02

Indices and Surds

Meaning of index :

1. $a \times a \times a \times a$ ... to m terms is written as a^m. It is read as a raise to power m.

   Clearly $(a \times a \times a \times \ldots$ to m term$) \times (a \times a \times a \ldots$to n term$)$

   $= a \times a \times a \times \ldots$ to $(m + n)$ terms

   $\therefore \boxed{a^m \times a^n = a^{m+n}}$

   $\dfrac{a \times a \times a \times \ldots \text{ to } m \text{ terms}}{a \times a \times a \times \ldots \text{ to } n \text{ terms}} = a \times a \times a \times \ldots$ to $m - n$ terms

   $\therefore \boxed{\dfrac{a^m}{a^n} = a^{m-n}}$ etc.

   Similarly more formulae can be established on index which are given below.

2. Important formulae about indices :

   If $a > 0, a \neq 1$, m and n are integers then

   2.1. $a^m \times a^n = a^{m+n}$

   2.2. $a^m \times a^n \times a^p = a^{m+n+p}$

   2.3. $(a^m)^n = a^{mn}$

   2.4. $\dfrac{a^m}{a^n} = a^{m-n}$

   2.5. $a^0 = 1$

   2.6. $a^{-m} = \dfrac{1}{a^m}$

   2.7. (i) $a^{m^n} = a^{(m^n)}$ (ii) $a^{m^{n^p}} = a^{m^{(n^p)}} = a^{\left(m^{(n^p)}\right)}$

   2.8. $(ab)^n = a^n b^n$, $(abc)^n = a^n b^n c^n$

3. Surd : If square root, cube root etc. of a number cannot be expressed in the rational form $\left(\dfrac{p}{q}, q \neq 0\right)$ then it is called a surd.

   e.g. $\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{18}$, etc are surds as they cannot be expressed as a rational number. Contrary to this $\sqrt[3]{27}, \sqrt[2]{25}, \sqrt[4]{\dfrac{162}{32}}$ etc. are not surds as their values are respectively 3, 5, $\dfrac{3}{2}$ and they are rational numbers. Clearly every number expressed in a surd is an irrational number $\sqrt{2}$ is also written as $2^{1/2}$, $\sqrt[3]{4}$ is written as $(4)^{1/3}$, $\sqrt[4]{18}$ is written $(18)^{1/4}$ etc.

4. Type of surds :

4.1. Pure Surd : $\sqrt{7}, \sqrt[3]{11}, \sqrt[4]{25}$ etc. are pure surds.

4.2. Mixed Surd : $3\sqrt{2}, 7\sqrt[2]{11}, \sqrt{32} = 4\sqrt{2}$ etc. are mixed surds.

4.3. Similar Surds: Two or more surds are said to be similar surds if their surd part (irrational part) are same. e.g. $\sqrt{27}$ and $\sqrt{75}$ are similar surds as $\sqrt{27} = 3\sqrt{3}$ and $\sqrt{75} = 5\sqrt{3}$ and their surd part '$\sqrt{3}$' is same.

$\sqrt{12}$ and $\sqrt{8}$ are distinct surds as $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{8} = 2\sqrt{2}$ and their surds part are respectively $\sqrt{3}$ and $\sqrt{2}$.

5. Order of the surd : $\sqrt{7}, \sqrt[3]{8}, \sqrt[4]{9}, \sqrt[5]{15}$ etc. are respectively surds of order two three, four, five.

$3^{3/2}$ is a surd of order 2 while $3^{2/3}$ is a surd of order 3.

6. Conjugate of surds : If a and b are rational numbers where b is not a perfect square then conjugate of $a + \sqrt{b}$ is $a - \sqrt{b}$.

e.g. conjugate of $3 + \sqrt{5}$ is $3 - \sqrt{5}$; conjugate of $\sqrt{6} - 4$ is $-\sqrt{6} - 4$.

Be careful that sign of surd part should be changed while writing conjugate.

7. Condition for two surds to be equal : If a, b, c, d are all rational numbers and b and d are not perfect square then $\boxed{a + \sqrt{b} = c + \sqrt{d} \Leftrightarrow a = c \text{ and } b = d}$

Thus two surds are said to be equal if their rational parts are equal and their irrational part are also equal.

8. Rationalization of Denominator of a surd :

To rationalize a surd, whose denominator is the of the form $a + \sqrt{b}$

9. Square root of surd of $a + \sqrt{b}$ form :

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

and $$\sqrt{a-\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

10. Some important formulae for surd : **Formula of surd is same as that of Indices. If p, q, r, s are rational numbers then**

10.1. $a^p \times a^q = a^{p+q}$

10.2. $\frac{a^p}{a^q} = a^{p-q}$

10.3. $\left(a^p\right)^q = a^{pq}$

10.4. $a^{-p} = \frac{1}{a^p}$

10.5. **If** $a^n = y$ **then** $a = y^{1/n}$

10.6. **If** $a^x = b^y$ **then** $a = b^{y/x}$

10.7. **If** $a^x = b^y$ **then** $a^{1/y} = b^{1/x}$ **etc.**

11. Formulae in Radical Notations :

11.1. $x^n = a \Leftrightarrow x = \sqrt[n]{a}, (a \in R, a \geq 0)$

11.2. **If n is an odd positive integer and $a > 0$ then** $\sqrt[n]{-a} = -\sqrt[n]{a}$
(here $m, n \geq 2$, and $a, b > 0$ then)

11.3. $\sqrt[n]{a} = a^{\frac{1}{n}}$

11.4. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$

11.5. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

11.6. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

11.7. $\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$

11.8. $\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \times a^{\frac{1}{m}} = a^{\frac{1}{n}+\frac{1}{m}} = a^{\frac{m+n}{mn}} = \sqrt[mn]{a^{m+n}}$

11.9. $\frac{\sqrt[n]{a}}{\sqrt[m]{a}} = \frac{a^{\frac{1}{n}}}{a^{\frac{1}{m}}} = a^{\frac{1}{n}-\frac{1}{m}} = a^{\frac{m-n}{mn}} = \sqrt[mn]{a^{m-n}}$

12. A special property : **If x is a positive integer th...** 1

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12.1. See solved example 15 to see the application of above property.

12.2. If x is a negative quantity then $x + \frac{1}{x} \le -2$

Solved Examples

1. Find the least positive integer x, y, z greater than one for which $\sqrt{xy^{-2}z^{3}} \div \left(\sqrt[3]{x^3y^2z^{-3}}\right)^{-1}$ is an integer.

Solution : Given expression $= \dfrac{(xy^{-2}z^{3})^{\frac{1}{2}}}{\left(\left(x^3y^2z^{-3}\right)^{\frac{1}{3}}\right)^{-1}} = \dfrac{x^{\frac{1}{2}}y^{-1}z^{\frac{3}{2}}}{(xy^{2/3}z^{-1})^{-1}}$

$$= (x^{1/2}y^{-1}z^{3/2}) \times (xy^{2/3}z^{-1})$$

$$= x^{\frac{1}{2}+1}y^{-1+\frac{2}{3}}z^{\frac{3}{2}-1}$$

$$= x^{\frac{3}{2}}y^{\frac{-1}{3}}z^{\frac{1}{2}} = \frac{\left(x^3\right)^{\frac{1}{2}}\left(z^{\frac{1}{2}}\right)}{y^{\frac{1}{3}}}$$

Clearly it is an integer if least value of x, y, z (> 1) are respectively $x = 4, z = 4, y = 8$.

For these values $= \dfrac{(64)^{\frac{1}{2}}4^{\frac{1}{2}}}{8^{\frac{1}{3}}} = \dfrac{8\times 2}{2} = 8$ which is an integer.

