Chapter 21

MEASURES OF CENTRAL TENDENCY

Exercise 21.1

1. (a) Calculate the arithmetic mean of 5.7, 6.6, 7.2, 9.3, 6.2.
(b) The weights (in kg) of 8 new born babies are 3, 3.2, 3.4, 3.5, 4, 3.6, 4.1, 3.2. Find the mean weight of the babies.

Solution:
(a) Given observations are 5.7, 6.6, 7.2, 9.3, 6.2.
Number of observations = 5
Mean = sum of observations / number of observations
∴ Mean = (5.7+6.6+7.2+9.3+6.2)/5
= 35/5 = 7
Hence the mean of the given observations is 7.

(b) Given weight of babies are 3, 3.2, 3.4, 3.5, 4, 3.6, 4.1, 3.2
Number of observations = 8
Mean = sum of observations / number of observations
∴ Mean = (3+3.2+3.4+3.5+4+3.6+4.1+3.2)/8
= 28/8 = 3.5 kg
Hence the mean of the weight of babies is 3.5 kg.

2.The marks obtained by 15 students in a class test are 12, 14, 07, 09, 23, 11, 08, 13, 11, 19, 16, 24, 17, 03, 20 find
(i) the mean of their marks.
(ii) the mean of their marks when the marks of each student are increased by 4.
(iii) the mean of their marks when 2 marks are deducted from the marks of each student.
(iv) the mean of their marks when the marks of each student are doubled.

Solution:
(i) Marks obtained by students are 12, 14, 07, 09, 23, 11, 08, 13, 11, 19, 16, 24, 17, 03, 20.
Number of students = 15
Mean = sum of observations / number of observations
= 12+14+07+09+23+11+08+13+11+19+16+24+17+03+20
= 207/15
= 13.8
Hence the mean of their marks is 13.8.

(ii) If mark of each student is increased by 4, total increased marks = 4×15 = 60
Total increase in sum of marks = 207+60 = 267
∴ mean = sum of marks/number of students
∴ mean = 267/15 = 17.8
Hence the mean is 17.8.

(iii) If mark of each student is deducted by 2, total deducted marks = 2×15 = 30
Total decrease in sum of marks = 207-30 = 177
∴ mean = sum of marks/number of students
∴ mean = 177/15 = 11.8

Hence the mean is 11.8.

(iv) If mark of each student is doubled , then new sum of marks = 2×207 = 414
∴mean = new sum of marks/number of students
∴mean = 414/15 = 27.6
Hence the mean is 27.6.

3. (a) The mean of the numbers 6, y, 7, x, 14 is 8. Express y in terms of x.
(b) The mean of 9 variates is 11. If eight of them are 7, 12, 9, 14, 21, 3, 8 and 15 find the 9th variate.

Solution:
(a)Given observations are 6, y, 7, x, 14.
Mean = 8
Number of observations = 5
Mean = Sum of observations/number of observations
∴8 = (6+y+7+x+14)/5
∴40 = 27+x+y
40-27 = x+y
13 = x+y
∴y = 13-x
Hence the answer is y = 13-x.

(b)Given mean = 11
Number of variates = 9
Variates are 7, 12, 9, 14, 21, 3, 8 ,15
Let the 9^{th} variate be x.
Sum of variates = 7+12+9+14+21+3+8+15+x
= 89+x
Mean = Sum of variates/number of variates
∴11 = (89+x)/9
11×9 = 89+x
99 = 89+x
∴x = 99-89 = 10
Hence the 9^{th} variate is 10.

4. (a) The mean age of 33 students of a class is 13 years. If one girl leaves the class, the mean becomes $12\frac{15}{16}$ years. What is the age of the girl ?
(b) In a class test, the mean of marks scored by a class of 40 students was calculated as 18.2. Later on, it was detected that marks of one student was wrongly copied as 21 instead of 29. Find the correct mean.

Solution:
(a)Given mean age = 13
Number of students = 33
Sum of ages = mean ×number of students
= 13×33
= 429

After a girl leaves, the mean of 32 students becomes $12\frac{15}{16}$ = 207/16

Now sum of ages = 32×207/16
= 414
So the age of the girl who left = 429-414 = 15 years.
Hence the age of the girl who left is 15 years.

(b)Mean of marks = 18.2
Number of students = 40
Total marks of 40 students = 40×18.2 = 728
Difference of marks when copied wrongly = 29-21 = 8
So total marks = 728+8 = 736
∴mean = 736/40
= 18.4
Hence the correct mean is 18.4.

5. Find the mean of 25 given numbers when the mean of 10 of them is 13 and the mean of the remaining numbers is 18.

Solution:
Mean of 10 numbers = 13
∴Sum of numbers = 13×10 = 130
Mean of remaining 15 numbers = 18
∴Sum of numbers = 15×18 = 270
∴Sum of all numbers = 130+270 = 400
∴Mean = sum of numbers/25 = 400/25 = 16
Hence the mean of 25 numbers is 16.

6. Find the mean of the following distribution:

Number	**5**	**10**	**15**	**20**	**25**	**30**	**35**
Frequency	**1**	**2**	**5**	**6**	**3**	**2**	**1**

Solution:

Number (x)	Frequency (f)	fx
5	1	5×1 = 5
10	2	10×2 = 20
15	5	15×5 = 75
20	6	20×6 = 120
25	3	25×3 = 75
30	2	30×2 = 60
35	1	35×1 = 35
Total	Σf = 20	Σfx = 390

∴Mean = Σfx/Σf

= 390/20 = 19.5
Hence the mean is 19.5.

7. The contents of 100 match boxes were checked to determine the number of matches they contained

No. of matches	**35**	**36**	**37**	**38**	**39**	**40**	**41**
No. of boxes	**6**	**10**	**18**	**25**	**21**	**12**	**8**

(i) Calculate, correct to one decimal place, the mean number of matches per box.
(ii) Determine how many extra matches would have to be added to the total contents of the 100 boxes to; bring the mean up to exactly 39 matches. (1997)

Solution:
(i)

No. of matches (x)	Number of boxes (f)	fx
35	6	35×6 = 210
36	10	36×10 = 360
37	18	37×18 = 666
38	25	38×25 = 950
39	21	39×21 = 819
40	12	40×12 = 480
41	8	41×8 = 328
Total	Σf = 100	Σfx = 3813

Mean = Σfx/Σf
= 3813/100
= 38.13
= 38.1
Hence the mean is 38.1.

