Class XI Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer(VSA) questions of 2 marks each.
4. Section C has 6 Short Answer(SA) questions of 3 marks each.
5. Section D has 4 Long Answer(LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternatives in all such questions.

Section A

1. A digit is selected at random from either of the two sets {1, 2, 3, 4, 5, 6, 7, 8, 9} and {1, 2, 3, 4, 5, 6, 7, 8, 9}. What is the chance that the sum of the digits selected is 10? **[1]**

   a) $\frac{1}{9}$ b) $\frac{2}{9}$

   c) $\frac{10}{18}$ d) $\frac{10}{81}$

2. The mean of 20 observations is 15. On checking, it was found that two observations were wrongly copied as 3 and 6. If wrong observations are replaced by correct values 8 and 4, then the correct mean is **[1]**

   a) 15.35 b) 15

   c) 15.15 d) 16

3. A retailer purchases a fan for ₹1500 from a wholesaler and sells it to a consumer at 10% profit. If the sales are intra-state and the rate of GST is 12%, the selling price of the fan by the retailer (excluding tax) is: **[1]**

   a) ₹ 1800 b) ₹ 1650

   c) ₹ 1848 d) ₹ 1500

4. If $\log_{\sqrt{3}} 27$ = x, then the value of x is **[1]**

   a) 3 b) 9

   c) 4 d) 6

5. If set A = {1, 2} and set B = {a, b}, then cartesian product of set and A set B is given by **[1]**

a) $A \times B = \{(a, 1), (b, 1), (2, a), (2, b)\}$

b) $A \times B = \{(a, 1), (1, b), (2, a), (2, b)\}$

c) $A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

d) $A \times B = \{(a, 1), (b, 1), (2, a), (b, 2)\}$

6. If $5^{x+2} = 625$ then value of x is **[1]**

a) -2

b) 1

c) 0

d) 2

7. A bag contains 5 brown and 4 black socks. A man pulls out two socks. The probability that these are of the same colour is: **[1]**

a) $\frac{30}{108}$

b) $\frac{18}{108}$

c) $\frac{5}{108}$

d) $\frac{48}{108}$

8. If (x, 3) and (3, 5) are the extremities of a diameter of a circle with centre at (2, y), then the values of x and y are **[1]**

a) None of these

b) (3, 1)

c) x = 4, y = 1

d) x = 8, y = 2

9. Pointing to a woman in a photograph, Ramesh said **She is the daughter of the father of the sister of my brother**. How is that woman related to Ramesh? **[1]**

a) Wife

b) Mother

c) Sister

d) Daughter

10. If m is the geometric mean of **[1]**
$\left(\frac{y}{z}\right)^{\log(yz)}, \left(\frac{z}{x}\right)^{\log(zx)}$ and $\left(\frac{x}{y}\right)^{\log(z))}$
then what is the value of m?

a) 6

b) 9

c) 3

d) 1

11. Standard form of 0.0029 is **[1]**

a) 2.9×10^{-3}

b) 29×10^{2}

c) 29×10^{-2}

d) 2.9×10^{-4}

12. Relationship between annual nominal rate of interest and annual effective rate of interest, if frequency of compounding is greater than one: **[1]**

a) Effective rate < Nominal rate

b) Effective rate <= Nominal rate

c) Effective rate > Nominal rate

d) Effective rate = Nominal rate

13. A shopkeeper bought a TV from a distributor at a discount of 25% of the listed price of ₹ 32000. The shopkeeper sells the TV to a consumer at the listed price. If the sales are intra-state and the rate of GST is 18%, the price including tax (under GST) of the TV paid by the consumer is: **[1]**

a) ₹ 34880

b) ₹ 28320

c) ₹ 32000

d) ₹ 37760

14. Three identical dice are rolled. The probability that the same number will appear on each of them is **[1]**

a) $\frac{1}{6}$

b) $\frac{1}{18}$

c) $\frac{1}{36}$ d) $\frac{3}{28}$

15. A biased coin with probability p, 0 < p < 1, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals **[1]**

a) $\frac{2}{3}$ b) $\frac{2}{5}$

c) $\frac{1}{3}$ d) $\frac{3}{5}$

16. Mahesh invested an amount of ₹ 10000 in a fixed deposit scheme for 2 years at a compound interest rate 8% per annum. How much amount will Mahesh get on the maturity of the deposit? **[1]**

a) ₹ 11664 b) ₹ 11644

c) ₹ 11446 d) ₹ 11466

17. How many even numbers can be formed by using all the digits 2, 3, 4, 5, 6? **[1]**

a) 72 b) 36

c) 120 d) 24

18. The domain of the relation, R = {(x, y) : x , y ∈ Z , zxy = 4} is **[1]**

a) {-2, -1, 1, 2, 4} b) {-2, -1, 1, 2}

c) {1, 2, 4} d) {-4, -2, -1, 1, 2, 4}

19. **Assertion (A):** If each of the observations x_1, x_2, ..., x_n is increased by a, where a is a negative or positive number, then the variance remains unchanged. **[1]**