2. If $a^b = b^a$ then prove that $\left(\frac{a}{b}\right)^{\frac{a}{b}} = a^{\frac{a}{b}-1}$

Solution : $a^b = b^a \Rightarrow a = \left(b^a\right)^{\frac{1}{b}} \Rightarrow a = b^{\frac{a}{b}}$ ... (i)

Now, $\left(\frac{a}{b}\right)^{\frac{a}{b}} = \dfrac{a^{\frac{a}{b}}}{b^{\frac{a}{b}}} = \dfrac{a^{\frac{a}{b}}}{a} = a^{\frac{a}{b}-1}$ $(\because$ from (i) $b^{\frac{a}{b}} = a)$

3. Prove that $\dfrac{1}{1+a^{x-y}+a^{x-z}} + \dfrac{1}{1+a^{y-z}+a^{y-x}} + \dfrac{1}{1+a^{z-x}+a^{z-y}} = 1$

Solution : First term $= \dfrac{1}{1+a^{x-y}+a^{x-z}} \times \dfrac{a^{-x}}{a^{-x}} = \dfrac{a^{-x}}{a^{-x}+a^{-y}+a^{-z}}$

Second term $= \dfrac{1}{1+a^{y-z}+a^{y-x}} \times \dfrac{a^{-y}}{a^{-y}} = \dfrac{a^{-y}}{a^{-y}+a^{-z}+a^{-x}}$

Third term $= \dfrac{1}{1+a^{z-x}+a^{z-y}} \times \dfrac{a^{-z}}{a^{-z}} = \dfrac{a^{-z}}{a^{-z}+a^{-x}+a^{-y}}$

Here, denominator of each term is same, so adding them

Given expression $= \dfrac{a^{-x}+a^{-y}+a^{-z}}{a^{-x}+a^{-y}+a^{-z}} = 1$

4. If $x^{\frac{1}{m}} = y^{\frac{1}{n}} = z^{\frac{1}{p}}$ and $xyz = 1$ then prove that $m + n + p = 0$

Solution : Let $x^{\frac{1}{m}} = y^{\frac{1}{n}} = z^{\frac{1}{p}} = k$

Then $x = k^m, y = k^n, z = k^p$

Now, from $xyz = 1$

$$k^m \times k^n \times k^p = 1$$

or, $k^{m+n+p} = 1 = k^0$

$\therefore \quad m + n + p = 0$

5. If $x = \dfrac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}}$ then prove that $bx^2 - ax + b = 0$

Solution : On rationalization,

$$x = \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}} \times \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} + \sqrt{a-2b}}$$

$$= \frac{(a+2b) + (a-2b) + 2\sqrt{(a+2b)(a-2b)}}{(a+2b) - (a-2b)}$$

$$= \frac{2a + 2\sqrt{a^2 - 4b^2}}{4b} = \frac{a + \sqrt{a^2 - 4b^2}}{2b}$$

or, $2bx = a + \sqrt{a^2 - 4b^2}$

or, $2bx - a = \sqrt{a^2 - 4b^2}$

Squaring both sides

$$4b^2x^2 + a^2 - 2.2bx.a = a^2 - 4b^2$$

or, $4b^2x^2 - 4abx = -4b^2$

or, $4(b^2x^2 - abx + b^2) = 0$

or, $4b(bx^2 - ax + b) = 0$

or, $bx^2 - ax + b = 0$

6. If $\dfrac{A}{a} = \dfrac{B}{b} = \dfrac{C}{c} = \dfrac{D}{d}$ then prove that

$$\sqrt{Aa} + \sqrt{Bb} + \sqrt{Cc} + \sqrt{Dd} = \sqrt{(a+b+c+d)(A+B+C+D)}$$

Solution : Let $\dfrac{A}{a} = \dfrac{B}{b} = \dfrac{C}{c} = \dfrac{D}{d} = k$ then

$$A = ak, B = bk, C = ck \text{ and } D = dk$$

$\therefore \quad \sqrt{Aa} + \sqrt{Bb} + \sqrt{Cc} + \sqrt{Dd} = \sqrt{aka} + \sqrt{bkb} + \sqrt{ckc} + \sqrt{dkd}$

$= \sqrt{k}\ (a + b + c + d)$ ... (i

Again, $\sqrt{(a+b+c+d)(A+B+C+D)} = \sqrt{(a+b+c+d)(ak+bk+ck+dk)}$

$$= \sqrt{(a+b+c+d)\,k\,(a+b+c+d)}$$

$$= \sqrt{k}\ (a+b+c+d)$$

From (i) and (ii) $\sqrt{Aa}+\sqrt{Bb}+\sqrt{Cc}+\sqrt{Dd} = \sqrt{(a+b+c+d)(A+B+C+D)}$

7. If $x = \frac{1}{2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)$ then prove that $\frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}} = a+b$

Solution : $\frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}} = \frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}} \times \frac{x-\sqrt{1+x^2}}{x-\sqrt{1+x^2}}$

$$= \frac{2a\sqrt{1+x^2}\left(x-\sqrt{1+x^2}\right)}{x^2-(1+x^2)}$$

$$= -2ax\sqrt{1+x^2}+2a(1+x^2)$$

But squaring $x = \frac{1}{2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)$

$$x^2 = \frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}-2\sqrt{\frac{a}{b}}\sqrt{\frac{b}{a}}\right) = \frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}-2\right)$$

$\therefore\ \ 1+x^2 = 1+\frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}-2\right) = \frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}+2\right) = \frac{1}{4}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)^2$

or, $\sqrt{1+x^2} = \frac{1}{2}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)$

Now from (i)

Given expression $= -2a\,\frac{1}{2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)\frac{1}{2}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)+2a\,\frac{1}{4}\cdot\left(\frac{a}{b}+\frac{b}{a}+2\right)$

$$= -2a\,\frac{1}{4}\left(\frac{a}{b}-\frac{b}{a}\right)+2a\cdot\frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}+2\right) = 2a\left(\frac{1}{2}+\frac{1}{2}\cdot\frac{b}{a}\right) = a$$

8. Simplify $\frac{(\sqrt{3}+\sqrt{5})(\sqrt{5}+\sqrt{2})}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

Solution : Given expression $= \frac{(\sqrt{3}+\sqrt{5})(\sqrt{5}+\sqrt{2})}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

$$= \frac{(\sqrt{3}+\sqrt{5}+\sqrt{2}-\sqrt{2})(\sqrt{5}+\sqrt{2})}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = \left(1-\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}\right)(\sqrt{5}+\sqrt{2})$$

$$= \left\{1-\frac{\sqrt{2}}{(\sqrt{5}+\sqrt{2})+\sqrt{3}} \times \frac{(\sqrt{5}+\sqrt{2})-\sqrt{3}}{(\sqrt{5}+\sqrt{2})-\sqrt{3}}\right\}(\sqrt{5}+\sqrt{2})$$

$$= \left\{1 - \frac{\sqrt{2} \times ((\sqrt{5} + \sqrt{2}) - \sqrt{3})}{(\sqrt{5} + \sqrt{2})^2 - (\sqrt{3})^2}\right\} (\sqrt{5} + \sqrt{2})$$

$$= \left\{1 - \frac{\sqrt{2}\,((\sqrt{5} + \sqrt{2}) - \sqrt{3})}{5 + 2 + 2\sqrt{10} - 3}\right\} (\sqrt{5} + \sqrt{2})$$

$$= \left\{1 - \frac{\sqrt{2}\,((\sqrt{5} + \sqrt{2}) - \sqrt{3})}{4 + 2\sqrt{10}}\right\} (\sqrt{5} + \sqrt{2})$$

$$= \left\{\frac{4 + 2\sqrt{10} - \sqrt{2}\,((\sqrt{5} + \sqrt{2}) - \sqrt{3}}{4 + 2\sqrt{10}}\right\} (\sqrt{5} + \sqrt{2})$$

$$= \left\{\frac{4 + 2\sqrt{10} - \sqrt{10} - 2 + \sqrt{6}}{2\sqrt{2}\,(\sqrt{2} + \sqrt{5})}\right\} (\sqrt{5} + \sqrt{2})$$

$$= \frac{2 + \sqrt{10} + \sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{2}\,(\sqrt{2} + \sqrt{5} + \sqrt{3})}{2\sqrt{2}} = \frac{1}{2}(\sqrt{2} + \sqrt{3} + \sqrt{5})$$

9. Find the square root of $6 - \sqrt{35}$

Solution : First method, $6 - \sqrt{35} = 6 - \sqrt{5}\,\sqrt{7} = \frac{1}{2}(12 - 2\sqrt{5}\,\sqrt{7})$

$$= \frac{1}{2}(5 + 7 - 2\sqrt{5}\,\sqrt{7}) = \frac{1}{2}(\sqrt{7} - \sqrt{5})^2$$

$\therefore$ Required square root $= \pm \frac{1}{\sqrt{2}}(\sqrt{7} - \sqrt{5})$

Second Method :

$$\therefore \quad a - \sqrt{b} = \left(\sqrt{\frac{a + \sqrt{a^2 - b}}{2}} - \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}\right)^2$$

$$\therefore \quad 6 - \sqrt{35} = \left(\sqrt{\frac{6 + \sqrt{36 - 35}}{2}} - \sqrt{\frac{6 - \sqrt{36 - 35}}{2}}\right)^2 = \left(\sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}\right)^2$$

$\therefore$ Square root $6 - \sqrt{35} = \pm\left(\sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}\right)$

10. Find the value of $\sqrt{6 - \sqrt{35}}$

Solution : [illegible]

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12. Find the square root of $21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}$

Solution : Let $21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}=(\sqrt{x}+\sqrt{y}-\sqrt{z})^2$

$\therefore \quad 21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}=x+y+z+2\sqrt{xy}-2\sqrt{xz}-2\sqrt{yz}$

$\therefore \quad x+y+z=21$

$2\sqrt{xy}=8\sqrt{3}$

$2\sqrt{yz}=4\sqrt{5}$

$2\sqrt{zx}=4\sqrt{15}$

Multiplying these three terms

$2\times2\times2\,xyz=8\times4\times4\sqrt{3}\sqrt{5}\sqrt{15}$

$xyz=4\times4\times15=240$

$\Rightarrow \sqrt{xyz}=\sqrt{240}=4\sqrt{15}$

$\therefore \quad \sqrt{x}=\frac{4\sqrt{15}}{\sqrt{yz}}=2\sqrt{3} \qquad \sqrt{y}=\frac{4\sqrt{15}}{\sqrt{zx}}=2 \qquad \sqrt{z}=\frac{4\sqrt{15}}{\sqrt{xy}}=\sqrt{5}$

These values also satisfy $x+y+z=21$

Hence required square root $\pm(2\sqrt{3}+2-\sqrt{5})$

13. Find the cube root of $72-32\sqrt{5}$

Solution : Let $(72-32\sqrt{5})^{1/3}=x-\sqrt{y}$

$\therefore \quad (72+32\sqrt{5})^{1/3}=x+\sqrt{y}$ *(conjugate proper*

On multiplication $(72^2-(32\sqrt{5})^2)^{1/3}=x^2-y$

or, $(5184-1024\times5)^{1/3}=x^2-y$

or, $(64)^{1/3}=x^2-y$

or, $x^2-y=4$ ...

