Sample Question Paper - 21
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes   **Maximum Marks: 40**

General Instructions:
1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.
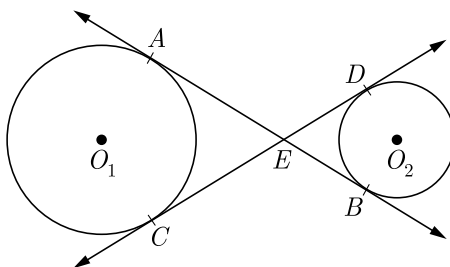

SECTION A

1. Find the values of k for which the quadratic equation $x^2 + 2\sqrt{2k}\,x + 18 = 0$ has equal roots.

OR

If α and β are the roots of $ax^2 - bx + c = 0\,(a \neq 0)$, then calculate $\alpha + \beta$.

2. Find the AP whose third term is 5 and seventh term is 9.

3. In Figure, common tangents AB and CD to the two circle with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.

4. A cone of height 24 cm and radius of base 6 cm is made up of clay. If we reshape it into a sphere, find the radius of sphere.

5. Find the mean of the following frequency distribution:

Class	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

6. The mean of the following frequency distribution is 25. Find the value of p.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	p

The mode of the following frequency distribution is 36. Find the missing frequency f.

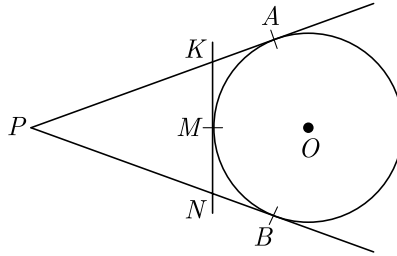| Class	0- 10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	f	16	12	6	7

Section B

7. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28th term of AP.

8. A man on the top of a vertical tower observes a car moving at a uniform speed towards him. If it takes 12 min. for the angle of depression to change from $30°$ to $45°$, how soon after this, the car will reach the tower ?

9. Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

10. The rain water from 22m $\times$ 20 m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

OR

The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. Use $\pi = \frac{22}{7}$

Section C

11. In given figure, PA and PB are tangents from a point P to the circle with centre O. At the point M, other tangent to the circle is drawn cutting PA and PB at K and N. Prove that the perimeter of $\triangle PNK = 2PB$.

12. Monthly expenditures on milk in 100 families of a housing society are given in the following frequency distribution:

Monthly expenditure (in Rs.)	0- 175	175-350	350-525	525-700	700-875	875-1050	1050-1125
Number of families	10	14	15	21	28	7	5

Find the mode and median for the distribution.

OR

The mode of the following data is 67. Find the missing frequency x.

Class	40-50	50-60	60-70	70-80	80-90
Frequency	5	x	15	12	7

13. Braking Distance : The distance that a car travels between the time the driver makes the decision to hit the brakes and the time the car actually stops is called the braking distance. For a certain car traveling v, the braking distance d is given by $d = v + \frac{1}{20}v^2$.

(i)   Find the braking distance when v is 50 km/h.

(ii) If a driver decides to brake 120 metre from a stop sign, how fast can the car be going and still stop by the time it reaches the sign?

14. Eiffel Tower : The Eiffel Tower is a landmark and an early example of wrought-iron construction on a gigantic scale. The lower section consists of four immense arched legs set on masonry piers. The legs curve inward until they unite in a single tapered tower. Platforms, each with an observation deck, are at three levels; on the first is also a restaurant.

The tower, constructed of about 7000 tons of iron, has stairs and elevators. A meteorological station, a radio communications station, and a television transmission antenna, as well as a suite of rooms that were used by Eiffel are located near the top of the tower.

(i)   For a person standing 324 m from the center of the base of the Eiffel Tower, the angle of elevation to the top of the tower is $45°$. How tall is the Eiffel Tower?

(ii) A car is moving at uniform speed towards the Eiffel tower. It takes 15 minutes for the angle of depression from the top of tower to the car to change from $30°$ to $60°$. After how much time after this, the car will reach the base of the tower?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Time Allowed: 120 minutes   **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
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4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. Find the values of k for which the quadratic equation $x^2 + 2\sqrt{2k}\,x + 18 = 0$ has equal roots.

Ans :

We have $x^2 + 2\sqrt{2k}\,x + 18 = 0$

Comparing it by $ax^2 + bx + c$, we get $a = 1$, $b = 2\sqrt{2k}$ and $c = 18$.

