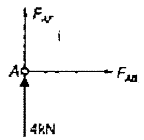
Assuming tension as positive and compression as negative.

**Considering joint $A$:**

$\Sigma F_x = 0$ $\qquad\qquad$ $\Sigma F_y = 0$

$\Rightarrow F_{AB} = 0$ $\qquad\qquad$ $\Rightarrow F_{AF} + 4 = 0$

$\Rightarrow F_{AF} = -4$ kN (Compression)



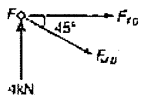

**Considering joint $F$:**

$\Sigma F_y = 0$ $\qquad\qquad$ $\Sigma F_x = 0$

$\Rightarrow 4 - F_{FB}\sin 45° = 0$ $\qquad\qquad$ $\Rightarrow F_{FG} + F_{FB}\cos 45° = 0$

$\Rightarrow \dfrac{F_{FB}}{\sqrt{2}} = 4$ $\qquad\qquad$ $\Rightarrow F_{FG} = -4\sqrt{2} \times \dfrac{1}{\sqrt{2}}$

$\Rightarrow F_{FB} = 4\sqrt{2}$ kN (Tension) $\qquad\qquad$ $\Rightarrow F_{FG} = -4$ kN (Compression)

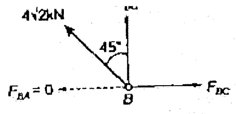



**Considering joint $B$:**

$\Sigma F_x = 0$ $\qquad\qquad$ $\Sigma F_y = 0$

$\Rightarrow F_{BC} - 4\sqrt{2}\sin 45° = 0$ $\qquad\qquad$ $\Rightarrow F_{BG} + 4\sqrt{2}\cos 45° = 0$

$\Rightarrow F_{BC} = 4$ kN (Tension) $\qquad\qquad$ $\Rightarrow F_{BG} = -4$ kN (Compression)



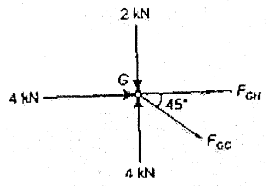

**Considering joint $G$:**

$\Sigma F_y = 0$ $\qquad\qquad$ $\Sigma F_x = 0$

$\Rightarrow 4 - 2 - F_{GC}\sin 45° = 0$ $\qquad\qquad$ $\Rightarrow F_{GH} + F_{GC}\cos 45° + 4 = 0$

$\Rightarrow \dfrac{F_{GC}}{\sqrt{2}} = 2$ $\qquad\qquad$ $\Rightarrow F_{GH} = -2\sqrt{2} \times \dfrac{1}{\sqrt{2}} - 4$

$\Rightarrow F_{GC} = 2\sqrt{2}$ kN (Tension) $\qquad\qquad$ $\Rightarrow F_{GH} = -6$ kN (Compression)

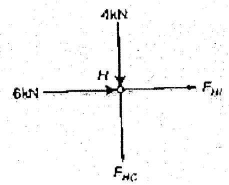



**Considering joint $H$:**

$\Sigma F_x = 0$ $\qquad\qquad$ $\Sigma F_y = 0$

$\Rightarrow F_{HI} + 6 = 0$ $\qquad\qquad$ $\Rightarrow F_{HC} + 4 = 0$

$\Rightarrow F_{HI} = -6$ kN (Compression) $\qquad\qquad$ $\Rightarrow F_{HC} = -4$ kN (Compression)




Since, the truss is symmetrically loaded, the forces in the members which are right of $H$ will be same as their corresponding members which are left of $H$.

$\therefore$ $F_{AB} = F_{ED} = 0$

$F_{AF} = F_{EJ} = -4$ kN (Compression) $\qquad$ $F_{FG} = F_{JI} = -4$ kN (Compression)

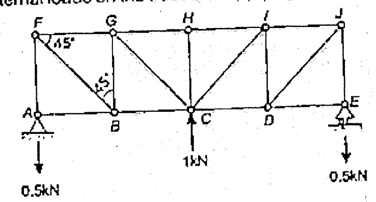
$F_{FB} = F_{JD} = 4\sqrt{2}$ kN (Tension) $\qquad$ $F_{BG} = F_{DI} = -4$ kN (Compression)

$F_{BC} = F_{DC} = 4$ kN (Tension) $\qquad$ $F_{GH} = F_{HI} = -6$ kN (Compression)

$F_{GC} = F_{IC} = 2\sqrt{2}$ kN (Tension) $\qquad$ $F_{HC} = -4$ kN (Compression)

**K-system of forces:**

Now removing all the external loads on the truss and applying a unit force in the upward direction at $C$.

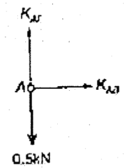



**Considering Joint $A$:**

$\Sigma F_x = 0$ $\qquad\qquad$ $\Sigma F_y = 0$

$\Rightarrow K_{AB} = 0$ $\qquad\qquad$ $\Rightarrow K_{AF} - 0.5 = 0$

$\Rightarrow K_{AF} = 0.5$ kN (Tension)



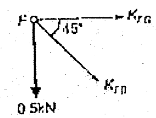

**Considering joint $F$:**

$\Sigma F_y = 0$ $\qquad\qquad$ $\Sigma F_x = 0$

$\Rightarrow K_{FB}\sin 45° + 0.5 = 0$ $\qquad\qquad$ $\Rightarrow K_{FG} + K_{FB}\cos 45° = 0$

$\Rightarrow \dfrac{K_{FB}}{\sqrt{2}} = -0.5$ $\qquad\qquad$ $\Rightarrow K_{FG} - 0.5\sqrt{2} \times \dfrac{1}{\sqrt{2}} = 0$

$\Rightarrow K_{FB} = -0.5\sqrt{2}$ (Comp.) $\qquad\qquad$ $\Rightarrow K_{FG} = 0.5$ kN (Tension)

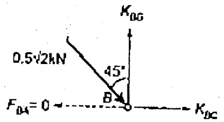


Considering joint $B$:

$\Sigma F_y = 0$ $\qquad$ $\Sigma F_x = 0$

$\Rightarrow$ $K_{BG} - 0.5\sqrt{2}\cos 45° = 0$ $\qquad$ $\Rightarrow$ $K_{BC} + 0.5\sqrt{2}\sin 45° = 0$

$\Rightarrow$ $K_{BG} = 0.5$ kN (Tension) $\qquad$ $\Rightarrow$ $K_{BC} = -0.5$ kN (Compression)



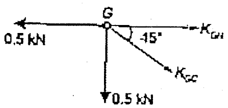

Considering joint $G$:

$\Sigma F_y = 0$ $\qquad$ $\Sigma F_x = 0$

$\Rightarrow$ $K_{GC}\sin 45° + 0.5 = 0$ $\qquad$ $\Rightarrow$ $K_{GH} + K_{GC}\cos 45° - 0.5 = 0$

$\Rightarrow$ $K_{GC} = -0.5\sqrt{2}$ kN (Comp) $\qquad$ $\Rightarrow$ $K_{GH} = 0.5 - \left(-0.5\sqrt{2} \times \frac{1}{\sqrt{2}}\right)$

$\Rightarrow$ $K_{GH} = 1$ kN (Tension)

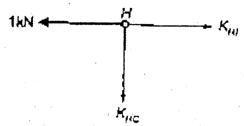



Considering joint $H$:

$\Sigma F_x = 0$ $\qquad$ $\Sigma F_y = 0$

$\Rightarrow$ $K_{HI} - 1 = 0$ $\qquad$ $\Rightarrow$ $K_{HC} = 0$

$\Rightarrow$ $K_{HI} = 1$ kN (Tension)




Similarly, forces in other members will be given as

$K_{ED} = K_{AB} = 0$ kN $\qquad$ $K_{EJ} = K_{AF} = 0.5$ kN (Tension)

$K_{JI} = K_{FG} = 0.5$ kN (Tension) $\qquad$ $K_{JD} = K_{FB} = -0.5\sqrt{2}$ kN (Compression)

$K_{DI} = K_{BG} = 0.5$ kN (Tension) $\qquad$ $K_{DC} = K_{BC} = -0.5$ (Compression)

$K_{IH} = K_{GH} = 1$ kN (Tension) $\qquad$ $K_{IC} = -K_{GC} = -0.5\sqrt{2}$kN (Compression)

$K_{HC} = 0$

Taking length of vertical and horizontal members as '$a$' and '$a$' respectively.

| Members | P | K | L | PKL | K²L | P+KX |
|---|---|---|---|---|---|---|
| AB | 0 | 0 | a | 0 | 0 | 0 |
| AF | -4 | 0.5 | a | -2a | 0.25a | -0.71 |
| FG | -4 | 0.5 | a | -2a | 0.25a | -0.71 |
| FB | $4\sqrt{2}$ | $-0.5\sqrt{2}$ | $\sqrt{2}$a | $-4\sqrt{2}$a | $0.5\sqrt{2}$a | 1 |
| BG | -4 | 0.5 | a | -2a | 0.25a | -0.71 |
| BC | 4 | -0.5 | a | -2a | 0.25a | 0.71 |
| GH | -6 | 1 | a | -6a | a | 0.58 |
| GC | $2\sqrt{2}$ | $-0.5\sqrt{2}$ | $\sqrt{2}$a | $-2\sqrt{2}$a | $0.5\sqrt{2}$a | -1.82 |
| ED | 0 | 0 | a | 0 | 0 | 0 |
| EJ | -4 | 0.5 | a | -2a | 0.25a | -0.71 |
| JI | -4 | 0.5 | a | -2a | 0.25a | -0.71 |
| JD | $4\sqrt{2}$ | $-0.5\sqrt{2}$ | $\sqrt{2}$a | $-4\sqrt{2}$a | $0.5\sqrt{2}$a | 1 |
| DI | -4 | 0.5 | a | -2a | 0.25a | -0.71 |
| DC | 4 | -0.5 | a | -2a | 0.25a | 0.71 |
| HI | -6 | 1 | a | -6a | a | 0.58 |
| IC | $2\sqrt{2}$ | $-0.5\sqrt{2}$ | $\sqrt{2}$a | $-2\sqrt{2}$a | $0.5\sqrt{2}$a | -1.82 |
| HC | -4 | 0 | a | 0 | 0 | -4 |

$\therefore$ $$\Sigma PKL = -\left(28a + 12\sqrt{2}a\right)$$

and $$\Sigma K^2L = 4a + 2\sqrt{2}a$$

The reaction at $C$ may be given as

$$X = -\frac{\Sigma \frac{PKL}{AE}}{\Sigma \frac{K^2L}{AE}}$$

If the truss members have same axial stiffness, then

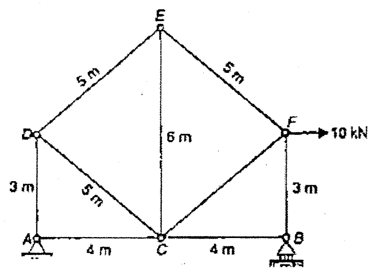
$$X = -\frac{\frac{1}{AE}\Sigma PKL}{\frac{1}{AE}\Sigma K^2L} \quad \Rightarrow \quad X = -\frac{\Sigma PKL}{\Sigma K^2L}$$

$$\Rightarrow \quad X = -\frac{-\left(28a + 12\sqrt{2}a\right)}{4a + 2\sqrt{2}a} \quad \Rightarrow \quad X = \frac{28 + 12\sqrt{2}}{4 + 2\sqrt{2}} = 6.58 \text{ kN}$$

The forces in each member can now be given as

$$S = P + KX \text{ (shown in table)}$$

**Example 3.16** Find the forces in all members of the truss if support $B$ moves horizontally to the right by 5 mm. Take $A = 2500$ mm² and $E = 200$ GPa.

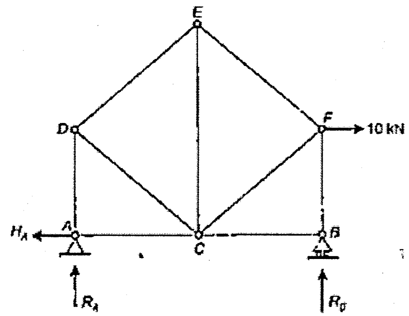



***Solution:***

$D_{Se} = r_e - 3$
$= 4 - 3 = 1$

$D_{Si} = m - (2j - 3)$
$= 9 - (2 \times 6 - 3)$
$= 0$

Thus the given truss is externally indeterminate to 1st degree. Let $H_B$ is redundant reaction. Remove redundant reaction, now remaining truss is determinate.




