Time Domain Specification:

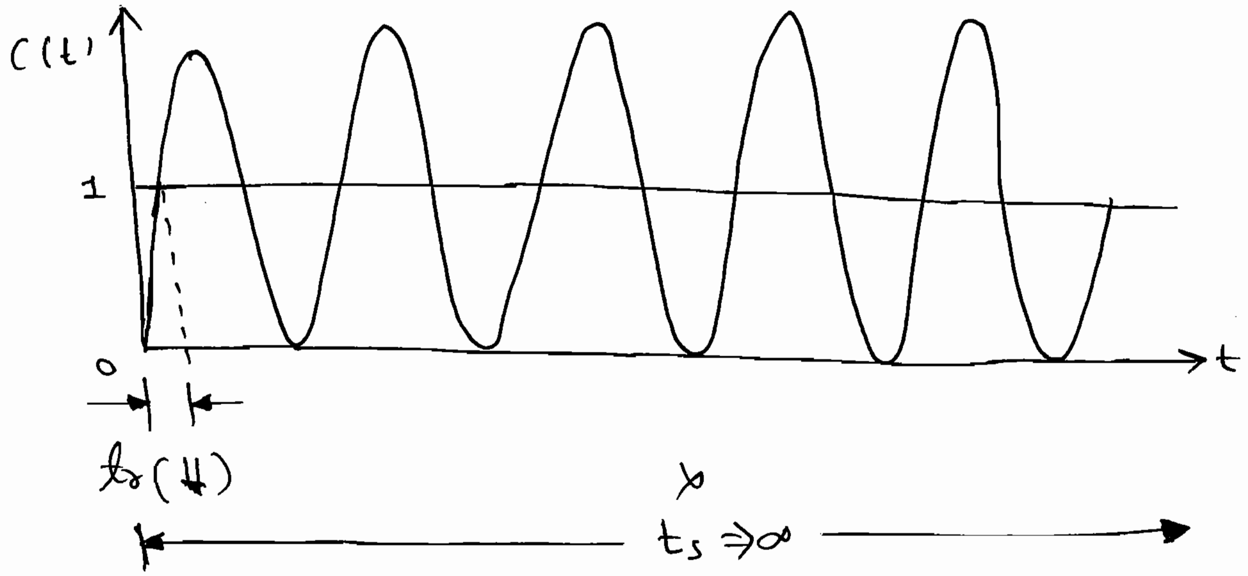
⇒ For time domain specification select a unit step input and underdamped system.

	tr.	ss.	S	
Impulse	✓	×	✓	Practically not exist.
step	✓	✓	✓	Practically exist
ramp	✓	✓	×	Unbounded I/p's
Parabolic	✓	✓	×.	

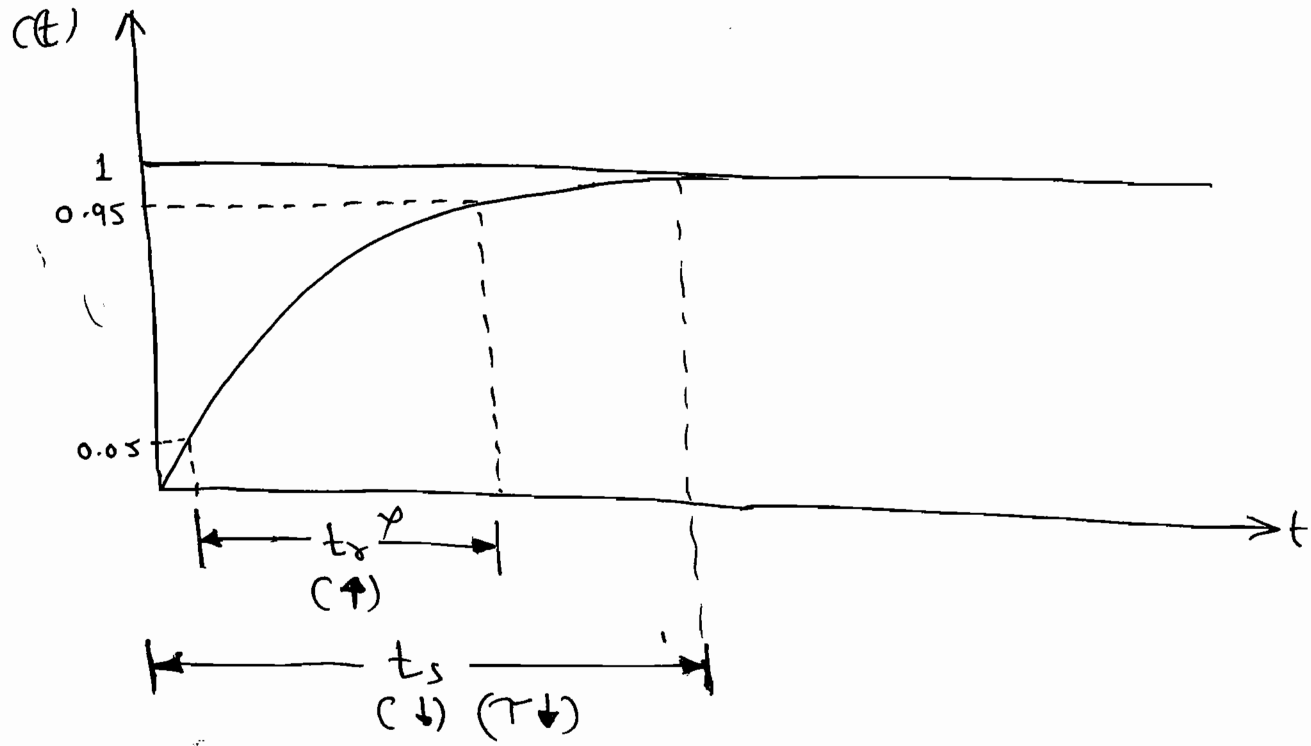
⇒ For system behaviour impulse is used.

⇒ For analysis w.r.t. time step is used.

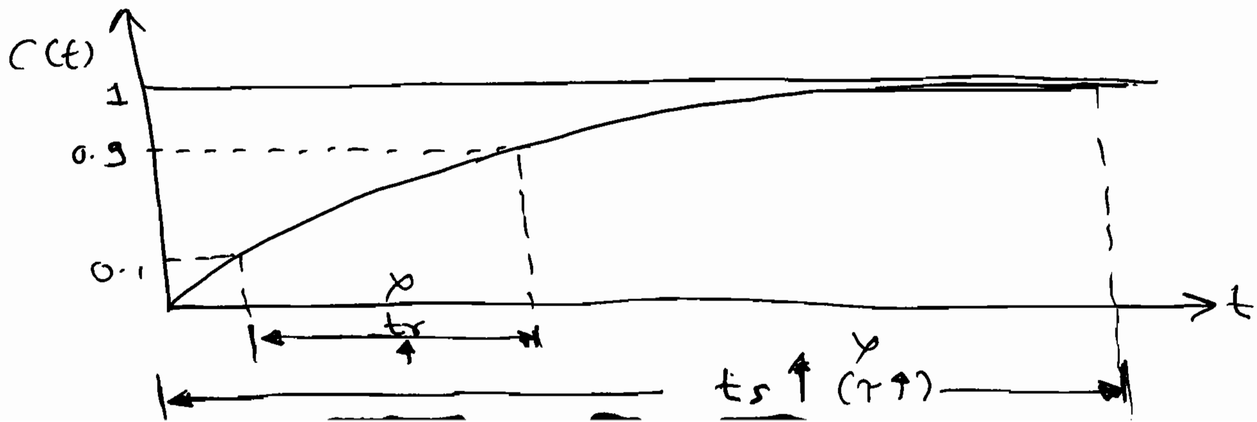
① $\xi = 0$: Undamped system:



② $\xi = 1$: Critical Damped (t_r 5% to 95%).

