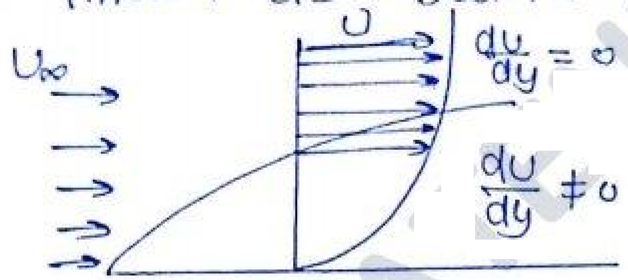
Boundary layer Theory (Not in ESE)

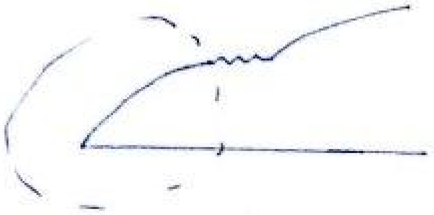
When viscous fluid flows over the solid surface near the surface large velocity gradient occurs the ~~reason~~ region where large velocity gradient occur known as boundary layer region.

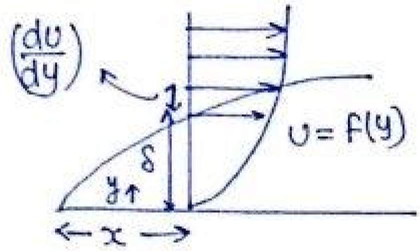


Factor affecting transition from laminar to turbulant:-

① Velocity of flow
② Viscosity of fluid.
③ density of the Fluid
④ Geometry of surface
⑤ surface roughness.
⑥ Pressure gradient $\left(\frac{\partial P}{\partial x}\right)$
⑦ External disturbance
⑧ Temp. of fluid or plate. $\left(\mu \propto \frac{1}{T}\right)$

Boundary conditions for laminar flow over flat plate:-





$x = 0, \quad y = 0 \rightarrow$ Not for turb

(i) $y = 0 \qquad u = 0$

(ii) $y = \delta \qquad u = U_\infty$

(iii) $y = \delta \qquad \frac{du}{dy} = 0$

(iv) $y = 0 \qquad \frac{d^2u}{dy^2} = 0$

$y = 0 \qquad \tau = \tau_{max}$

$$\tau = \mu \frac{du}{dy}$$

$$\frac{d\tau}{dy} = \mu \frac{d^2u}{dy^2}$$

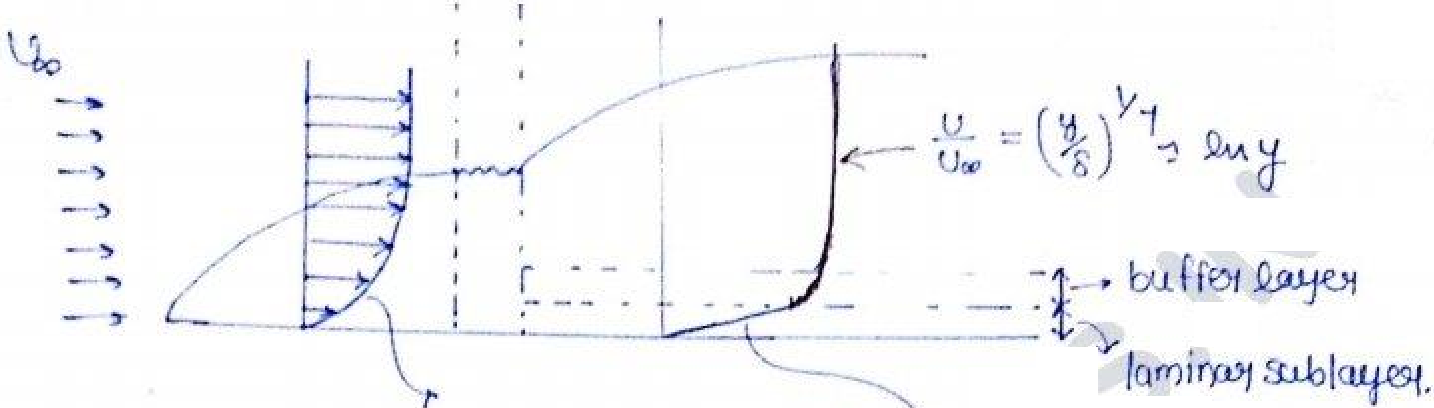
$$\text{max} \quad \frac{d\tau}{dy} = 0, \quad \frac{d^2u}{dy^2} = 0$$

$$\frac{u}{U_\infty} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2 + d\left(\frac{y}{\delta}\right)^3$$

using boundry Condition

$a = 0, \quad b = 3/2, \quad c = 0, \quad d = -\frac{1}{2}$

$$\boxed{\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3}$$



$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

linear
velocity profile.
(~~Petu~~ In actual it is parabolic but due to less thickness assumed linear.)