Again from (i), $72-32\sqrt{5}=(x-\sqrt{y})^3$

or, $72-32\sqrt{5}=x^3-3x^2\sqrt{y}+3xy-y\sqrt{y}$

or, $72-32\sqrt{5}=x^3+3xy-(3x^2+y)\sqrt{y}$ ...

From irrational part $y=5$

From (ii), $x^2-5=4\Rightarrow x^2=9\Rightarrow x=\pm3$

Here, $x=3,\ y=5$, also satisfy equation (iii).

$\therefore$ Required cube root $=3-\sqrt{5}$

14. Express $\dfrac{(4+\sqrt{15})^{3/2}+(4-\sqrt{15})^{3/2}}{(6+\sqrt{35})^{3/2}-(6-\sqrt{35})^{3/2}}$ in rational form.

Solution : $\because\ 4+\sqrt{15}=\frac{1}{2}(3+5+2\sqrt{15})=\frac{1}{2}\ (3+5+2\sqrt{15})=\frac{1}{2}\left(\sqrt{3}+\sqrt{5}\right)^2$

$6+\sqrt{35}=\frac{1}{2}(12+2\sqrt{35})=\frac{1}{2}(7+5+2\sqrt{7}\sqrt{5})=\frac{1}{2}(\sqrt{7}+\sqrt{5})^2$ etc.

Hence, Given expression $=\dfrac{\left\{\frac{1}{2}(\sqrt{5}+\sqrt{3})^2\right\}^{\frac{3}{2}}+\left\{\frac{1}{2}(\sqrt{5}-\sqrt{3})^2\right\}^{\frac{3}{2}}}{\left\{\frac{1}{2}\left(\sqrt{7}+\sqrt{5}\right)^2\right\}^{\frac{3}{2}}-\left\{\frac{1}{2}\left(\sqrt{7}-\sqrt{5}\right)^2\right\}^{\frac{3}{2}}}$

$=\dfrac{(\sqrt{5}+\sqrt{3})^3+(\sqrt{5}-\sqrt{3})^3}{(\sqrt{7}+\sqrt{5})^3-(\sqrt{7}-\sqrt{5})^3}$

$=\dfrac{2((\sqrt{5})^3+3\sqrt{5}\,(\sqrt{3})^2)}{2(3\left(\sqrt{7}\right)^2\sqrt{5}+\left(\sqrt{5}\right)^3)}$

$(\because\ (a+b)^3+(a-b)^3=2\,(a^3+3ab^2)$ etc)

$=\dfrac{5\sqrt{5}+9\sqrt{5}}{21\sqrt{5}+5\sqrt{5}}=\dfrac{5+9}{21+5}=\dfrac{14}{26}=\dfrac{7}{13}$

15. Solve the equation $\sqrt{3-x^4+2x^2}=x^2+\dfrac{1}{x^2}$ for real value of x.

Solution : $\because$ Given equation is $\sqrt{3-x^4+2x^2}=x^2+\dfrac{1}{x^2}$

or, $\sqrt{4-\left(x^4-2x^2+1\right)}=x^2+\dfrac{1}{x^2}$

or, $\sqrt{4-\left(x^2-1\right)^2}=x^2+\dfrac{1}{x^2}$

Since value of $(x^2-1)^2$ is zero or greater than zero

therefore $\sqrt{4-\left(x^2-1\right)^2}\le\sqrt{4-0}$

i.e. LHS $\le\sqrt{4}\le 2$

Also, RHS $=x^2+\dfrac{1}{x^2}=\left(x-\dfrac{1}{x}\right)^2+2$

$\because\ \left(x-\dfrac{1}{x}\right)^2\ge 0$

$\therefore$ RHS ≥ 2

Hence both LHS and RHS are equal if each of them is 2, which occurs at $x=\pm 1$, which is required solution

Exercise—2A

1. Simplest form of $\sqrt[4p]{z^6} \div \sqrt[2p]{z^{-5}}$ is.

(a) $\sqrt[4]{z^p}$ (b) $\sqrt[p]{z^4}$ (c) $\sqrt{z^{4p}}$ (d) $\sqrt{\frac{p}{z^4}}$

2. Which among the following is simplest form of $\sqrt[4]{x^{-3}} \div \left(\sqrt[8]{x}\right)^{-12}$?

(a) $x^{\frac{4}{3}}$ (b) $\sqrt[4]{x^5}$ (c) $\sqrt[4]{x^9}$ (d) $\sqrt[4]{x^3}$

3. For what value of a, b, c expression $\sqrt{a^3b^{-2/3}c^{-7/6}} \div \sqrt[3]{a^4b^{-1}c^{5/4}}$ is a integer ?

(a) $a = 16, b = 2, c = 4$ (b) $a = 32, b = 3, c = \frac{1}{2}$

(c) $a = 729, b = \frac{1}{3}, c = \frac{1}{6}$ (d) $a = 64, b, = 4, c = 3$

4. If $a^b = b^a$ and $a = 2b$ then value of b is

(a) 1 (b) –1 (c) 2 (d) –2

5. On simplification $\left(\frac{a^p}{a^q}\right)^{p+q} \times \left(\frac{a^q}{a^r}\right)^{q+r} \times \left(\frac{a^r}{a^p}\right)^{r+p}$ yeilds

(a) a^{p+q+r} (b) $a^{pq+qr+rp}$ (c) 0 (d) 1

6. Expression $\left(\frac{a^x}{a^y}\right)^{x^2+xy+y^2} \times \left(\frac{a^y}{a^z}\right)^{y^2+yz+z^2} \times \left(\frac{a^z}{z^x}\right)^{z^2+zx+x^2}$ in its simplest form is

(a) a (b) $\frac{1}{a}$ (c) 0 (d) 1

7. Expression $\dfrac{\left(a+\frac{1}{b}\right)^{\frac{1}{x}}\left(a-\frac{1}{b}\right)^{\frac{1}{x}}}{\left(b+\frac{1}{a}\right)^{\frac{1}{x}}\left(b-\frac{1}{a}\right)^{\frac{1}{x}}}$ is an integer if

(a) $a = 4, b = 2, x = 4$ (b) $a = 16, b = 2, x = 4$

(c) $a = 64, b = 2, x = 8$ (d) $a = 16, b = 4, x = 4$

8. The value of

$$\frac{x^d}{x^d + x^{d+b-a} + x^{d+c-a}} + \frac{x^d}{x^d + x^{d+a-b} + x^{d+c-b}} + \frac{x^d}{x^d + x^{d+a-c} + x^{d+b-c}}$$ is

(a) x^d (b) x^{a+b+c} (c) x^{-d} (d) 1

9. If $x = \sqrt[3]{a} - \frac{1}{\sqrt[3]{a}}$ then value of $x^3 + 3x$ is

(a) $a + a^{-1}$ (b) $a - a^{-1}$ (c) $a^2 + a^{-2}$ (d) 0

10. If $a = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$ then value of $a^3 - 6a^2 + 6a$ is

(a) 2 (b) –2 (c) 4 (d) –4

11. If $a^x = b^y = c^z$ and $abc = 1$ then value of $xy + yz + zx$ is

(a) abc (b) 1 (c) 0 (d) –1

12. Value of $\sqrt[yz]{\frac{a^{\frac{y}{z}}}{a^{\frac{z}{y}}}} \times \sqrt[zx]{\frac{a^{\frac{z}{x}}}{a^{\frac{x}{z}}}} \times \sqrt[xy]{\frac{a^{\frac{y}{x}}}{a^{\frac{x}{y}}}}$ is

(a) a^{xyz} (b) $a^{\frac{1}{xyz}}$ (c) 1 (d) a

13. If $a^{m^n} = \left(a^m\right)^n$ then which of the following relation is true ?

(a) $m = n^{\frac{1}{n-1}}$ (b) $n = m^{\frac{1}{m-1}}$ (c) $m = n^{n-1}$ (d) $n = m^{m-1}$