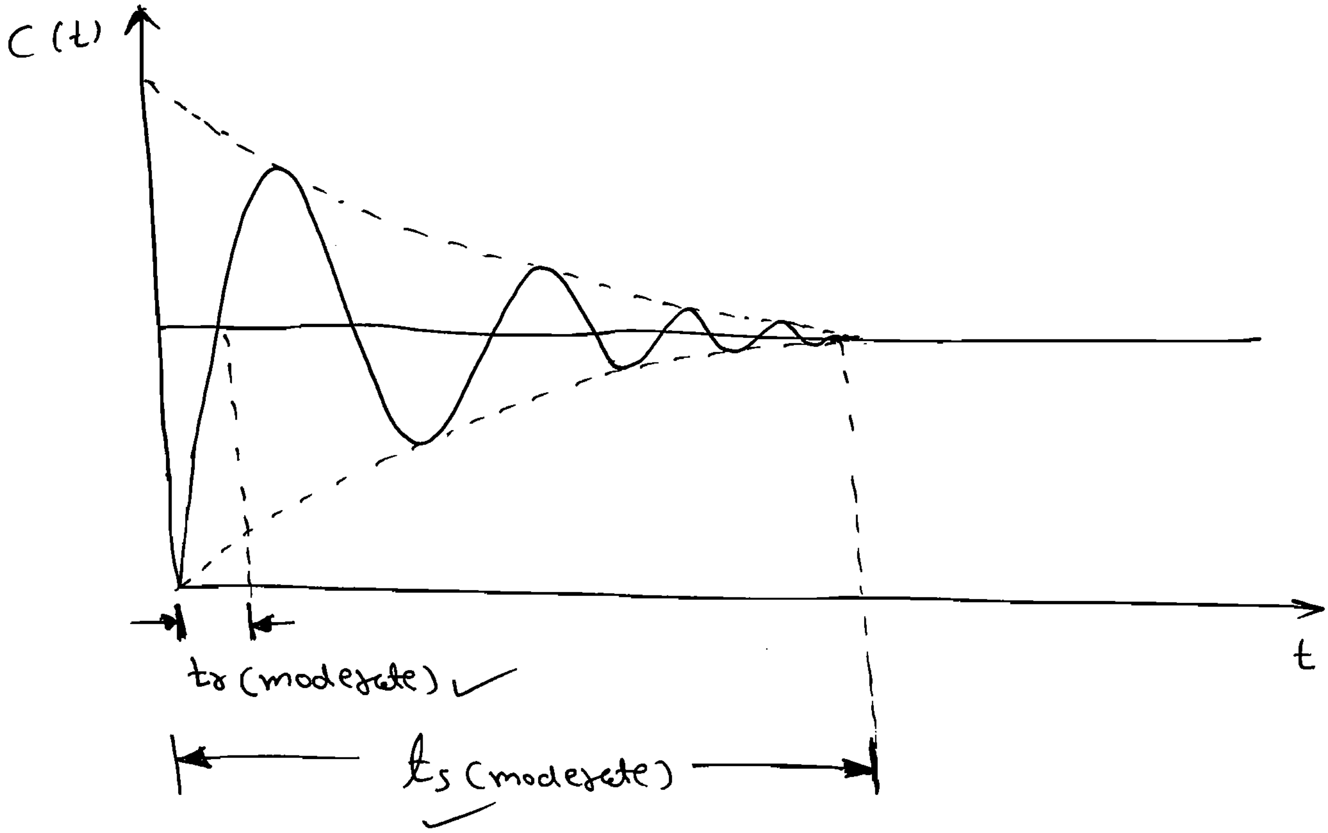


③ $\xi > 1$: overdamped (t_r : 10% to 90%).



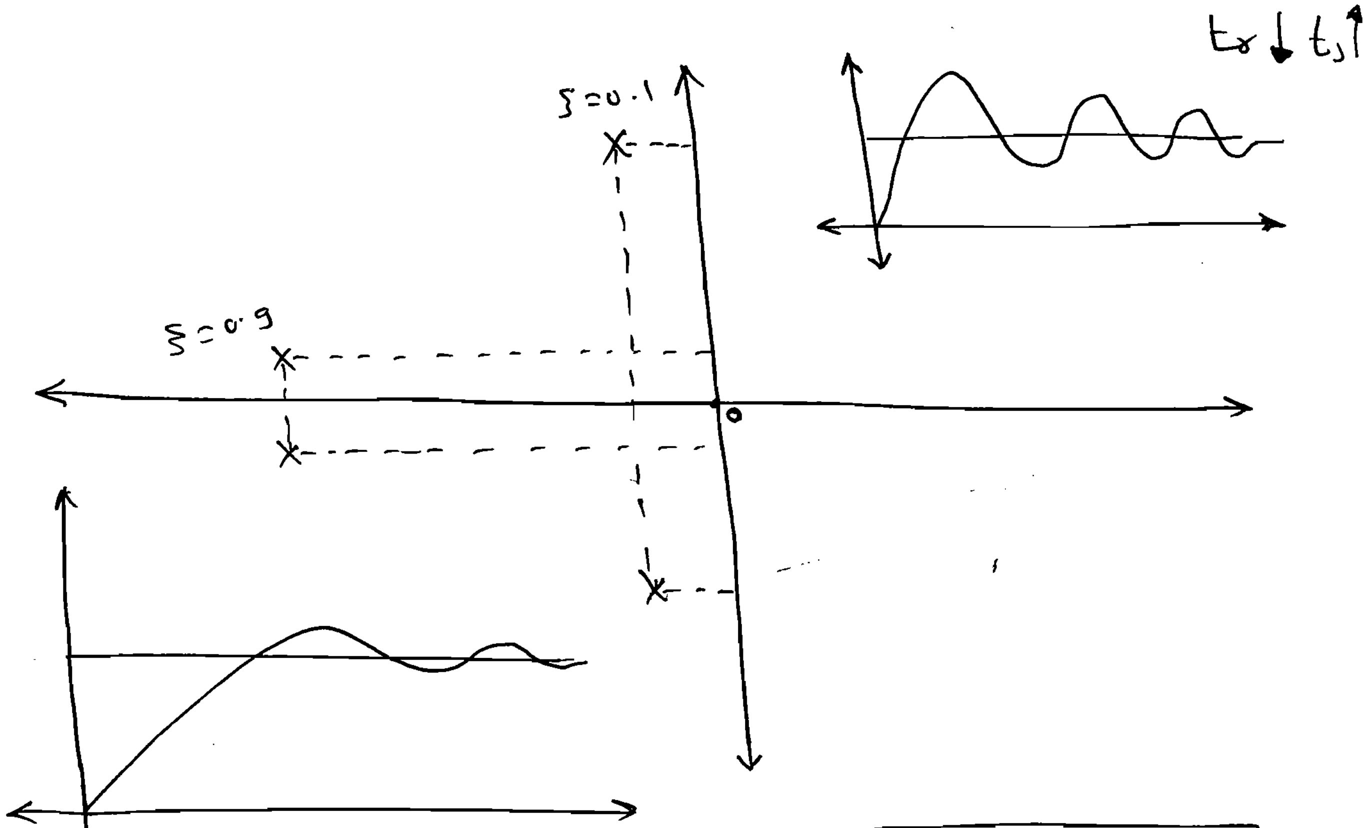
④ $0 < \xi < 1$: **Underdamped System:**

$0.4 < \xi < 0.7$.



* Optimum Value of ξ:

⇒



So, $\boxed{0.4 < \xi < 0.7}$

⇒ For time domain Specification select the underdamped system because.

① If select the undamped system the rise time is very small. The setteling time is infinity whereas if select the critical damping system rise time is large but setteling time is very small.

② If select the overdamped system the rise time is large and setting time also large.

⇒ Practically for any system, required smallest rise time and smallest setting time.

⇒ In underdamped system we can get the moderate values of rise time and setting time.