Reason (R): Adding or subtracting a positive or negative number to (or from) each observation of a group does not affect the variance.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** If the sum of first two terms of an infinite GP is 5 and each term is three times the sum of the succeeding terms, then the common ratio is $\frac{1}{4}$. **[1]**

Reason (R): In an AP 3, 6, 9, 12 ..... the 10th term is equal to 33.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. A and B together can build a wall in 30 days. If A is twice as good a workman as B, in how many days will A alone finish the work? **[2]**

22. If **RAHUL** is coded as 22 - 5 - 12 - 25 - 16, then how will you code **VIRAT**? **[2]**

OR

A and B are brothers. C and D are sisters. A's son is D's brother. How B is related to C?

23. A clock gains 5 seconds in 2 minutes and was set right at 9:00 a.m. If it shows 2:30 in the afternoon on the same day. What is the correct time? **[2]**

24. If y = x + $\frac{1}{x}$, prove that $x^2\frac{dy}{dx}$ - xy + 2 = 0. **[2]**

OR

Differentiate the given function w.r.t. x: $\frac{\sqrt{(x-3)(x^2+4)}}{3x^2+4x+5}$

25. Convert the decimal number to the binary number: 639 **[2]**

Section C

26. If a, b, c, d are in G.P, show that $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$ are in G.P. **[3]**

OR

If A_1 and A_2 are two A.M.'s between a and b, prove that

i. $A_1 + A_2 = a + b$

ii. $(2A_1 - A_2)(2A_2 - A_1) = ab$

27. Rohit is the husband of Vanshika. Sumita is the sister of Rohit. Anushka is the sister of Vanshika. How Anushka is related to Rohit? **[3]**

28. Find the domain of the function f defined by f(x) = $\sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$ **[3]**

29. A machinery can be purchased by paying ₹ 600,000 now or ₹ 200,000 initially and two instalments of ₹ 300,000 at the end of second year and ₹ 190,000 at the end of 4th year respectively. To pay cash now, the buyer would have to withdraw the money from an investment earning interest at 8 % compounded annually. Which option is better and by how much in present value terms? **[3]**

30. Mr. Sharma in Delhi is using an MTNL connection with a monthly plan of ₹ 240, which has no free calls. The call charges are ₹ 1 per minute. Calculate the telephone bill payable for Mr. Sharma if he talks for 470 minutes in a month. Assume the broadband charges to be Nil. A GST of 18% is levied on the total bill amount. **[3]**

31. In a class of 50 students, 30 students like Mathematics, 25 like Science, and 16 like both. Find the number of students who like **[3]**

i. either Mathematics or Science

ii. neither Mathematics nor Science.

Section D

32. How many 3-digit numbers are there which have exactly one of their digits as 6? **[5]**

OR

In how many ways can 9 examination papers be arranged so that the best and the worst papers are never together?

33. Discuss the continuity of the function f(x) at x = 1, defined by $f(x) = \begin{cases} \frac{3}{2} - x, & \text{if } \frac{1}{2} \leq x < 1 \\ \frac{3}{2}, & \text{if } x = 1 \\ \frac{3}{2} + x, & \text{if } 1 < x \leq 2 \end{cases}$ **[5]**

34. Find Cov (X, Y) between X and Y, if **[5]**

X:	1	2	3	4	5
Y:	2	4	6	8	10

OR

Find the mean deviation about the median for the following data:

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

35. Mr. Rishabh lives in Bengaluru, Karnataka. He consumed 37 kL of water in one month. Calculate his water bill for that month. Water tariff plan is given below. **[5]**

Units of consumption (in kL)	upto 8	8 to 25	25 - 50	>50

Price per unit (in ₹)	7	11	25	45

Maintenance charge = ₹ 56 per month

Sewerage charges flat ₹ 14 for those whose consumption is upto 8 kL and 25% of water charges whose consumption is more than 8 kL.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Children always keep of thinking something new, sometimes reaching a destination they say we will go by longer sometimes by shorter route, etc.

A child is standing at the point A(2, 3) and far away he can notice a straight road path represented by the equation 3x - 4y - 12 = 0. He wants to reach path in the minimum possible time.

(a) Find the slope of the path followed by child?

(b) Find the Equation of the path followed by child?

(c) Find the Distance covered by child in reaching the path?

OR

If child wants to reach at point (4, 0) on the straight road, then find the equation of path he should follow.

37. **Read the text carefully and answer the questions:** **[4]**

Different organisations collect the data and analyse it quantitatively. During one such analysis some mistake crept in. The result given was that mean and variance of 100 observations as 40 and 5.1 but later on rechecking it was found that one observation was mistakenly taken as 50 instead of 40.

(a) What is incorrect sum of variates?

(b) What is correct sum of observations?