**Reactions:**

$\Sigma F_x = 0$: $\qquad$ $H_A = 10$ kN ($\leftarrow$)

$\Sigma F_y = 0;$ $\qquad R_A + R_B = 0$ $\qquad$ ...(i)

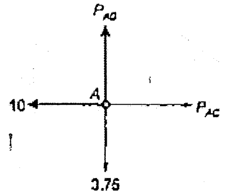
$\Sigma M_B = 0;$ $\qquad R_A \times 8 + 10 \times 3 = 0$

$$R_A = -3.75 \text{ kN } (\downarrow)$$

$\therefore$ $\qquad R_B = 3.75 \text{ kN } (\uparrow)$

**P-system of Forces:**

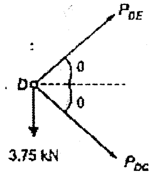
**Joint A:**




$\Sigma F_x = 0;$ $\qquad P_{AC} = 10 \text{ kN}$ (Tension)

$\Sigma F_y = 0;$ $\qquad P_{AD} = 3.75 \text{ kN}$ (Tension)

**Joint D:**




$\Sigma F_x = 0;$ $\qquad P_{DE}\cos\theta + P_{DC}\cos\theta = 0$

$$P_{DE} + P_{DC} = 0 \qquad \text{...(i)}$$

$\Sigma F_y = 0;$ $\qquad 3.75 + P_{DC}\sin\theta = P_{DE}\sin\theta$

$$3.75 + \frac{3P_{DC}}{5} = \frac{3P_{DE}}{5}$$

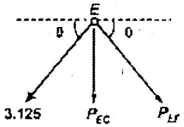
$$3P_{DC} - 3P_{DE} + 18.75 = 0 \qquad \text{...(ii)}$$

Solving, (i) and (ii) we get

$$P_{DE} = 3.125 \text{ kN} \quad \text{(Tension)}$$

$$P_{DC} = -3.125 \quad \text{(Compression)}$$

**Joint E:**




$\Sigma F_x = 0;$ $\qquad P_{EF}\cos\theta = 3.125\cos\theta$

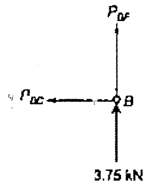
$$P_{EF} = 3.125 \text{ kN} \quad \text{(Tension)}$$

$\Sigma F_y = 0;$ $\qquad P_{EC} + 3.125\sin\theta + P_{EF}\sin\theta = 0$

$$P_{EC} = -\left(3.125 \times \frac{3}{5} + 3.125 \times \frac{3}{5}\right)$$

$$= -3.75 \text{ kN} \quad \text{(Compression)}$$

**Joint B:**




$\Sigma F_y = 0;$ $\qquad P_{BF} + 3.75 = 0$

$$P_{BF} = -3.75 \text{ kN} \quad \text{(Compression)}$$

$\Sigma F_x = 0;$ $\qquad P_{BC} = 0$

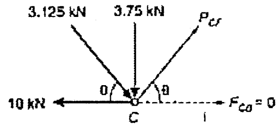
**Joint C:**

$\Sigma F_x = 0;$ $\qquad 10 = 3.125\cos\theta + P_{CF}\cos\theta$

$$10 = 3.125 \times \frac{4}{5} + P_{CF} \times \frac{4}{5}$$

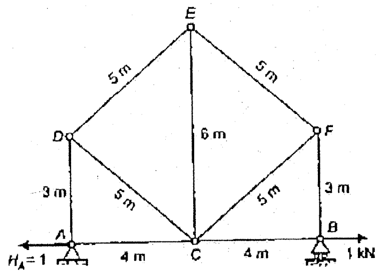
$$50 = 12.5 + 4P_{CF}$$

$$P_{CF} = 9.375 \text{ kN} \quad \text{(Tension)}$$

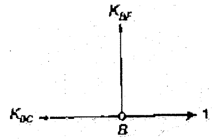



**K-system of Forces:**

Remove all external loading and apply unit load in the direction of redundant reaction.




**Joint B:**



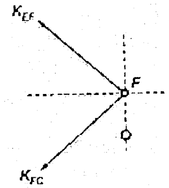

$\Sigma F_x = 0;$ $\qquad K_{BC} = 1$ (Tension)

$\Sigma F_y = 0;$ $\qquad K_{BF} = 0$

**Joint F:**

Since no load at joint F. Hence,

$$K_{FC} = K_{EF} = 0$$




**Joint D:**

Since no load at D. Hence,

$$K_{DA} = K_{DC} = 0$$

**Joint A:**

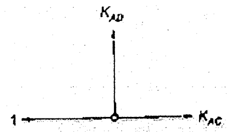
$\Sigma F_x = 0;$ $\qquad K_{AC} = 1 \text{ kN}$ (Tension)

$\Sigma F_y = 0;$ $\qquad K_{AD} = 0$

**Joint E:**

$$\Sigma F_y = 0$$

$$K_{EC} = 0$$




| Member | P | K | L | PKL | $K^2L$ | $S = P + XK$ |
|---|---|---|---|---|---|---|
| AD | 3.75 | 0 | 3 | 0 | 0 | 3.75 |
| DC | −3.125 | 0 | 5 | 0 | 0 | −3.125 |
| DE | 3.125 | 0 | 5 | 0 | 0 | 3.125 |
| EC | −3.75 | 0 | 6 | 0 | 0 | −3.75 |
| EF | 3.125 | 0 | 5 | 0 | 0 | 3.125 |
| AC | 10 | 1 | 4 | 40 | 4 | −297.5 |
| FB | −3.75 | 0 | 3 | 0 | 0 | −3.75 |
| FC | 9.375 | 0 | 5 | 0 | 0 | 9..75 |
| CB | 0 | 1 | 4 | 0 | 4 | −307.5 |
| | | | | $\Sigma PKL = 40$ | $\Sigma K^2L = 8$ | |

$$\frac{\Sigma PKL}{AE}\begin{pmatrix}\text{Movement due to}\\ \text{load toward right}\end{pmatrix} - X\frac{\Sigma K^2L}{AE}\begin{pmatrix}\text{Movement due to}\\ \text{reaction towards left}\end{pmatrix} = 0.005\text{ m}$$

$$\therefore \quad \left(\frac{40\times10^6}{2500\times200\times10^6}\right) - X\left(\frac{8\times10^6}{2500\times200\times10^6}\right) = 0.005$$

$$8 - 1.6X = 500$$

$$X = -307.5\text{ kN}$$

$$H_B = -307.5\text{ kN}$$

The final force in members (S-system of forces) are tabulated above.

### 3.3.1.1 Effect Due to Lack of FIT

If members are fabricated in the factory but due to some error one of the members say $AC$ is fabricated $\Delta$ to short or $\Delta$ too long. If this member is fitted forcefully, it will induce forces in all members. If $AC$ is $\Delta$ too short, then it will be pulled by a force $X_0$ and fitted.

Therefore a tensile force act in the direction of $AC$. Due to this $X_0$ force, forces in all members may be calculated. Similarly if AC is $\Delta$ too long, then it will be compressed and at $A$ and $C$ a compressive force $X_0$ will act in the direction of $AC$.

Let axial forces in various members due to force $X_0$ in $AC$ are $Q_1, Q_2 ..... Q_n$.

Hence, the total strain energy $U$ will be

$$U = \frac{Q_1^2 L_1}{2A_1E_1} + ........ + \frac{X_0^2 L_{AC}}{2AE}$$

For true value of $X_0$,

$$\therefore \quad \frac{\partial U}{\partial X_0} = \Delta$$

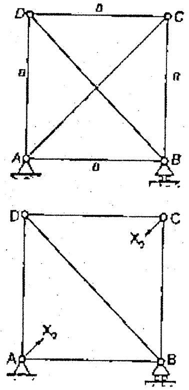


Fig. 3.10

If a redundant frame subjected to external loading and some members are too long or too short the final forces in members will be

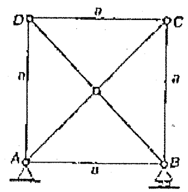
$$S_0 = S + kX_0$$

where, $S$ = S-system of force (i.e. final forces in members due to loading)

$K$ = K-system to force (i.e. force induces in members due to unit loads)

$X_0$ = Force due to lack of fit

**Example 3.17** If a square $ABCD$ shown below having constant axial rigidity is to a assembled in the field. If one of the member (Say $AC$) is $\Delta$ too short, then find forces developed in truss members due to forceful filling of member $AC$.

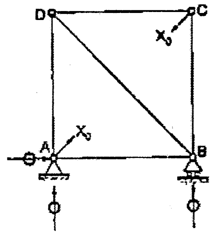



***Solution:***

$$D_S = m + r_e - 2j$$
$$= 6 + 3 - 2\times4$$
$$= 1$$

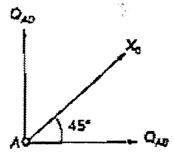
Thus the given frame is indeterminate.

Hence due to lack of fit forces will induced in all members of frame.




Q-system of forces:

Joint $A$:



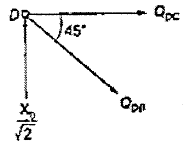

$\Sigma F_x = 0$: $\quad Q_{AB} + X_0\cos45° = 0$

$$Q_{AB} = -\frac{X_0}{\sqrt2} \quad \text{(Compressive)}$$

$\Sigma F_y = 0$: $\quad Q_{AD} + X_0\sin45° = 0$

$$Q_{AD} = -\frac{X_0}{\sqrt2} \quad \text{(Compressive)}$$

Joint $D$:




$\Sigma F_y = 0$: $\quad Q_{BD}\sin45° = \frac{X_0}{\sqrt2}$

$$\frac{Q_{BD}}{\sqrt2} = \frac{X_0}{\sqrt2}$$

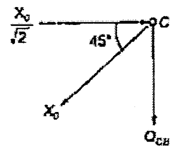
$$\therefore \quad Q_{BD} = X_0 \quad \text{(Tensile)}$$

$\Sigma F_x = 0$: $\quad Q_{DC} + Q_{BD}\cos45° = 0$

$$Q_{DC} + \frac{X_0}{\sqrt2} = 0$$

$$\therefore \quad Q_{DC} = -\frac{X_0}{\sqrt2} \text{ (Compressive)}$$

Joint $C$:




$\Sigma F_y = 0$; $\quad X_0\sin45° + Q_{CB} = 0$

$$Q_{CB} = -X_0\times\frac{1}{\sqrt2}$$

$$Q_{CB} = -\frac{X_0}{\sqrt2} \text{ (Compressive)}$$

Total strain energy stored in members due to $X_0$ is given by

$$U = \sum\frac{Q^2L}{2AE} = \frac{1}{2AE}\cdot 2X_0^2\, a(\sqrt2+1)$$

$$U = \frac{X_0^2 a(\sqrt2+1)}{AE}$$

| Member | $Q$ | $L$ | $S_0 = Q$ | $Q^2L$ |
|---|---|---|---|---|
| **AB** | $-\frac{X_0}{\sqrt{2}}$ | a | $-\frac{AE\Delta}{2\sqrt{2}\cdot a(\sqrt{2}+1)}$ | $\frac{X_0^2 a}{2}$ |
| **AC** | $X_0$ | $a\sqrt{2}$ | $+\frac{AE\Delta}{2a(\sqrt{2}+1)}$ | $X_0^2 a\sqrt{2}$ |
| **AD** | $-\frac{X_0}{\sqrt{2}}$ | a | $-\frac{AE\Delta}{a2\sqrt{2}(\sqrt{2}+1)}$ | $\frac{X_0^2 a}{\sqrt{2}}$ |
| **BC** | $-\frac{X_0}{\sqrt{2}}$ | a | $-\frac{AE\Delta}{2\sqrt{2}a(\sqrt{2}+1)}$ | $\frac{X_0^2 a}{\sqrt{2}}$ |
| **BD** | $X_0$ | $a\sqrt{2}$ | $+\frac{AE\Delta}{2a(\sqrt{2}+1)}$ | $\frac{X_0^2 a}{\sqrt{2}}$ |
| **CD** | $-\frac{X_0}{\sqrt{2}}$ | a | $-\frac{AE\Delta}{2\sqrt{2}a(\sqrt{2}+1)}$ | $\frac{X_0^2 a}{\sqrt{2}}$ |
| | | | | $\Sigma Q^2L = 2X_0^2 a(\sqrt{2}+1)$ |

For true value of $X_0$,

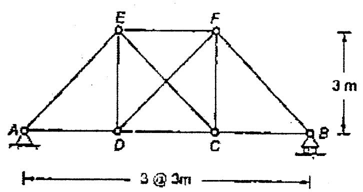
$$\therefore \quad \frac{\partial U}{\partial X_0} = \Delta$$

$$\frac{2X_0 \cdot a(\sqrt{2}+1)}{AE} = \Delta$$

$$\therefore \quad X_0 = \frac{AE\Delta}{2a(\sqrt{2}+1)}$$

The final forces induced in member are tabulated above.

**Example 3.18** In the truss shown in figure the member *EC* was the last to fitted in the truss. While fitting it was noticed that the member was 5 mm longer than the length required. Find the force developed in all the members. *AE* is constant.



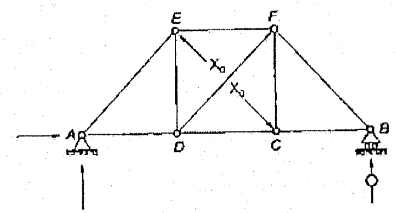

***Solution:***

$$D_S = m + r_e - 2j$$

Here, $m = 10$, $r_e = 3$, $j = 6$

$$\therefore \quad D_S = 10 + 3 - 2\times 6 = 1$$

Hence above truss is indeterminate to 1[st] degree. When the member *EC* fitted forcefully then forces will induced in members. As the member *EC* is too long the member will be subjected to compression (say $X_0$ which will acts at joints *E* and *C* as shown in figure).