(ii)New mean = 39
∴Σfx = 39×100 = 3900
So number of extra matches to be added = 3900-3813 = 87
Hence the number of extra matches to be added is 87.

8. Calculate the mean for the following distribution :

Pocket money (in Rs)	**60**	**70**	**80**	**90**	**100**	**110**	**120**
No. of students	**2**	**6**	**13**	**22**	**24**	**10**	**3**

Solution:

Pocket money in Rs (x)	Number of students (f)	fx
60	2	60×2 = 120
70	6	70×6 = 420
80	13	80×13 = 1040
90	22	90×22 = 1980
100	24	100×24 = 2400
110	10	110×10 = 1100
120	3	120×3 = 360
Total	Σf = 80	Σfx = 7420

Mean = Σfx/Σf
= 7420/80
= 92.75
Hence the mean is 92.75.

9. Six coins were tossed 1000 times, and at each toss the number of heads were counted and the results were recorded as under :

No. of heads	**6**	**5**	**4**	**3**	**2**	**1**	**0**
No. of tosses	**20**	**25**	**160**	**283**	**338**	**140**	**34**

Calculate the mean for this distribution.

Solution:

No. of heads (x)	No. of tosses (f)	fx
6	20	6×20 = 120
5	25	5×25 = 125
4	160	4×160 = 640
3	283	3×283 = 849
2	338	2×338 = 676
1	140	1×140 =140
0	34	0×34 = 0
Total	Σf = 1000	Σfx = 2550

Mean = Σfx/Σf
= 2550/1000
= 2.55
Hence the mean is 2.55.

10. Find the mean for the following distribution.

Numbers	**60**	**61**	**62**	**63**	**64**	**65**	**66**

Cumulative frequency	8	18	33	40	49	55	60

Solution:

Numbers (x)	Cumulative frequency	Frequency (f)	fx
60	8	8	60×8 = 480
61	18	18-8 = 10	61×10 = 610
62	33	33-18 = 15	62×15 = 930
63	40	40-33 = 7	63×7 = 441
64	49	49-40 = 9	64×9 = 576
65	55	55-49 = 6	65×6 = 390
66	60	60-55 = 5	66×5 = 330
Total		Σf = 60	Σfx = 3757

∴ Mean = Σfx/Σf
= 3757/60
= 62.616
= 62.62
Hence the mean is 62.62.

11.

Category	A	B	C	D	E	F	G
Wages (in Rs) per day	50	60	70	80	90	100	110
No. of workers	2	4	8	12	10	6	8

(i) Calculate the mean wage correct to the nearest rupee (1995)
(ii) If the number of workers in each category is doubled, what would be the new mean wage ?

Solution:

Category	Wages in Rs. (x)	No. of workers f	fx
A	50	2	100
B	60	4	240
C	70	8	560
D	80	12	960
E	90	10	900
F	100	6	600
G	110	8	880
Total		Σf = 50	Σfx = 4240

∴ Mean = Σfx/Σf
= 4240/50

= 84.8
= 85
Hence the mean is 85.

(ii)If number of workers is doubled, then total number of workers = 50×2 = 100
So wages will be doubled.
∴Total wages = 4240×2 = 8480
∴Mean = Σfx/Σf
= 8480/100
= 84.8
= 85
Hence the mean is 85.

12.If the mean of the following distribution is 7.5, find the missing frequency " f ".

Variate	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Frequency	**20**	**17**	**f**	**10**	**8**	**6**	**7**	**6**

Solution:

Variate (x)	Frequency (f)	fx
5	20	100
6	17	102
7	f	7f
8	10	80
9	8	72
10	6	60
11	7	77
12	6	72
Total	Σf = 74+f	Σfx = 563+7f

Given mean = 7.5
∴Mean = Σfx/Σf
7.5= (563+f)/(74+7f)
7.5×(74+f) = 563+7f
555+7.5f = 563+7f
7.5f-7f = 563-555
0.5f = 8
∴f = 8/0.5 = 16
Hence the value of missing frequency f is 16.

13. Find the value of the missing variate for the following distribution whose mean is 10

Variate (x_i)	**5**	**7**	**9**	**11**	**_**	**15**	**20**
Frequency (f_i)	**4**	**4**	**4**	**7**	**3**	**2**	**1**

Solution:

Let the missing variate be x.

Variate (x_i)	Frequency (f_i)	f_ix_i
5	4	20
7	4	28
9	4	36
11	7	77
x	3	3x
15	2	30
20	1	20
Total	$\Sigma f_i = 25$	$\Sigma f_ix_i = 211+3x$

Given mean = 10

Mean = $\Sigma f_ix_i/\Sigma f_i$

$\therefore 10 = (211+3x)/25$

$10\times25 = 211+3x$

$250 = 211+3x$

$250-211 = 3x$

$39 = 3x$

$x = 39/3 = 13$

Hence the missing variate is 13.

14. Marks obtained by 40 students in a short assessment are given below, where a and b are two missing data.

Marks	**5**	**6**	**7**	**8**	**9**
No. of students	**6**	**a**	**16**	**13**	**b**

If the mean of the distribution is 7.2, find a and b.

Solution:

Marks (x)	No. of students (f)	fx
5	6	30
6	a	6a
7	16	112
8	13	104
9	b	9b
Total	$\Sigma f = 35+a+b$	$\Sigma fx = 246+6a+9b$

Given number of students = 40

$\therefore \Sigma f = 35+a+b = 40$

$a+b = 40-35 = 5$

$a = 5-b$ ……(i)

Mean $= \Sigma fx/\Sigma f$

Given mean = 7.2

$\therefore (246+6a+9b)/40 = 7.2$

$\therefore$ (246+6a+9b) = 40×7.2
$\therefore$ (246+6a+9b) = 288
$\therefore$ 6a+9b = 288-246
$\therefore$ 6a+9b = 288-246
$\therefore$ 6a+9b = 42
2a+3b = 14 …..(ii)
Substitute (i) in (ii)
2(5-b)+3b = 14
10-2b+3b = 14
10+b = 14
$\therefore$ b = 14-10 = 4
$\therefore$ a = 5-4 = 1
Hence the value of a and b is 1 and 4 respectively.