(c) What is incorrect Σx^2?

OR

What is corrected variance?

38. **Read the text carefully and answer the questions:** **[4]**

Rohit and Rakesh planned to play Business Board game with two dice. In which they will get the turn one by one, they roll the dice and continue the game.

(a) Rakesh got first chance to roll the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is 8?

(b) Rohit got next chance. What is the probability that he got the sum of the two number appearing on the top face of the dice is 13?

(c) Rohit got next change. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is equal 7?

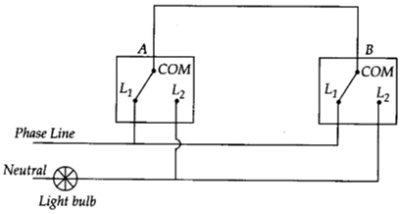
OR

Read the text carefully and answer the questions: **[4]**

Following is a circuit diagram showing an electronic assembly consists of two sub-systems say A and B:

From previous testing procedures, the following probabilities are assumed to be known.

P(A fails) = 0.2; P(B fails alone) = 0.15; P(A and B fail) = 0.15.



(a) What is probability that A fails alone?

(b) What is probability that neither A nor B fails?

(c) What is conditional probability that A fails given that B has failed?

Solution

Section A

1. **(a)** $\frac{1}{9}$

   Explanation: Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9} then, n (A $\times$ A) = g^2

   Let B be the event that sum of the digits is 10. Then,

   B = {(1, 9), (9, 1), (4,6), (6, 4), (8, 2), (2, 8), (7, 3), (3, 7), (5, 5)}

   $\therefore$ Required probability $= \frac{n(B)}{n(A \times A)} = \frac{9}{9^2} = \frac{1}{9}$

2. **(c)** 15.15

   Explanation: Sum of all observations = 20 $\times$ 15 = 300

   Sum of correct observations = 300 - (3 + 6) + (8 + 4) = 303

   Correct mean = $\frac{303}{20}$ = 15.15

3. **(b)** ₹ 1650

   Explanation: If a retailer purchases a fan for ₹ 1500 from a wholesaler and sells it to a consumer at 10% profit and the rate of GST is 12% then, including tax (under GST) the selling price would be ₹$\frac{1500 \times 110}{100} = 1650$

4. **(d)** 6

   Explanation: $\log_{\sqrt{3}} 27 = x \Rightarrow (\sqrt{3})^x = 27$

   $\Rightarrow 3^{\frac{x}{2}} = 3^3 \Rightarrow \frac{x}{2} = 3 \Rightarrow x = 6$

5. **(c)** A $\times$ B = {(1, a), (1, b), (2, a), (2, b)}

   Explanation: The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called cartesian product of sets A and B.

   $\therefore$ A $\times$ B = {(1, a), (1, b), (2, a), (2, b)}

6. **(d)** 2

   Explanation: as $5^{x+2} = 625 \Rightarrow 5^{x+2} = 5^4$

   $\Rightarrow$ x + 2 = 4 $\Rightarrow$ x = 2

7. **(d)** $\frac{48}{108}$

   Explanation: P(same coloured socks) = P(both brown) + P(both white)

   = $\frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8}$

   = $\frac{20}{72} + \frac{12}{72}$

   = $\frac{32}{72}$

   = $\frac{4}{9} = \frac{48}{108}$

8. **(a)** None of these

   Explanation: The endpoints of the diameter of a circle are (x, 3) and (3, 5).

   According to the question, we have:

   centre is midpoint of the endpoints of diameters.

   $\frac{x+3}{2}$ = 2, y = $\frac{5+3}{2}$

   $\Rightarrow$ x = 1, y = 4

9. **(c)** Sister

   Explanation: Father of sister of my brother is my father C also.

   So, daughter of my father is my sister.

10.

(d) 1

Explanation: Here, m = $\left[\left(\frac{y}{z}\right)^{\log(yz)} \times \left(\frac{z}{x}\right)^{\log(zx)} \times \left(\frac{x}{y}\right)^{\log(xy)}\right]^{1/3}$

$\therefore m^3 = x^{\log(xy) - \log(zx)} \times y^{\log(yz) - \log(xy)} \times x^{\log(zx) - \log(yx)}$

$\Rightarrow m^3 = x^{\log\left(\frac{y}{z}\right)} \times y^{\log\left(\frac{z}{x}\right)} \times z^{\log\left(\frac{x}{y}\right)}$

Taking log on both sides, we get

$3 \log m = \log\left(\frac{y}{z}\right)\log x + \log\left(\frac{z}{x}\right)\log y + \log\left(\frac{x}{y}\right)\log z$

$\Rightarrow 3 \log m = \log y \log x - \log z \log x + \log z \log y - \log x \log y + \log x \log z - \log y \log z$

$\Rightarrow 3 \log m = 0 \Rightarrow \log m = 0 \Rightarrow m = e^0 = m = 1$

11. **(a)** 2.9×10^{-3}

Explanation: Standard form of

0.0029

$= 2.9 \times 10^{-3}$

12.