Given that, equation $x^2 + 2\sqrt{2}\,kx + 18 = 0$ has equal roots.

$$b^2 - 4ac = 0$$
$$(2\sqrt{2}\,k)^2 - 4 \times 1 \times 18 = 0$$
$$8k^2 - 72 = 0$$
$$8k^2 = 72$$
$$k^2 = \frac{72}{8} = 9$$
$$k = \pm\,3$$

OR

If α and β are the roots of $ax^2 - bx + c = 0\,(a \neq 0)$, then calculate $\alpha + \beta$.

Ans :

We know that

Sum of the roots $= -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Thus $\quad \alpha + \beta = -\left(\dfrac{-b}{a}\right) = \dfrac{b}{a}$

2. Find the AP whose third term is 5 and seventh term is 9.

Ans :

Let the first term be a, common difference be d and nth term be a_n.

Now $\qquad a_3 = a + 2d = 5 \qquad\qquad$ ...(1)

and $\qquad a_7 = a + 6d = 9 \qquad\qquad$ ...(2)
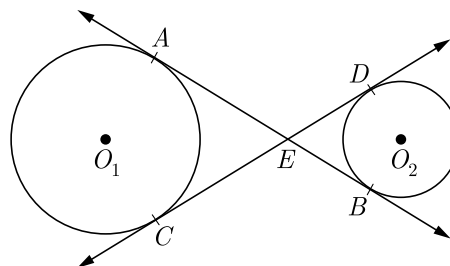
Subtracting (2) from (1) we have

$$4d = 4 \Rightarrow d = 1$$

Substituting this value of d in (1) we get

$$a = 3$$

Hence AP is 3, 4, 5, 6, ...

3. In Figure, common tangents AB and CD to the two circle with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.

Ans :

Since EA and EC are tangents from point E to the circle with centre Q_1

$$EA = EC \qquad\qquad ...(1)$$

and EB and ED are tangents from point E to the circle with centre O_2

$$EB = ED \qquad\qquad (2)$$

Adding eq (1) and (2) we have

$$EA + BE = CE + ED$$
$$AB = CD \qquad\qquad \text{Hence Proved}$$

4. A cone of height 24 cm and radius of base 6 cm is

made up of clay. If we reshape it into a sphere, find the radius of sphere.

Ans :

Volume of sphere $=$ Volume of cone

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\frac{4}{3} \times r_1^3 = (6)^2 \times \frac{24}{3}$$

$$4r_1^3 = 36 \times 24$$

$$r_1^3 = 6^3$$

$$\Rightarrow \qquad r_1 = 6 \text{ cm}$$

Hence, radius of sphere is 6 cm.

5. Find the mean of the following frequency distribution:

Class	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

Ans :

We prepare following table to find mean.

Class-Interval	Mid-Point x_i	f_i	$f_i x_i$
0-6	3	7	21
6-12	9	5	45
12-18	15	10	150
18-24	21	12	252
24-30	27	6	162
Total		$\sum f_i = 40$	$\sum f_i x_i = 630$

Mean $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{630}{40} = 15.75$

6. The mean of the following frequency distribution is 25. Find the value of p.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	p

Ans :

We prepare following table to find mean.

Class-Interval	Mid-Point x_i	f_i	$f_i x_i$
0-10	5	4	20
10-20	15	6	90
20-30	25	10	250
30-40	35	6	210
40-50	45	p	$45\,p$
		$26 + p$	$570 + 45p$

We have $M = \dfrac{\sum f_i x_i}{\sum f_i}$

$$25 = \frac{570 + 45p}{26 + p}$$

$$650 + 25p = 570 + 45p$$

$$650 - 570 = 45p - 25p$$

Thus $p = 4$

OR

The mode of the following frequency distribution is 36. Find the missing frequency f.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	f	16	12	6	7

Ans :

Mode is 36 which lies in class 30-40, therefore this is model class.

Here, $f_0 = f$, $f_1 = 16$, $f_2 = 12$, $l = 30$ and $h = 10$

Mode, $M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$36 = 30 + \frac{16 - f}{2 \times 16 - f - 12} \times 10$$

$$6 = \frac{16 - f}{20 - f} \times 10$$

$$120 - 6f = 160 - 10f$$

$$4f = 40$$

$$\Rightarrow \qquad f = 10$$

Section B

7. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28th term of AP.