**Q-system of Forces:**

*Joint A:*

Since no external load at joint *A*.

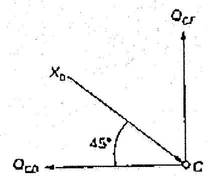
$$\therefore \quad Q_{AD} = Q_{AE} = 0$$

*Joint B:*

Since no external force at *B*.

$$\therefore \quad Q_{BC} = Q_{BF} = 0$$

*Joint C:*

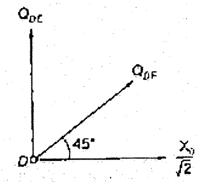



$\Sigma F_x = 0;$ $\quad Q_{CD} = X_0\cos 45°$

$$\therefore \quad Q_{CD} = \frac{X_0}{\sqrt{2}} \quad \text{(Tension)}$$

$\Sigma F_y = 0$ $\quad Q_{CF} = X_0 \sin 45° = \frac{X_0}{\sqrt{2}}$ (Tension)

*Joint D:*




$\Sigma F_x = 0;$ $\quad Q_{DF}\cos 45° + \frac{X_0}{\sqrt{2}} = 0$

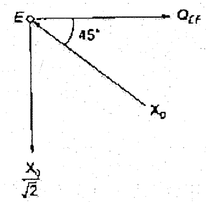
$$\frac{Q_{DF}}{\sqrt{2}} + \frac{X_0}{\sqrt{2}} = 0$$

$$\therefore \quad Q_{DF} = -X_0 \quad \text{(Compression)}$$

$\Sigma F_y = 0;$ $\quad Q_{DE} + Q_{DF}\sin 45° = 0$

$$Q_{DE} = -\left(\frac{-X_0}{\sqrt{2}}\right) = \frac{X_0}{\sqrt{2}} \quad \text{(Tension)}$$

*Joint E:*




$\Sigma F_x = 0;$ $\quad Q_{EF} - X_0\cos 45° = 0$

$$Q_{EF} = \frac{X_0}{\sqrt{2}} \quad \text{(Tension)}$$

$$\Sigma Q^2L = 0 + 0 + \frac{3}{2}X_0^2 + 3\sqrt{2}X_0^2 + \frac{3X_0^2}{2} + 3\sqrt{2}X_0^2 + \frac{3X_0^2}{2} + \frac{3X_0^2}{2}$$

$$= \frac{4\times 3}{2}X_0^2 + 6\sqrt{2}X_0^2$$

$$= 6X_0^2 + 6\sqrt{2}X_0^2 = 6(1+\sqrt{2})X_0^2$$

| Member | $Q$ | $L$ | $Q^2L$ |
|---|---|---|---|
| AD | 0 | 3 | 0 |
| AE | 0 | $3\sqrt{2}$ | 0 |
| ED | $\frac{X_0}{\sqrt{2}}$ | 3 | $\frac{3X_0^2}{2}$ |
| EC | $-X_0$ | $3\sqrt{2}$ | $3\sqrt{2}X_0^2$ |
| EF | $\frac{X_0}{\sqrt{2}}$ | 3 | $\frac{3X_0^2}{2}$ |
| DF | $-X_0$ | $3\sqrt{2}$ | $3\sqrt{2}X_0^2$ |
| FC | $\frac{X_0}{\sqrt{2}}$ | 3 | $\frac{3X_0^2}{2}$ |
| DC | $\frac{X_0}{\sqrt{2}}$ | 3 | $\frac{3X_0^2}{2}$ |
| FB | 0 | $3\sqrt{2}$ | 0 |
| CB | 0 | 3 | 0 |
| | | | $\Sigma Q^2L = 6(1+\sqrt{2})X_0^2$ |

$\therefore$ Total strain energy, $U = \sum \frac{Q^2L}{2AE} = \frac{6(1+\sqrt{2})X_0^2}{2AE}$

For true value of $X_0$,

$$\frac{\partial U}{\partial X_0} = \frac{6 \times 2(1+\sqrt{2})X_0}{2AE} = \Delta$$

$$\therefore \quad \frac{6(1+\sqrt{2})X_0}{AE} = \Delta \quad \Rightarrow \quad X_0 = \frac{AE\Delta}{6(1+\sqrt{2})}$$

Hence force is the member of truss are tabulated below.

| Member | AD | AE | ED | EC | EF | DF | FC | DC | FB | CB |
|---|---|---|---|---|---|---|---|---|---|---|
| Force | 0 | 0 | $+\frac{AE\Delta}{6(2+\sqrt{2})}$ | $-\frac{AE\Delta}{6(1+\sqrt{2})}$ | $+\frac{AE\Delta}{6(2+\sqrt{2})}$ | $-\frac{AE\Delta}{6(1+\sqrt{2})}$ | $+\frac{AE\Delta}{6(2+\sqrt{2})}$ | $+\frac{AE\Delta}{6(2+\sqrt{2})}$ | 0 | 0 |

### 3.6.1.2 Effect of Temperature on Redundant Frames

Consider a redundant frame shown in figure.

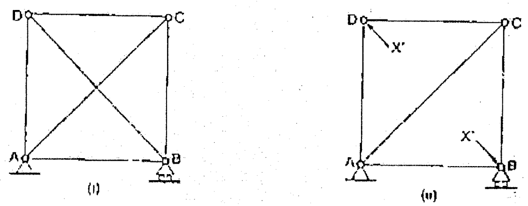



*Fig. 3.11*

Let the temperature of member $BD$ rise by T°C. Then the free expansion of $BD$ is given by $\Delta = L\alpha T$, but free expansion is not permissible. Hence a compressive force $X'$ develops in member which press the joint $D$ and $B$.

Let axial forces in various members due to force $X$ in $BD$ are $Q_1, Q_2, Q_3, \ldots\ldots Q_n$. Hence the total strain energy is given by

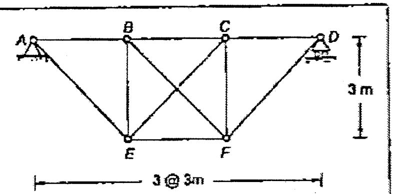
$$U = \frac{\Sigma Q^2 L}{2AE}$$

For true value of $X'$; $\quad \frac{\partial U}{\partial X'} = L\alpha T$

**NOTE:** If temperature of member lowered by T°C, then a tensile force $X'$ develops in member $BD$.

**Example 3.19** Find the forces developed in all the members of the truss shown.

If the temperature of member $EC$ rise by 20°C. Cross-sectional area of all the members is 2500 mm² and young's modulus is 200 kN/mm². Take coefficient of thermal expansion (α) = 12 × 10⁻⁶/°C.




***Solution:***

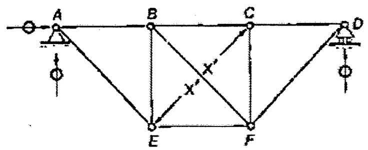
$$D_S = m + r_e - 2j$$
$$= 10 + 3 - 2 \times 6$$
$$= 1$$

Thus the given truss is redundant. Hence on rising the temperature of member $EC$. The axial forces develops in all the member of truss.

Free expansion, $\quad \Delta = L\alpha T$

$$= 3\sqrt{2} \times 12 \times 10^{-6} \times 20$$
$$= 1.018 \text{ mm}$$

As free expansion is prevented, a compressive force $X$ develops in the member $EC$. It compress the joint $E$ and $C$ as shown figure.




Q-system of Forces:

Joint $A$:

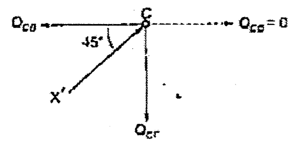
Since no external force at $A$.

Hence, $\quad Q_{AB} = Q_{AE} = 0$

Joint $D$:

Since no external force at $D$

Hence, $\quad Q_{CD} = Q_{DF} = 0$




Joint $C$:

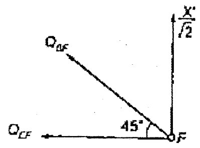
$\Sigma F_x = 0$: $\quad Q_{CB} - X'\cos 45° = 0$

$$Q_{CB} = \frac{X'}{\sqrt{2}} \quad \text{(Tension)}$$

$\Sigma F_y = 0;$ $\quad Q_{CF} - X' \sin 45° = 0$

$$Q_{CF} = \frac{X'}{\sqrt{2}} \quad \text{(Tension)}$$

Joint *F*:



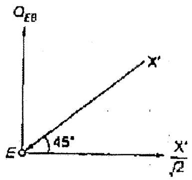

$\Sigma F_y = 0;$ $\quad Q_{BF} \sin 45° = -\frac{X'}{\sqrt{2}}$

$\therefore$ $\quad Q_{BF} = -X'$ (Compression)

$\Sigma F_x = 0;$ $\quad Q_{EF} + Q_{BF} \cos 45° = 0$

$$Q_{EF} = -\frac{Q_{BF}}{\sqrt{2}} = \frac{X'}{\sqrt{2}} \quad \text{(Tension)}$$

Joint *E*:




$\Sigma F_y = 0;$ $\quad Q_{ED} = X' \sin 45° = \frac{X'}{\sqrt{2}}$ (Tension)

| Member | Q (kN) | L(m) | $Q^2L$ | Final force |
|---|---|---|---|---|
| AB | 0 | 3 | 0 | 0 |
| AE | 0 | $3\sqrt{2}$ | 0 | 0 |
| BC | $\frac{X'}{\sqrt{2}}$ | 3 | $1.5X'^2$ | 24.84 |
| BE | $\frac{X'}{\sqrt{2}}$ | 3 | $1.5X'^2$ | 24.84 |
| CD | 0 | 3 | 0 | 0 |
| CF | $\frac{X'}{\sqrt{2}}$ | 3 | $1.5X'^2$ | 24.84 |
| EC | $-X'$ | $3\sqrt{2}$ | $3\sqrt{2}X'^2$ | −35.13 |
| EF | $\frac{X'}{\sqrt{2}}$ | 3 | $1.5X'^2$ | 24.84 |
| FD | 0 | 3 | 0 | 0 |
| FB | $-X'$ | 3 | $3\sqrt{2}X'^2$ | −35.13 |
| | | | $\Sigma Q^2L = 6(1+\sqrt{2})X'^2$ | |

The total strain energy, $\quad U = \frac{\Sigma Q^2 L}{2AE} = \frac{6(1+\sqrt{2})X'^2}{2AE}$

For true value of $X$,

$$\frac{\partial U}{\partial X'} = L\alpha T$$

$$\therefore \quad \frac{6 \times 2(1+\sqrt{2})X'}{2AE} = 1.018$$

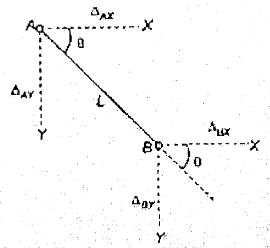
$$X' = \frac{1.018 \times 2 \times 2500 \times 2 \times 10^5}{12(1+\sqrt{2})} \times 10^{-3} \text{ kN} = 35.13 \text{ kN}$$

The final forces in members are tabulate above.

### 3.6.2 Stiffness Method

This method is suitable when number of unknown displacement at joint are least possible.

**Force Displacement Relation for Truss**




Consider a member *AB* which is incline at an angle θ with horizontal. Let horizontal and vertical component of displacement at joint *A* are $\Delta_{AX}$ and $\Delta_{AY}$ and horizontal and vertical displacement at joint *B* are $\Delta_{BX}$ and $\Delta_{BY}$. Consider joint *A*, the deflection of joint *A* in the direction of *AB* is given by,

$$\Delta_A = \Delta_{AX}\cos\theta + \Delta_{AY}\sin\theta \quad \ldots(i)$$

The deflection of joint *B* in the direction of *AB* is given by

$$\Delta_B = \Delta_{BX}\cos\theta + \Delta_{BY}\sin\theta \quad \ldots(ii)$$

Hence, the net deflection of *AB*,

$$\Delta_{AB} = \Delta_A - \Delta_B$$

From equation (*i*) and (*ii*),

$$\Delta_{AB} = (\Delta_{AX} - \Delta_{BX})\cos\theta + (\Delta_{AY} - \Delta_{BY})\sin\theta \quad \ldots(iii)$$

we know that, $\quad \Delta_{AB} = \frac{P_{AB}(L)}{AE}$

where, $\quad A$ = Area cross-section of *AB*

$E$ = Young's modulus

$\therefore$ From equation (*iii*),

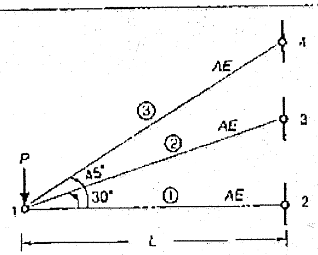
$$\frac{P_{AB} \cdot L}{AE} = (\Delta_{AX} - \Delta_{BX})\cos\theta + (\Delta_{AY} - \Delta_{BY})\sin\theta$$

Thus the force in member *AB*,

$$P_{AB} = \frac{AE}{L}[(\Delta_{AX} - \Delta_{BX})\cos\theta + (\Delta_{AY} - \Delta_{BY})\sin\theta]$$

**Example 3.20** For a three bar truss shown in the figure, compute the vertical displacement of node 1 by displacement (stiffness/equilibrium) method.