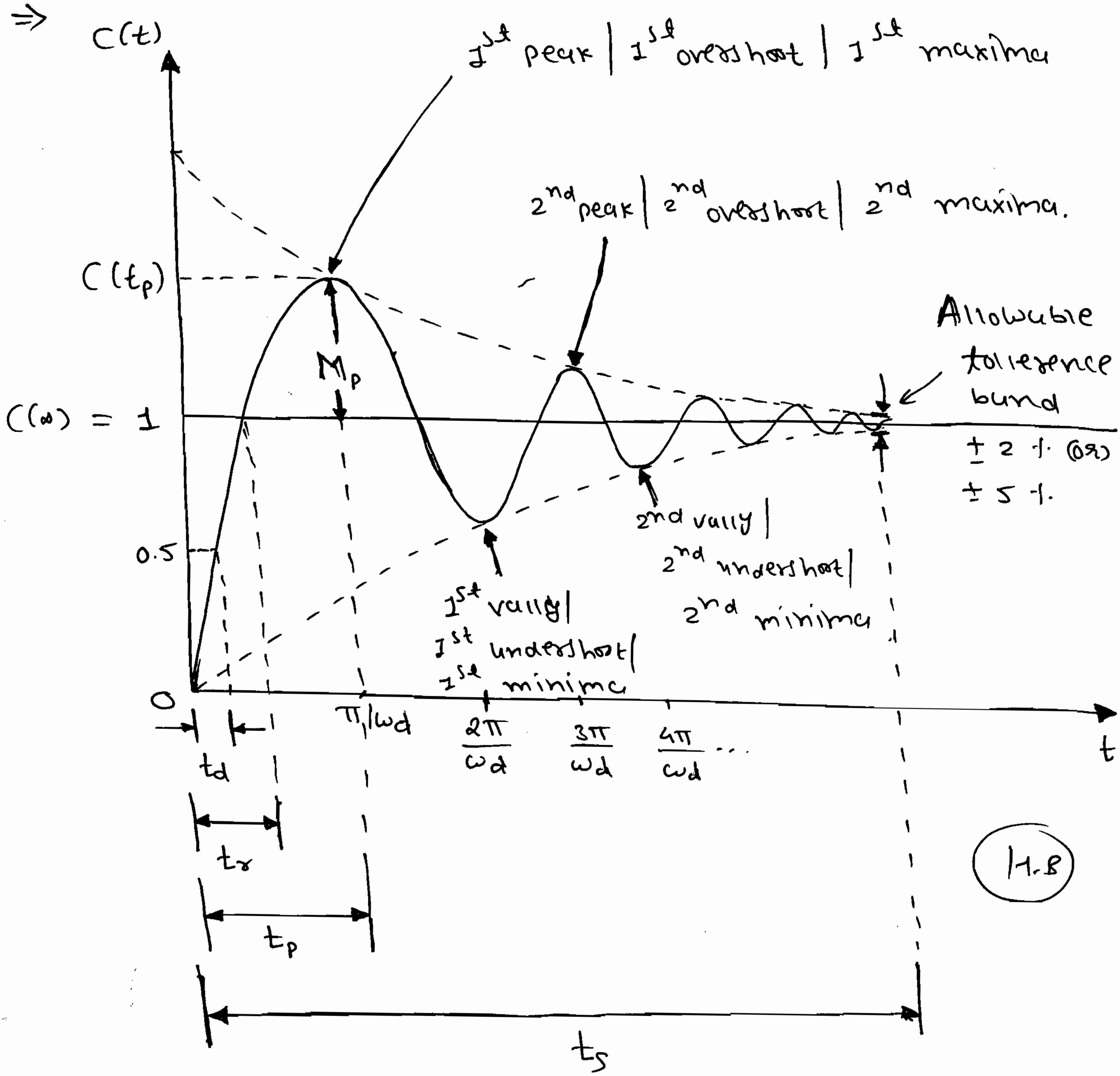
⇒ In underdamped system the best range of ξ is $0.4 < \xi < 0.7$.

⇒ when $0 < \xi < 1$, the unit step response of the system is:

$$\Rightarrow \quad c(t) = \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left(\omega_n\sqrt{1-\xi^2} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right].$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$\Rightarrow$



① Delay time :: (t_d):

$\Rightarrow$ It is the time required for the response to rise from 0 to 50 % of the final value is called the delay time.

→ denoted by t_d.

$$\boxed{t_d = \frac{1 + 0.75}{\omega_n} \text{ sec.}}$$ ⇐ H.B.

② Rise time (t_r):

⇒ It is the time required for the response to rise from 0 to 100 % of its final value is ~~caused~~ for underdamped system, 5 % to 95 % for critical damped system and 10 % to 90 % for over damped system.

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d}.$$

$$\therefore \boxed{t_r = \frac{\pi - \cos^{-1}(\xi)}{\omega_d}}$$

③ Peak - time (t_p):

⇒ It is the time required for the system to rise from 0 to peaks of the time response.

$$\Rightarrow \boxed{t_p = \frac{n\pi}{\omega_d}}$$

$n = 1 \rightarrow$ by default $\rightarrow$ 1st peak.

$$\therefore \boxed{t_p = \frac{\pi}{\omega_d}}$$

$\rightarrow$ For 2nd peak,

$$\boxed{t_p = \frac{3\pi}{\omega_d}}$$

$\Rightarrow$ For 1st vally, $\boxed{t_p = \frac{2\pi}{\omega_d}}$

④ **Peak- overshoot (M_p):-**

$\Rightarrow$ It is the difference betn the time response peak to the steady state vame.

$$\boxed{M_p = c(t_p) - c(\infty)}$$

⑤ **% of Peak - overshoot (% M_p):-**

$\Rightarrow$ It is the normalized difference betn time response peak to steady state.

$$\% M_p = \frac{c(t_p) - \cancel{c(\infty)}^{1}}{\cancel{c(\infty)}^{1}} \times 100\%$$

$$\therefore \% M_p = [c(t_p) - 1] \times 100\%$$

$$\Rightarrow \boxed{\% M_p = e^{-n\pi\xi/\sqrt{1-\xi^2}} \times 100\%}$$

$n=1 \Rightarrow$ by defaul $\rightarrow$ 1^{st} peak

$$\Rightarrow \boxed{\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%}$$

$\Rightarrow$ The n- value is similar to peak time.

$\Rightarrow$ The undershoot to the first vally point is.

$$\boxed{\% M_v = e^{-2\pi\xi/\sqrt{1-\xi^2}} \times 100\%}$$

⑥ Setteliing time (T_s):-

$\Rightarrow$ It is the time required for the response to rise from 0 to specified tolerance band usually $\pm 2\%$ (or) 5%.

$\Rightarrow \pm 5\%$, $t_s = 3\tau = \frac{3}{\xi\omega_n}$ sec.

$\pm 2\%$, $\boxed{t_s = 4\tau = \frac{4}{\xi\omega_n} \text{ sec.}} \longrightarrow$ default.

$\pm 0\%$, $t_s = 5\tau = \frac{5}{\xi\omega_n}$ sec. (SS value).

⑦ Time Period of Oscillation:

$\Rightarrow$ It is the time required to complete one cycle,

$$\boxed{\tau_{osc} = \frac{2\pi}{\omega_d} = 2\, T_p.}$$

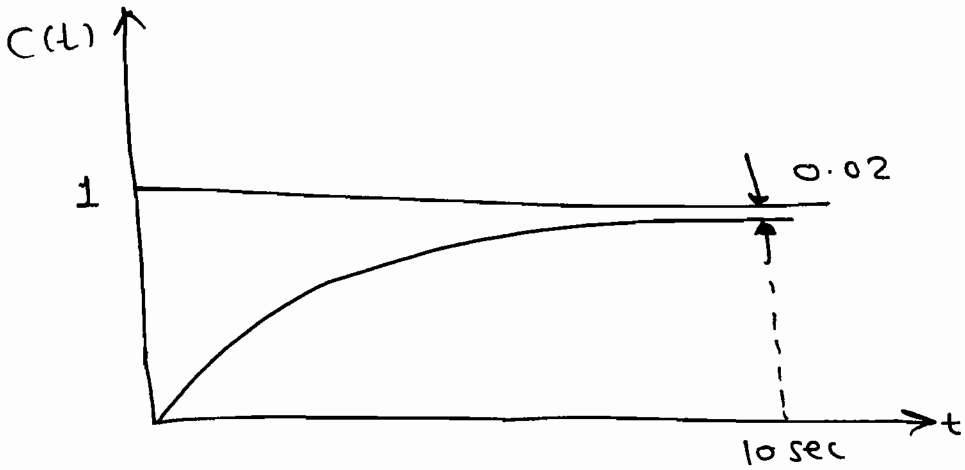
$\Rightarrow$ No. of oscillation before reaching steady state.

$$N = \frac{t_s \; (\pm 2\% \text{ (or) } \pm 5\%).}{\tau_{osc}.}$$

$$\therefore \quad N = \frac{t_s}{2\pi/\omega_d} = \frac{t_s \cdot \omega_d}{2\pi}.$$

$$\therefore \quad \boxed{N = \frac{t_s}{2 t_p}.}$$

$\boxed{Q}$ The unit step response of the system is shown in fig. ~~and~~ Find the following factors. (1) τ (2) t_d (3) t_r (4) t_p & M_p.