(c) Effective rate > Nominal rate

Explanation: If interest is compounded more than once a year the effective interest rate for a year exceeds the per annum nominal interest rate i.e., effective rate > nominal rate

13.

(d) ₹ 37760

Explanation: ₹ 37760

14.

(c) $\frac{1}{36}$

Explanation: Since throwing a single die three times is equivalent to throw three dice at a time.

$\therefore$ Sample space = {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6), ...}

Here, $n(5) = 6^3$

$\therefore$ Required Probability $= \frac{6}{6^3} = \frac{1}{6^2} = \frac{1}{36}$

15.

(c) $\frac{1}{3}$

Explanation: p is the probability of getting head.

q = 1 - p is the probability of getting tail.

The number of tosses required is even.

$= qp + q^3p + q^5p + q^7p + q^9p$ .....

$= qp\left(\frac{1}{1-q^2}\right)$

$= \frac{(1-p)p}{1-(1-p)^2}$

$= \frac{(1-p)p}{1-\left(1-2p+p^2\right)}$

$= \frac{1-p}{2-p}$

Given $\frac{1-p}{2-p} = \frac{2}{5}$

$\Rightarrow p = \frac{1}{3}$

16. **(a)** ₹ 11664

Explanation: Amount = $= 10000\left(1 + \frac{8}{100}\right)^2 = \frac{10000 \times 108 \times 108}{100 \times 100} = ₹11664$.

17. **(a)** 72

Explanation: To form an even number the last number can only be an even digit, therefore the number of impossibility for the last digit of number = 3

Now the ten's place can be filled by any of the remaining 4 digits, and hence the no. of ways for ten's place = 4

Then there remain three digits, so no. of ways of filling hundred's place = 3

Similarly no. of ways of filling thousand's place = 2 and of ten thousand = 1

Therefore, the total possibilities are = $3 \times 4 \times 3 \times 2 \times 1 = 72$

18.

(d) {-4, -2, -1, 1, 2, 4}

Explanation: Given, R = {(x, y) : x, y ∈ Z, zxy = 4}

= {(-4, -1), (-2, -2),

(-1, -4), (1, 4), (2, 2), (4, 1)}

Therefore, domain of R = {-4, -2, -1, 1, 2, 4}

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: Let $\bar{x}$ be the mean of x_1, x_2 ..., x_n. Then, variance is given by

$\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

If a is added to each observation, the new observations will be

$y_i = x_i + a$

Let the mean of the new observations be $\bar{y}$.

Then,

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}(x_i + a)$

$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a\right]$

$= \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{na}{n} = \bar{x} + a$

i.e. $\bar{y} = \bar{x} + a$ ...(ii)

Thus, the variance of the new observations is $\sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + a - \bar{x} - a)^2$ [using Eqs. (i) and (ii)]

$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sigma_1^2$

Thus, the variance of the new observations is same as that of the original observations.

Reason: We may note that adding (or subtracting) a positive number to (or from) each observation of a group does not affect the variance.

20.

(c) A is true but R is false.

Explanation: Assertion Let a be the first term and r(|r| < 1) be the common ratio of the GP.

∴ The GP is a, ar, ar^2,...

According to the question,

$T_1 + T_2 = 5 \Rightarrow a + ar = 5 \Rightarrow a(1 + r) = 5$

and $T_n = 3(T_{n+1} + T_{n+2} + T_{n+3} + ...)$

$\Rightarrow ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + ...)$

$\Rightarrow ar^{n-1} = 3ar^n(1 + r + r^2 + ...)$

$\Rightarrow 1 = 3r(\frac{1}{1-r})$

$\Rightarrow 1 - r = 3r$

$\Rightarrow r = \frac{1}{4}$

Reason: Given, 3, 6, 9, 12 ...

Here, a = 3, d = 6 - 3 = 3

∴ $T_{10} = a + (10 - 1)d$

$= 3 + 9 \times 3$

$= 3 + 27 = 30$

Section B

21. Since A is twice as good a workman as B,

A's one day work = B's 2 days work

$\Rightarrow$ B's one day work = A's $\frac{1}{2}$ day work ..(i)

Since A and B together can build a wall in 30 days,

∴ A's one day work + B's one day work = $\frac{1}{30}$

⇒ A's one day work + A's $\frac{1}{2}$ day work = $\frac{1}{30}$ [using (i)]

⇒ A's 1 + $\frac{1}{2}$ i.e. $\frac{3}{2}$ days work = $\frac{1}{30}$

⇒ A's one day work = $\frac{2}{3} \times \frac{1}{30} = \frac{1}{45}$

∴ An alone can complete the work in 45 days.

22. Here R is coded as 22 which is its actual positions 18 + 4.

Similarly, A is coded as 1 + 4 i.e. 5.