<u>laminar</u>

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}$$

$$\tau_w = \mu \cdot U_\infty \cdot \frac{3}{2} \cdot \frac{1}{\delta}$$

$$\tau_w \propto \frac{1}{\delta} \qquad x\uparrow \rightarrow \delta\uparrow \rightarrow \tau_w \downarrow$$

<u>Turbulent</u>

$$u = U_\infty \left(\frac{y}{\delta}\right)^{1/7} \qquad \tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}$$

$$\tau_w = \mu \cdot \frac{U_\infty}{(\delta)^{1/7}} \cdot \frac{1}{7} \left.\frac{1}{(y)^{6/7}}\right|_{y=0}$$

$$\text{at } y \rightarrow 0,\ \tau_w \rightarrow \infty$$

So it is not possible at wall
bcoz $\tau_w \rightarrow \underline{\infty}$

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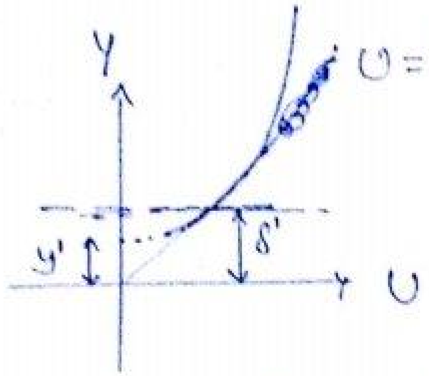


$y = y'$, $U = 0$

$C = ?$

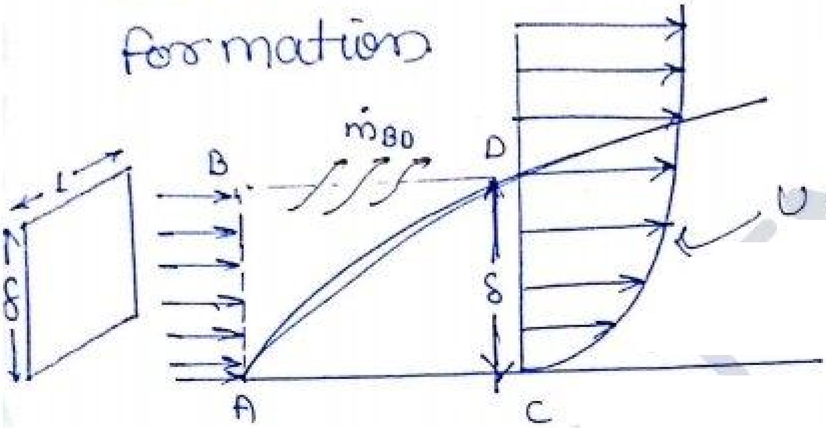
Pipe Flow

Smooth pipe	Rough pipe.
$y' = \frac{\delta'}{107}$	$y' = \frac{k}{30}$
$y' = \frac{11.6\,\nu}{107\,V_*}$	$y' = \frac{k}{30}$

In laminar sub-layer the velocity profile is parabolic but the thickness of laminar Sublayer is very less so for all practicle purpose it assumed to be linear this profile is tengent to (1/7) power law profile.

Displacement thickness:-

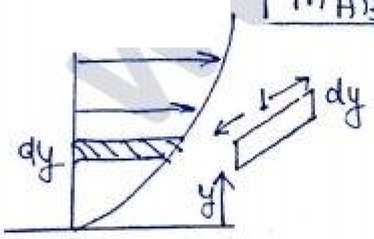
it is the distance by which the boundary should be displaced in order to compensate the loss in mass flow rate due to boundary layer formation



$$\dot{m}_{AB} = \rho A U_\infty \qquad \dot{m}_{BD} = \dot{m}_{AB} - \dot{m}_{DC}$$