Ans :

Let the first term be a, common difference be d, n th term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}\big[2a + (n-1)d\big]$$

Now, $S_7 = 63$

$$\frac{7}{2}\big[2a + 6d\big] = 63$$

$$2a + 6d = 18 \qquad \qquad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$

$$\frac{14}{2}[2a + 13d] = 224$$

$$2a + 13d = 32 \qquad \qquad (2)$$

Subtracting equation (1) form (2) we get

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of d in (1) we get

$$a = 3$$

Now
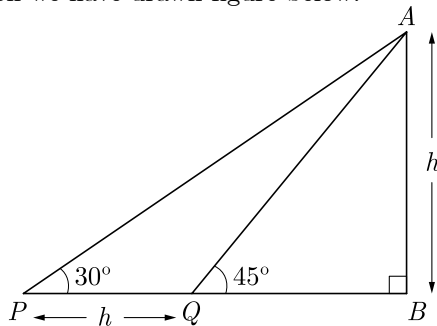$$a_n = a + (n - 1)d$$

$$a_{28} = 3 + 2 \times (27)$$

$$= 57$$

Thus 28th term is 57.

8. A man on the top of a vertical tower observes a car moving at a uniform speed towards him. If it takes 12 min. for the angle of depression to change from $30°$ to $45°$, how soon after this, the car will reach the tower ?

Ans :

Let AB be the tower of height h. As per given in question we have drawn figure below.

Car is at P at $30°$ and is at Q at $45°$ elevation.

Here $\qquad \angle AQB = 45°$

Now, in right $\triangle ABQ$ we have,

$$\tan 45° = \frac{AB}{BQ}$$

$$1 = \frac{h}{BQ}$$

$$BQ = h$$

In right $\triangle APB$ we have,

$$\tan 30° = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x + h}$$

$$x + h = h\sqrt{3}$$

$$x = h(\sqrt{3} - 1)$$

Thus, $\qquad$ Speed $= \dfrac{h(\sqrt{3} - 1)}{12}$ m/min

Time for remaining distance,

$$t = \frac{\dfrac{h}{\quad}}{\dfrac{h(\sqrt{3} - 1)}{12}} = \frac{12}{(\sqrt{3} - 1)}$$

$$= \frac{12(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{12(\sqrt{3} + 1)}{3 - 1}$$

$$= \frac{12}{2}(\sqrt{3} + 1) \quad = 6(\sqrt{3} + 1)$$

$$t = 6 \times 2.73 = 16.38$$

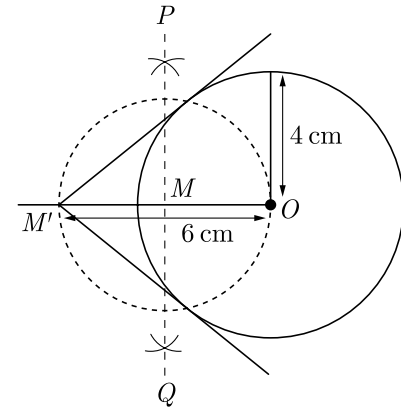Hence, time taken by car is 16.38 minutes.

9. Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

Sol :

Given, a point M' is at a distance of 6 cm from the centre of a circle of radius 4 cm.

Steps of Construction :
1. Draw a circle of radius 4 cm with centre O.
2. Join OM' and bisect it. Let M be mid-point of OM'.

3. Taking M as centre and MO as radius, draw a circle to intersect circle $(O, 4)$ at two points, P
4. Join PM' and QM'. PM' and QM' are the required tangents from M' to circle $C(O, 4)$.

10. The rain water from 22m $\times$ 20 m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

Ans :

Let h be the rainfall.
Volume of water collected in cylindrical vessel,

$$\frac{4}{5}\pi r^2 h = \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right) \text{ m}^3 = \frac{44}{5} \text{ m}^3$$

Rain water from roof $= 22 \times 20 \times h$ m³

Now $22 \times 20 \times h = \dfrac{44}{5}$

$$h = \dfrac{44}{5} \times \dfrac{1}{22 \times 20} = \dfrac{1}{50} \text{ m}^3$$

$$= \dfrac{1}{50} \times 100 = 2 \text{ cm}$$

OR

The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. Use $\pi = \frac{22}{7}$

Ans :

The diameter of the largest possible sphere is the side of the cube.