15. Find the mean of the following distribution.

Class interval	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
Frequency	**10**	**6**	**8**	**12**	**5**

Solution:

Class mark, x_i = (upper class limit + lower class limit)/2

Class interval	Frequency f_i	Class mark x_i	f_ix_i
0-10	10	5	50
10-20	6	15	90
20-30	8	25	200
30-40	12	35	420
40-50	5	45	225
Total	$\Sigma f_i = 41$		$\Sigma f_ix_i = 985$

Mean = $\Sigma f_ix_i / \Sigma f_i$
= 985/41
= 24.024
= 24.02 (approx)
Hence the mean of the distribution is 24.02.

16. Calculate the mean of the following distribution:

Class interval	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
Frequency	**8**	**5**	**12**	**35**	**24**	**16**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

Class interval	Frequency f_i	Class mark x_i	f_ix_i

0-10	8	5	40
10-20	5	15	75
20-30	12	25	300
30-40	35	35	1225
40-50	24	45	1080
50-60	16	55	880
Total	$\Sigma f_i = 100$		$\Sigma f_i x_i = 3600$

Mean = $\Sigma f_i x_i / \Sigma f_i$
= 3600/100
= 36
Hence the mean of the distribution is 36.

17. Calculate the mean of the following distribution using step deviation method:

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
No. of students	**10**	**9**	**25**	**30**	**16**	**10**

Solution:
Class mark (x_i) = (upper limit + lower limit)/2
Let assumed mean (A) = 25
Class size (h) = 10

Class Interval	No. of students (f_i)	Class mark (x_i)	$d_i = x_i - A$	$u_i = d_i/h$	f_iu_i
0-10	10	5	-20	-2	-20
10-20	9	15	-10	-1	-9
20-30	25	25	0	0	0
30-40	30	35	10	1	30
40-50	16	45	20	2	32
50-60	10	55	30	3	30
Total	$\sum f_i = 100$				$\sum f_iu_i = 63$

By step deviation method, Mean = $\bar{x} = A + h\sum f_iu_i / \sum f_i$
= 25+10(63/100)
= 25+10×0.63
= 25+6.3
=31.3
Hence the mean of the distribution is 31.3.

18. Find the mean of the following frequency distribution:

Class intervals	**0-50**	**50-100**	**100-150**	**150-200**	**200-250**	**250-300**
frequency	**4**	**8**	**16**	**13**	**6**	**3**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

Class interval	Frequency f_i	Class mark x_i	f_ix_i
0-50	4	25	100
50-100	8	75	600
100-150	16	125	2000
150-200	13	175	2275
200-250	6	225	1350
250-300	3	275	825
Total	$\Sigma f_i = 50$		$\Sigma f_ix_i = 7150$

Mean = $\Sigma f_ix_i / \Sigma f_i$
= 7150/50
=143
Hence the mean of the distribution is 143.

19. The following table gives the daily wages of workers in a factory:

Wages in Rs.	**45-50**	**50-55**	**55-60**	**60-65**	**65-70**	**70-75**	**75-80**
No. of workers	**5**	**8**	**30**	**25**	**14**	**12**	**6**

Calculate their mean by short cut method.

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2
Assumed mean, A = 62.5

Wages in Rs.	No. of workers (f_i)	Class mark (x_i)	$d_i = x_i - A$	f_id_i
45-50	5	47.5	-15	-75
50-55	8	52.5	-10	-80
55-60	30	57.5	-5	-150
60-65	25	62.5	0	0
65-70	14	67.5	5	70
70-75	12	72.5	10	120
75-80	6	77.5	15	90
Total	$\sum f_i = 100$			$\sum f_id_i = -25$

By short cut method, Mean = $\bar{x} = A + \sum f_id_i / \sum f_i$
= 62.5+-25/100
= 62.5-0.25
= 62.25
Hence the mean of the distribution is Rs.62.25.

20. Calculate the mean of the distribution given below using the short cut method.

Marks	**11-20**	**21-30**	**31-40**	**41-50**	**51-60**	**61-70**	**71-80**
No. of students	**2**	**6**	**10**	**12**	**9**	**7**	**4**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2
Assumed mean, A = 45.5

Marks	No. of students (f_i)	Class mark (x_i)	$d_i = x_i - A$	$f_i d_i$
11-20	2	15.5	-30	-60
21-30	6	25.5	-20	-120
31-40	10	35.5	-10	-100
41-50	12	45.5	0	0
51-60	9	55.5	10	90
61-70	7	65.5	20	140
71-80	4	75.5	30	120
Total	$\sum f_i = 50$			$\sum f_i d_i = 70$

By short cut method, Mean = $\bar{x} = A + \sum f_i d_i / \sum f_i$
= 45.5+70/50
= 45.5+1.4
= 46.9
Hence the mean of the distribution is Rs.46.9.

21. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

No. of days	**0-6**	**6-10**	**10-14**	**14-20**	**20-28**	**28-38**	**38-40**
No. of students	**11**	**10**	**7**	**4**	**4**	**3**	**1**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

No. of days	Frequency f_i	Class mark x_i	$f_i x_i$
0-6	11	3	33
6-10	10	8	80
10-14	7	12	84
14-20	4	17	68
20-28	4	24	96
28-38	3	33	99
38-40	1	39	39
Total	$\Sigma f_i = 40$		$\Sigma f_i x_i = 499$

Mean = $\Sigma f_i x_i / \Sigma f_i$
= 499/40
=12.475
Hence the mean number of days a student was absent is 12.475.

22. The mean of the following distribution is 23.4. Find the value of p.

Class intervals	**0-8**	**8-16**	**16-24**	**24-32**	**32-40**	**40-48**
Frequency	**5**	**3**	**10**	**P**	**4**	**2**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

Class intervals	Frequency f_i	Class mark x_i	$f_i x_i$
0-8	5	4	20
8-16	3	12	36
16-24	10	20	200
24-32	P	28	28P
32-40	4	36	144
40-48	2	44	88
Total	Σf_i = 24+P		$\Sigma f_i x_i$ = 488+28P

Given mean = 23.4
$\therefore$ Mean = $\Sigma f_i x_i / \Sigma f_i$
$\therefore 23.4 = (488+28P)/(24+P)$
$\therefore 23.4 \times (24+P) = 488+28P$
$\therefore 561.6+23.4P = 488+28P$
$\therefore 561.6-488 = 28P - 23.4P$
$73.6 = 4.6P$
$\therefore P = 73.6/4.6 = 16$
Hence the value of P is 16.

23. The following distribution shows the daily pocket allowance for children of a locality. The mean pocket allowance is Rs. 18. Find the value of f.