[IES : 2006]




***Solution:***

The axial compression (Δ) due to movement of node 2

$$= \Delta_{2x}\cos\theta + \Delta_{2y}\sin\theta$$

$\Delta_{2x} \rightarrow$ displacement of node 2 in x-direction

The axial elongation (Δ') due to movement of node 1

$$= \Delta_{1x}\cos\theta + \Delta_{1y}\sin\theta$$

$\therefore$ Net elongation of 12 $\quad = (\Delta_{1x} - \Delta_{2x})\cos\theta + (\Delta_{1y} - \Delta_{2y})\sin\theta$

But, Net elongation of 12 = $\frac{P_{12}L}{AE}$

$$\Rightarrow \quad P_{12} = \frac{AE}{L}[(\Delta_{1x}-\Delta_{2x})\cos\theta + (\Delta_{1y}-\Delta_{2y})\sin\theta]$$

Now from the above figure we have

$$\cos 30^\circ = \frac{L_{12}}{L_{13}} \quad \text{and} \quad \cos 45^\circ = \frac{L_{12}}{L_{14}}$$

$$\Rightarrow \quad L_{13} = \frac{2L_{12}}{\sqrt{3}} \quad \Rightarrow \quad L_{14} = \frac{\sqrt{2}L_{12}}{1}$$

$$\Rightarrow \quad L_{13} = \frac{2L}{\sqrt{3}} \qquad [\because L_{12} = L]$$

$$\Rightarrow \quad L_{14} = \sqrt{2}L$$

For member 12, the value of $\theta = 0^\circ$

$$\therefore \quad P_{12} = \frac{AE}{L}\left[(\Delta_{1x}-0)\cos 0^\circ + (\Delta_{1y}-0)\sin 0^\circ\right]$$

$$\Rightarrow \quad P_{12} = \frac{AE}{L}\Delta_{1x} \qquad \text{...(i)}$$

Similarly,

$$P_{13} = \frac{AE}{2L}\times\sqrt{3}\,[(\Delta_{1x}-\Delta_{3x})\cos 30^\circ + (\Delta_{1y}-\Delta_{3y})\sin 30^\circ]$$

$$\Rightarrow \quad P_{13} = \frac{\sqrt{3}}{2}\frac{AE}{L}\left[\Delta_{1x}\times\frac{\sqrt{3}}{2} + \Delta_{1y}\times\frac{1}{2}\right]$$

$$\Rightarrow \quad P_{13} = \frac{\sqrt{3}}{4}\frac{AE}{L}\left[\sqrt{3}\,\Delta_{1x} + \Delta_{1y}\right] \qquad \text{...(ii)}$$

$$\Rightarrow \quad P_{14} = \frac{AE}{\sqrt{2}L}\left[(\Delta_{1x}-\Delta_{4x})\cos 45^\circ + (\Delta_{1y}-\Delta_{4y})\sin 45^\circ\right]$$

$$\Rightarrow \quad P_{14} = \frac{AE}{\sqrt{2}L}\left[\Delta_{1x}\times\frac{1}{\sqrt{2}} + \Delta_{1y}\times\frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow \quad P_{14} = \frac{AE}{\sqrt{2}L}\times\frac{1}{\sqrt{2}}\,(\Delta_{1x} + \Delta_{1y})$$

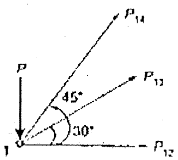
$$\Rightarrow \quad P_{14} = \frac{AE}{2L}\,(\Delta_{1x} + \Delta_{1y}) \qquad \text{...(iii)}$$

Equilibrium equation at node 1

$\Sigma F_x = 0$

$$\Rightarrow \quad P_{12} + P_{13}\cos 30^\circ + P_{14}\cos 45^\circ = 0$$

$$\Rightarrow \quad \frac{AE}{L}\Delta_1 x + \frac{\sqrt{3}}{4}\left(\sqrt{3}\,\Delta_{1x} + \Delta_{1y}\right)\cos 30^\circ + \frac{AE}{2L}\left(\Delta_{1x} + \Delta_{1y}\right)\cos 45^\circ = 0$$




$$\Rightarrow \quad \frac{AE}{L}\left[\Delta_1 x + \frac{3}{8}\left(\sqrt{3}\Delta_{1x} + \Delta_{1y}\right) + \frac{1}{2\sqrt{2}}\left(\Delta_{1x} + \Delta_{1y}\right)\right] = 0$$

$$\Rightarrow \quad \Delta_{1x} + \frac{3\sqrt{3}}{8}\Delta_{1x} + \frac{\Delta_{1x}}{2\sqrt{2}} + \frac{3}{8}\Delta_{1y} + \frac{\Delta_{1y}}{2\sqrt{2}} = 0$$

$$\Rightarrow \quad 2\Delta_{1x} + 0.73\,\Delta_{1y} = 0 \qquad \text{...(iv)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad P_{13}\sin 30^\circ + P_{14}\sin 45^\circ - P = 0$$

$$\Rightarrow \quad \frac{\sqrt{3}}{8}\frac{AE}{L}\left(\sqrt{3}\,\Delta_{1x} + \Delta_{1y}\right) + \frac{1}{2\sqrt{2}}\frac{AE}{L}\left(\Delta_{1x} + \Delta_{1y}\right) = P$$

$$\Rightarrow \quad \frac{3}{8}\Delta_{1x} + \frac{1}{2\sqrt{2}}\Delta_{1x} + \frac{\sqrt{3}}{8}\Delta_{1y} + \frac{1}{2\sqrt{2}}\Delta_{1y} = \frac{PL}{AE}$$

$$\Rightarrow \quad 0.73\,\Delta_{1x} + 0.57\,\Delta_{1y} = \frac{PL}{AE}$$

From (iv), we have

$$\therefore \quad \Delta_{1x} = -\frac{0.73\Delta_{1y}}{2}$$

Substituting in (v), we get

$$0.73\times\left(\frac{-0.73\Delta_{1y}}{2}\right) + 0.57\,\Delta_{1y} = \frac{PL}{AE}$$

$$\Rightarrow \quad 0.30355\,\Delta_{1y} = \frac{PL}{AE}$$

$$\Rightarrow \quad \Delta_{1y} = \frac{3.29\,PL}{AE}$$

## 3.7 Deflection of Truss Joints

The deflection of truss joint can be found by following methods:

1. **Force methods:** Maxwell's method/unit load method and strain energy methods. These methods can be used to find deflection of beam, rigid frame or trusses.
2. **Stiffness methods/Displacement methods:** These methods can also be used to find deflection of beam, rigid frame or truss.
3. **Graphical methods:** Bar chain method and Williot Mohr's diagram. These methods used only for truss.

### 3.7.1 Maxwell's Method/Unit Load Method

Case-1: Due to external loading

To find the deflection of joint *C* following steps may be used.

Step-1: Due to given load system calculate the forces in all members say $P_1, P_2, P_3 \ldots\ldots\ldots P_n$ (P-system of forces). The forces (P-system of forces) can be calculated by using method of joint.

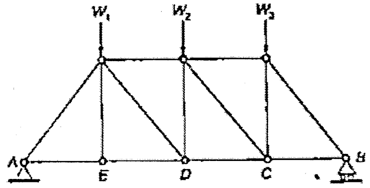



*Fig.3.12*

**Step-2:** Remove all given external loading and apply unit load at $C$ in the direction of desired deflection. Due to unit load alone find forces in member say $K_1, K_2, \ldots\ldots K_n$ ($K$-system of forces).

**Step-3:** The deflection of joint $C$ in the direction of applied unit load is given by

$$\Delta_C = \frac{\Sigma PKL}{AE}$$

If $D$ comes out positive the deflection of joint $C$ will be in the direction of unit load and vice-versa.

The calculation of deflection of truss joint can be done in tabular manner.

$$\therefore \quad \Delta_C = \sum \frac{PKL}{AE} \quad \text{or} \quad \sum K \cdot \frac{PL}{AE}$$

$$\therefore \quad \Delta_C = \sum_{i=1}^{n} K_i \Delta_i \quad \ldots(i)$$

where, $\Delta_i = \frac{PL}{AE}$ (Due to axial forces in members)

| Member | P | K | L | PKL |
|---|---|---|---|---|
| 1. | — | — | — | — |
| 2. | ..... | ..... | ..... | ..... |
| 3. | ..... | ..... | ..... | ..... |
| 4. | ..... | ..... | ..... | ..... |
| . | . | . | . | . |
| . | . | . | . | . |
| . | . | . | . | . |
| | | | | ΣPKL = |

**Case-2: Due to Variation of Temperature**

If temperature changes occurs then deflection of each member will be

$$\Delta = L\alpha T$$

where, $T$ is change in temperature. $T$ is taken positive if there is rise in temperature and taken negative if there is fall in temperature. Hence from ($i$), the deflection of joint $C$ due to change in temperature in various member is given by

$$\Delta_C = \sum_{i=1}^{n} K_i (L\alpha T)$$

**Case-3: Due to Both External and Temperature**

$$\Delta_C = \sum_{i=1}^{n} K_i \left[\frac{PL}{AE} + L\alpha T\right]$$

where, $P$ = +ve (Tension) $\quad$ $P$ = −ve (Compression)

$T$ = +ve (Rise) $\quad$ $T$ = −ve (Fall)

**Case-4: Due to Lack of Fit of some Members**

Consider a truss shown in figure.

If member $BC$ is $\Delta$ to long, then the deflection of joint $C$ may be given as

$$\Delta_C = \sum_{i=1}^{n} K_i \Delta_i$$

where, $K_i$ = Force induced in member due to unit load

$\Delta_i$ = Discrepancy in length ($\Delta$)

= −ve for too short

= +ve for too long

Hence deflection of joint $C$,

$$\Delta_C = \sum_{i=1}^{n} K_i \Delta_i$$

where, $\Delta_i$ = Axial deflection of member due to external loading

= $L\alpha T$, due to temperature effect

= $\Delta$, due to some other effect such as lack of fit.

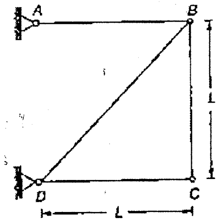
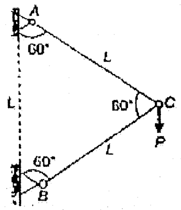



*Fig. 3.13*

**Example 3.21** The vertical deflection of node $C$ for the truss shown in figure is

(a) $\frac{PL}{AE}$ $\quad$ (b) $\frac{2PL}{AE}$

(c) $\frac{3PL}{AE}$ $\quad$ (d) $\frac{PL}{2AE}$




***Ans.* (b)**

P-system of forces:

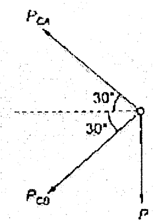
Consider joint $C$.

$\Sigma F_x = 0; \quad -P_{CA}\cos 30° - P_{CB}\cos 30° = 0$

$\therefore \quad P_{CA} = -P_{CB}$

$\Sigma F_y = 0; \quad P_{CA}\sin 30° - P - P_{CB}\sin 30° = 0$

$\therefore \quad P_{CA} = +P$ and $P_{CB} = -P$




K-system of Forces:

Remove load $P$ and apply unit load at $P$ in downward direction.