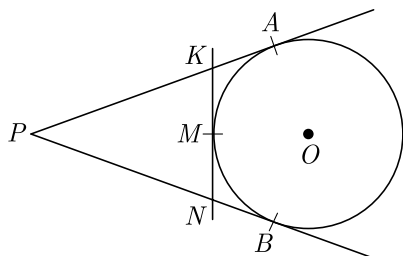Side of cube $\qquad a = 7$ cm

Thus radius of sphere $\qquad r = \dfrac{7}{2}$ cm.

Volume of the wood left,

$$V_{\text{cube}} - V_{\text{sphere}} = a^3 - \dfrac{4}{3}\pi r^3$$

$$= 7^3 - \dfrac{4}{3} \times \dfrac{22}{7} \times \left(\dfrac{7}{2}\right)^3$$

$$= 7^3 \left[1 - \dfrac{4}{3} \times \dfrac{22}{7} \times \left(\dfrac{1}{2}\right)^3\right]$$

$$= 7^3 \left[1 - \dfrac{4}{3} \times \dfrac{22}{7} \times \dfrac{1}{8}\right]$$

$$= 7^3 \left[1 - \dfrac{11}{21}\right] = 7^3 \times \dfrac{10}{21} = \dfrac{490}{3}$$

Hence, volume of wood $= 163.3$ cm³.

Section C

11. In given figure, PA and PB are tangents from a point P to the circle with centre O. At the point M, other tangent to the circle is drawn cutting PA and PB at K and N. Prove that the perimeter of $\triangle PNK = 2PB$.

Ans :

Since length of tangents from an external point to a circle are equal,

$$PA = PB$$
$$KM = KA$$
$$MN = BN$$

Now $\qquad KN = KM + MN$
$$= KA + BN$$

Now perimeter of $\triangle PNK$

$$p = PN + KN + PK$$
$$= PN + BN + KA + PK$$
$$= PB + PA$$
$$= 2PB \qquad (PA = PB)$$

12. Monthly expenditures on milk in 100 families of a housing society are given in the following frequency distribution :

Monthly expenditure (in Rs.)	0-175	175-350	350-525	525-700	700-875	875-1050	1050-1125
Number of families	10	14	15	21	28	7	5

Find the mode and median for the distribution.

Ans :

We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-175	10	10
157-350	14	24
350-525	15	39
525-700	21	60
700-875	28	88
875-1050	7	95
1050-1225	5	100
	$N = 100$	

We have $\qquad N = 100$; $\dfrac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 525-700. Thus median class is 525-700.

Median, $\qquad M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right) h$

$$= 525 + \dfrac{50 - 39}{21} \times 175$$

$$= 525 + \frac{11}{21} \times 175$$

$$= 525 + 91.6 = 616.6$$

Class 700-875 has the maximum frequency 28, therefore this is model class.

Here $l = 700$, $f_0 = 21$, $f_1 = 28$ $f_2 = 7$, $h = 175$

Mode, $\qquad M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$$= 700 + \left(\frac{28 - 21}{2 \times 28 - 21 - 7}\right) \times 175$$

$$= 700 + \frac{7}{28} \times 175$$

$$= 700 + 43.75$$

$$= 743.75$$

OR

The mode of the following data is 67. Find the missing frequency x.

Class	40-50	50-60	60-70	70-80	80-90
Frequency	5	x	15	12	7

Ans :

Given, mode of the given data is 67. Since 67 lies in interval 60-70, so 60-70 is modal class.

So, $\qquad l = 60$, $f_1 = 15$, $f_0 = x$, $f_2 = 12$

and $\qquad h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$67 = 60 + \left(\frac{15 - x}{2 \times 15 - x - 12}\right) \times 10$$

$$7 = \frac{15 - x}{30 - x - 12} \times 10$$

$$7(18 - x) = 10(15 - x)$$

$$126 - 7x = 150 - 10x$$

$$10x - 7x = 150 - 126$$

$$3x = 24 \Rightarrow x = 8$$

Hence, missing frequency is 8.

13. Braking Distance : The distance that a car travels between the time the driver makes the decision to hit the brakes and the time the car actually stops is called the braking distance. For a certain car traveling v, the braking distance d is given by $d = v + \frac{1}{20}v^2$.
(i) Find the braking distance when v is 50 km/h.
(ii) If a driver decides to brake 120 metre from a stop sign, how fast can the car be going and still stop by the time it reaches the sign?