Daily pocket allowance in Rs.	**11-13**	**13-15**	**15-17**	**17-19**	**19-21**	**21-23**	**23-25**
No. of children	**3**	**6**	**9**	**13**	**f**	**5**	**4**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

Daily pocket	No. of children f_i	Class mark x_i	$f_i x_i$

allowance in Rs.			
11-13	3	12	36
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96
Total	$\Sigma f_i = 40+f$		$\Sigma f_i x_i = 704+20f$

Given mean = 18
$\therefore$ Mean = $\Sigma f_i x_i / \Sigma f_i$
$\therefore 18 = (704+20f)/(40+f)$
$\therefore 18\times(40+f) = 704+20f$
$\therefore 720 + 18f = 704+20f$
$720-704 = 20f-18f$
$16 = 2f$
$f = 16/2 = 8$
Hence the value of f is 8.

24. The mean of the following distribution is 50 and the sum of all the frequencies is 120. Find the values of p and q.

Class intervals	**0-20**	**20-40**	**40-60**	**60-80**	**80-100**
Frequency	**17**	**P**	**32**	**q**	**19**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

Class intervals	Frequency f_i	Class mark x_i	$f_i x_i$
0-20	17	10	170
20-40	P	30	30P
40-60	32	50	1600
60-80	q	70	70q
80-100	19	90	1710
Total	$\Sigma f_i = 68+P+q$		$\Sigma f_i x_i = 3480+30P+70q$

Given sum of all frequencies, $\Sigma f_i = 120$
$\therefore 68+P+q = 120$
$\therefore P+q = 120-68 = 52$
$\therefore P+q = 52$
$\therefore P = 52-q$ …(i)

Given mean = 50
∴ Mean = $\Sigma f_i x_i / \Sigma f_i$
∴ 50 = (3480+30P+70q)/120
∴ 50×120 = 3480+30P+70q
∴ 6000 = 3480+30P+70q
∴ 6000- 3480 = 30P+70q
∴ 2520 = 30P+70q
∴ 252 = 3P+7q …(ii)

Substitute (i) in (ii)
252 = 3(52-q)+7q
252 = 156-3q+7q
252-156 = 4q
4q = 96
q = 96/4 = 24
∴ P = 52-24 = 28
Hence the value of P and q is 28 and 24 respectively.

25.The mean of the following frequency distribution is 57.6 and the sum of all the frequencies is 50. Find the values of p and q.

Class intervals	**0-20**	**20-40**	**40-60**	**60-80**	**80-100**	**100-120**
Frequency	**7**	**P**	**12**	**q**	**8**	**5**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

Class intervals	Frequency f_i	Class mark x_i	$f_i x_i$
0-20	7	10	70
20-40	P	30	30P
40-60	12	50	600
60-80	q	70	70q
80-100	8	90	720
100-120	5	110	550
Total	Σf_i = 32+P+q		$\Sigma f_i x_i$ = 1940+30P+70q

Given sum of all frequencies, Σf_i = 50
∴ 32+P+q = 50
∴ P+q = 50-32= 18
∴ P+q = 18
∴ P = 18-q …(i)

Given mean = 57.6
∴ Mean = $\Sigma f_i x_i / \Sigma f_i$

$\therefore 57.6 = (1940+30P+70q)/50$
$\therefore 57.6 \times 50 = 1940+30P+70q$
$\therefore 2880 = 1940+30P+70q$
$\therefore 2880 - 1940 = 30P+70q$
$\therefore 940 = 30P+70q$
$\therefore 94 = 3P+7q$ …(ii)

Substitute (i) in (ii)
$94 = 3(18-q)+7q$
$94 = 54-3q+7q$
$94-54 = 4q$
$40 = 4q$
$q = 40/4 = 10$
$\therefore P = 18-10 = 8$

Hence the value of P and q is 8 and 10 respectively.

26. The following table gives the life time in days of 100 electricity tubes of a certain make :

Life time in days	**No. of tubes**
Less than 50	**8**
Less than 100	**23**
Less than 150	**55**
Less than 200	**81**
Less than 250	**93**
Less than 300	**100**

Find the mean life time of electricity tubes.

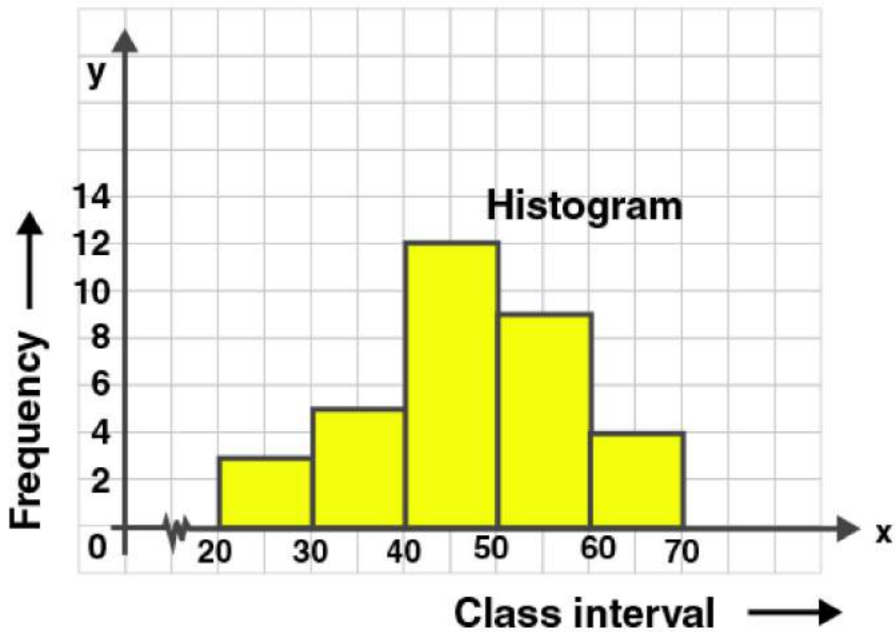
Solution:
Class mark (x_i) = (upper limit + lower limit)/2
Let assumed mean (A) = 175
Class size (h) = 50

Class Interval	No. of tubes (cf)	Class mark (x_i)	$d_i = x_i - A$	$u_i = d_i/h$	Frequency (f_i)	f_iu_i
0-50	8	25	-150	-3	8	-24
50-100	23	75	-100	-2	15	-30
100-150	55	125	-50	-1	32	-32
150-200	81	175	0	0	26	0
200-250	93	225	50	1	12	12
250-300	100	275	100	2	7	14
Total					$\sum f_i = 100$	$\sum f_iu_i = -60$

By step deviation method, Mean = $\bar{x} = A + h\sum f_iu_i / \sum f_i$
$= 175+50(-60/100)$

= 175+50×-0.60
= 175-30
= 145
Hence the mean of the electricity tubes is 145.