$\therefore \quad K_{CA} = +1$ kN

$K_{CB} = -1$ kN

The deflection at $C$,
$$\Delta_C = \frac{\Sigma KPL}{AE} = \frac{(1 \times P \times L) + (-1 \times -P \times L)}{AE} = \frac{2PL}{AE}$$

Alternately:

$P_{CA} = P$ and $P_{CB} = P$

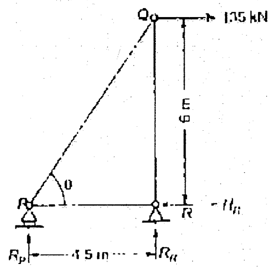
The strain energy stored in truss, $U = U_{AC} + U_{BC} = \frac{P^2L}{2AE} + \frac{P^2L}{2AE} = \frac{P^2L}{AE}$

the deflection at $C$ will be, $\Delta_C = \frac{\partial U}{\partial P} = \frac{2PL}{AE}$

**Example 3.22** The right triangular truss is made of members having equal cross-sectional area of 1550 mm² and young's modulus is 2 × 10⁵ MPa. The horizontal deflection of the joint $Q$ is

(a) 2.47 mm $\quad$ (b) 10.25 mm

(c) 14.31 mm $\quad$ (d) 15.68 mm




***Ans.* (d)**

$\Sigma F_x = 0; \quad H_R - 135 = 0$

$\therefore \quad H_R = 135$ kN

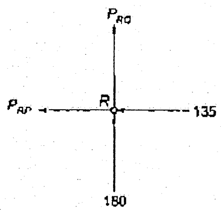
$\Sigma M_R = 0; \quad +R_P \times 4.5 + 135 \times 6 = 0$

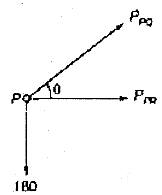
$\therefore \quad R_P = -180$ kN

and $\quad R_R = +180$ kN

P-system of Forces:

Joint $R$ | Joint $P$




$\Sigma F_x = 0;\ P_{RP} = -135$ kN

$\Sigma F_y = 0;\ P_{RQ} = -180$

$\Sigma F_y = 0;\quad P_{PQ}\sin\theta = 180$

$P_{PQ}\cdot\dfrac{6}{7.5} = 180$

$P_Q = 225$ kN

K-system of Forces:

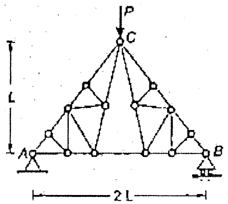
$K = \dfrac{P}{135}$ for each members

| Member | $P$ | $K$ | $L$ | $PKL$ |
|---|---|---|---|---|
| $PQ$ | 225 | 1.67 | 7.5 | 2818.125 |
| $QR$ | –180 | –1.33 | 6 | 1436.4 |
| $RP$ | –135 | –1 | 4.5 | 607.5 |
| | | | | $\Sigma PKL = 4862.02$ |

$$\therefore\quad \Delta_Q = \frac{\Sigma PKL}{AE} = \frac{4862.02\times10^{-6}\times10^{3}}{1550\times10^{-6}\times2\times10^{5}\times10^{-3}} = 15.68 \text{ mm}$$

**Example 3.23** A truss is shown in the figure members are of equal cross-section A and same modulus of elasticity $E$. A vertical force is applied at $C$.

The deflection of point $C$ is




(a) $\left(\dfrac{2\sqrt{2}+1}{2}\right)\dfrac{PL}{AE}$

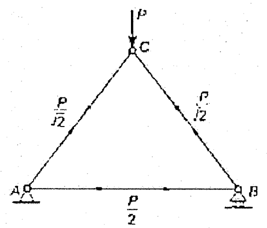
(b) $\sqrt{2}\,\dfrac{PL}{AE}$

(c) $(2\sqrt{2}+1)\dfrac{PL}{AE}$

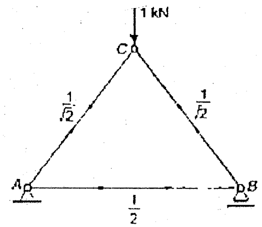
(d) $(\sqrt{2}+1)\dfrac{PL}{AE}$

*Ans. (a)*

It is evident from the diagram that all the interior members will carry zero forces.




P– system of forces

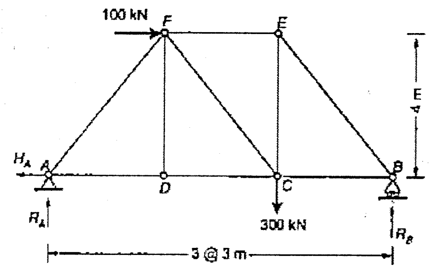



K– system of forces

Assuming tensile forces as positive and compressive forces as negative.

$$\therefore\quad \text{Deflection at } C,\qquad \Delta_C = \frac{\Sigma PKL}{AE} = \frac{(2\sqrt{2}+1)PL}{2AE}$$

**Example 3.24** Determine the horizontal displacement of roller support of the truss shown in figure. The cross-section of each member is 3000 mm². Take $E = 200$ kN/mm².




***Solution:***

Reactions:

$\Sigma F_x = 0;\qquad H_A = 100$ kN

$\Sigma F_y = 0;\qquad R_A + R_B = 300$ kN

$\Sigma M_B = 0;\qquad R_A\times9 + 100\times4 - 300\times3 = 0$

$$R_A = \frac{900-400}{9} = 55.55 \text{ kN}$$

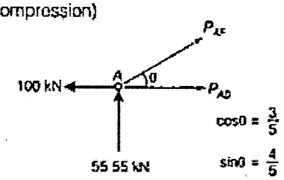
$$R_B = 244.45 \text{ kN}$$

P-system of Forces:

Joint $A$:

$\Sigma F_y = 0;\qquad P_{AF}\sin\theta + 55.55 = 0$

$$P_{AF} = -\frac{100}{\sin\theta} = -69.44 \text{ kN (Compression)}$$




$\Sigma F_x = 0;\qquad P_{AF}\cos\theta + P_{AD} = 100$

$P_{AD} = 100 - P_{AF}\cos\theta$

$P_{AD} = 100 + 69.44\times\dfrac{3}{5}$

$= 141.66$ kN (Tension)

Joint $D$:

Since member $AD$ and $DC$ are collinear. Hence,

$P_{DF} = 0$

and $\quad P_{DC} = P_{AD} = 141.66$ kN (Tension)

**Joint F:**

$\Sigma F_y = 0;$ $P_{FC}\sin\theta = 69.44\sin\theta$

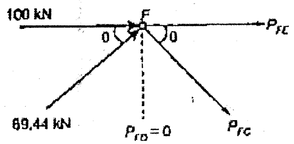
$P_{FC} = 69.44$ kN (Tension)

$\Sigma F_x = 0;$

$100 + 69.44\cos\theta + P_{FE} + P_{FC}\cos\theta = 0$

$$100 + 69.44\times\frac{3}{5} + P_{FE} + 69.44\times\frac{3}{5} = 0$$

$\therefore$ $P_{EF} = -183.33$ kN (Compression)




**Joint B:**

$\Sigma F_y = 0;$ $P_{BE}\sin\theta + 244.45 = 0$

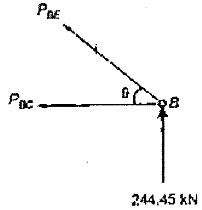
$$P_{BE} = -\frac{244.45}{\sin\theta}$$

$= -305.56$ (Compression)

$\Sigma F_x = 0;$ $P_{BC} + P_{BE}\cos\theta = 0$

$$P_{BC} - 305.56\times\frac{3}{5} = 0$$

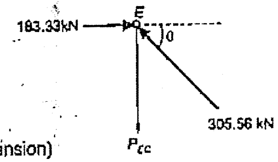
$\therefore$ $P_{BC} = 183.34$ kN (Tension)




**Joint E:**

$\Sigma F_x = 0;$ $P_{EC} = 305.56\sin\theta$

$$P_{EC} = 250\times\frac{4}{5} = 244.45 \text{ kN (Tension)}$$



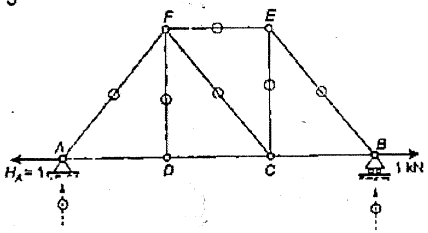

**K-system of Forces:**

Remove all external loading and apply horizontal unit load at roller support as shown in figure and final K-system of forces

It is clear that,

$K_{BC} = K_{CD} = K_{DA} = 1$ kN (Tension)

The other members will carry zero forces.

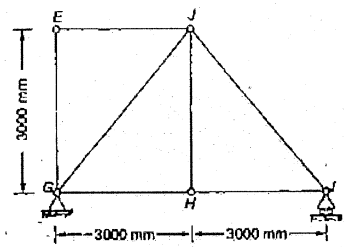



| Member | P (kN) | K (kN) | L(m) | PKL |
|---|---|---|---|---|
| AD | +141.66 | +1 | 3 | 424.98 |
| AF | −69.44 | 0 | 5 | 0 |
| FD | 0 | 0 | 4 | 0 |
| EF | −183.33 | 0 | 3 | 0 |
| FC | +69.44 | 0 | 5 | 0 |
| DC | +141.66 | +1 | 3 | 424.98 |
| CE | +244.45 | 0 | 4 | 0 |
| CB | +183.34 | +1 | 3 | 550.02 |
| EB | −305.56 | 0 | 5 | 0 |
| | | | | ΣPKL = 1399.98 |

$\therefore$ Horizontal movement of roller support is given by,

$$\Delta_{HB} = \sum\frac{PKL}{AE} = \frac{1399.98\times10^3}{3000\times200} = 2.33 \text{ mm}$$

**Example 3.25** The member *EJ* and *IJ* of a steel truss shown in the figure below are subjected to a temperature rise of 30°C. The coefficient of thermal expansion of steel is 0.000012 /°C per unit length. Determine the displacement (mm) of the joint *E* relative to joint *H* along the direction *HE* of the truss.




***Solution:***

Apply unit load at *E* in the direction of *HE* and find k-system of forces.

$\Sigma F_x = 0;$ $H_G = \frac{1}{\sqrt{2}} (\rightarrow)$

$\Sigma F_y = 0;$ $R_G + R_I + \frac{1}{\sqrt{2}} = 0$

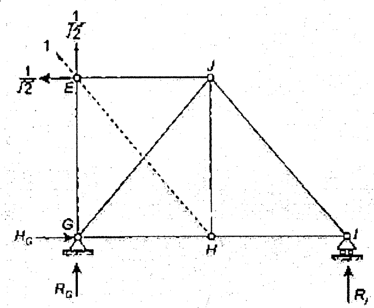
Also, $\Sigma M_G = 0;$ $R_I\times6 + \frac{1}{\sqrt{2}}\times3 = 0$

$\therefore$ $R_I = -\frac{3}{\sqrt{2}\times6}$

$\therefore$ $R_I = -\frac{1}{2\sqrt{2}}$

Hence, $R_G = -\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\therefore$ $R_G = -\frac{1}{2\sqrt{2}}$

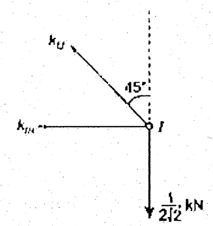



k-system of forces:

Joint *I*:

$\Sigma F_y = 0;$ $K_{IJ}\sin45° - \frac{1}{2\sqrt{2}} = 0$

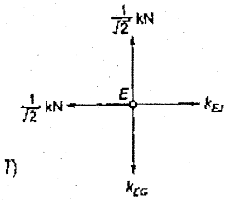
$\therefore$ $K_{IJ} = \frac{1}{2}$ kN (Tension)




Joint *E*:

$\Sigma F_x = 0;$ $K_{EJ} - \frac{1}{\sqrt{2}} = 0$

$\therefore \qquad K_{EJ} = \dfrac{1}{\sqrt{2}}$




The deflection of $E$ in the direction of $HE$ is given by

$$\Delta_E = \Sigma K_i\,(L_i\,\alpha\,T)$$

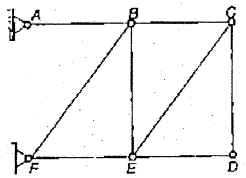
But only $EJ$ and $IJ$ are subjected to temperature change.

$$\therefore \quad \Delta_E = K_{EJ}\,(L_{EJ}\times\alpha\times T) + K_{IJ}\,(L_{IJ}\times\alpha\times T)$$

$$= \frac{1}{\sqrt{2}}\times(3000\times0.000012\times30)+\frac{1}{2}\times(3000\sqrt{2}\times0.000012\times30)$$

$$= 1.527 \text{ mm}$$

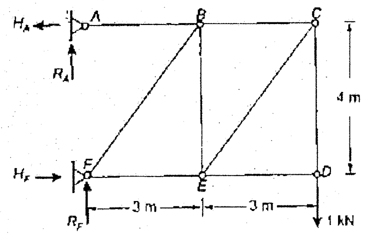
**Example 3.26** The three members of truss $AB$, $BC$ and $EF$ as shown in figure have been cut either too long or too short as given below. Calculate vertical displacement of joint $D$ due to discrepancy in the length of these members. All the members have same area cross-section.




$\delta_{AB} = -3$ mm, $\quad \delta_{BC} = 2$ mm, $\quad \delta_{EF} = 4$ mm

***Solution:***

Apply vertically downward unit load at $D$ and find K-system of forces.