Ans :

We have $\qquad d = v + \frac{1}{20}v^2$
Substituting $v = 50$ we have

$$d = 50 + \frac{1}{20} \times 50^2$$

$$= 50 + 125 = 175 \text{ m}$$

If a driver decides to brake 120 metre from a stop sign, substituting $v = 120$ in $d = v + \frac{1}{20}v^2$ we have

$$120 = v + \frac{1}{20}v^2$$

$$2400 = 20v + v^2$$

$$v^2 + 20v - 2400 = 0$$

$$v^2 + 60v - 40v - 2400 = 0$$

$$v(v + 60) - 40(v - 60) = 0$$

$$(v + 60)(v - 40) = 0$$

$$v = 40, -60$$

(i) Braking distance is 175 m when v is 50 km/h.
(ii) If a driver decides to brake 120 metre from a stop sign, the car be going by 40 km/h and still stop by the time it reaches the sign.

14. Eiffel Tower : The Eiffel Tower is a landmark and an early example of wrought-iron construction on a gigantic scale. The lower section consists of four immense arched legs set on masonry piers. The legs curve inward until they unite in a single tapered tower. Platforms, each with an observation deck, are at three levels; on the first is also a restaurant. The tower, constructed of about 7000 tons of iron, has stairs and elevators. A meteorological station, a radio communications station, and a television transmission antenna, as well as a suite of rooms that were used by Eiffel are located near the top of the tower.
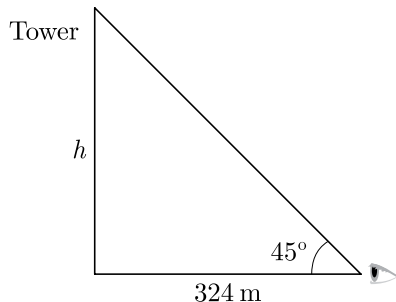
(i) For a person standing 324 m from the center of the base of the Eiffel Tower, the angle of elevation to the top of the tower is $45°$. How tall is the Eiffel Tower?

(ii) A car is moving at uniform speed towards the Eiffel tower. It takes 15 minutes for the angle of depression from the top of tower to the car to change from $30°$ to $60°$. After how much time after this, the car will reach the base of the tower?
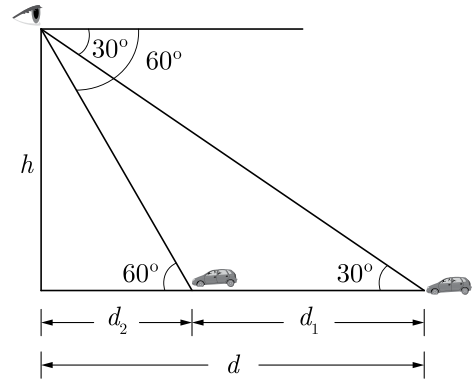
Ans :

Let h be the height of Eiffel tower. We draw a diagram of the situation as shown below.

Now
$$\tan 45° = \frac{h}{324}$$
$$1 = \frac{h}{324}$$
$$h = 324 \text{ m}$$

Let d be the initial distance of car from Eiffel tower. At this point A the angle of depression of car from top of tower is $30°$. After 15 minute car reaches at point B. At this point the angle of depression of car from top of tower is $60°$. We draw a diagram of the situation as shown below.

Now
$$\tan 30° = \frac{h}{d}$$
$$\frac{1}{\sqrt{3}} = \frac{324}{d}$$
$$d = 324\sqrt{3} \text{ m}$$

Now
$$\tan 60° = \frac{h}{d_2}$$
$$\sqrt{3} = \frac{324}{d_2}$$
$$d_2 = \frac{324}{\sqrt{3}} = \frac{324 \times \sqrt{3}}{3}$$
$$= 108\sqrt{3} \text{ m}$$

Now
$$d_1 = d - d_2$$
$$= 324\sqrt{3} - 108\sqrt{3} = 216\sqrt{3}$$

Here $d_2 = \frac{1}{2}d_1$. Thus time to cover d_2 is half of time to cover d_1 which is $\frac{15}{2} = 7.5 \min$.

(i) Height of Eiffel tower is 324 m.

(ii) After 7.5 minute, the car will reach the base of the tower.