14. Simplified value of $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ is

(a) $\sqrt{2} - \sqrt{3} + \sqrt{6}$ (b) $4\sqrt{3} - 6\sqrt{2}$

(c) 1 (d) None of these

15. Value of $\sqrt{4+\sqrt{15}}$ is

(a) $\frac{\sqrt{5}-\sqrt{3}}{2}$ (b) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}}$

(c) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{5}+\sqrt{3}}{2}$

16. If $n = 7 + 4\sqrt{3}$ then value of $\left(\sqrt{n} + \frac{1}{\sqrt{n}}\right)$ is

(a) 2 (b) $2\sqrt{3}$ (c) 4 (d) $4\sqrt{3}$

17. Square root of $\frac{3}{2}(x-1) + \sqrt{2x^2 - 7x - 4}$ is

(a) $\frac{1}{2}\left(\sqrt{2x+1} + \sqrt{x-4}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\sqrt{2x+1} + \sqrt{x-4}\right)$

(c) $\frac{1}{\sqrt{2}}\left(\sqrt{2x-1} + \sqrt{x+4}\right)$ (d) $\frac{1}{2}\left(\sqrt{2x-1} + \sqrt{x+4}\right)$

18. Square root of $2a - \sqrt{3a^2 - 2ab - b^2}$, $(a > b > 0)$ is

(a) $\frac{1}{\sqrt{2}}\left(\sqrt{3a+b} + \sqrt{a-b}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\sqrt{3a-b} + \sqrt{a+b}\right)$

(c) $\frac{1}{\sqrt{2}}\left(\sqrt{3a+b} - \sqrt{a-b}\right)$ (d) $\frac{1}{\sqrt{2}}\left(\sqrt{3a-b} - \sqrt{a+b}\right)$

19. Value of $\sqrt{28 - 6\sqrt{3}} + \sqrt{28 + 6\sqrt{3}}$ is

(a) $6\sqrt{3}$ (b) 2 (c) $-6\sqrt{3}$ (d) -2

20. Value of $\sqrt{1+x^2+(1+x^2+x^4)^{\frac{1}{2}}}$ is

(a) $\frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}+\sqrt{1-x+x^2}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)$

(c) $\frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}-\sqrt{1+x-x^2}\right)$ (d) None of these

21. Square root of $x+y+z+2\sqrt{xz+yz}$ is

(a) $\sqrt{x}+\sqrt{y+z}$ (b) $\sqrt{x+y+z}$

(c) $\sqrt{x+y}+\sqrt{z}$ (d) None of these

22. If $\sqrt{3x-7}+\sqrt{3x+7}=4+\sqrt{2}$ then value of $x+\frac{1}{x}$ is

(a) $\frac{82}{9}$ (b) $\frac{10}{3}$ (c) $\frac{5}{2}$ (d) $\frac{4}{3}$

23. Square root of $6+\sqrt{12}-\sqrt{24}-\sqrt{8}$ is

(a) $\sqrt{3}+\sqrt{2}-1$ (b) $\sqrt{3}+1-\sqrt{2}$

(c) $1+\sqrt{2}-\sqrt{3}$ (d) None of these

24. If $a=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ then value of $\sqrt{3a^2-5ab+3b^2}$ is

(a) 289 (b) 17 (c) $\sqrt{17}$ (d) $17\sqrt{17}$

25. Simplest form of $\left(\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}\right)^2$ is

(a) 3 (b) 9 (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

26. Expression $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to

(a) $-\sqrt{5}+\sqrt{2}+\sqrt{10}$ (b) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$

(c) $1+\sqrt{5}-\sqrt{2}-\sqrt{10}$ (d) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

27. Number of real root of the equation $x^2+\frac{1}{\sqrt{x^2-x+1}}+\sqrt{x^2-x+1}=2$

(a) 0 (b) 1 (c) 2 (d) 4

28. If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then x^2+xy+y^2 is a multiple of

(a) 11 (b) 3

(c) 9 (d) All (a), (b) and (c)

29. If $(\sqrt{5}-\sqrt{2})p = \sqrt{5}+\sqrt{2}$ and $pq = (pq)^3$, then the value of $3p^2+4pq-3q^2$ is $(pq \neq 0)$

(a) $\frac{1}{9}(12+56\sqrt{10})$
(b) $\frac{1}{9}(12-56\sqrt{10})$
(c) $\frac{1}{3}(12-56\sqrt{10})$
(d) $\frac{1}{3}(12+56\sqrt{10})$

30. If $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}} = \sqrt{p}+\sqrt{q}+\sqrt{r}$ then value of $p+q+r$ is

(a) $\sqrt{10}$ (b) 10 (c) 11 (d) $\sqrt{10}$

31. The irrational part of cube root $72-32\sqrt{5}$ is

(a) $-2\sqrt{5}$ (b) $-\sqrt{5}$ (c) $2\sqrt{5}$ (d) $\sqrt{5}$

32. Value of $\frac{1}{\sqrt{11-2\sqrt{30}}} - \frac{3}{7-2\sqrt{10}} - \frac{4}{\sqrt{8-4\sqrt{3}}}$ is

(a) 0 (b) 1 (c) 2 (d) 4

33. Value of $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{1}{3}$ (d) 3

34. If $a = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$ then $\sqrt{a^2-a-1}$ is equal to

(a) 5 (b) $\sqrt{5}$ (c) 0 (d) 1

35. If $\left(a+\sqrt{a^2+b^3}\right)^{\frac{1}{3}} + \left(a-\sqrt{a^2+b^3}\right)^{\frac{1}{3}}$ then what is the value of $x^3+3bx-2a$?

(a) $2a^2$ (b) $-2a^3$ (c) 1 (d) 0

36. Value of $\sqrt{139-80\sqrt{3}}$ is

(a) $5\sqrt{3}-8$ (b) $8-5\sqrt{3}$ (c) $\pm(8-5\sqrt{3})$ (d) $(16-5\sqrt{3})$

37. If $(a+3)\sqrt{2}+3 = b\sqrt{8}+a-1$ then value of $a+b$ is

(a) 3 (b) 6 (c) $\frac{13}{2}$ (d) $\frac{15}{2}$

38. If $x>2$ then what is the value of $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$?

(a) 2 (b) $2\sqrt{x-1}$ (c) -2 (d) $-2\sqrt{x-1}$

39. If $1<x<2$ then what is the value of $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$?

(a) 2 (b) $2\sqrt{x-1}$ (c) $2+2\sqrt{x-1}$ (d) $2-2\sqrt{x-1}$

? is

40. If $x = \frac{\sqrt{3}}{2}$ then what is the value of $\frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}}$?

(a) 1 (b) 2 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

41. If $\frac{x}{y} = \frac{y}{z} = \frac{z}{w}$ then $\frac{x^m + y^m + z^m + w^m}{x^{-m} + y^{-m} + z^{-m} + w^{-m}}$ is equals to—

(a) 0 (b) 1 (c) $(xyzw)^m$ (d) $(xyzw)^{m/2}$

42. Which of the following quantity is an integer?

(a) $[(\sqrt{2}+\sqrt{3})/(\sqrt{3}-\sqrt{2})] + \sqrt{6}$ (b) $[(\sqrt{2}+\sqrt{3})/(\sqrt{3}-\sqrt{2})] + 2\sqrt{6}$

(c) $[(\sqrt{2}+\sqrt{3})/(\sqrt{2}-\sqrt{3})] + 2\sqrt{6}$ (d) $[(\sqrt{2}+\sqrt{3})/(\sqrt{2}-\sqrt{3})] + \sqrt{6}$

43. What is the real value of $(256)^{0.16} \times (16)^{0.18}$?

(a) 2 (b) 4 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

44. $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}}$ is equals to—

(a) 2 (b) $2-\sqrt{2}$ (c) $4+\sqrt{2}$ (d) $2\sqrt{2}$

45. If $x = \frac{2ab}{b^2+1}$ then value of $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$ is

(a) $a+b$ (b) $a-b$ (c) a (d) b

Answers—2A

1. (b)	2. (d)	3. (c)	4. (c)	5. (d)	6. (d)	7. (d)	8. (d)
9. (a)	10. (a)	11. (c)	12. (c)	13. (a)	14. (d)	15. (c)	16. (c)
17. (b)	18. (c)	19. (a)	20. (a)	21. (c)	22. (b)	23. (b)	24. (b)
25. (d)	26. (b)	27. (b)	28. (d)	29. (d)	30. (b)	31. (b)	32. (a)
33. (b)	34. (c)	35. (d)	36. (a)	37. (d)	38. (b)	39. (a)	40. (a)
41. (d)	42. (c)	43. (b)	44. (a)	45. (d)			

Explanation

1. (b) Given Expression $= \frac{\left(z^6\right)^{\frac{1}{4p}}}{\left(z^{-5}\right)^{\frac{1}{2p}}} = z^{\frac{6}{4p}} \times z^{\frac{5}{2p}} = z^{\frac{3+5}{2p}} = z^{\frac{4}{p}} = \left(z^4\right)^{\frac{1}{p}} = \sqrt[p]{z^4}$