Soln: $t_s = 10$ sec

$t_s = \frac{4\tau}{1}.$ $(\because \pm 2\%)$.

$\therefore\; t_s = 4\tau \Rightarrow \tau = t_s/4$

$$\boxed{\tau = 10/4 = 2.5 \text{ sec}.}$$

⇒ The standard form of the unit step response is,

$$\boxed{c(t) = k(1 - e^{-t/\tau}).}$$

k ↑ S.S. value

$$\therefore \quad c(t) = (1 - e^{-t/\tau})$$

$$\therefore \quad c(t) = \left(1 - e^{-t/2.5}\right).$$

① $\tau = 2.5$ sec.

② t_d.

→ at $t = t_d \Rightarrow c(t) = 0.5$

$$\therefore \quad 0.5 = 1 - e^{-t_d/2.5}.$$

$$\therefore \quad e^{-t_d/2.5} = 0.5$$

$$\therefore \quad \boxed{t_d = 1.733 \text{ s}}$$

③ t_r

→ ~~at $t = t_r \Rightarrow c(t_r) = 1$.~~

→ For rise time consider the time duration from 10 % to 90 % of the final value,

at $t = t_{r1} \Rightarrow c(t) = 0.1$.

$$\therefore \quad 0.1 = 1 - e^{-t_{r1}/2.5} \Rightarrow \boxed{t_{r1} = 0.263}$$

at $t = t_{r2} \Rightarrow c(t) = 0.9.$

$\therefore \quad 0.9 = 1 - e^{-t_{r2}/2.5} \Rightarrow \boxed{t_{r2} = 5.76\,s}$

$\therefore \quad t_r = t_{r2} - t_{r1} = 2.2\tau.$

$\therefore \quad \boxed{t_r = 2.2\tau}$

$\therefore \quad t_r = 2.2 \times 2.5$

$\boxed{t_r = 5.5 \text{ sec}}$

$\Rightarrow$ (4) t_p & M_p.

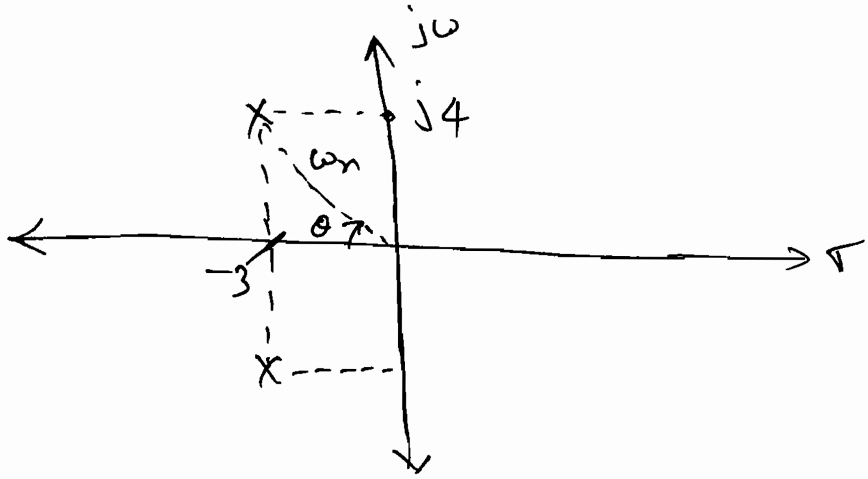
→ there is no peaks are exist hence no. peak time and no peak overshoot.

$\boxed{Q}$ The impulse response of a system is $c(t) = k \cdot e^{-3t} \cdot \sin 4t$. Find the following factors:

(1) t_s (2) ω_n (3) ξ (4) M_p (5) t_d (6) t_r (7) t_p.

Sol^n^:

$c(t) = k \cdot e^{-\underbrace{3}_{1/\tau}t} \cdot \sin \underbrace{4}_{\omega_d} t$



⇒ ① $\tau = 1/3$ sec.

$\omega_d = 4$ rad/sec.

$\omega_n = \sqrt{3^2 + 4^2}$

⇒ $\boxed{\omega_n = 5 \text{ rad/sec.}}$

② $\boxed{\omega_n = 5 \text{ rad/sec}}$

③ $\underline{\xi}$:

$\therefore \omega_d = \omega_n \sqrt{1-\xi^2}$. (or)

$\therefore \left(\frac{4}{5}\right)^2 = 1 - \xi^2$

$\therefore 1 - \frac{16}{25} = \xi^2$

$\xi = 3/5$

$\boxed{\xi = 0.6}$

$\xi \neq \theta$

$\theta = \cos^{-1}\xi$

$\xi = \cos\theta$

$\xi = \cos\left[\tan^{-1}\left(\frac{4}{3}\right)\right]$

$\boxed{\xi = 0.6}$

④ $\underline{M_p}$:

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 \%$$

$$= e^{-\frac{3.14 \times 0.6}{\sqrt{1-0.36}}} \times 100 \%$$

⇒ $\boxed{\% M_p = 9.5 \%}$

⑤ $\underline{t_d}$:

$$t_d = \frac{1 + 0.7\xi}{\omega_n}.$$

$$\Rightarrow \quad t_d = \frac{1 + (0.7)(0.6)}{5}$$

$$\boxed{t_d = 0.284 \text{ sec}}$$

⑥ t_r:

$$t_r = \frac{\pi - \cos^{-1}(\zeta)}{\omega_d}$$

$$= \frac{3.14 - \cos^{-1}(0.6)}{\omega_d}$$

$$= \frac{3.14 - 53.13^\circ}{4}$$

$$= \frac{3.14 - \left(53.13^\circ \times \frac{\pi}{180^\circ}\right)}{4}$$

$$\therefore \boxed{t_r = 0.553 \text{ sec}}$$

⑦ t_p:

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4}$$

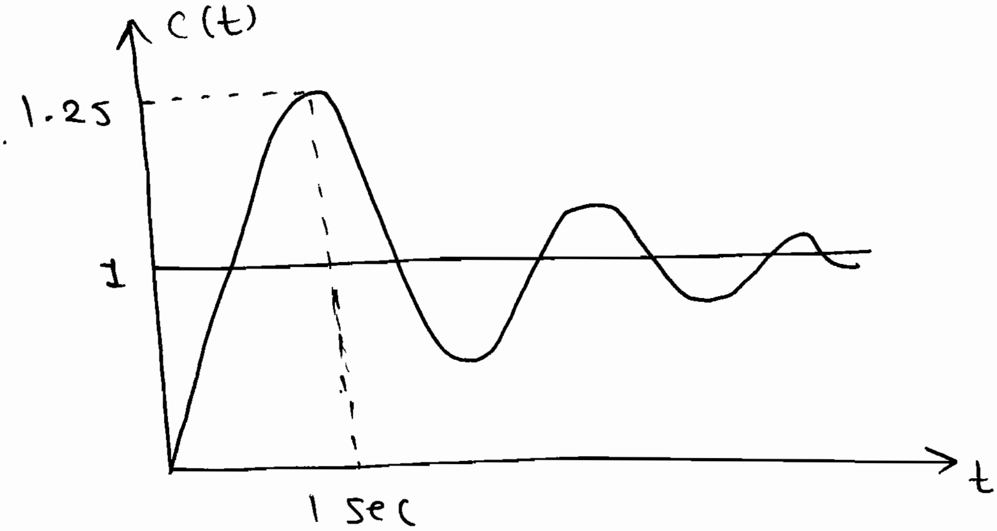
$$\therefore \boxed{t_p = 0.785 \text{ sec.}}$$

$\boxed{Q}$ The unit step response of the system is shown in figure. Find the following factor.