$$m_{AB} = \rho(\delta \times 1) U_\infty$$

$$\boxed{\dot{m}_{AB} = \rho \delta U_\infty}$$

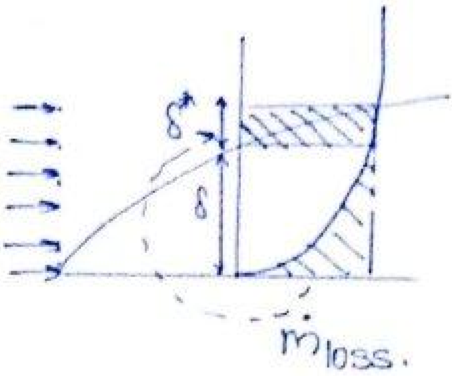


$$d\dot{m} = \rho dA \cdot U$$

$$d\dot{m} = \rho \int_0^\delta u \cdot (dy \times 1)$$

$$* \boxed{\dot{m}_{DC} = \rho \int_0^\delta u dy}$$

$$\dot{m}_{BD} = \rho \delta U_\infty - \rho \int_0^\delta u dy$$



$$\dot m_{loss} = \rho(\delta^* \times 1) U_\infty$$

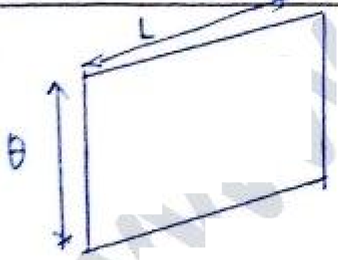
$$\dot m_{loss} = \rho \delta^* U_\infty$$

$$\rho \delta^* U_\infty = \rho \delta U_\infty - \rho \int_0^y u\, dy \Rightarrow \delta^* = \int_0^\delta dy - \int_0^\delta \frac{u}{U_\infty} dy$$

$$\delta^* = \delta - \int_0 \frac{u\,dy}{U_\infty} \qquad \int_0^\delta dy$$

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy}$$ Displacement thickness.

Momentum thickness:-



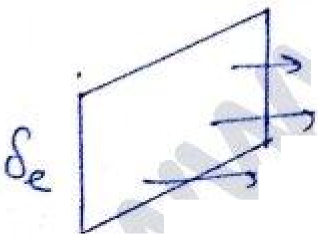
$$\dot p = \dot m V$$

$$\dot p = (\rho A V) \cdot V$$

$$\dot p_{loss} = \rho(\theta \times 1) U_\infty^2$$

$$\rho\, \cancel{\theta U_\infty^2} = \boxed{\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy}$$

Kinetic energy thickness.



$$KE = \frac{1}{2}(\rho A V) V^2$$

$$(KE)_{loss} = \frac{1}{2}\rho(\delta_e \times 1) U_\infty^3$$

$$\boxed{\delta_e = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy}$$

Drag Force

$$F_D = \dot{P}_{loss} = \rho(\theta \times 1) \cdot U_\infty^2$$

$$* \quad \boxed{F_D = \dot{P}_1 - \dot{P}_2 = \rho \theta U_\infty^2}$$

Q.28

$$\dot{m}_{DC} = \rho \int_0^\delta U_\infty \left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right) dy$$