27. Using the information given in the adjoining histogram, calculate the mean correct to one decimal place.



Solution:
Class mark, x_i = (upper class limit + lower class limit)/2
Assumed mean, A = 45

Class interval	frequency (f_i)	Class mark (x_i)	$d_i = x_i - A$	$f_i d_i$
20-30	3	25	-20	-60
30-40	5	35	-10	-50
40-50	12	45	0	0
50-60	9	55	10	90
60-70	4	65	20	80
	$\sum f_i = 33$			$\sum f_i d_i = 60$

By short cut method, Mean = $\bar{x} = A+\sum f_i d_i / \sum f_i$
= 45+60/33

= 45+1.81
= 46.81
= 46.8 [corrected to one decimal place]
Hence the mean is Rs.46.8.

Exercise 21.2

1. A student scored the following marks in 11 questions of a question paper : 3, 4, 7, 2, 5, 6, 1, 8, 2, 5, 7 Find the median marks.

Solution:
Arranging the data in the ascending order
1,2,2,3,4,5,5,6,7,7,8
Here number of terms, n = 11
Here n is odd.
So median = $[(n+1)/2]^{th}$ observation
= (11+1)/2
= 12/2
= 6^{th} observation
Here 6^{th} observation is 5.
Hence the median is 5.

2. (a) Find the median of the following set of numbers : 9, 0, 2, 8, 5, 3, 5, 4, 1, 5, 2, 7 (1990)
(b)For the following set of numbers, find the median: 10, 75, 3, 81, 17, 27, 4, 48, 12, 47, 9, 15.

Solution:
(a) Arranging the numbers in ascending order :
0, 1, 2, 2, 3, 4, 5, 5, 5, 7, 8, 9
Here, n = 12 which is even
Median = ½ ($n/2^{th}$ term + $((n/2)+1)^{th}$ term)
= ½ ($12/2^{th}$ term + $((12/2)+1)^{th}$ term)
= ½ (6^{th} term + $(6+1)^{th}$ term)
= ½ (6^{th} term + 7^{th} term)
= ½ (4+5)
= 9/2
= 4.5
Hence the median is 4.5.

(b) Arranging the numbers in ascending order :
3, 4, 9, 10, 12, 15, 17, 27, 47, 48, 75, 81
Here, n = 12 which is even
Median = ½ ($n/2^{th}$ term + $((n/2)+1)^{th}$ term)
= ½ ($12/2^{th}$ term + $((12/2)+1)^{th}$ term)
= ½ (6^{th} term + $(6+1)^{th}$ term)
= ½ (6^{th} term + 7^{th} term)
= ½ (15+17)
= ½ ×32
= 16
Hence the median is 16.

3. Calculate the mean and the median of the numbers : 2, 1, 0, 3, 1, 2, 3, 4, 3, 5

Solution:

Arranging the numbers in ascending order :
0, 1, 1, 2, 2, 3, 3, 3, 4, 5
Here, n = 10 which is even
Median = ½ (n/2 th term + $((n/2)+1)^{th}$ term)
= ½ (10/2 th term + $((10/2)+1)^{th}$ term)
= ½ (5 th term + $(5+1)^{th}$ term)
= ½ (5 th term + 6^{th} term)
= ½ (2+3)
= ½ ×5
= 2.5
Hence the median is 2.5.

Mean = sum of the observations/ number of observations
= $\Sigma x_i/n$
= (0+1+1+2+2+3+3+3+4+5)/10
= 24/10
= 2.4
Hence the mean is 2.4.

4.The median of the observations 11, 12, 14, (x – 2), (x + 4), (x + 9), 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean.

Solution:
Observation are as follows :
11, 12, 14, (x-2), (x+4), (x+9), 32, 38, 47
n = 9
Here n is odd. So median = $((n+1)/2)^{th}$ term
= $(9+1)/2\ )^{th}$ term
= 5^{th} term
= x+4
Given median = 24
∴ x+4 = 24
∴ x = 24 -4 = 20

Sum of observations = 11+12+14+(x-2)+(x+4)+(x+9)+32+38+47
= 165+3x
Substitute x = 20
Sum of observations = 165+3×20
= 165+60
= 225
Mean = Sum of observations /number of observations
= 225/9 = 25
Hence the value of x is 20 and mean is 25.

5.The mean of the numbers 1, 7, 5, 3, 4, 4, is m. The numbers 3, 2, 4, 2, 3, 3, p have mean m-1 and median q. Find
(i) p
(ii) q
(iii) the mean of p and q.

Solution:
(i) Mean of 1, 7, 5, 3, 4, 4 is m.
Here n = 6
∴Mean, m = (1+7+5+3+4+4)/6
∴m = 24/6
∴m = 4

Given the numbers 3, 2, 4, 2, 3, 3, p have mean m-1.
So m-1 = (3+2+4+2+3+3+p)/7
∴4-1 = (17+p)/7
∴3 = (17+p)/7
∴3×7 = 17+p
∴21 = 17+p
∴p = 21-17
∴p = 4
Hence the value of p is 4.

(ii) Given the numbers have median q.
Arranging them in ascending order
2, 2, 3, 3, 3, 4, 4
Here n = 7 which is odd
So median = $((n+1)/2)^{th}$ term
q = $((7+1)/2)^{th}$ term
q = $(8/2)^{th}$ term
q = 4^{th} term
q = 3
So value of q is 3.

(iii)mean of p and q = (p+q)/2
= (4+3)/2
= 7/2
= 3.5
Hence the mean of p and q is 3.5.

6. Find the median for the following distribution:

Wages per day in Rs.	**38**	**45**	**48**	**55**	**62**	**65**
No. of workers	**14**	**8**	**7**	**10**	**6**	**2**

Solution:
We write the distribution in cumulative frequency table.

Wages per day in Rs.	No. of workers (f)	Cumulative frequency
38	14	14
45	8	22
48	7	29

55	10	39
62	6	45
65	2	47

Here total number of observations, n = 47 which is odd.
So median =((n+1)/2)th term
= ((47+1)/2)th term
= (48/2)th term
= 24th term
= 48 [Since 23rd to 29th observation is 48]
Hence the median is 48.