2. (d) Given Expression $= \frac{\left(x^{-3}\right)^{\frac{1}{4}}}{\left(x^{\frac{1}{8}}\right)^{-12}} = \frac{x^{-\frac{3}{4}}}{x^{-\frac{3}{2}}} = x^{-\frac{3}{4}+\frac{3}{2}} = x^{\frac{3}{4}} = \left(x^3\right)^{\frac{1}{4}} = \sqrt[4]{x^3}$

3. (c) Given Expression $= \dfrac{\left(a^3 b^{-\frac{2}{3}} c^{-\frac{7}{6}}\right)^{\frac{1}{2}}}{\left(a^4 b^{-1} c^{\frac{5}{4}}\right)^{\frac{1}{3}}}$

$$= \frac{a^{\frac{3}{2}} b^{-\frac{1}{3}} c^{-\frac{7}{12}}}{a^{\frac{4}{3}} b^{-\frac{1}{3}} c^{\frac{5}{12}}}$$

$$= a^{\frac{3}{2}-\frac{4}{3}} c^{-\frac{7}{12}-\frac{5}{12}}$$

$$= a^{\frac{9-8}{6}} c^{\frac{-7-5}{12}} = a^{\frac{1}{6}} c^{-1} = \frac{a^{1/6}}{c},$$

putting values given in each option one by one, we find that (c) is the correct option

4. (c) Given, $a^b = b^a$

or, $(2b)^b = b^{2b}$ $\quad (\because a = 2b)$

or, $(2b)^b = (b^2)^b$

$\therefore \quad 2b = b^2$

or, $\quad b = 2$

5. (d) Given Expression $= \left(a^{p-q}\right)^{p+q} \times \left(a^{q-r}\right)^{q+r} \times \left(a^{r-p}\right)^{r+p}$

$$= a^{p^2-q^2} \times a^{q^2-r^2} \times a^{r^2-p^2}$$

$$= a^{\left(p^2-q^2\right)+\left(q^2-r^2\right)+\left(r^2-p^2\right)}$$

$$= a^0 = 1$$

6. (d) Solved as in above question $(x-y)(x^2+xy+y^2) = x^3 - y^3$

7. (d) Given Expression $= \dfrac{\left(\frac{ab+1}{b}\right)^{\frac{1}{x}}\left(\frac{ab-1}{b}\right)^{\frac{1}{x}}}{\left(\frac{ab+1}{a}\right)^{\frac{1}{x}}\left(\frac{ab-1}{a}\right)^{\frac{1}{x}}}$

$$= \left(\frac{a}{b}\right)^{\frac{1}{x}}\left(\frac{a}{b}\right)^{\frac{1}{x}} - \left(\frac{a}{b}\right)^{\frac{2}{x}}$$

Clearly it is integer at $a = 16, b = 4, x = 4$

Required value $= \left(\frac{16}{4}\right)^{\frac{2}{4}} = 4^{\frac{1}{2}} = 2$

8. (d) Do as in solved example (3). Multiply numerator and Denominator of first term by x^a that of second term by x^b and that of third by x^c etc.

9. (a) Cube both sides

10. (a) Given $a = 2^{\frac{1}{3}} + 2^{\frac{2}{3}} + 2$ or, $a - 2 = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

Cubing both sides, $a^3 - 6a^2 + 12a - 8 = 2 + 4 + 3 2^{\frac{1}{3}} 2^{\frac{2}{3}}\left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$

or, $a^3 - 6a^2 + 12a - 8 = 6 + 3.2(a - 2)$

or, $a^3 - 6a^2 + 12a - 8 = 6 + 6a - 12$

or, $a^3 - 6a^2 + 6a = -6 + 8 = 2$

11. (c) Do as in solved example (4)

Here, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$\Rightarrow \frac{yz + zx + xy}{xyz} = 0$

$\Rightarrow xy + yz + zx = 0$

13. (a) $a^{m^n} = \left(a^m\right)^n \Rightarrow a^{m^n} = a^{mn}$

$\Rightarrow m^n = mn$

$\Rightarrow \frac{m^n}{m} = n$

$\Rightarrow m^{n-1} = n$

$\Rightarrow m = n^{\frac{1}{n-1}}$

14. (d) Required value $= \frac{3\sqrt{2}}{\sqrt{3} + \sqrt{6}} \times \frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} - \sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6} + \sqrt{2}} \times \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

$= \frac{3\sqrt{12} - 3\sqrt{6}}{6 - 3} - \frac{4\sqrt{18} - 4\sqrt{6}}{6 - 2} + \frac{\sqrt{18} - \sqrt{12}}{3 - 2}$

$= \left(\sqrt{12} - \sqrt{6}\right) - \left(\sqrt{18} - \sqrt{6}\right) + \left(\sqrt{18} - \sqrt{12}\right) = 0$

15. (c) $\sqrt{4 + \sqrt{15}} = \frac{1}{\sqrt{2}}\sqrt{8 + 2\sqrt{15}}$

$= \frac{1}{\sqrt{2}}\left(5 + 3 + 2\sqrt{15}\right)$

$= \frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{5} + \sqrt{3}\right)^2} = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{2}}$

16. (c)

∴

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16. (c) $n = 7 + 4\sqrt{3} = 4 + 3 + 2\cdot 2\cdot\sqrt{3} = \left(2+\sqrt{3}\right)^2$

$\therefore \quad \sqrt{n} = 2 + \sqrt{3}$

$\frac{1}{n} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{49-48}$

$= 7 - 4\sqrt{3} = \left(2-\sqrt{3}\right)^2$

$\therefore \quad \frac{1}{\sqrt{n}} = 2 - \sqrt{3}$

Hence, $\sqrt{n} + \frac{1}{\sqrt{n}} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

17. (b) Given expression $= \frac{1}{2}\left\{(3x-3) + 2\sqrt{(2x+1)(x-4)}\right\}$

$= \frac{1}{2}\left\{(2x+1) + (x-4) + 2\sqrt{(2x+1)(x-4)}\right\}$

$= \frac{1}{2}\left(\sqrt{2x+1} + \sqrt{x-4}\right)^2$

$\therefore$ Required square root $= \pm\frac{1}{\sqrt{2}}\left(\sqrt{2x+1} + \sqrt{x-4}\right)$

18. (c) Given expression $= 2a - \sqrt{3a^2 - 2ab - b^2}$

$= 2a - \sqrt{(3a+b)(a-b)}$

$= \frac{1}{2}\left\{4a - 2\sqrt{(3a+b)(a-b)}\right\}$

$= \frac{1}{2}(3a+b) + (a-b) - 2\sqrt{(3a+b)(a-b)}$

$= \frac{1}{2}\left(\sqrt{3a+b} - \sqrt{a-b}\right)^2$

$\therefore$ Required square root $= \pm\frac{1}{\sqrt{2}}\left(\sqrt{3a+b} - \sqrt{a-b}\right)$

19. (a) $28 - 6\sqrt{3}$

$= 1 + 3\sqrt{3} - 2\cdot 1\cdot 3\sqrt{3}$

$= \left(3\sqrt{3} - 1\right)^2$ etc.

$\therefore \quad \sqrt{28 - 6\sqrt{3}} + \sqrt{28 + 6\sqrt{3}}$

$= \left(3\sqrt{3} - 1\right) + 3\sqrt{3} + 1 = 6\sqrt{3}$

20. (a) $1 + x^2 + \left(1 + x^2 + x^4\right)^{\frac{1}{2}}$

$= 1 + x^2 + \left(\left(1+x^2\right)^2 - x^2\right)^{\frac{1}{2}}$

$$= 1 + x^2 + \left(\left(1+x^2+x\right)\left(1+x^2-x\right)\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(2+2x^2+2\sqrt{1+x^2+x}\sqrt{1+x^2-x}\right)$$

$$= \frac{1}{2}\left\{\left(1+x^2+x\right)+\left(1+x^2-x\right)+2\sqrt{1+x^2+x}\times\sqrt{1+x^2-x}\right\}$$

$$= \frac{1}{2}\left(\sqrt{1+x+x^2}+\sqrt{1+x^2-x}\right)^2$$

$$\therefore \sqrt{1+x^2+\left(1+x^2+x^4\right)^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}+\sqrt{1+x^2-x}\right)$$