$$\dot{m}_{DC} = \frac{2}{3} \rho U_\infty \delta$$

$$\dot{m}_{BD} = \rho \delta U_\infty - \frac{2}{3} \rho U_\infty \delta$$

$$m_{BD} = \frac{1}{3} \rho \delta U_\infty$$

$$\frac{m_{BC}}{\rho \delta U_\infty} = \frac{1}{3}$$

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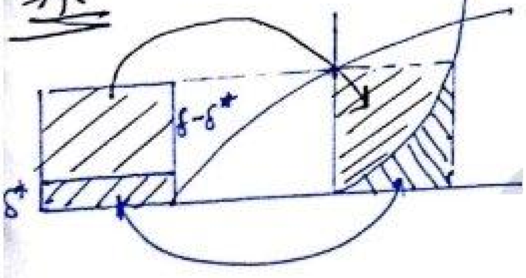
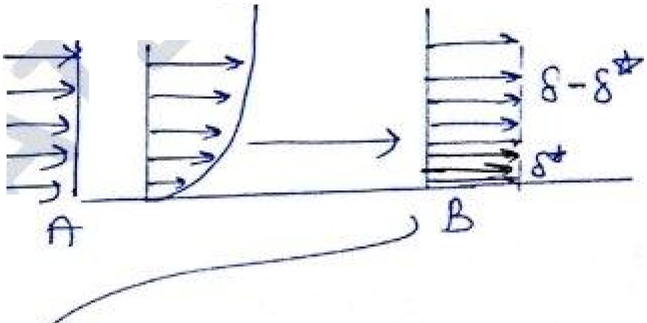
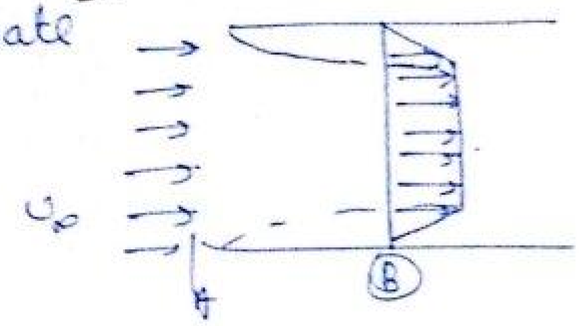


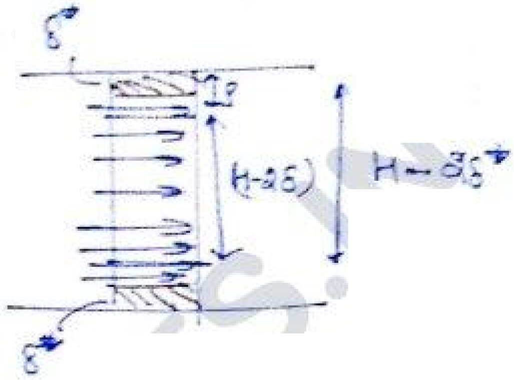


Now we can apply B.equation b/w A & B $(\delta-\delta^*)$

Q. 26/26

Gate





$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

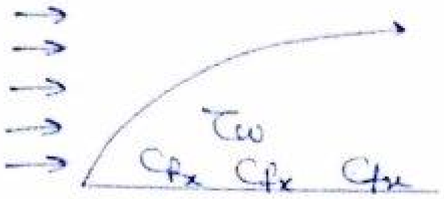
$$\delta^* = \delta/2$$

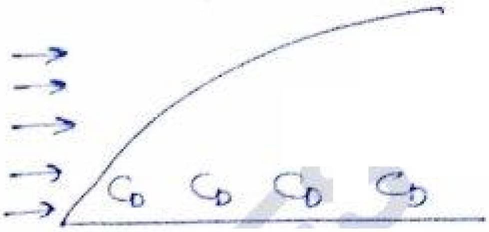
$$\dot{m}_A = \dot{m}_B$$

$$\rho (H \times 1) U_\infty = (H - 2\delta^*) \times U_m$$

$$\frac{U_m}{U_\infty} = \frac{H}{H-\delta} = \frac{1}{\left(1 - \delta/H\right)}$$

Skin friction coefficient :-





$$\boxed{C_{fx} = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}}$$

$x\uparrow \rightarrow \tau_w \downarrow$

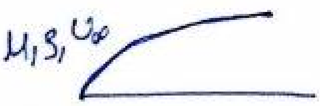
$$C_D = \frac{1}{L}\int_0^L C_{fx}\, dx$$

$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\rho A U_\infty^2}}$$

Coefficient of drag

Boundary layer thickness.

Concept
Prandtl
(1904)

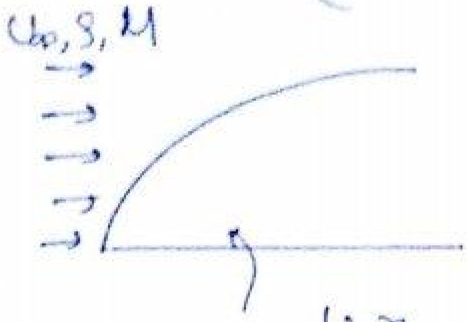


Blasius (1908)
Analytical
exact soln

(Moment integral eqn)
Vonkarman
(Approximate)