Reactions:

$\Sigma F_x = 0; \qquad H_A = H_F \qquad \ldots(i)$

$\Sigma F_y = 0; \qquad R_A + R_F = 1 \qquad \ldots(ii)$

$MB = 0$ (Hinged)

$\therefore \qquad R_A \times 3 = 0$

$\therefore \qquad R_A = 0$

$\therefore \qquad R_F = 1$ kN (↑)

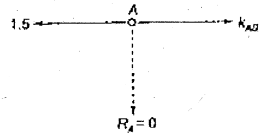
Also, $\Sigma M_F = 0; \quad -H_A \times 4 + 1 \times 6 = 0$

$\Rightarrow \qquad H_A = 1.5$ kN

Hence, $\qquad H_F = +1.5$ kN

K-system of Forces:

Consider joint $A$:

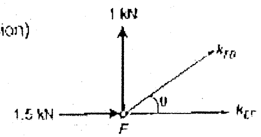



$\Sigma F_y = 0, \qquad K_{AB} = 1.5$ kN (Tension)

Consider joint $F$:

$\Sigma F_y = 0; \qquad 1 + K_{FB}\sin\theta = 0$

$$K_{FB} = -\frac{1}{\sin\theta} = -1.25 \text{ kN} \quad \text{(Compression)}$$

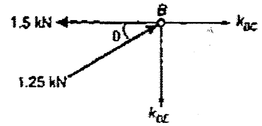



$\Sigma F_x = 0; \qquad 1.5 + K_{EF} + K_{FB}\cos\theta = 0$

$$1.5 + K_{EF} - 1.25\times\frac{3}{5} = 0$$

$\therefore \qquad K_{EF} = -0.75$ kN (Compression)

Consider joint $B$:




$\Sigma F_x = 0; \quad -1.5 + K_{BC} + 1.25\cos\theta = 0$

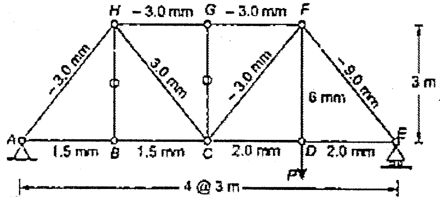
$K_{BC} = +0.75$ kN (Tension)

| Member | K (kN) | Δ (mm) | KΔ |
|---|---|---|---|
| *AB* | +1.5 | −3 | −4.5 |
| *BC* | +0.75 | +2 | 1.5 |
| *EF* | −0.75 | +4 | −3 |
| Other | – | 0 | 0 |
| | | | ΣKΔ = −6.0 |

Deflection of joint $D$, $\qquad \Delta_D = \sum_{i=1}^{n} K_i\,\Delta_i$

$\Delta_C = -6.0$ mm

**Example 3.27** Compute the vertical deflection of joint $C$ due to a point load $P$ acts at $D$. The actual changes in length due to point load $P$ are shown.




***Solution:***

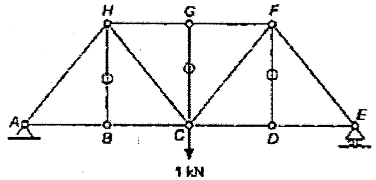
We know deflection of truss joint is given by,

$$\delta = \sum_{i=1}^{n} K\Delta$$

where, $\Delta$ = Axial deformations in members

$K$ = Forces in members due to unit load at that joint where deflection is required

$\therefore$ Remove load $P$ and apply unit load at $C$ and find K-system of forces.




By symmetry, $\qquad R_A = R_E = 0.5$ kN (↑)

Joint $A$:

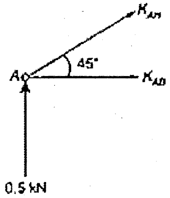
$\Sigma F_y = 0;$

$$K_{AH}\sin45^\circ + 0.5 = 0$$

$$\therefore \quad K_{AH} = -\frac{0.5}{\sin45^\circ} = -0.5\sqrt{2} \text{ kN} \quad \text{(Compression)}$$

$\Sigma F_x = 0, \qquad K_{AB} + K_{AH}\cos45^\circ = 0$

$$K_{AB} = -K_{AH}\cos 45° = +0.5\sqrt{2} \times \frac{1}{\sqrt{2}}$$




$\therefore$ $K_{AB} = +0.5$ kN (Tension)

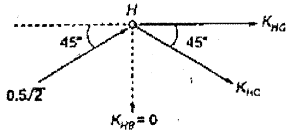
**Joint B:**

Since member $AB$ and $BC$ are collinear.

Hence, $K_{BH} = 0$

and $K_{BC} = K_{AB} = +0.5$ kN (Tension)

**Joint H:**




$\Sigma F_y = 0$; $K_{HC} = +0.5\sqrt{2}$ kN (Tension)

$\Sigma F_x = 0$;

$$K_{HG} + K_{HC}\cos 45° + 0.5\sqrt{2}\cos 45° = 0$$

$$K_{HG} + 0.5\sqrt{2} \times \frac{1}{\sqrt{2}} + 0.5\sqrt{2} \times \frac{1}{\sqrt{2}} = 0$$

$$K_{HG} + 0.5 + 0.5 = 0$$

$\Rightarrow$ $K_{HG} = -1$ kN (Compression)

By symmetry,

$K_{GF} = K_{HG} = -1$ kN (Compression) $\quad$ $K_{FC} = K_{HC} = +0.5\sqrt{2}$ kN (Tension)

$K_{ED} = K_{AB} = +0.5$ kN (Tension) $\quad$ $K_{CD} = K_{DE} = 0.5$ kN (Tension)

| Member | Δ (mm) | K (kN) | KΔ |
|---|---|---|---|
| AB | +1.50 | +0.5 | 0.75 |
| BC | +1.50 | +0.5 | 0.75 |
| CD | +2.00 | +0.5 | 1.00 |
| DE | +2.00 | +0.5 | 1.00 |
| EF | −9.00 | $-0.5\sqrt{2}$ | 6.364 |
| FG | −3.00 | −1.0 | 3.0 |
| GH | −3.00 | −1.0 | 3.0 |
| HA | −3.00 | $-0.5\sqrt{2}$ | 2.122 |
| HB | 0 | 0 | 0 |
| GC | 0 | 0 | 0 |
| FD | +6.00 | 0 | 0 |
| HC | +3.00 | $0.5\sqrt{2}$ | 2.122 |
| FC | −3.00 | $0.5\sqrt{2}$ | −2.122 |
| | | | ΣKΔ = 17.986 |

$\therefore$ Deflection of join $C$ is,

$$\Delta_C = \Sigma K\Delta = 17.986 \text{ mm}$$

## 3.8 Influence Line Diagram for Trusses

The influence line diagram unit load more from one end to other. To obtain ILD the whole truss is divided into three zones.

Zone-1: From left support to the joint just left to the section i.e. $L_1L_2$.

Zone-2: That panel of truss in which section lies.

Zone-3: From the just to the right of section to the right support i.e. $L_3L_5$.

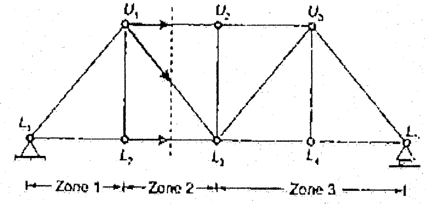


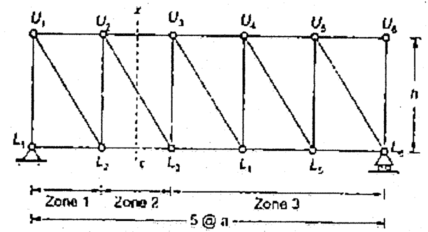

*Fig. 3.14*

**Procedure to Draw ILD**

Step-1: Find the support reaction as a function of position of unit load i.e. $x$.

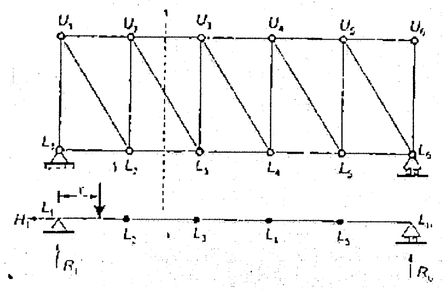
Step-2: When load is on left side of section (within zone-1), then consider right portion of truss and find value of member force by considering equilibrium of that portion. If load is on right side of section (within zone-3), then consider equilibrium of left portion of truss.

Step-3: The influence line when the unit load is within zone-2 is obtained by joining the ordinates of ILD at the end of zone 1 to the ordinate of ILD at zone-3 by straight line.




ILD for $U_2U_3$ (Upper Chord Member), $U_2L_3$ (Incline Member), and $L_2L_3$ (Lower Chord Member):

**Case-1: When unit load in zone 1**




$\Sigma F_x = 0$; $\quad H_1 = 0$

$\Sigma F_y = 0$; $\quad R_1 + R_6 = 1 \quad$ ...(a)

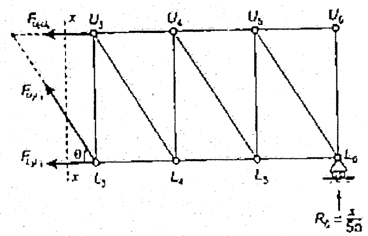
Also, $\Sigma M_6 = 0$; $\quad R_1 \times L - 1(L-x) = 0$

∴ $R_1 = \frac{(L-x)}{L} = \frac{(5a-x)}{5a}$ (Linear)

Hence from equation (a), we get,

$R_6 = \frac{x}{L} = \frac{x}{5a}$ (Linear)

Since unit load is in zone 1. Therefore consider right portion of truss.




ILD for $F_{U_2U_3}$

To find $F_{U_2U_3}$ take $\Sigma M_{L_3} = 0$

$-R_6 \times (L_3L_6) - F_{U_2U_3} \times (h) = 0$

∴ $F_{U_2U_3} = -R_6 \times \frac{(L_3L_6)}{h} = -\left(\frac{x}{5a}\right) \times \frac{3a}{h}$

∴ $F_{U_2U_3} = -\frac{3x}{5h}$ ...(i)

It is also noted that, $R_6 \times (L_3L_6)$ is BM at $L_3$.

∴ $F_{U_2U_3} = -\frac{M_{L_3}}{h}$ (Compression)

From equation (i),

if unit load at $L_1$ i.e. $x = 0$, then $F_{U_2U_3} = 0$

if unit load at $L_2$ i.e. $x = a$, then $F_{U_2U_3} = -\frac{3a}{5h}$ (Compression)

ILD for $F_{L_2L_3}$ and $F_{U_2L_3}$:

Resolve $F_{U_2L_3}$ in horizontal and vertical direction.

Horizontal component of $F_{U_2L_3} = F_{U_2L_3}\cos\theta(\leftarrow)$

Vertical component of $F_{U_2L_3} = F_{U_2L_3}\sin\theta(\uparrow)$

To find $F_{U_2L_3}$ take,

$\Sigma F_y = 0$

$F_{U_2L_3}\sin\theta + R_6 = 0$

∴ $F_{U_2L_3}\sin\theta = -R_6$

$F_{U_2L_3} = -\frac{R_6}{\sin\theta} = -\frac{x}{5a\sin\theta}$

∴ $F_{U_2L_3} = -\frac{x\,\text{cosec}\,\theta}{5}$

if unit load is at $L_1$, then $F_{U_2L_3} = 0$

if unit load is at $L_2$, then $F_{U_2L_3} = -\frac{a\,\text{cosec}\,\theta}{5a} = -\frac{\text{cosec}\,\theta}{5}$ (Compression)

For ILD of $F_{L_2L_3}$ take,

$\Sigma M_{U_2} = 0$

$F_{L_2L_3} \times (h) - R_6 \times (L_2L_6) = 0$

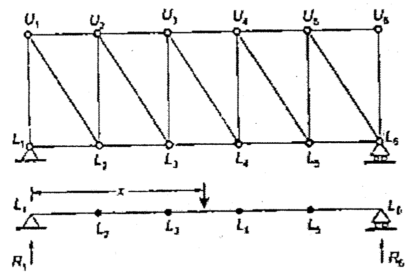
$F_{L_2L_3} = \frac{R_6 \times (L_2L_6)}{h} = +\frac{M_{L_2}}{h}$ (Tension)

$= \frac{\left(\frac{x}{5a}\right) \times 4a}{h} = \frac{4x}{5h}$ (Linear)

if unit load at $L_1$ i.e. $x = 0$, then $F_{L_2L_3} = 0$

if unit load at $L_2$ i.e. $x = a$, then $F_{L_2L_3} = +\frac{4a}{5h}$ (Tension)

Case-2: When unit load in zone 3




$R_1 + R_6 = 1$

Also, $\Sigma M_6 = 0$

∴ $R_1 \times (5a) - 1 \times (5a - x) = 0$

$R_1 = \frac{(5a-x)}{5a}$ (Tension)

If unit load at $L_3$ i.e. $x = 2a$, then $F_{L_2L_3} = \frac{3a}{5h}$ (Tension)

if unit load at $L_6$ i.e. $x = 5a$, then $F_{L_2L_3} = 0$

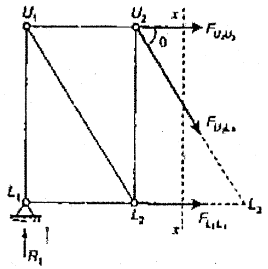
ILD for $F_{U_2U_3}$ and $F_{U_2L_3}$:

Resolve $F_{U_2L_3}$ in horizontal and vertical direction.