① M_p
② $\% M_p$
③ ζ
④ ω_n
⑤ delay time.
⑥ t_r
⑦ t_s
⑧ OLTF
⑨ CLTF.

assume UFB System.

→



Sol^n:

① M_p:

$$c(t_p) = 1.25 \quad , \quad c(\infty) = 1$$

$$\therefore M_p = c(t_p) - c(\infty) = 1.25 - 1 = 0.25$$

$$\therefore \boxed{M_p = 0.25}$$

② % M_p:

$$\therefore \% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100 \%$$

$$\boxed{\% M_p = 25 \%}$$

③ ξ:

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100.$$

$$\therefore \frac{25}{100} = e^{-\pi\xi/\sqrt{1-\xi^2}}.$$

$$\therefore \frac{-\pi\xi}{\sqrt{1-\xi^2}} = -1.386$$

$$\therefore \quad \frac{\xi}{\sqrt{1-\xi^2}} = 0.441$$

$$\therefore \quad \xi^2 = (0.195)\,(1-\xi^2).$$

$$\therefore \quad 1.195\,\xi^2 = 0.195$$

$$\Rightarrow \quad \boxed{\xi = 0.404.}$$

④ $\underline{\underline{\omega_n}}$:

$$\cancel{t_s = 1\,sec.}$$

$$\cancel{t_s = 4T.}$$

$$\cancel{t_s = \frac{4}{\xi\omega_n} \Rightarrow \omega_n = \frac{4}{\xi\, t_s} = \frac{4}{0.4}}$$

$$\therefore \quad t_p = \frac{\pi}{\omega_d}.$$

$$\therefore \quad \omega_d = 3.14.$$

$$\Rightarrow \quad \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore \quad \omega_n = \frac{3.14}{\sqrt{1-0.16}}$$

$$\Rightarrow \quad \boxed{\omega_n = 3.43 \text{ rad/sec}}$$

⑤ $\underline{\underline{t_d}}$:

$$\therefore \quad t_d = \frac{1 + 0.7\xi}{\omega_n}.$$

$$= \frac{1 + (0.7)(0.4)}{3.43}$$

$$\therefore \quad \boxed{t_d = 0.373 \text{ sec}}$$

⑥ $\underline{t_r}$:

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$$= \frac{3.14 - \cos^{-1}(0.4)}{3.14}$$

$$\therefore \boxed{t_r = 0.63 \text{ sec}}$$

⑦ $\underline{t_s}$:

$$t_s = 4\tau = \frac{4}{\xi\omega_n}$$

$$\therefore t_s = \frac{4}{0.4 \times 3.43}$$

$$\therefore \boxed{t_s = 2.915 \text{ sec}}$$

⑧ OLTF:

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$$

$$= \frac{(3.43)^2}{s(s + 2(0.4)(3.43))}$$

$$\boxed{G(s) = \frac{11.765}{s(s + 2.744)}}$$

⑨ CLTF:

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore \boxed{\frac{C(s)}{R(s)} = \frac{11.765}{s^2 + 2.744s + 11.765}}$$

Q Find the $\% M_p$ to the following systems to the unit step input.

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+25}.$$

Soln:

Here, $\omega_n = 5$ rad/sec

but $\xi = 0$.

So, $\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \cdot 100\%$.

$\% M_p = e^{-\pi \cdot 0/\sqrt{1-0}} \cdot 100\%$.

$\therefore \boxed{\% M_p = 100\%}$

Q find M_p of $\dfrac{C(s)}{R(s)} = \dfrac{100}{s^2+20s+100}$.

Soln: Here, $\omega_n^2 = 100$.

$\Rightarrow \boxed{\omega_n = 10 \text{ rad/sec}}$

$\cancel{2}\xi\omega_n = \cancel{20}\ 10$

$\boxed{\xi = 1} \rightarrow \boxed{CD}$

$\therefore M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$.

$= e^{-\pi\xi/\sqrt{1-1}} \times 100\%$.

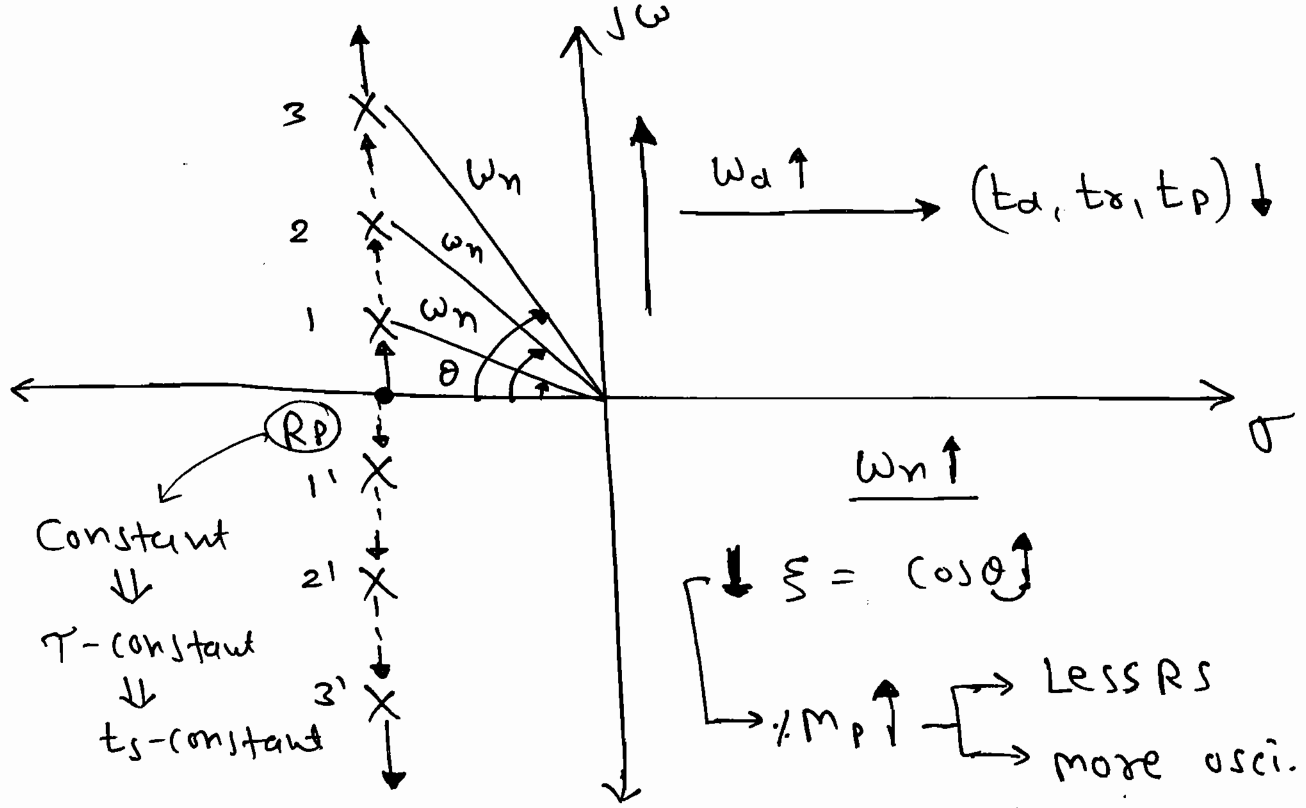
$M_p = \cancel{100}\ 0\%$. $\boxed{M_p = 0\%}$

Note: When ξ increases from 0 to 1, % M_p is decreases from 100 % to 0 %.