21. (c) Given expression $= x+y+z+2\sqrt{(x+y)z}$

$$= (x+y)+z+2\sqrt{x+y}\sqrt{z}$$

$$= \left(\sqrt{x+y}+\sqrt{z}\right)^2 \text{ etc.}$$

22. (b) $\sqrt{3x-7}+\sqrt{3x+7} = 4+\sqrt{2}$

squaring both sides,

$$3x-7+3x+7+2\sqrt{3x-7}\sqrt{3x+7} = 16+2+8\sqrt{2}$$

or, $6x+2\sqrt{9x^2-49} = 18+8\sqrt{2}$

equation rational parts, $6x = 18$

$$\Rightarrow x = 3$$

at, $x = 3$, irrational part

$= 2\sqrt{9\times9-49}$

$= 2\sqrt{32} = 8\sqrt{2}$ which is correct

$\therefore x+\frac{1}{x} = 3+\frac{1}{3} = \frac{10}{3}$

23. (b) $6+\sqrt{12}-\sqrt{24}-\sqrt{8} = 6+2\sqrt{3}-2\sqrt{6}-2\sqrt{2}$

$$= 6+2\sqrt{3}-2\sqrt{3}\sqrt{2}-2\sqrt{2}$$

$$= 3+2+1+2\sqrt{3}-2\sqrt{3}\sqrt{2}-2\sqrt{2}$$

$$= \left(\sqrt{3}+1-\sqrt{2}\right)^2 \text{ etc.}$$

24. (b) $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= 5-2\sqrt{6}$ and $b = 5+2\sqrt{6}$

$\therefore ab = 25-24 = 1$

Now, $3a$ ... $= 3(a-b)^2 + ab$

$$= 3\left(5-2\sqrt{6}-5-2\sqrt{6}\right)^2+1$$

$$= 3(96) + 1 = 289$$

$$\therefore \quad \sqrt{3a^2-5ab+3b^2} = \sqrt{289} = 17$$

25. (d) $\sqrt{26-15\sqrt{3}} = \sqrt{\dfrac{52-30\sqrt{3}}{2}}$

$$= \sqrt{\frac{\left(3\sqrt{3}-5\right)^2}{2}} = \frac{3\sqrt{3}-5}{\sqrt{2}}$$

$$\sqrt{38+5\sqrt{3}} = \sqrt{\frac{76+10\sqrt{3}}{2}}$$

$$= \sqrt{\frac{\left(5\sqrt{3}+1\right)^2}{2}} = \frac{5\sqrt{3}+1}{\sqrt{2}}$$

Hence given expression $= \left(\dfrac{\dfrac{3\sqrt{3}-5}{\sqrt{2}}}{5\sqrt{2}-\dfrac{5\sqrt{3}+1}{\sqrt{2}}}\right)^2 = \left(\dfrac{3\sqrt{3}-5}{9-5\sqrt{3}}\right)^2$

$$= \left(\frac{3\sqrt{3}-5}{\sqrt{3}\left(3\sqrt{3}-5\right)}\right)^2 = \frac{1}{3}$$

26. (b) Given expression $= \dfrac{12}{3+\sqrt{5}+2\sqrt{2}} \times \dfrac{\left(3+2\sqrt{2}\right)-\sqrt{5}}{\left(3+2\sqrt{2}\right)-\sqrt{5}}$

$$= \frac{12}{\left(3+2\sqrt{2}\right)^2-\left(\sqrt{5}\right)^2} \times \left(\left(3+2\sqrt{2}\right)-\sqrt{5}\right)$$

$$= \frac{12}{12+12\sqrt{2}}\left(3+2\sqrt{2}-\sqrt{5}\right)$$

$$= \frac{1}{\sqrt{2}+1}\left(3+2\sqrt{2}-\sqrt{5}\right) \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= 3\sqrt{2}-3+4-2\sqrt{2}-\sqrt{10}+\sqrt{5}$$

$$= \sqrt{2}+1-\sqrt{10}+\sqrt{5}$$

27. (b) $\dfrac{1}{\sqrt{x^2-x+1}} + \sqrt{x^2-x+1} = 2-x^2$

Here, LHS is sum of a positive quantity and its reciprocal

$\therefore \quad$ LHS ≥ 2

And RHS $= 2 - x^2 \le 2$

$\therefore$ Both sides are equal $\frac{1}{\sqrt{x^2-x+1}} + \sqrt{x^2-x+1} = 2$ and $2x^2 = 2$

clearly, $2 - x^2 = 2 \Rightarrow x = 0$

and at, $x = 0$, LHS $= 2$

Hence, $x = 0$ is only solution of the equation.

Thus equation has only one real root

28. (d) Do as in question no. 21. Required value is 99, which is a multiple of all 11, 3, 9

29. (d) $(pq) = (pq)^3$

$\Rightarrow pq = 1$

$\because \; p = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{7+2\sqrt{10}}{3}$

$\therefore \; q = \frac{1}{p} = \frac{\sqrt{5}-\sqrt{2}}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{7-2\sqrt{10}}{3}$

Now $3p^2 + 4pq - 3q^2 = 3(p+q)(p-q) + 4pq$

$$= 3 \times \frac{14}{3} \times \frac{4\sqrt{10}}{3} + 4 \times 1$$

$$= \frac{1}{3}(12 + 56\sqrt{10})$$

30. (b) $10 + \sqrt{24} + \sqrt{40} + \sqrt{60}$

$$= 10 + 2\sqrt{2}\sqrt{3} + 2\sqrt{2}\sqrt{5} + 2\sqrt{3}\sqrt{5}$$

$$= (\sqrt{2})^2 + (\sqrt{3})^2 + (\sqrt{5})^2 + 2\sqrt{2}\sqrt{3} + 2\sqrt{2}\sqrt{5} + 2\sqrt{3}\sqrt{5}$$

$$= (\sqrt{2} + \sqrt{3} + \sqrt{5})^2$$

$\therefore \; \sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}} = \sqrt{2} + \sqrt{3} + \sqrt{5}$

$\therefore \; p + q + r = 2 + 3 + 5 = 10$

31. (b) Do as in solved example 13. Required cube root is $3 - \sqrt{5}$.

32. (a) $11 - 2\sqrt{30} = (\sqrt{6} - \sqrt{5})^2$, $7 - 2\sqrt{10}$

$$= (\sqrt{5} - \sqrt{2})^2, \; 8 + 4\sqrt{3}$$

$$= (\sqrt{6} + \sqrt{2})^2 \text{ etc.}$$

33. (b) Given expression $= \frac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}} = \frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

$= \frac{\sqrt{3}(\sqrt{3}+\sqrt{2})}{\sqrt{3}+\sqrt{2}} = \sqrt{3}$

34. (c) $a = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}}$

$= \sqrt{\frac{(\sqrt{5}+1)^2}{5-1}} = \frac{\sqrt{5}+1}{2}$

$\frac{1}{a} = \frac{2}{\sqrt{5}+1}$

$= \frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$

$= \frac{\sqrt{5}-1}{2}$

Now, $a^2 - a - 1 = a\left(a - 1 - \frac{1}{a}\right)$

$= \frac{\sqrt{5}+1}{2}\left(\frac{\sqrt{5}+1}{2} - 1 - \frac{\sqrt{5}-1}{2}\right)$

$= \frac{\sqrt{5}+1}{2}\left(\frac{1}{2} - 1 - \frac{1}{2}\right) = 0$

$\therefore \quad \sqrt{a^2 - a - 1} = \sqrt{0} = 0$

35. (d) Given $x = (a + \sqrt{a^2+b^3})^{1/3} + (a - \sqrt{a^2+b^3})^{1/3}$

cubing both sides,

$$x^3 = (a + \sqrt{a^2+b^3}) + (a - \sqrt{a^2+b^3}) + 3(a + \sqrt{a^2+b^3})^{1/3}(a - \sqrt{a^2+b^3})^{1/3}\{(a + \sqrt{a^2+b^3})^{1/3} + (a - \sqrt{a^2+b^3})^{1/3}\}$$

$\Rightarrow \quad x^3 = 2a - 3b\,(x)$

$\Rightarrow \quad x^3 + 3bx - 2a = 0$

36. (a) $139 - 80\sqrt{3} = 139 - 2 \times 8 \times 5\sqrt{3}$

$= 8^2 + (5\sqrt{3})^2 - 2 \times 8 \times 5\sqrt{3}$

$= (5\sqrt{3} - 8)^2$

Note that : $5\sqrt{3} > 8$

$\therefore \quad \sqrt{139 - 80\sqrt{3}} = 5\sqrt{3} - 8$

37. (d) $(a+3)\sqrt{2}+3=b2\sqrt{2}+a-1$

equation rational and irrational part,

$a+3=2b$ and $a-1=3$

$\therefore$ $a=2b-3$ and $a=4$

$\therefore$ $4=2b-3$ and $a=4$

or, $b=\frac{7}{2}$ and $a=4$

Hence, $a+b=\frac{15}{2}$

38. (b) $\sqrt{x+2\sqrt{x-1}}=\sqrt{(x-1)+1+2\sqrt{x-1}}=\sqrt{\left(\sqrt{x-1}+1\right)^2}=\sqrt{x-1}+1$

$\sqrt{x-2\sqrt{x-1}}=\sqrt{(x-1)+1-2\sqrt{x-1}}=\sqrt{\left(\sqrt{x-1}-1\right)^2}=\sqrt{x-1}-1$