H is coded as 8 + 4 = 12, U is coded as 21 + 4 = 25 and L is coded as 12 + 4 = 16.

V is equivalent to 22 + 4 = 26

I is equivalent to 9 + 4 = 13

R is equivalent to 18 + 4 = 22

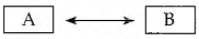
A is equivalent to 1 + 4 = 5

and T is equivalent to 20 + 4 = 24

∴ 'VIRAT' will be coded as 26 - 13 - 22 - 5 - 24

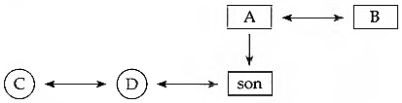
OR

A and B are brothers



C and D are sisters

A's son is D's brother



∴ B is C's uncle

23. Given that, the clock gains 5 seconds in 2 minutes

⇒ it gains 12 × 5 = 60 seconds i.e. 1 minute in 12 × 2 = 24 minutes

⇒ when the incorrect clock moves 25 minutes, the correct clock moves 24 minutes.

Now, from 9:00 a.m. to 2:30 p.m. on the same day the time passed by incorrect clock

= 5 hours 30 minutes = 330 minutes.

When an incorrect clock moves 330 minutes, the correct clock moves

= $\frac{24}{25}\times$ 330 minutes = 316 minutes 48 seconds

= 5 hours 16 minutes 48 seconds

Hence, the correct time is 2:6:48 p.m.

24. Let assume: $y = x + \frac{1}{x}$ ..... (1)

Diff. (1), w.r.t. 'x', we get

$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = 1 + \left(\frac{-1}{x^2}\right)$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$ ..... (2)

We have to prove

$x^2\frac{dy}{dx} - xy + 2 = 0$

L.H.S. $x^2\frac{dy}{dx} - xy + 2$

$= x^2\left(1 - \frac{1}{x^2}\right) - x\left(\frac{x+1}{x}\right) + 2$

Using (1) and (2)

= x^2 - 1 - x^2 - 1 + 2

= 0 = R.H.S

Hence proved.

OR

Let y = $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$, taking logarithm of both sides and applying the properties of logarithm, we get

log y = $\frac{1}{2}$ [log (x - 3) + log (x^2 + 4) - log ($3x^2$ + 4x + 5)]

Differentiating both sides w.r.t. x, we get

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-3} \cdot 1 + \frac{1}{x^2+4} \cdot 2x - \frac{1}{3x^2+4x+5} \cdot (3 \cdot 2x+4)\right]$

$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left(\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right)$

$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left[\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$

25. Given decimal number is 639

2	639	
2	319	1
2	159	1
2	79	1
2	39	1
2	19	1
2	9	1
2	4	1
2	2	0
2	1	0
2	0	1

Required binary number is 1001111111

Section C

26. From the question, it is given that a, b, c, d are in G.P

So, bc = ad

$b^2 = ac$

$c^2 = bd$

We have to show that,

$a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$ are in G.P.

Then, $(b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$

Consider the LHS = $(b^2 + c^2)^2$

$= b^4 + c^4 + 2b^2 c^2$

From the equation (ii) and equation (iii)

$= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$

$= c^2(a^2 + b^2) + d^2(a^2 + b^2)$

$= (a^2 + b^2)(c^2 + d^2)$

Now consider the

RHS = $(a^2 + b^2)(c^2 + d^2)$

By comparing the LHS and RHS

LHS = RHS

Hence it is proved that, $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$ are in G.P.

OR

Let A_1, A_2 be two A.M.'s between a and b, then a, A_1, A_2, b are in A.P.

As a, A_1, A_2 are in A.P.

$\therefore 2A_1 = a + A_2$

$\Rightarrow 2A_1 - A_2 = a$ ...(1)

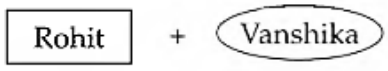
Similarly, A_1, A_2, b are in A.P.

$\Rightarrow 2A_2 - A_1 = b$ ...(2)

i. Adding (1) and (2), we get $A_1 + A_2 = a + b$

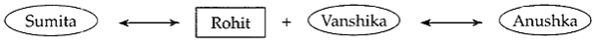
ii. Multiplying (1) and (2), we get $(2A_1 - A_2)(2A_2 - A_1) = ab$

27. Rohit is the husband of Vanshika



Sumita is the sister of Rohit

Anushka is the sister of Vanshika



So Anushka is Rohit's wife's sister

Anushka is the sister-in-law of Rohit.