→ When $\xi \geq 1$, % M_p = 0 %. because no ~~the~~ oscillation are exist in the system.

$\boxed{Q}$ Find the Variation in time domain specification to the given Poles Path in the S-plane.

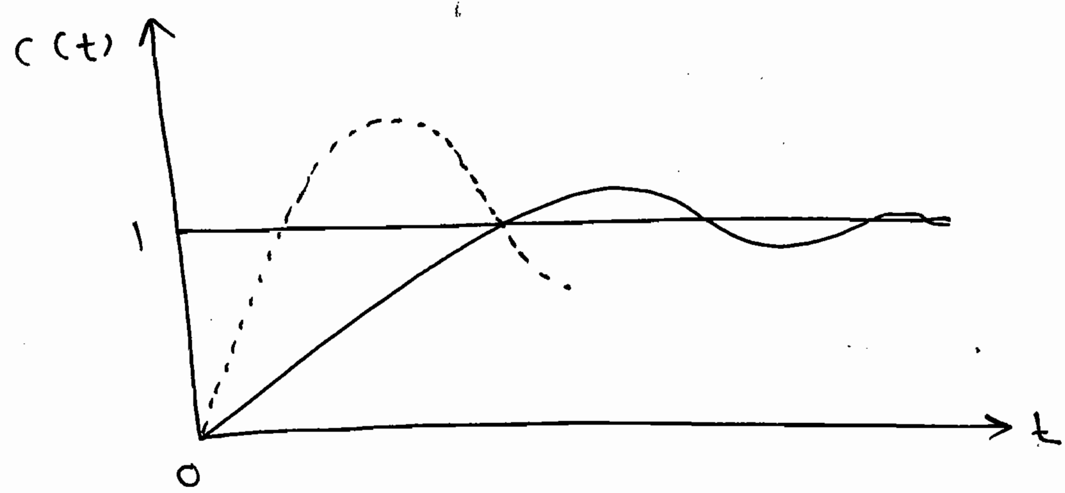
Sol^n.



⇒ As real part is constant, time constant is constant hense setteling time is constant.

⇒ As imaginary part increases the damped oscillation ω_d increases. As ω_d

increases the time specification t_d, t_r & t_p decreases.

=> 

$\boxed{\text{optimum value of } \% M_p = 5\% \text{ to } 25\%}$

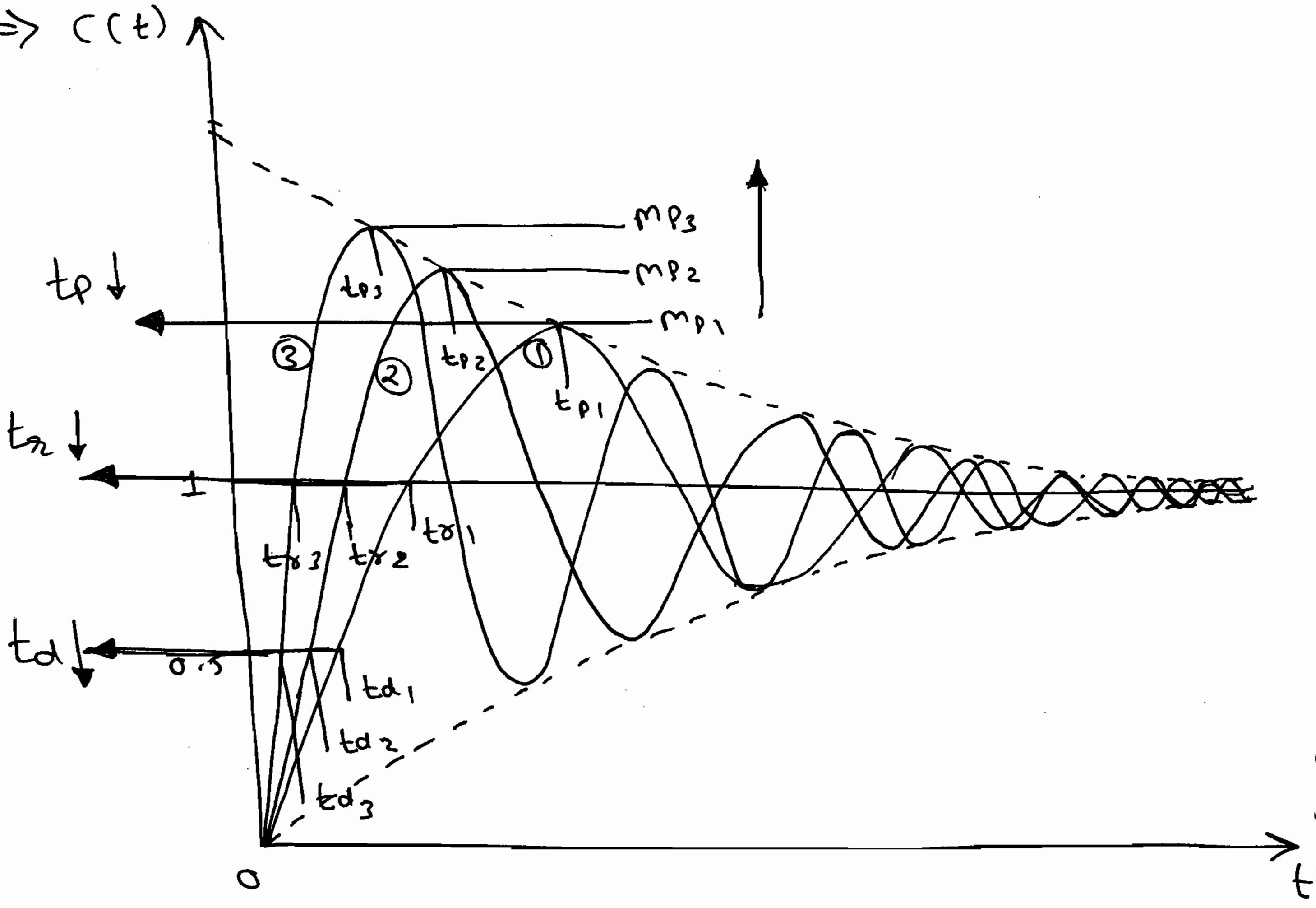
=> As the inclination of the pole θ increases the damping ratio ξ decreases. Hence, the % of M_p increases.

=> The large M_p make the system less RS & more oscillatory.

=> The optimum range of the % M_p is 5% to 25%.