7. Find the median for the following distribution.

Marks	**35**	**45**	**50**	**64**	**70**	**72**
No. of students	**3**	**5**	**8**	**10**	**5**	**5**

Solution:
We write the distribution in cumulative frequency table.

Marks	No. of students (f)	Cumulative frequency
35	3	3
45	5	8
50	8	16
64	10	26
70	5	31
72	5	36

Here total number of observations, n = 36 which is even.
Median = ½ (n/2 th term + ((n/2)+1)th term)
= ½ (36/2 th term + ((36/2)+1)th term)
= ½ (18 th term + (18+1)th term)
= ½ (18 th term + 19th term)
= ½ (64+64) [Since 17th to 26th observation is 64]
= ½ ×128
= 64
Hence the median is 64.

8.Marks obtained by 70 students are given below :

Marks	**20**	**70**	**50**	**60**	**75**	**90**	**40**
No. of students	**8**	**12**	**18**	**6**	**9**	**5**	**12**

Calculate the median marks.

Solution:
We write the marks in ascending order in cumulative frequency table.

Marks	No. of students (f)	Cumulative frequency
20	8	8
40	12	20
50	18	38
60	6	44
70	12	56
75	9	65
90	5	70

Here total number of observations, n = 70 which is even.
Median = ½ ($n/2^{th}$ term + $((n/2)+1)^{th}$ term)
= ½ ($70/2^{th}$ term + $((70/2)+1)^{th}$ term)
= ½ (35^{th} term + $(35+1)^{th}$ term)
= ½ (35^{th} term + 36^{th} term)
= ½ (50+50) [Since all observations from 21^{st} to 38^{th} are 50]
= ½ ×100
= 50
Hence the median is 50.

9. Calculate the mean and the median for the following distribution :

Number	**5**	**10**	**15**	**20**	**25**	**30**	**35**
Frequency	**1**	**2**	**5**	**6**	**3**	**2**	**1**

Solution:
We write the numbers in cumulative frequency table.

Marks (x)	No. of students (f)	Cumulative frequency	fx
5	1	1	5
10	2	3	20
15	5	8	75
20	6	14	120
25	3	17	75
30	2	19	60
35	1	20	35
Total	Σf = 20		Σfx = 390

Mean = Σfx/Σf

= 390/20
= 19.5
Hence the mean is 19.5.

Here number of observations, n = 20 which is even.
So median = = ½ (n/2 th term + ((n/2)+1)th term)
= ½ (20/2 th term + ((20/2)+1)th term)
= ½ (10 th term + (10+1)th term)
= ½ (10 th term + 11th term)
= ½ (20+20) [Since all observations from 9th to 14th are 20]
= ½ ×140
= 20
Hence the median is 20.

10. The daily wages in (rupees of) 19 workers are
41, 21, 38, 27, 31, 45, 23, 26, 29, 30, 28, 25, 35, 42, 47, 53, 29, 31, 35.
find :
(i) the median
(ii) lower quartile
(iii) upper quartile
(iv) inter quartile range

Solution:
Arranging the observations in ascending order
21, 23, 25, 26, 27, 28, 29, 29, 30, 31, 31, 35, 35, 38, 41, 42, 45, 47, 53
Here n = 19 which is odd.
(i)Median = ((n+1)/2)th term
= (19+1)/2
= 20/2
= 10th term
= 31
Hence the median is 31.

(ii) Lower quartile, Q_1 = ((n+1)/4) th term
= (19+1)/4
= 20/4
= 5 th term
= 27
Hence the lower quartile is 27.

(iii)Upper quartile, Q_3 = (3(n+1)/4) th term
= (3×(19+1)/4) th term
= (3×(20/4)) th term
= (3×5) th term
= 15 th term
= 41
Hence the upper quartile is 41.

(iv)Interquartile range = Q_3-Q_1
= 41-27

= 14
Hence the Interquartile range is 14.

11.From the following frequency distribution, find :
(i) the median
(ii) lower quartile
(iii) upper quartile
(iv) inter quartile range

Variate	**15**	**18**	**20**	**22**	**25**	**27**	**30**
Frequency	**4**	**6**	**8**	**9**	**7**	**8**	**6**

Solution:
We write the variates in cumulative frequency table.

Variate	Frequency (f)	Cumulative frequency
15	4	4
18	6	10
20	8	18
22	9	27
25	7	34
27	8	42
30	6	48

(i) Here number of observations, n = 48 which is even.
So median $= \frac{1}{2} (n/2^{th} \text{ term} + ((n/2)+1)^{th} \text{ term})$
$= \frac{1}{2} (48/2^{th} \text{ term} + ((48/2)+1)^{th} \text{ term})$
$= \frac{1}{2} (24^{th} \text{ term} + (24+1)^{th} \text{ term})$
$= \frac{1}{2} (24^{th} \text{ term} + 25^{th} \text{ term})$
$= \frac{1}{2} (22+22)$ [Since all observations from 19^{th} to 27^{th} are 22]
$= \frac{1}{2} \times 44$
= 22
Hence the median is 22.

(ii) Lower quartile, $Q_1 = (n/4)^{th}$ term
= (48)/4
$= 12^{th}$ term
= 20
Hence the lower quartile is 20.

(iii)Upper quartile, $Q_3 = (3n/4)^{th}$ term
$= (3\times48/4)^{th}$ term
$= (3\times12)^{th}$ term
$= 36^{th}$ term
= 27
Hence the upper quartile is 27.

(iv)Interquartile range = Q_3-Q_1
= 27-20
= 7
Hence the Interquartile range is 7.

12. For the following frequency distribution, find :
(i) the median
(ii) lower quartile
(iii) upper quartile

Variate	**25**	**31**	**34**	**40**	**45**	**48**	**50**	**60**
Frequency	**3**	**8**	**10**	**15**	**10**	**9**	**6**	**2**

Solution:
We write the variates in cumulative frequency table.

Variate	Frequency (f)	Cumulative frequency
25	3	3
31	8	11
34	10	21
40	15	36
45	10	46
48	9	55
50	6	61
60	2	63

(i) Here number of observations, n = 63 which is odd.
Median = $((n+1)/2)^{th}$ term
= (63+1)/2
= 64/2
= 32^{th} term
= 40
Hence the median is 40.

(ii) Lower quartile, Q_1 = $((n+1)/4)^{th}$ term
= (63+1)/4
= 64/4
= 16^{th} term
= 34
Hence the lower quartile is 34.