28. Let f = g + h, then $g(x) = \sqrt{4 - x}$ and $h(x) = \frac{1}{\sqrt{x^2 - 1}}$

For D_g, g(x) must be a real number

$\Rightarrow 4 - x \geq 0$

$\Rightarrow 4 \geq x \Rightarrow x \leq 4 \Rightarrow D_g = (-\infty, 4]$

For D_h, h(x) must be a real number

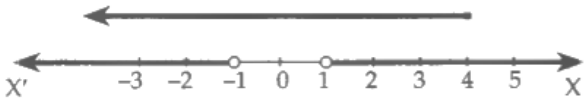
$\Rightarrow x^2 - 1 > 0$

$\Rightarrow (x + 1)(x - 1) > 0$

$\Rightarrow x < -1$ or $x > 1$

$\Rightarrow D_h = (-\infty, -1) \cup (1, \infty)$

As f = g + h, so $D_f = D_g \cap D_h$



From fig. it is clear that $D_g \cap D_h = (-\infty, -1) \cup (1, 4]$

Hence, the domain of the function $f = (-\infty, -1) \cup (1, 4]$

29. Let P_1 be the present value of ₹ 300,000 due 2 years hence at $i = \frac{8}{100} = 0.08$ compounded annually. Also, let P_2 be the present value of ₹ 190,000 due 4 years hence at $i = \frac{8}{100} = 0.08$ compounded annually. Then,

$P_1 = 300{,}000(1 + i)^{-2}$ and $P_2 = 190{,}000(1 + i)^{-4}$

$\Rightarrow P_1 = 300{,}000(1.08)^{-2}$ and $P_2 = 190{,}000(1.08)^{-4}$

$\Rightarrow P_1 = 300{,}000 \times 0.85733882$ and $P_2 = 190{,}000 \times 0.73502985$

$\Rightarrow P_1 = 257{,}201.446$ and $P_2 = 139{,}655.6715$

$\Rightarrow P_1 + P_2 = 396{,}857.3175 \simeq 396{,}857.32$

Thus, in the second option the machinery costs ₹ (200,000 + 396,857.32) = ₹ 596,857.32

Clearly, in the second option the machinery costs less. Hence, second option is better and it is better by ₹ (600,000 - 596,857.32) = ₹ 3142.68

30. Here, the monthly charges are ₹ 240

Since the tariff plan chosen by Mr. Sharma has no free calls and he talks for 470 minutes

∴ Call charges @ ₹ 1 per minute = ₹ (470×1) = ₹ 470

Broband charges = Nil

∴ Total Bill Amount = Monthly Charges + Call Charges

= ₹ 240 + ₹ 470 = ₹710

A GST of 18% is levied on the total bill amount

GST= 18% of 710

= ₹ 127.80

∴ Total Bill Payable = ₹ 710 + ₹ 127.80

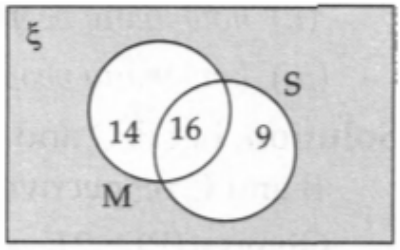
= ₹ 837.80

31. We draw a Venn diagram to solve the problem.

Here ξ = all the students of the class,

M = students who like Mathematics and

S = students who like Science.



Since 16 students like both Mathematics and Science, we mark 16 in the common region of M and S. Then, as 30 students like Mathematics and of these 16 students have already been marked, therefore, 14 is marked in the remaining portion of M. Also, as 25 students like Science and of these 16 have already been marked, therefore, 9 is marked in the remaining portion of S.

i. The number of students who like either Mathematics or Science

= 14 + 16 + 9 = 39

ii. The number of students who like neither Mathematics nor Science (shown shaded in the diagram)

= 50 - 39 = 11

Section D

32. i. When six is at unit's place then possible numbers = $8 \times 9 \times 1 = 72$.

ii. When six is at ten's place then possible numbers = $8 \times 1 \times 9 = 72$.

iii. When six is at hundred's place then possible numbers = $1 \times 9 \times 9 = 81$.

Required number of ways = 72 + 72 + 81 = 225.

(in case I and II hundred place cannot have zero and six).

OR

The number of arrangements in which the best and the worst papers never come together can be obtained by subtracting from the total number of arrangements, the number of arrangements in which the best and worst come together.

The total number of arrangements of 9 papers = ${}^{9}P_{9} = 9!$

Considering the best and the worst papers as one paper, we have 8 papers which can be arranged in ${}^{8}P_{8} = 8!$ ways. But, the best and worst papers can be put together in 2! ways. So, the number of permutations in which the best and the worst papers can be put together = $(2! \times 8!)$.

Hence, the number of ways in which the best and the worst papers never come together = $9! - 2! \times 8! = 9 \times 8! - 2 \times 8! = 7 \times 8!$ = 282240

33. $\underset{x=1}{LHL} = \lim\limits_{x \to 1^-} \left(\frac{3}{2} - x\right) = \frac{3}{2} - 1 = \frac{1}{2}$

$\underset{x=1}{RHL} = \lim\limits_{x \to 1^+} \left(\frac{3}{2} + x\right) = \frac{3}{2} + 1 = \frac{5}{2}$

$\underset{x=1}{LHL} \neq \underset{x=1}{RHL}$. Hence, the function is discontinuous at x = 1.