Horizontal component of $F_{U_2L_3} = F_{U_2L_3}\cos\theta\,(\rightarrow)$

Vertical component of $F_{U_2L_3} = F_{U_2L_3}\sin\theta\,(\downarrow)$

To find $F_{U_2U_3}$ take,




$$\Sigma M_{L_3} = 0$$

$$R_1 \times 2a + F_{U_2U_3} \times (h) = 0$$

$$F_{U_2U_3} = -\frac{R_1 \times 2a}{h} = -\frac{M_{L_3}}{h}$$

$$\therefore \quad F_{U_2U_3} = -\frac{(5a-x)\cdot 2}{5h} \quad \text{(Compression)}$$

If unit load is at $L_3$ i.e. $x = 2a$, then $F_{U_2U_3} = -\frac{6a}{5h}$ (Compression)

If unit load is at $L_6$ i.e. $x = 5a$, then $F_{U_2U_3} = 0$

To find $F_{U_2L_3}$ take,

$$\Sigma F_y = 0$$

$$\therefore \quad F_{U_2L_3}\sin\theta = R_1$$

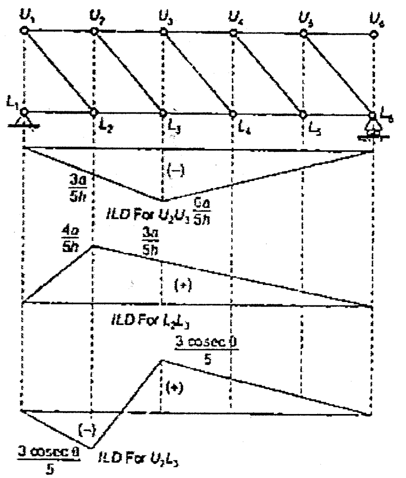
$$F_{U_2L_3} = \frac{(5a-x)}{5a(\sin\theta)}$$

$$F_{U_2L_3} = \frac{(5a-x)\,\text{cosec}\theta}{5a} \quad \text{(Tension)}$$

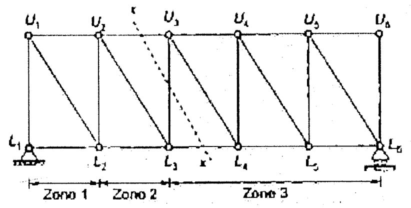
If unit load is at $L_3$ i.e. $x = 2a$, then

$$F_{U_2L_3} = \frac{3\,\text{cosec}\theta}{5} \quad \text{(Tension)}$$

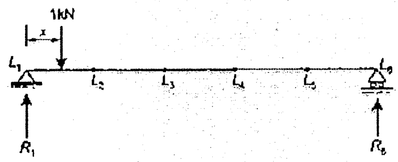
If unit load is at $L_6$ i.e. $x = 5a$, then $F_{U_2L_3} = 0$




ILD for $F_{U_3L_3}$ (Vertical Member):




Case-1: When unit load is in zone 1

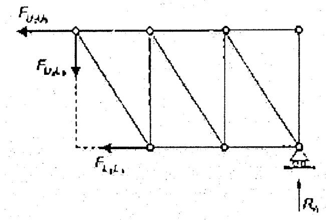



$$R_1 + R_6 = 1$$

Also, $\Sigma M_6 = 0$: $\quad R_1 \times 5a - 1\,(5a - x) = 0$

$$\therefore \quad R_1 = \frac{(5a-x)}{5a}$$

Hence, $\quad R_6 = \frac{x}{5a}$




Since unit load is in zone 1.

Hence consider right portion of truss.

To find $F_{U_3L_3}$ take,

$$\Sigma F_y = 0$$

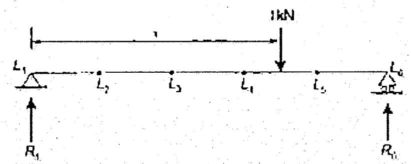
$$\therefore \quad R_6 - F_{U_3L_3} = 0$$

$$\therefore \quad F_{U_3L_3} = R_6 = \frac{x}{5a} \quad \text{(Linear)}$$

If unit load is at $L_1$ i.e. $x = 0$, then $F_{U_3L_3} = 0$

If unit load is at $L_2$ i.e. $x = 0$, then $F_{U_3L_3} = \frac{1}{5}$ (Tension)

Case-2: When unit load is in zone 3




$$\therefore \quad R_1 = \frac{5a-x}{5a}$$

$$R_6 = \frac{x}{5a}$$

Since unit load is in zone 3.

Hence consider left portion of truss.

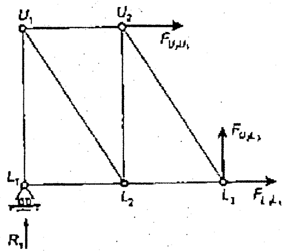
To find $F_{U_3L_3}$ take,

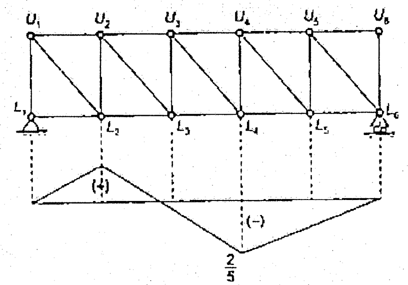
$$\therefore \quad R_1 + F_{U_3L_3} = 0$$

$$\therefore \quad F_{U_3L_3} = -R_1 = -\frac{(5a - x)}{5a} \quad \text{(Linear)}$$

If unit load is at $L_3$ i.e. $x = 3a$, then $F_{U_3L_3} = -\frac{2}{5}$ (Compression)

If unit load is at $L_5$ i.e. $x = 5a$, then $F_{U_3L_3} = 0$



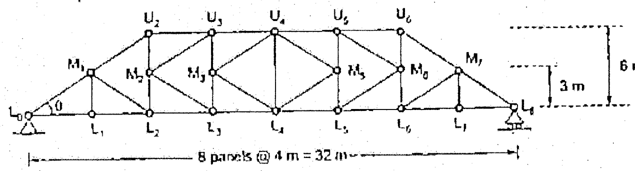

ILD for $FU_3L_3$ (Vertical member)

**Example 3.28** A K-truss consisting of 8 panels of 4 m each is as shown in the figure below. Given, when the unit load is at $L_2$, $L_3$ and $L_4$ then the force in the member $M_2U_3$ is 0.028 kN (tensile), 0.52 kN (compression) and 0.416 kN (compression) respectively.

Use sign convention as mentioned below:

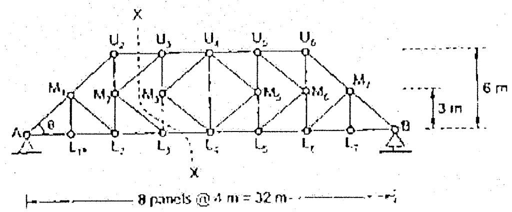
Positive → tension

Negative → compression




Draw the influence line diagram for force in member $M_3L_3$.

*Solution:*




Pass a section X-X. Cutting members $U_2U_3$, $M_2U_3$, $M_3L_3$ and $L_3L_4$ and consider the equilibrium of the left portion.

(i) When the unit load is at $L_2$

$$R_A = \frac{1 \times 24}{32} = 0.75 \text{ kN and } P_{M_2U_3} = 0.208 \text{ kN (tension)}$$

$$\therefore \quad P_{M_2U_3}\sin\theta + P_{M_3L_3} = 1 - R_A = 1 - 0.75 = 0.25 \text{ kN}$$

$$\therefore \quad P_{M_3L_3} = 0.25 - P_{M_2U_3}\sin\theta = 0.25 - (0.208 \times 0.6) = 0.125 \text{ kN (tension)}$$

(ii) When the unit load is at $L_3$

$$R_A = \frac{1 \times 20}{32} = 0.625 \text{ and } P_{M_2U_3} = 0.52 \text{ kN (compression)}$$

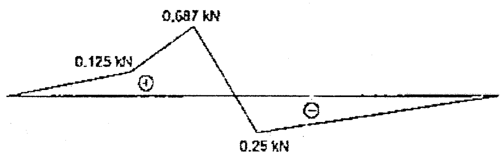
$$\therefore \quad -P_{M_2U_3}\sin\theta + P_{M_3L_3} = 1 - R_A = 1 - 0.625 = 0.375 \text{ kN}$$

$$\text{or} \quad P_{M_3L_3} = 0.375 + (0.52 \times 0.6) = 0.687 \text{ kN (tension)}$$

(iii) When the unit load is at $L_4$

$$R_A = \frac{1 \times 16}{32} = 0.5 \text{ and } P_{M_2U_3} = 0.416 \text{ kN (compression)}$$

$$P_{M_3L_3} = R_A - P_{M_2U_3}\sin\theta = 0.5 - (0.416 \times 0.6) = 0.25 \text{ kN (compression)}$$

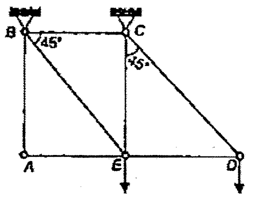



**Summary**

- A simple truss is composed from basic triangular units.
- A compound truss is formed when two or more simple trusses are joined either by hinge or additional members.
- A plane truss is externally stable if minimum three independent support reactions are available which are nonparallel and concurrent.
- A truss is internally stable is minimum number of member are equals to $m = 2j - 3$.
- To analyse statically determinate truss there are two methods:
  (i) methods of section
  (ii) method of joint
- If at a joint three members meet, out of three two are collinear then third member will carry zero force. If there is no force at joint
- If at a joint two members meet and no external load is there then both member will carry zero force.

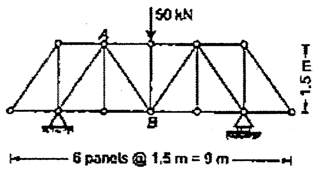
## Objective Brain Teasers

**Q.1** The cantilever frame shown in the given figure is supported by vertical links at *B* and *C* and carries loads as shown. The force in the bar *AE* is



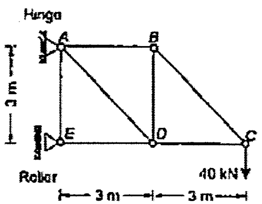

(a) 500 kg (b) 1000 kg
(c) zero (d) 2500 kg

**Q.2** The force in the member *AB* of the truss shown in the given figure is



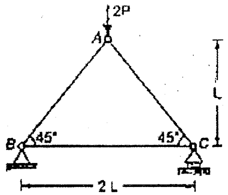

(a) 25 kN compression
(b) $25\sqrt{2}$ kN tension
(c) $25\sqrt{2}$ kN compression
(d) 25 kN tension

**Q.3** The pin-jointed cantilever truss is loaded as shown in the given figure. The force in member *ED* is



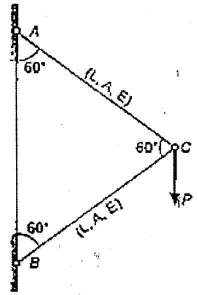

(a) 40 kN (Compressive)
(b) 80 kN (Tensile)
(c) 80 kN (Compressive)
(d) 120 kN (Compressive)

**Q.4** A simple plane truss acted upon by a load 2*P* at the apex *A* is shown below. The axial force in the member *AB* is



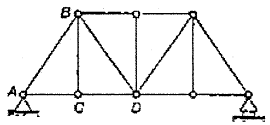

(a) $P$ (b) $\sqrt{2}P$
(c) $\frac{\sqrt{3}}{2}P$ (d) $\sqrt{3}P$

**Q.5** What is the vertical deflection of joint *C* of the frame shown below?

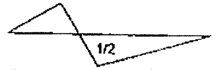



(a) *PL/AE* (b) 2 *PL/AE*
(c) *PL/2AE* (d) 3 *PL/AE*

**Q.6** The given figure shows a portal truss:




The influence line for force in member *BC* will be

(a)



(b)



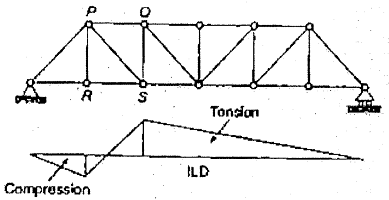
(c)



(d)

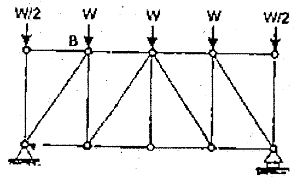


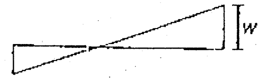
**Q.7** The influence line diagram (ILD) shown is for the member

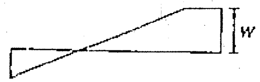


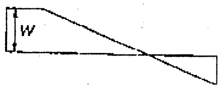

(a) *PS* (b) *RS*
(c) *PQ* (d) *QS*

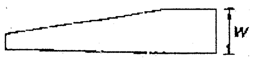
**Q.8** The correct shape of the force polygon for the joint *B* of the truss shown in the given figure is



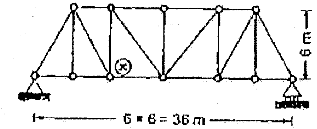

(a)



(b)



(c)


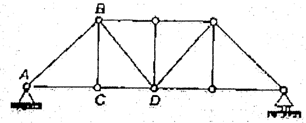

(d)



**Q.9** What is the maximum ordinate for influence line for the force in the member marked *X*?




(a) 1.0 (b) 1.33
(c) 1.50 (d) 2.50

**Q.10** The influence line for force in member *BC* is

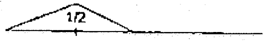


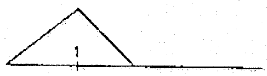

(a)



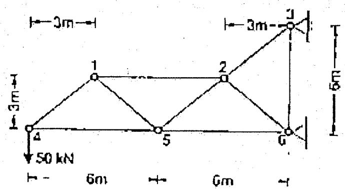
(b)



(c)