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2}$$

Blasius (1908) Analytical (Remember)
(Exact sol^n)



$$Re_x = \frac{U_\infty x}{\nu}$$

$$C_{fx} = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$$

$$\boxed{\delta = \frac{5x}{\sqrt{Re_x}}}$$

$$\boxed{C_{fx} = \frac{0.664}{\sqrt{Re_x}}}$$

$$\boxed{C_D = \frac{1.328}{\sqrt{Re_L}}}$$

Von Karmann

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2} \Bigg\} \text{ laminar \& turb}$$

laminar flat plate

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$$

No need to remember

$$\delta = \frac{4.64x}{\sqrt{Re_x}}, \quad C_{fx} = \frac{0.646}{\sqrt{Re_x}}$$

$$C_{fx} = \frac{0.0592}{(Re_x)^{1/5}}$$

turbulant

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

$$\delta = \frac{0.375\,x}{(Re_x)^{1/5}} \quad , \quad C_D = \frac{0.074}{(Re_L)^{1/5}}$$

$$C_{fx} = \frac{0.0592}{(Re_x)^{1/5}}$$

Q.27

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U_\infty^2} \qquad C_D = \frac{1.328}{\sqrt{Re_L}}$$

$$\cancel{C_D = \frac{1}{L}\int_0^L C_{fx}\,dx} \qquad Re_x = \frac{U_\infty x}{\nu}$$

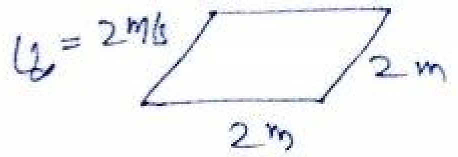
$$\cancel{= \frac{1}{L}\int_0^L \frac{1.328\,(x)^{-1/2}}{\sqrt{U_\infty/\nu}}} \qquad Re_L = 2\times10^5 < 5\times10^5$$

$$\cancel{C_D =} 1.32$$

$$\frac{1.328}{\sqrt{\frac{2\times 2.5}{2\times10^{-5}}}} = \frac{F_D}{\frac{1}{2}\times1.2\times2.5\times1\times4}$$

$$F_D = 0.0159\,\underline{N}$$

Q.25
P.50



$Re = \frac{\rho V L}{\mu} = \frac{u_\infty L}{\nu}$

$\cancel{\delta_x = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 2}{\sqrt{\frac{2 \times 2}{1 \times 10^{-6}}}}}$

$\cancel{\delta_L = \frac{5 \times 2}{2} \times 10^3}$

$\cancel{\delta_L = 5}$

$Re_{lam} = Re_x$

$\cancel{Re_L} =$

$5 \times 10^5 = \frac{u_\infty \times x}{10^{-6}}$

$5 \times \cancel{10^5} = u_\infty \times x \times \cancel{10^6}$

$\frac{2.5}{10} = x \Rightarrow x = 0.25$

$\cancel{\delta_x = \frac{5 \times 0.25}{\sqrt{\frac{2 \times 0.25}{10^{-6} \times 10^{4+2}}}} = \sqrt{2 \times 5 \times 0.25} \times 10^3} =$

$\delta_x = \frac{5 \times 0.25}{\sqrt{5 \times 10^5}} = \underline{1.76 \text{ mm}}$

Q. 30

$$\frac{u}{U_\infty} = \frac{y}{\delta}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$= \delta - \frac{\delta^2}{2\delta} =$$

$$\delta^* = \delta/2$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dx = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

$$\theta = \delta/6$$

$$\delta_e = \int_0^\infty \frac{u}{U_\infty}\left(1 - \left(\frac{u}{U_\infty}\right)^2\right) dy = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y^2}{\delta^2}\right) dy$$

$$\delta_e = \delta/4$$

$$\delta^* = \frac{\delta}{2}, \quad \delta_e = \frac{\delta}{4}, \quad \theta = \frac{\delta}{6}$$

$$\boxed{\delta > \delta^* > \delta_e > \theta}$$ Valid for any Profile

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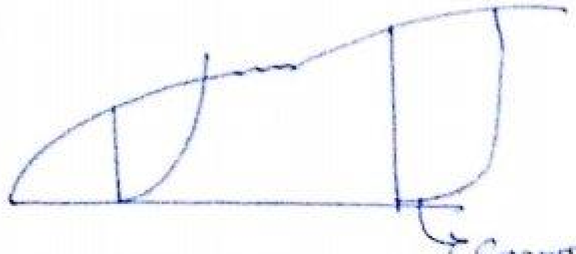

$$\boxed{\delta > \delta^* > \delta_e > \theta}$$

Valid for any profile

shape factor $\boxed{H = \dfrac{\delta^*}{\theta}}$