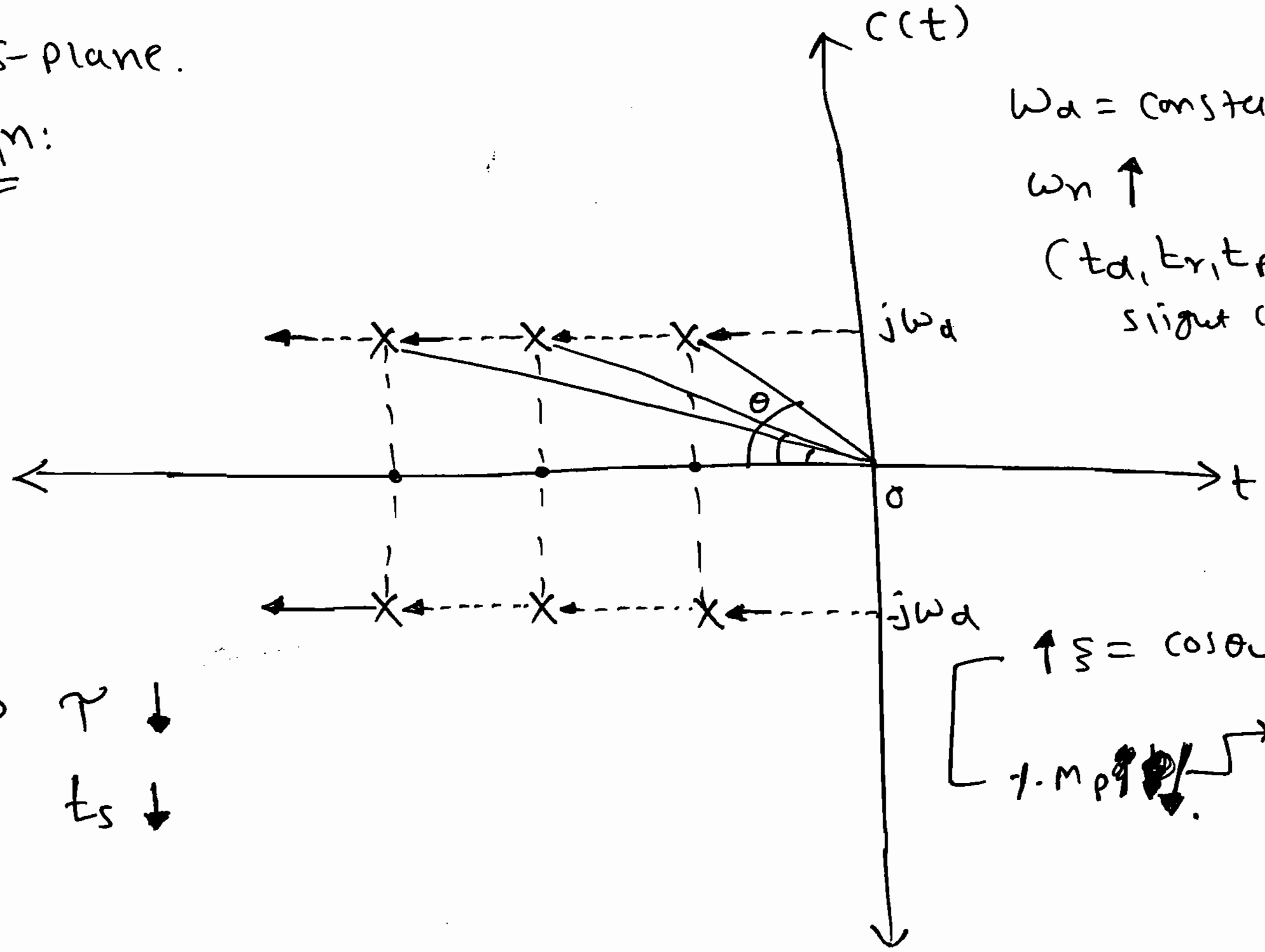
=> If the peak overshoot is more than 25% the system is less relative stable.

=> If the peak overshoot is $M_p < 5\%$ the system is slow response.

⇒ 

$\boxed{Q}$ Find the Variation in time domain specification to the given poles pctn in s-plane.

Soln:



ω_d = constant.

$\omega_n \uparrow$

(t_d, t_r, t_p slight change)

$\uparrow \xi = \cos\theta \downarrow$

$\%M_p \downarrow$ → more RL Ⓢ

⇒ $\tau \downarrow$

$t_s \downarrow$

$\Rightarrow$ Pole moving towards the left side and t_s & τ both decreases.

$\Rightarrow$ Imaginary part is constant.

so, $\omega_d =$ constant.

$$t_p = \frac{\pi}{\omega_d} = \text{Constant.}$$

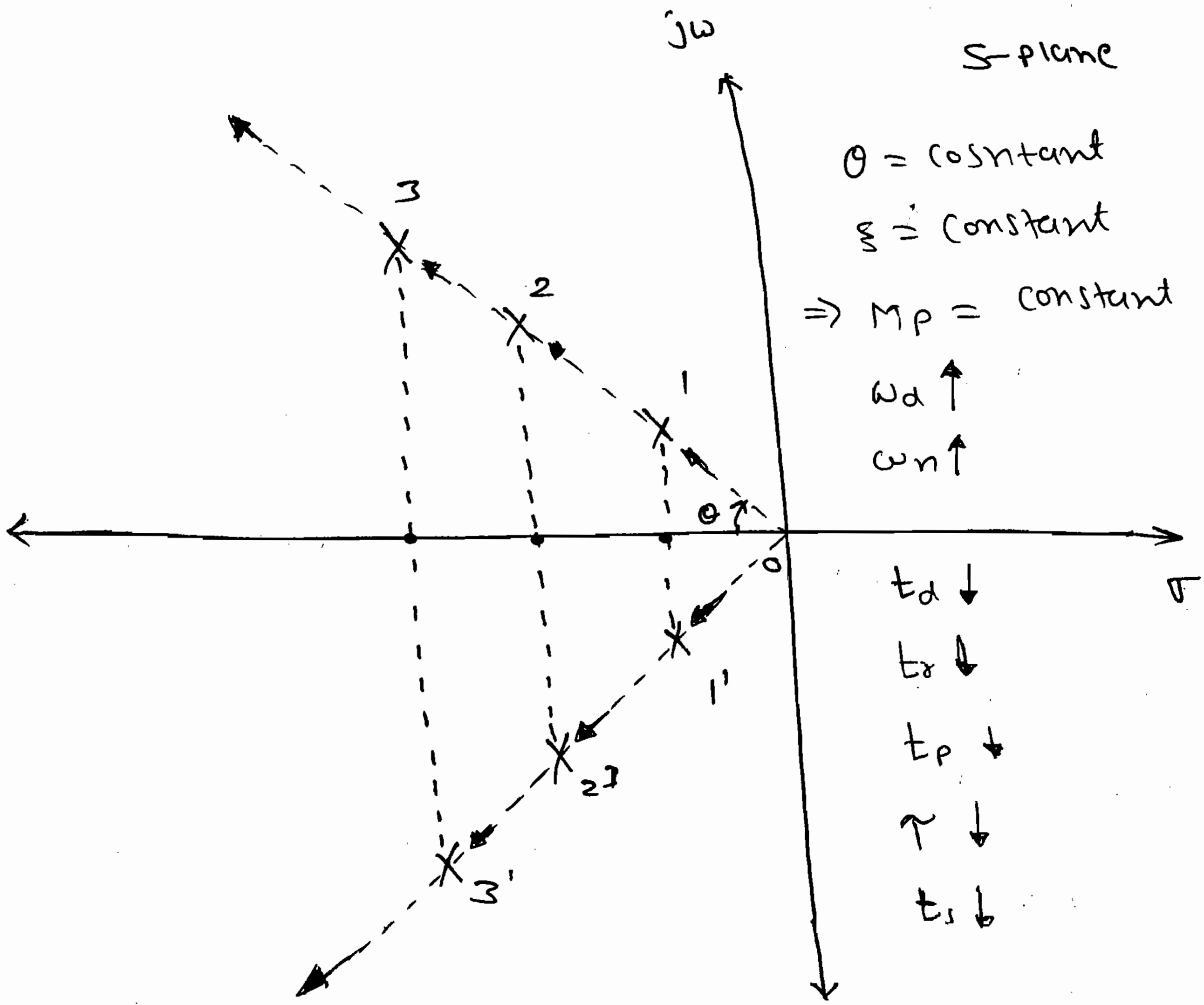
$\Rightarrow$ $t_p =$ constant.

$\Rightarrow$ As imaginary part is constant the damped oscillations ω_d constant but there exist a slight variation in t_d and t_r.

$\Rightarrow$ As the inclination at the pole 'θ' increases. decreases the damping ratio ξ Hence the % M_p ~~increases~~ decreases. The system become more relative stable.

Q Find the variation in the time specification when location of poles moves (or) change as shown in fig.

⇒



⇒ As the incination of the pole θ = constant the damping ratio ξ is constant and hence % M_p is also constant.

⇒ As the poles moves towards the left time constant τ decreases hence settelling time decreases.

⇒ As imaginary part is increases damped oscillation ω_d increases hence $(t_d, t_r, t_p)\downarrow$ hence $\omega_n \downarrow$.