$= \lim\limits_{h \to 0} \frac{\sqrt{1+k(-h)} - \sqrt{1-k(-h)}}{-h}$

$= \lim\limits_{h \to 0} \frac{(\sqrt{1-kh} - \sqrt{1+kh})(\sqrt{1-kh} + \sqrt{1+kh})}{-h[\sqrt{1-kh} + \sqrt{1+kh}]}$

$= \lim\limits_{h \to 0} \frac{1-kh-1-kh}{-h[\sqrt{1-kh} + \sqrt{1+kh}]}$

$= \lim\limits_{h \to 0} \frac{-2kh}{-h[\sqrt{1-kh} + \sqrt{1+kh}]}$

$= \lim\limits_{h \to 0} \frac{2k}{\sqrt{1-kh} + \sqrt{1+kh}} = \frac{2k}{1+1} = k$ ...(iii)

$\underset{x=0}{RHL} = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} f(h)$

$= \lim\limits_{h \to 0} \frac{2(h)+1}{h-1} = \frac{0+1}{0-1} = -1$ ...(iv)

From (i), (ii), (iii) and (iv), we get

k = -1, for function to be continuous at x = 0.

34. We have,

x_i	y_i	$x_i - \bar{X}$	$y_i - \bar{Y}$	$(x_i - \bar{X})(y_i - \bar{Y})$
1	2	-2	-4	8
2	4	-1	-2	2
3	6	0	0	0
4	8	1	2	2
5	10	2	4	8
$\Sigma x_i = 15$	$\Sigma y_i = 30$		$\Sigma (x_i - \bar{X})(y_i - \bar{Y}) = 20$	

We have, n = 5, $\Sigma x_i = 15$ and $\Sigma y_i = 30$

$\therefore \bar{X} = \frac{1}{n} \sum x_i = \frac{15}{5} = 3$, $\bar{Y} = \frac{1}{n} \sum y_i = \frac{30}{5} = 6$, $\Sigma (x_i - \bar{X})(y_i - \bar{Y}) = 20$ and, n = 5

$\therefore$ Cov (X, Y) = $\frac{1}{n} \sum (x_i - \bar{X})(y_i - \bar{Y}) = \frac{20}{5} = 4$

It is clear from the above illustration that if $\bar{X}$ and $\bar{Y}$ are not integers, then the calculations for the covariance by using formula (i) will be cumbersome and time consuming. We therefore develop an alternate formula as discussed below.

We have,

Cov (X, Y) = $\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{X})(y_i - \bar{Y})$

$\Rightarrow$ Cov (X, Y) = $\frac{1}{n} \sum_{i=1}^{n} \{x_i y_i - x_i \bar{Y} - y_i \bar{X} + \bar{X}\bar{Y}\}$

$\Rightarrow$ Cov (X, Y) = $\frac{1}{n} \left\{ \sum_{i=1}^{n} x_i y_i - \bar{Y} \sum_{i=1}^{n} x_i - \bar{X} \sum_{i=1}^{n} y_i + n\bar{X}\bar{Y} \right\}$

$\Rightarrow$ Cov (X, Y) = $\frac{1}{n} \left\{ \sum_{i=1}^{n} x_i y_i - \bar{Y}(n\bar{X}) - \bar{X}(n\bar{Y}) + n\bar{X}\bar{Y} \right\}$ [$\because \sum_{i=1}^{n} x_i = n\bar{X}, \sum_{i=1}^{n}$ y = n$\bar{Y}$]

$\Rightarrow$ Cov (X, Y) = $\frac{1}{n} \left\{ \sum_{i=1}^{n} x_i y_i - n\bar{X}\bar{Y} \right\}$

= $\frac{1}{n} \sum_{i=1}^{n} x_i y_i - \bar{X}\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} x_i y_i - \left\{ \frac{1}{n} \sum_{i=1}^{n} x_i \right\} \left\{ \frac{1}{n} \sum_{i=1}^{n} y_i \right\}$

Hence, Cov (X, Y) = $\frac{1}{n} \sum_{i=1}^{n} x_i y_i - \left\{ \frac{1}{n} \sum_{i=1}^{n} x_i \right\} \left\{ \frac{1}{n} \sum_{i=1}^{n} y_i \right\}$ ...(ii)

i.e., Cov (X, Y) = (Mean of the product of values of (X and Y) - (Product of means of X and Y).

OR

Table for mean deviation about median

x_i	**f_i**	**cf**	**$\lvert d_i \rvert = \lvert x_i - 13 \rvert$**	**$f_i \lvert d_i \rvert$**
3	3	3	10	30
6	4	7	7	28
9	5	12	4	20
12	2	14	1	2
13	4	18	0	0
15	5	23	2	10
21	4	27	8	32
22	3	30	9	27
	$\sum_{i=1}^{n} f_i = 30$			$\sum_{i=1}^{8} f_i \lvert d_i \rvert = 149$

N = 30, N is even, $\frac{N}{2} = 15$ or $\frac{2(30+1)}{4} = 15.5$ lies between 15 and 16.