$\therefore$ Required value $=\left(\sqrt{x-1}+1\right)+\left(\sqrt{x-1}-1\right)=2\sqrt{x-1}$

39. (a) $\sqrt{x+2\sqrt{x-1}}=\sqrt{(x-1)+1+2\sqrt{x-1}}=\sqrt{\left(\sqrt{x-1}+1\right)^2}=\sqrt{x-1}+1$

$\sqrt{x-2\sqrt{x-1}}=\sqrt{(x-1)+1-2\sqrt{x-1}}=\sqrt{\left(1-\sqrt{x-1}\right)^2}=1-\sqrt{x-1}$

($\because 1<x<2$ hence value of $x-1$ is less then 1. Therefore $1-\sqrt{x-1}$ is a positive quantity)

Hence, Required sum $=\sqrt{x-1}+1+1-\sqrt{x-1}=2$

40. (a) $\sqrt{1+x}=\sqrt{1+\frac{\sqrt{3}}{2}}=\sqrt{\frac{2+\sqrt{3}}{2}}=\sqrt{\frac{2+\sqrt{3}}{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=\sqrt{\frac{4+2\sqrt{3}}{4}}=\frac{1}{2}\left(\sqrt{3}+1\right)^2=\frac{1}{2}(\sqrt{3}+1)$

$\sqrt{1-x}=\sqrt{1-\frac{\sqrt{3}}{2}}=\sqrt{\frac{2-\sqrt{3}}{2}}=\sqrt{\frac{4-2\sqrt{3}}{4}}=\frac{1}{2}\left(\sqrt{3}-1\right)$

$\therefore$ $\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$

$$=\frac{1+\frac{\sqrt{3}}{2}}{1+\frac{1}{2}\left(\sqrt{3}+1\right)}+\frac{1-\frac{\sqrt{3}}{2}}{1-\frac{1}{2}\left(\sqrt{3}-1\right)}$$

$$=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{3-\sqrt{3}}$$

$$=\frac{6-2\sqrt{3}+3\sqrt{3}-3+6+2\sqrt{3}-3\sqrt{3}-3}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)}$$

$$=\frac{6-3+6-3}{9-3}=\frac{6}{6}=1$$

41. (d) Let $\frac{x}{y} = \frac{y}{z} = \frac{z}{w} = k$

then, $x = y\,k,\ y = zk,\ z = wk$

$\therefore \quad x = y\,k = (zk)k = (wk)kk = wk^3$

$y = zk = (wk)k = wk^2$

$$\text{Hence Given expression} = \frac{(wk^3)^m + (wk^2)^m + (wk)^m + w^m}{(wk^3)^{-m} + (wk^2)^{-m} + (wk)^{-m} + w^{-m}}$$

$$= \frac{w^m\left(k^{3m} + k^{2m} + k^m + 1\right)}{w^{-m}\left(k^{-3m} + k^{-2m} + k^{-m} + 1\right)}$$

$$= \frac{w^{2m}\left(k^{3m} + k^{2m} + k^m + 1\right)}{k^{-3m}\left(1 + k^m + k^{2m} + k^{3m}\right)}$$

$$= w^{2m}\,k^{3m} = \left(w^4k^6\right)^{\frac{m}{2}}$$

$$= \left(wk^3 \cdot wk^2 \cdot wk \cdot w\right)^{\frac{m}{2}} = \left(xyzw\right)^{\frac{m}{2}}$$

42. (c) (a) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}} + \sqrt{6} = \frac{(\sqrt{2}+\sqrt{3})^2}{3-2} + \sqrt{6}$

$= 2 + 3 + 2\sqrt{6} + \sqrt{6}$

$= 5 + 3\sqrt{6}$

Which is not an integer,

(b) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}} + 2\sqrt{6} = \frac{(\sqrt{2}+\sqrt{3})^2}{3-2} + 2\sqrt{6}$

$= 2 + 3 + 2\sqrt{6} + 2\sqrt{6}$

$= 5 + 4\sqrt{6}$

Which is not an integer,

(c) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} + 2\sqrt{6} = \frac{(\sqrt{2}+\sqrt{3})^2}{2-3} + 2\sqrt{6}$

$= -(2 + 3 + 2\sqrt{6}) + 2\sqrt{6}$

$= -5 - 2\sqrt{6} + 2\sqrt{6}$

$= -5$

Which is an integer.

Hence, option (c) is correct

43. (b) $(256)^{0.16} \times (16)^{0.18} = (4^4)^{0.16} \times (4^2)^{0.18}$

$$= 4^{0.64} + 4^{0.36}$$

$$= 4^{0.64 + 0.36}$$

$$= 4^1 = 4$$

44. (a) $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}} = 2 + \sqrt{2} + \frac{2-\sqrt{2}-2-\sqrt{2}}{4-2}$

$$= 2 + \sqrt{2} + \frac{(-2\sqrt{2})}{2}$$

$$= 2 + \sqrt{2} - \sqrt{2} = 2$$

45. (d) $a + x = a + \frac{2ab}{b^2+1}$

$$= \frac{a\left(b^2+1\right)+2ab}{b^2+1}$$

$$= \frac{a\left(b^2+1+2b\right)}{b^2+1} = \frac{a\left(b+1\right)^2}{b^2+1} \text{ etc.}$$

Exercise—2B

If $a = \frac{2+\sqrt{3}}{2-\sqrt{3}}$ and $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$ then the value of $(a^2 + b^2 + ab)$ is

(a) 185 (b) 195 (c) 200 (d) 175

[SSC Tier-I 201

2. If $x = \frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ then the value of $\frac{x+\sqrt{2}}{x-\sqrt{2}} + \frac{x+\sqrt{3}}{x-\sqrt{3}}$ is

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{6}$ (d) 2

[SSC Tier-I 201

3. If $x = \frac{1}{2+\sqrt{3}}, y = \frac{1}{2-\sqrt{3}}$ then the value of $\frac{1}{x+1} + \frac{1}{y+1}$ is

(a) $\frac{1}{2}$ (b) $\sqrt{3}$ (c) 1 (d) $\frac{1}{\sqrt{3}}$

[SSC Tier-I 201

4. If $x = \frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+x} + \sqrt{1-x}$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) 2

[SSC Tier-I 201

5. If $a = \sqrt{2} + 1, b = \sqrt{2} - 1$, then the value of $\frac{1}{a+1} + \frac{1}{b+1}$ is

(a) 9 (b) 3 (c) 1 (d) 2

[SSC Tier-I 201

6. If $a = 3 + 2\sqrt{2}$, then the value of $\frac{a^6 + a^4 + a^2 + 1}{a^3}$ is

(a) 192 (b) 240 (c) 204 (d) 212

[SSC Tier-I 2012]

7. If $x = 1 + \sqrt{2} + \sqrt{3}$ then the value of $(2x^4 - 8x^3 - 5x^2 + 26x - 28)$ is

(a) $6\sqrt{6}$ (b) 0 (c) $3\sqrt{6}$ (d) $2\sqrt{6}$

[SSC Tier-I 2012]

8. If $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$ then the value of $\frac{x^2 + y^2}{x^3 + y^3}$ is

(a) $\frac{7}{38}$ (b) $\frac{7}{40}$ (c) $\frac{7}{19}$ (d) $\frac{7}{26}$

[SSC Tier-I 2012]

9. If $2\sqrt{x} = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} - \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$ then what is the value of x ?

(a) 30 (b) $\sqrt{15}$ (c) 15 (d) 6

[SSC Tier-I 2012]

10. If $a = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$, $b = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ then what is the value of $\frac{a^2}{b} + \frac{b^2}{a}$?

(a) 970 (b) 1030 (c) 930 (d) 900

[SSC Tier-I 2012]

11. If $x = 2 + \sqrt{3}$ then the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) 6 (d) $\sqrt{3}$

[SSC Tier-I 2012]

12. What is the value of $\sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$?

(a) 3 (b) 4 (c) 1 (d) 2

[SSC Tier-I 2012]

13. If $\sqrt{\frac{x-a}{x-b}} + \frac{a}{x} = \sqrt{\frac{x-b}{x-a}} + \frac{b}{x}$, $b \neq a$ then what is the value of x ?