$\boxed{a}$ Find the time domain specification of 151

$$G(s) = \frac{25}{s(s+4)}, \quad H(s) = 1.$$

Soln:

$$G(s) = \frac{25}{s(s+4)}.$$

$$\Rightarrow \quad \omega_n^2 = 25$$

$$\boxed{\omega_n = 5 \text{ rad/sec}}$$

$$\Rightarrow \quad \cancel{2}\xi\omega_n = \cancel{4}\,2$$

$$\xi \times 5 = 2$$

$$\boxed{\xi = 0.4}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5 \times \sqrt{1-0.16}$$

$$\boxed{\omega_d = 4.58 \text{ rad/sec}}$$

$$\Rightarrow \quad t_d = \frac{1+0.7\xi}{\omega_n}$$

$$t_d = \frac{1+(0.7\times 0.4)}{5}$$

$$\boxed{t_d = 0.256}$$

$$\Rightarrow \quad t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d}.$$

$$= \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$$\therefore \quad t_r = \frac{3.14 - \cos^{-1}(0.4)}{4.58}$$

$$\therefore \boxed{t_r = 0.4326 \text{ sec}}$$

$$\Rightarrow t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4.58}$$

$$\therefore \boxed{t_p = 0.686 \text{ sec}}$$

$$\Rightarrow M_p = e^{-\pi\xi / \sqrt{1-\xi^2}} \times 100 \%$$

$$= e^{-3.14 \times 0.4 / \sqrt{0.84}} \times 100 \%$$

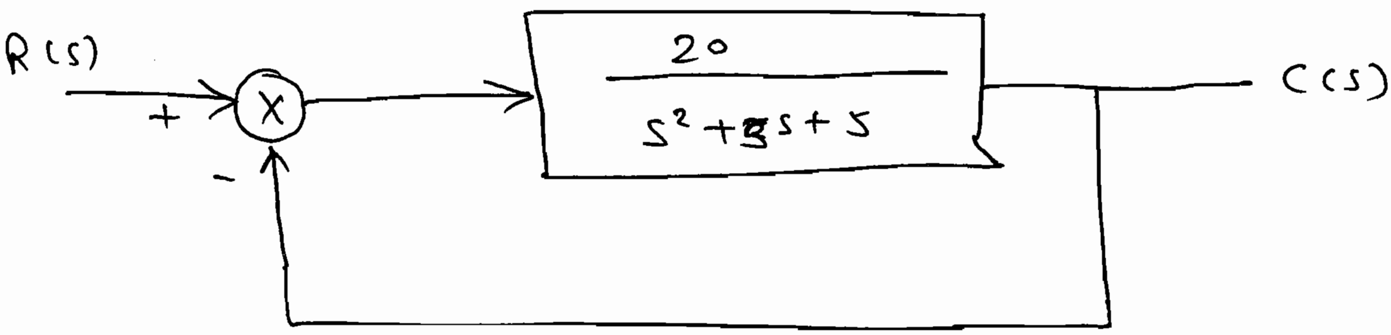
$$\boxed{M_p = 25.4 \%}$$

$$\Rightarrow t_s = 4\tau = \frac{4}{\xi \omega_n}$$

$$t_s = \frac{4}{0.4 \times 5}.$$

$$\therefore \boxed{t_s = 2 \text{ sec}}$$

$\boxed{Q}$ Repeat the above Problem,



Sol^n:

$$G(s) = \frac{20}{s^2+3s+5}.$$

$\Rightarrow$

$$\frac{C(s)}{R(s)} = \frac{20}{s^2 + 5s + 25}$$

$$\frac{C(s)}{R(s)} = \frac{20}{25}\left[\frac{25}{s^2 + 5s + 25}\right].$$

→ This change effect the ss value but do not effect the T.D. specification.

⇒ $\omega_n^2 = 25$

$\boxed{\omega_n = 5 \text{ rad/sec}}$

$2\xi\omega_n = 5$

$2 \times \xi \times \cancel{5} = \cancel{5}$

$\boxed{\xi = 0.5}$

⇒ $\tau = \frac{1}{\xi\omega_n} = \frac{1}{5 \times 0.5}$

$\boxed{\tau = 0.4 \text{ sec}}$

$\omega_d = \omega_n\sqrt{1-\xi^2}.$

$= 5\sqrt{1 - 0.25}$

$\boxed{\omega_d = 4.33 \text{ rad/sec}}$

⇒ $t_s = 4\tau$

$\boxed{t_s = 1.6 \text{ sec}}$

⇒ $t_d = \frac{1 + 0.7\xi}{\omega_n}.$

$= \frac{1 + (0.7 \times 0.5)}{5}$

∴ $\boxed{t_d = 0.27 \text{ sec}}$

⇒ $t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$

$t_r = \frac{\pi - \cos^{-1}(0.5)}{4.33}$

∴ $\boxed{t_r = 0.483 \text{ sec}}$

$\Rightarrow$ $t_p = \dfrac{\pi}{\omega_d}$

$= \dfrac{3.14}{4.33}$

$\boxed{t_p = 0.725 \text{ sec}}$

$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$

$= e^{-3.14\times0.5/\sqrt{0.75}} \times 100\%$

$\boxed{M_p = 16.31\%}$

$\Rightarrow$ the unit step response to the above system is,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t + \cos^{-1}\xi).$$

$$\boxed{c(t) = 1 - \frac{e^{-2.5t}}{\sqrt{0.75}} \cdot \sin(4.33t + \cancel{0.8}\, 60^\circ).}$$

$\Rightarrow$ Variation in time domain specification w.r.t. ξ (ω_n = constant).

$\Rightarrow$ As ξ increases from 0 to 1, the poles moves towards the left and near to the real axis.

$\Rightarrow$ In this case ① $\tau \downarrow$
② $t_s \downarrow$
③ $\omega_d \downarrow$

$\Rightarrow$ As ω_d decreases the time domain specification (t_d, t_r & t_p) $\uparrow$.

$\Rightarrow$ As ξ increase from 0 to 1, $\% M_p$ decreases and the system become more Relatively stable.

$\boxed{Q}$ Find the T.D. specification to the following system

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x.$$

Soln:

$$\frac{C(s)}{R(s)} = \frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}.$$

$\Rightarrow$ $\omega_n^2 = 8$

$\omega_n = 2\sqrt{2}$ rad/sec.

$\boxed{\omega_n = 2.83 \text{ rad/sec}}$

$2\xi\omega_n = \cancel{4}\ 2$

$\xi = \frac{2}{2.83}$.

$\xi = 1/\sqrt{2} = 0.707$.

$\boxed{\xi = 0.707}$

$\Rightarrow$ $\tau = \frac{1}{\xi\omega_n} = \frac{1}{\frac{1}{\sqrt{2}} \times 2\sqrt{2}} = \frac{1}{2 \times 2} = 0.5$

$\boxed{\tau = 0.5 \text{ sec}}$

$\Rightarrow$ $t_s = 4\tau = 4 \times 0.5$

$\boxed{t_s = 2 \text{ sec}}$

$$\Rightarrow \quad t_d = \frac{1 + 0.7\xi}{\omega_n}$$

$$= \frac{1 + (0.7 \times 0.707)}{2.83}$$

$$\therefore \boxed{t_d = 0.53 \text{ sec}}$$

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\boxed{\omega_d = 2 \text{ rad/sec}}$$

$$\therefore \quad t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$$

$$\therefore t_r = \frac{\pi - \cos^{-1} 0.707}{2}$$

$$\therefore \boxed{t_r = 1.177 \text{ sec}}$$

$$t_p = \pi / \omega_d$$

$$= 3.14 / 2$$

$$\boxed{t_p = 1.57 \text{ sec}}$$

$$M_p = e^{-\pi\xi / \sqrt{1-\xi^2}} \times 100\,\%$$

$$= e^{-3.14 \times 0.707 / \sqrt{1-0.707^2}} \times 100\,\%$$

$$\Rightarrow \boxed{M_p = 4.33\,\%}$$