Median = $\left(\frac{\text{Value of 15 th term} + \text{Value of 16 th term}}{2} \right) = \frac{13+13}{2} = 13$

Mean deviation = $\frac{\sum_{i=1}^{8} f_i|d_i|}{\sum_{i=1}^{8} f_i} = \frac{149}{30}$ = 4.97

35. Here, the consumption of water is given to be 37 kL,

According to the given tariff plan

Water consumption charge $= ₹(8 \times 7 + 17 \times 11 + 12 \times 25)$

= ₹ (56 + 187 + 300)

= ₹ (543)

Sewerage charge for consumption above 8 KL is 25% of the consumption charges.

∴ Sewerage charge = 25% of 543

= 135.75

Maintenance charges = ₹ 56 per month

Total water bill = consumption charge + Sewerage charge +Maintenance charge

= ₹ 543 + ₹ 135.75 + ₹ 56

= ₹ 734.75

Section E

36. **Read the text carefully and answer the questions:**

Children always keep of thinking something new, sometimes reaching a destination they say we will go by longer sometimes by shorter route, etc.

A child is standing at the point A(2, 3) and far away he can notice a straight road path represented by the equation 3x - 4y - 12 = 0. He wants to reach path in the minimum possible time.

(i) m $\times \frac{3}{4}$ = -1 $\Rightarrow$ m = $\frac{-4}{3}$

(ii) Equation of line through (2, 3) and having shope $\frac{-4}{3}$ is

y - 3 = $\frac{-4}{3}$(x - 2)

$\Rightarrow$ 3y - 9 = -4x + 8

$\Rightarrow$ 4x + 3y - 17 = 0

(iii) Distance = $\left|\frac{6-12-12}{\sqrt{9+16}}\right|$

= $\left|\frac{-18}{5}\right|$ = $\frac{18}{5}$units

OR

Equation of path joining (2, 3) and (4, 0) is

y - 0 = $\frac{0-3}{4-2}$(x - 4)

$\Rightarrow$ 2y = -3x + 12

$\Rightarrow$ 3x + 2y - 12 = 0

37. **Read the text carefully and answer the questions:**

Different organisations collect the data and analyse it quantitatively. During one such analysis some mistake crept in. The result given was that mean and variance of 100 observations as 40 and 5.1 but later on rechecking it was found that one observation was mistakenly taken as 50 instead of 40.

(i) Mean = 40, n = 100, sum = 100 $\times$ 40 = 4000

(ii) Corrected sum = 4000 - 50 + 40 = 3990

(iii) $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\Rightarrow (5.1)^2 = \frac{\Sigma x^2}{100} - (40)^2$

$\Rightarrow$ (26.01 + 1600)100 = Σx^2

OR

Corrected Σx^2 = 162601 - $(50)^2$ + $(40)^2$

= 162601 - 2500 + 1600 = 161701

Corrected $\sigma^2 = \frac{161701}{100} - (39.9)^2$

= 1617.01 - 1592.01 = 25

38. **Read the text carefully and answer the questions:**

Rohit and Rakesh planned to play Business Board game with two dice. In which they will get the turn one by one, they roll the dice and continue the game.

(i) Favourable cases (2, 6), (6, 2), (4, 4), (5, 3), (3, 5)

Probability of getting the sum as eight = $\frac{5}{36}$

(ii) As the sum of numbers on two dice is 13 will not be possible, therefore zero probability.

(iii) Favourable events of getting sum = 7

i.e, (1, 6), (6,1), (5, 2), (4, 3), (3, 4), (2, 5)

Required probability = $\frac{6}{36} = \frac{1}{6}$

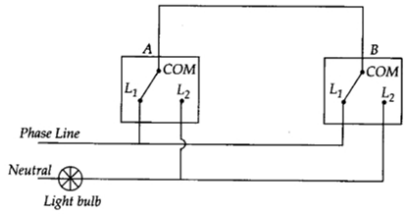
OR

Read the text carefully and answer the questions:

Following is a circuit diagram showing an electronic assembly consists of two sub-systems say A and B:

From previous testing procedures, the following probabilities are assumed to be known.

P(A fails) = 0.2; P(B fails alone) = 0.15; P(A and B fail) = 0.15.



(i) P(A fails alone) = P(A) - P(A ∩ B)

= 0.20 - 0.15 = 0.05

(ii) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

= 0.20 + 0.30 - 0.15 = 0.35

$\therefore P(\overline{A} \cap \overline{B}) = 1 - 0.35 = 0.65$

(iii) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.30} = \frac{1}{2} = 0.50$