(iii)Upper quartile, Q_3 = $(3(n+1)/4)^{th}$ term
= $(3\times(63+1)/4)^{th}$ term
= $(3\times(64/4))^{th}$ term
= $(3\times16)^{th}$ term

= 48 th term
= 48
Hence the upper quartile is 48.

Exercise 21.3

1.Find the mode of the following sets of numbers ;
(i) 3, 2, 0, 1, 2, 3, 5, 3
(ii) 5, 7, 6, 8, 9, 0, 6, 8, 1, 8
(iii) 9, 0, 2, 8, 5, 3, 5, 4, 1, 5, 2, 7

Solution:
Mode is the number which appears most often in a set of numbers.
(i)Given set is 3, 2, 0, 1, 2, 3, 5, 3.
In this set, 3 occurs maximum number of times.
Hence the mode is 3.

(ii) Given set is 5, 7, 6, 8, 9, 0, 6, 8, 1, 8.
In this set, 8 occurs maximum number of times.
Hence the mode is 8.

(iii) Given set is 9, 0, 2, 8, 5, 3, 5, 4, 1, 5, 2, 7.
In this set, 5 occurs maximum number of times.
Hence the mode is 5.

2. Calculate the mean, the median and the mode of the numbers : 3, 2, 6, 3, 3, 1, 1, 2

Solution:
We arrange given data in ascending order 1, 1, 2, 2, 3, 3, 3, 6
Mean = $\Sigma x_i/n$
= (1+1+2+2+3+3+3+6)/8
= 21/8
= 2.625
Hence the mean is 2.625.

Here number of observations, n = 8 which is even.
So median = ½ ($n/2^{th}$ term + $((n/2)+1)^{th}$ term)
= ½ ($8/2^{th}$ term + $((8/2)+1)^{th}$ term)
= ½ (4^{th} term + $(4+1)^{th}$ term)
= ½ (4^{th} term + 5^{th} term)
= ½ (2+3)
= ½ ×5
= 2.5
Hence the median is 2.5.

In the given set, 3 occurs maximum number of times.
Hence the mode is 3.

3. Find the mean, median and mode of the following distribution : 8, 10, 7, 6, 10, 11, 6, 13, 10

Solution:
We arrange given data in ascending order 6, 6, 7, 8, 10, 10, 10, 11, 13
Mean = $\Sigma x_i/n$
= (6+6+7+8+10+10+10+11+13)/9

= 81/9
= 9
Hence the mean is 9.

Here number of observations, n = 9 which is odd.
Median = $((n+1)/2)^{th}$ term
= (9+1)/2
= 10/2
= 5^{th} term
= 10
Hence the median is 10.

In the given set, 10 occurs maximum number of times.
Hence the mode is 10.

4. Calculate the mean, the median and the mode of the following numbers : 3, 1, 5, 6, 3, 4, 5, 3, 7, 2

Solution:
We arrange given data in ascending order 1, 2, 3, 3, 3, 4, 5, 5, 6, 7
Mean = $\Sigma x_i/n$
= (1+2+3+3+3+4+5+5+6+7)/10
= 39/10
= 3.9

Here number of observations, n = 10 which is even.
So median = ½ ($n/2^{\ th}$ term + $((n/2)+1)^{th}$ term)
= ½ ($10/2^{\ th}$ term + $((10/2)+1)^{th}$ term)
= ½ ($5^{\ th}$ term + $(5+1)^{th}$ term)
= ½ ($5^{\ th}$ term + 6^{th} term)
= ½ (3+4)
= ½ ×7
= 3.5
Hence the median is 3.5.

In the given set, 3 occurs maximum number of times.
Hence the mode is 3.

5. The marks of 10 students of a class in an examination arranged in ascending order are as follows: 13, 35, 43, 46, x, x +4, 55, 61,71, 80
If the median marks is 48, find the value of x. Hence, find the mode of the given data. (2017)

Solution:
Given data in ascending order: 13, 35, 43, 46, x, x +4, 55, 61,71, 80
Given median = 48
Number of observations, n = 10 which is even.
∴ median = ½ ($n/2^{\ th}$ term + $((n/2)+1)^{th}$ term)
48 = ½ ($10/2^{\ th}$ term + $((10/2)+1)^{th}$ term)
48 = ½ ($5^{\ th}$ term + $(5+1)^{th}$ term)
48 = ½ ($5^{\ th}$ term + 6^{th} term)
48 = ½ (x+x+4)

$48 = ½ \times (2x+4)$
$48 = x+2$
$\therefore x = 48-2 = 46$
$\therefore x+4 = 46+4 = 50$
So the distribution becomes
13, 35, 43, 46, 46, 50, 55, 61,71, 80
Here 46 occurs maximum number of times.
Hence the mode is 46.

6.A boy scored the following marks in various class tests during a term each test being marked out of 20:
15, 17, 16, 7, 10, 12, 14, 16, 19, 12, 16
(i) What are his modal marks ?
(ii) What are his median marks ?
(iii) What are his mean marks ?

Solution:
(i)We arrange given marks in ascending order
7, 10, 12, 12, 14, 15, 16, 16, 16, 17, 19
16 appears maximum number of times.
Hence his modal mark is 16.

(ii)Here number of observations, n = 11 which is odd.
So Median = $((n+1)/2)^{th}$ term
= (11+1)/2
= 12/2
= 6^{th} term
= 15
Hence the median is 15.

(iii) Mean = $\Sigma x_i/n$
= 7+ 10+12+12+14+ 15+16+16+16+ 17+ 19
= 154/11
= 14
Hence the mean is 14.

7. Find the mean, median and mode of the following marks obtained by 16 students in a class test marked out of 10 marks : 0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 8

Solution:
Given data is 0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 8
Number of observations, n = 16
Mean = $\Sigma x_i/n$
= (0+0+2+2+3+3+3+4+5+5+5+5+6+6+7+8)/16
= 64/16
= 4
Hence the mean is 4.

Here n = 16 which is even.
So median = ½ ($n/2^{th}$ term + $((n/2)+1)^{th}$ term)
= ½ ($16/2^{th}$ term + $((16/2)+1)^{th}$ term)

= ½ (8 th term + $(8+1)^{th}$ term)
= ½ (8 th term + 9^{th} term)
= ½ (4+5)
= 9/2
= 4.5
Hence the median is 4.5.