(a) $\frac{ab}{a+b}$ (b) 1 (c) $\frac{a}{a+b}$ (d) $\frac{b}{a+b}$

[SSC Tier-I 2012]

14. If $\sqrt{4x - 9} + \sqrt{4x + 9} = 5 + \sqrt{7}$ then what is the value of x ?

(a) 5 (b) 7 (c) 3 (d) 4

[SSC Tier-I 2012]

Answers—2B

1. (b) 2. (d) 3. (c) 4. (c) 5. (c) 6. (c) 7. (a) 8. (d)
9. (c) 10. (a) 11. (a) 12. (d) 13. (a) 14. (d)

Explanation

1. (b) $a = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = (2+\sqrt{3})^2$

$b = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = (2-\sqrt{3})^2$

$\therefore \quad a^2 + b^2 + ab = (a+b)^2 - ab = \left((2+\sqrt{3})^2 + (2-\sqrt{3})^2\right)^2 - 1$ $(\because ab = 1)$

$\left(2\left(2^2+(\sqrt{3})^2\right)\right)^2 - 1 = 14^2 - 1 = 196 - 1 = 195$

2. (d) $x = \frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ ...(i)

or, $\frac{x}{\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

By, Componendo-dividendo

$\frac{x+\sqrt{2}}{x-\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ ...(ii)

Again from (i), $\frac{x}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

or, $\frac{x+\sqrt{3}}{x-\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ ...(iii)

Adding (ii) and (iii),

$$\frac{x+\sqrt{2}}{x-\sqrt{2}} + \frac{x+\sqrt{3}}{x-\sqrt{3}} = \frac{(3\sqrt{3}+\sqrt{2})-(3\sqrt{2}+\sqrt{3})}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 2$$

3. (c) $x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

and $y = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

$\therefore \quad \frac{1}{x+1} + \frac{1}{y+1} = \frac{1}{2-\sqrt{3}+1} + \frac{1}{2+\sqrt{3}+1}$

$= \frac{1}{3-\sqrt{3}} + \frac{1}{3+\sqrt{3}}$

$= \frac{3+\sqrt{3}+3-\sqrt{3}}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{6}{9-3} = 1$

4. (c) $\sqrt{1+x}+\sqrt{1-x} = \sqrt{1+\frac{\sqrt{3}}{2}}+\sqrt{1-\frac{\sqrt{3}}{2}}$

$$= \sqrt{\frac{2+\sqrt{3}}{2}}+\sqrt{\frac{2-\sqrt{3}}{2}}$$

$$= \sqrt{\frac{4+2\sqrt{3}}{4}}+\sqrt{\frac{4-2\sqrt{3}}{4}}$$

$$= \sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2}+\sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2}$$

$$= \frac{1+\sqrt{3}}{2}+\frac{\sqrt{3}-1}{2}=\sqrt{3}$$

5. (c) $\frac{1}{a+1}+\frac{1}{b+1}=\frac{1}{\sqrt{2}+2}+\frac{1}{\sqrt{2}}$

$$=\frac{1}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}+\frac{\sqrt{2}}{2}$$

$$=\frac{2-\sqrt{2}}{4-2}+\frac{\sqrt{2}}{2}$$

$$=\frac{2-\sqrt{2}+\sqrt{2}}{2}=1$$

6. (c) $\frac{a^6+a^4+a^2+1}{a^3}=a^3+a+\frac{1}{a}+\frac{1}{a^3}$ ... (i)

$\because\ a=3+2\sqrt{2}$

$\therefore\ \frac{1}{a}=\frac{1}{3+2\sqrt{2}}=\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=\frac{3-2\sqrt{2}}{9-8}=3-2\sqrt{2}$

Hence, $a+\frac{1}{a}=3+2\sqrt{2}+3-2\sqrt{2}=6$

cubing, $a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)=6^3$

or, $a^3+\frac{1}{a^3}+3\times6=216$

or, $a^3+\frac{1}{a^3}=216-18=198$

From (i), Required value $=\left(a^3+\frac{1}{a^3}\right)+\left(a+\frac{1}{a}\right)$

$$=198+6=204$$

7. (a) $x-1=\sqrt{2}+\sqrt{3}$

Squaring both sides, $x^2-2x+1=2+3+2\sqrt{6}$ ... (i)

or, $x^2-2x-4=2\sqrt{6}$

Squaring both sides $x^4+4x^2+16-4x^3-8x^2+16x=24$

$$x^4-4x^3-4x^2+16x-8=0$$

(Multiply by...

$\therefore\ 2x^4 - 8x^3 - 8x^2 + 32x - 16 = 0$

or, $(2x^4 - 8x^3 - 5x^2 + 26x - 28) + (-3x^2 + 6x + 12) = 0$

$\therefore$ Required value $-3(x^2 - 2x - 4) = 0$

or, $2x^4 - 8x^3 - 5x^2 + 26x - 28 = -(-3x^2 + 6x + 12)$

or, Required value $= 3(x^2 - 2x - 4) = 3(2\sqrt{6})$

$= 6\sqrt{6}$

(From...

8. **(d)** Given, $x + y = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

and $xy = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$

$\therefore\ x^2 + y^2 = (x + y)^2 - 2xy = 4^2 - 2 = 14$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$= (x + y)\{(x + y)^2 - 3xy\}$

$= 4\{4^2 - 3 \times 1\} = 52$

$\therefore\ \dfrac{x^2+y^2}{x^3+y^3} = \dfrac{14}{52} = \dfrac{7}{26}$

. **(c)** $2\sqrt{x} = \dfrac{(\sqrt{5}+\sqrt{3})^2 - (\sqrt{5}-\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \dfrac{4(\sqrt{5}\sqrt{3})}{5-3}$

or, $\sqrt{x} = \dfrac{4\sqrt{15}}{2\times 2} = \sqrt{15}$

or, $x = 15$

10. **(a)** $a = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{(\sqrt{3}-\sqrt{2})^2}{3-2} = 5 - 2\sqrt{6}$

Similarly $b = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5 + 2\sqrt{6}$

and $a\cdot b = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 1$

$\therefore\ \dfrac{a^2}{b} + \dfrac{b^2}{a} = \dfrac{a^3+b^3}{ab}$

$= a^3 + b^3$

$= (5 - 2\sqrt{6})^3 + (5 + 2\sqrt{6})^3$

$= 2[5^3 + 3\cdot 5(2\sqrt{6})^2]$

$= 2(125 + 360)$

$= 970$

$(\because (x + y)^3 + (x - y)^3 = 2(x^3 + 3xy^2))$

ply by 2)

rom (i))

11. (a) Let $t = \sqrt{x} + \frac{1}{\sqrt{x}}$

then, $t^2 = x + \frac{1}{x} + 2$

$= 2 + \sqrt{3} + \frac{1}{2+\sqrt{3}} + 2$

$= 2 + \sqrt{3} + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + 2$

$= 2 + \sqrt{3} + \frac{2-\sqrt{3}}{4-3} + 2$

$= 2 + \sqrt{3} + 2 - \sqrt{3} + 2 = 6$

$\therefore \quad t = \sqrt{6}$

or, $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$

12. (d) $\sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$

$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{(2+\sqrt{3})^2}}}$ $\quad (\because (2+\sqrt{3})^2 = 2^2 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3})$

$= \sqrt{-\sqrt{3} + \sqrt{3 + 8(2+\sqrt{3})}}$

$= \sqrt{-\sqrt{3} + \sqrt{19 + 8\sqrt{3}}}$

$= \sqrt{-\sqrt{3} + \sqrt{(4+\sqrt{3})^2}}$ $\quad (\because (4+\sqrt{3})^2 = 16 + 3 + 8\sqrt{3})$

$= \sqrt{-\sqrt{3} + 4 + \sqrt{3}} = \sqrt{4} = 2$

13. (a) $\sqrt{\frac{x-a}{x-b}} + \frac{a}{x} = \sqrt{\frac{x-b}{x-a}} + \frac{b}{x}$

or, $\sqrt{\frac{x-a}{x-b}} - \sqrt{\frac{x-b}{x-a}} = \frac{b}{x} + \frac{-a}{x}$

$\frac{(x-a)-(x-b)}{\sqrt{x-b}\sqrt{x-a}} = \frac{b-a}{x}$

or, $\frac{-a+b}{\sqrt{x-b}\sqrt{x-a}} = \frac{b-a}{x}$

or, $\frac{1}{\sqrt{x-b}\sqrt{x-a}} = \frac{1}{x}$

Squaring both sides, $(x-b)(x-a) = x^2$

$x^2 - (a+b)x + ab = x^2$

$\therefore \quad x = \frac{ab}{a+b}$

$3xy^2$))

14. (d) By trial, $x = 4$ satisfies the equation. If can be solved as follows

$$\left(\sqrt{4x-9}+\sqrt{+x+9}\right)\left(\sqrt{4x-9}-\sqrt{4x+9}\right) = 4x-9-4x-9$$

or, $\left(5+\sqrt{7}\right)\left(4x-9-\sqrt{4x+9}\right) = -18$

or, $\sqrt{4x-9}-\sqrt{4x+9} = \dfrac{-18}{5+\sqrt{7}} \times \dfrac{5-\sqrt{7}}{5-\sqrt{7}}$

or, $\sqrt{4x-9}-\sqrt{4x+9} = \dfrac{-18\left(5-\sqrt{7}\right)}{25-7}$

or, $\sqrt{4x-9}-\sqrt{4x+9} = -\left(5-\sqrt{7}\right)$ ... (i)

Given, $\sqrt{4x-9}+\sqrt{4x+9} = 5+\sqrt{7}$ ... (ii)

Adding, $2\sqrt{4x-9} = 2\sqrt{7}$

or, $4x-9 = 7$

or, $4x = 16$

or, $x = 4$

★★★