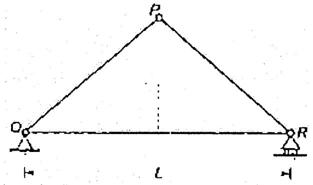

(d)



**Q.11** Axial forces in the members 1 – 2 and 1 – 5 of the truss shown in the given figure are respectively




(a) 50 kN (compressive) and 25 kN (tensile)
(b) 25 kN (tensile) and $50/\sqrt{2}$ kN (compressive)
(c) 100 kN (tensile) and $50\sqrt{2}$ (compressive)
(d) 25 kN (compressive) and $50/\sqrt{2}$ (tensile)

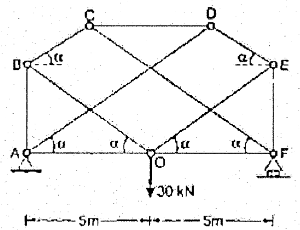
**Q.12** A statically determinate truss *PQR* is subjected to a temperature rise $\Delta T$. *A*, *E* and $\alpha$ are constant for all members. The force in member *QR* due to this temperature increase is




(a) $EA\alpha DT$ (b) $\sqrt{2}EA\alpha\Delta T$

(c) $EA\Delta T/L$ (d) None of these

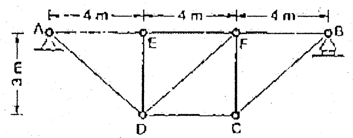
**Q.13** Axial force in the member *BC* of the truss shown in the given figure is (where $\alpha = 30°$)




(a) 15 kN (b) $10\sqrt{3}$kN

(c) $15\sqrt{3}$kN (d) 30 kN

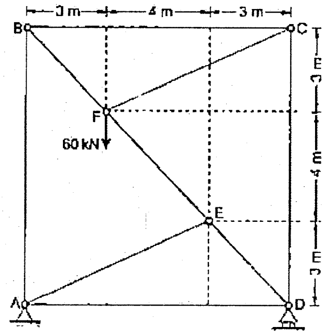
**Q.14** The vertical deflection of the joint 'C' of the frame shown in figure due to temperature rise of 60°F in the upper chord members only is (Given, the coefficient of expansion = $6 \times 10^{-6}$ per 1°F and $E = 2 \times 10^{-6}$ kg/cm²)




(a) 0.367 cm (↑) (b) 0.896 cm (↓)

(c) 0.256 cm (↑) (d) 0.901 cm (↓)

**Q.15** Consider the given truss shown below:

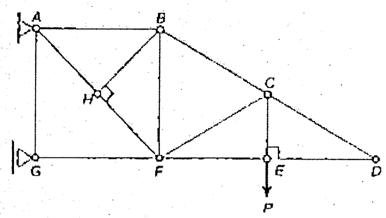



The number of zero force members in the given truss is

(a) 2 (b) 3

(c) 4 (d) 1

**Q.16** A pin-jointed frame is shown in the figure below. The number of member(s) that have zero force is/are




(a) 6 (b) 4

(c) 5 (d) 3

**Answers**

| | | | | |
|---|---|---|---|---|
| 1. (c) | 2. (b) | 3. (c) | 4. (b) | 5. (b) |
| 6. (c) | 7. (a) | 8. (a) | 9. (b) | 10. (d) |
| 11. (c) | 12. (d) | 13. (d) | 14. (c) | 15. (c) |
| 16. (c) | | | | |

## Hints and Explanations:

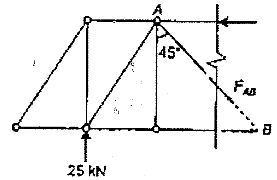
**1. (c)**

There is no force at joint *A*, so forces in members *AB* and *AE* will be zero.

**2. (b)**

The reaction at left support = 25 kN

Cutting a section through member *AB* and other horizontal members of the same panel and considering left part of the cut section. The force in the member *AB* will be tensile.




$\Sigma F_y = 0$;

$F_{AB} \times \cos 45° = 25$ kN

$\therefore$ $F_{AB} = 25\sqrt{2}$kN (tension)

**3. (c)**

Reaction at *E* is

$H_E \times 3 = 40 \times 6$

$\Rightarrow$ $H_E = 80$ kN

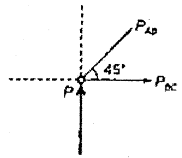
So force in member *ED* is 80 kN compressive.

**4. (b)**

Vertical Reaction at

$$B = \frac{2P}{2} = P$$

Considering joint *B*




$\Sigma F_y = 0$;

$\Rightarrow P_{AB} \sin 45° + P = 0$

$\Rightarrow$ $P_{AB} = -\sqrt{2}P$ (Compression)

**5. (b)**

Applying equilibrium equations at *C*.

$\Sigma F_x = 0$

$\Rightarrow -P_{AC}\cos 30° + P_{BC}\cos 30° = 0$

[Assuming *AC* in tension and *BC* in compression]

$\Rightarrow$ $P_{AC} = P_{BC}$ ...(i)

and $\Sigma F_y = 0$

$\Rightarrow P_{AC}\sin 30° + P_{BC}\sin 30° = P$

[From (i) $P_{AC} = P_{BC}$]

$\Rightarrow$ $2P_{AC}\sin 30° = P$

$\Rightarrow$ $P_{AC} = P$

Strain energy stored in AC,

$$U_{AC} = \frac{S^2L}{2AE} = \frac{P^2L}{2AE}$$

Strain energy stored in BC,

$$U_{BC} = \frac{S^2L}{2AE} = \frac{P^2L}{2AE}$$

$\therefore$ $U = U_{AC} + U_{BC}$

$\Rightarrow$ $U = \frac{P^2L}{2AE} + \frac{P^2L}{2AE}$

$\Rightarrow$ $U = \frac{P^2L}{AE}$

Vertical deflection of joint *C*,

$$\delta_v = \frac{\partial U}{\partial P} = \frac{2PL}{AE}$$

**6. (c)**

Force in member *BC*, when the load is right of *D*, will be zero. When the load is between *A* and *D*, the tensile force with maximum value of 1 unit will develop. So correct diagram is as shown in figure (c).

**8. (a)**

The force polygon will represent all forces at joint *B*. There are 4 members and load *W* at joint *B*. However force in horizontal member left of *B* is zero. Thus there are 3 members having non-zero force. So (a) is the force polygon showing 4 forces. Other figures show 5 forces.

**10. (d)**

When the unit load is anywhere on the right side of the joint *D*, no load will be transmitted to the joint *C*. Hence for this position of the unit load, there will be no force in the member *BC*. When the unit load is exactly at *C* the force in member *BC* will be equal to 1 (tensile). When the unit load is exactly at *A*, then also no load is transmitted to the joint *C* and hence for this position of the load also, there will be no force in the member *BC*.

**11. (c)**

Cutting a section through 1–2; 2–5; and 5–6. Taking moment of left part about 5.

$$F_{1-2} = \frac{50 \times 6}{3} = 100 \text{ kN (Tensile)}$$

Cut a section through 1–2, 1–5 and 4–5 and balance vertical force for left part only

$$\frac{F_{1-5}}{\sqrt{2}} = 50$$

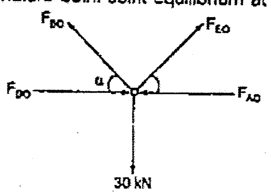
$$\therefore F_{1-5} = 50\sqrt{2} kN \text{ (Compressive)}$$

**12. (d)**

In a determinate structure no stresses or forces are generated due to temperature change.

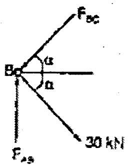
**13. (d)**

The given truss is symmetrical. So force in member $BO$ and $EO$ will be same in magnitude and nature both. Joint equilibrium at $O$.




$$F_{bo} = \frac{15}{\sin 30^\circ} = 30 \text{kN}$$

Joint equilibrium at $B$.




$$\therefore F_{BC} = 30 \text{ kN}$$

**14. (c)**

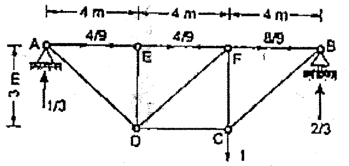
The vertical deflection of $C$ is.

$$\delta_v = \Sigma(k\Delta_i + L\alpha T)$$

Here, $\Delta_i = 0$ for all members

$\therefore$ $\delta_v = \Sigma k(L\alpha T)$ ...(i)

As the temperature rise is only in upper chord members, therefore change in length in all other member is zero




From eq. (i),

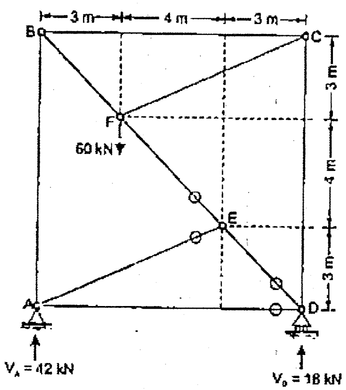
$$\delta_v = -\frac{4}{9} \times 400 \times 6 \times 10^{-6} \times 60$$

$$+\left(-\frac{4}{9} \times 400 \times 6 \times 10^{-6} \times 60\right)$$

$$+\left(-\frac{8}{9} \times 400 \times 6 \times 10^{-6} \times 60\right)$$

$$= -0.256 \text{ cm } (\downarrow) = 0.256 \text{ cm } (\uparrow)$$

**15. (c)**




Total number of members whose carrying zero forces are namely $AE$, $AD$, $DE$, $EF$ i.e., four.

**16. (c)**

At joint $D$, there is no external force and member are not collinear, so forces in both members will be 0.

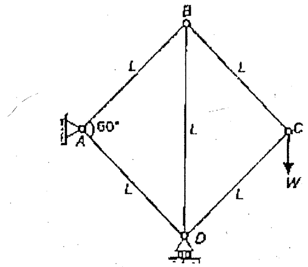
At joint $E$, force in member $EF$ will be 0 as force in member $ED$ is 0.

At joint $H$, force in member $HB$ will be 0.

At joint $G$, there is no vertical reaction so force in the member $GA$ will be 0.

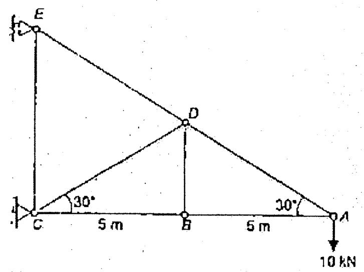
## Conventional Practice Questions

**Q.1** Find forces in all members of the truss shown in figure.



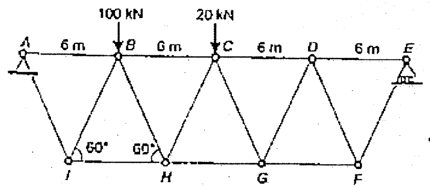

Ans. $P_{CB} = W$ (Tension), $P_{BD} = -W$ (Compression), $P_{DA} = -W$ (Compression), $P_{AB} = W$ (Tension)

**Q.3** Determine the deflection of the joint A of the truss shown in figure. Take the stress in tension members as 140 N/mm² and in compression member as 90 N/mm². Take $E = 2.0 \times 10^5$ N/mm².



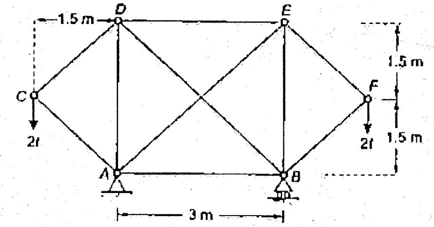

Ans. 26.55

**Q.2** Find forces in all members of the truss shown in figure.



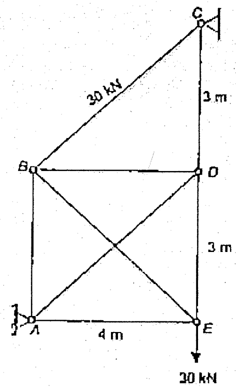

Ans. $F_{AB} = -10.1$ kN, $F_{AI} = +20$ kN, $F_{IB} = -20.20$ kN, $F_{IH} = +20.20$ kN, $F_{BH} = 8.66$ kN, $F_{BC} = -24.54$ kN, $F_{HC} = -8.66$ kN, $F_{HG} = +28.87$ kN, $F_{CG} = -14.43$ kN, $F_{DG} = +14.43$ kN, $F_{DF} = -14.43$ kN, $F_{CD} = -21.65$ kN, $F_{GF} = +14.43$ kN, $F_{DE} = -7.22$ kN

**Q.4** The pin-jointed truss shown in figure below is simply supported at $A$ and $B$ and carries load as shown. All members have a cross-sectional area of 10 cm² and the members are initially unstressed. Calculate vertical deflection of joint $C$ if $E = 1900$ t/cm².




Ans. 0.365 mm (↓)

**Q.5** Determine all member forces for truss shown in figure.




Ans. $S_{AB} = -22.505$ kN, $F_{AD} = -12.492$ kN, $F_{BD} = -25$ kN, $F_{CD} = +15$ kN, $F_{DE} = 22.495$ kN, $F_{AE} = -10.00$ kN, $F_{BC} = -25$ kN, $F_{BE} = 12.5$kN