Here 5 appears maximum number of times.
Hence mode is 5.

8. Find the mode and median of the following frequency distribution :

x	**10**	**11**	**12**	**13**	**14**	**15**
f	**1**	**4**	**7**	**5**	**9**	**3**

Solution:
We write the data in cumulative frequency table.

x	Frequency (f)	Cumulative frequency
10	1	1
11	4	5
12	7	12
13	5	17
14	9	26
15	3	29

Here number of observations, n = 29 which is odd.
Median = $((n+1)/2)^{th}$ term
= (29+1)/2
= 30/2
= 15^{th} term
= 13
Hence the median is 13.
Here the frequency corresponding to 14 is maximum.
Hence the mode is 14.

9. The marks obtained by 30 students in a class assessment of 5 marks is given below:

Marks	**0**	**1**	**2**	**3**	**4**	**5**
No. of students	**1**	**3**	**6**	**10**	**5**	**5**

Calculate the mean, median and mode of the above distribution.

Solution:
We write the data in cumulative frequency table.

Marks x	Frequency (f)	Cumulative frequency	fx
0	1	1	0
1	3	4	3
2	6	10	12

3	10	20	30
4	5	25	20
5	5	30	25
Total	Σf = 30		Σfx = 90

Mean = Σfx/Σf
= 90/30
= 3
Hence the mean is 3.
Here number of observations, n = 30 which is even.
So median = ½ (n/2 th term + ((n/2)+1)th term)
= ½ (30/2 th term + ((30/2)+1)th term)
= ½ (15 th term + (15+1)th term)
= ½ (15 th term + 16th term)
= ½ (3+3)
= 6/2
= 3
Hence the median is 3.
Here the mark 3 occurs maximum number of times.
Hence the mode is 3.

10. The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.

Marks obtained	**5**	**6**	**7**	**8**	**9**	**10**
No. of students	**3**	**9**	**6**	**4**	**2**	**1**

Solution:
We write the marks in cumulative frequency table.

Marks x	Frequency (f)	fx	Cumulative frequency
5	3	15	3
6	9	54	12
7	6	42	18
8	4	32	22
9	2	18	24
10	1	10	25
Total	Σf = 25	Σfx = 171	

Mean = Σfx/Σf
= 171/25
= 6.84
Hence the mean is 6.84.

Here number of observation, n = 25 which is odd.
Median = ((n+1)/2)th term

= (25+1)/2
= 26/2
= 13^{th} term
= 7
Hence the median is 7.

Here the frequency corresponding to 6 is maximum.
Hence the mode is 6.

11. At a shooting competition, the scores of a competitor were as given below :

Score	**0**	**1**	**2**	**3**	**4**	**5**
No. of shots	**0**	**3**	**6**	**4**	**7**	**5**

(i) What was his modal score ?
(ii) What was his median score ?
(iii) What was his total score ?
(iv) What was his mean ?

Solution:
We write the marks in cumulative frequency table.

Score x	No. of shots (f)	fx	Cumulative frequency
0	0	0	0
1	3	3	3
2	6	12	9
3	4	12	13
4	7	28	20
5	5	25	25
Total	$\Sigma f = 25$	$\Sigma fx = 80$	

(i) Here the frequency corresponding to 4 is maximum. 4 occurs 7 times.
Hence his modal score is 4.

(ii)Here number of observation, n = 25 which is odd.
Median = $((n+1)/2)^{th}$ term
= (25+1)/2
= 26/2
= 13^{th} term
= 3
Hence his median score is 3.

(iii)Total score = Σfx = 80
Hence his total score is 80.

(iv)Mean = $\Sigma fx/\Sigma f$
= 80/25
= 3.2

Hence his mean score is 3.2.

12. (i) Using step-deviation method, calculate the mean marks of the following distribution.
(ii) State the modal class.

Class interval	**50-55**	**55-60**	**60-65**	**65-70**	**70-75**	**75-80**	**80-85**	**85-90**
Frequency	**5**	**20**	**10**	**10**	**9**	**6**	**12**	**8**

Solution:
(i) Class mark (x_i) = (upper limit + lower limit)/2
Let assumed mean (A) = 67.5
Class size (h) = 5

Class Interval	Frequency (f_i)	Class mark (x_i)	$d_i = x_i$ - A	$u_i = d_i/h$	f_iu_i
50-55	5	52.5	-15	-3	-15
55-60	20	57.5	-10	-2	-40
60-65	10	62.5	5	-1	-10
65-70	10	67.5	10	0	0
70-75	9	72.5	15	1	9
75-80	6	77.5	20	2	12
80-85	12	82.5	25	3	36
85-90	8	87.5	30	4	32
Total	$\sum f_i = 80$				$\sum f_iu_i = 24$

By step deviation method, Mean = $\bar{x} = A+h\sum f_iu_i /\sum f_i$
= 67.5+5(24/80)
= 67.5+5×0.3
= 67.5+1.5
= 69
Hence the mean of the distribution is 69.

(ii) Modal class is the class with highest frequency.
Here the modal class is 55-60.

13. The following table gives the weekly wages (in Rs.) of workers in a factory :

Weekly wages (in Rs)	**50-55**	**55-60**	**60-65**	**65-70**	**70-75**	**75-80**	**80-85**	**85-90**
No. of workers	**5**	**20**	**10**	**10**	**9**	**6**	**12**	**8**

Calculate:
(i) The mean.
(ii) the modal class
(iii) the number of workers getting weekly wages below Rs. 80.
(iv) the number of workers getting Rs. 65 or more but less than Rs. 85 as weekly wages.

Solution:

We write the given data in cumulative frequency table.

Class Interval	Frequency (f_i)	Class mark (x_i)	Cumulative frequency	f_ix_i
50-55	5	52.5	5	262.5
55-60	20	57.5	25	1150
60-65	10	62.5	35	625
65-70	10	67.5	45	675
70-75	9	72.5	54	652.5
75-80	6	77.5	60	465
80-85	12	82.5	72	990
85-90	8	87.5	80	700
Total	$\sum f_i = 80$			$\sum f_ix_i = 5520$

(i) Mean = $\sum f_ix_i / \sum f_i$

= 5520/80

= 69

Hence the mean is 69.

(ii) Modal class is the class with highest frequency.

Here the modal class is 55-60.

(iii) The number of workers getting weekly wages below Rs. 80 is 60.

[Check the cumulative frequency column and class interval column. 60 workers get below Rs. 80]

(iv) The number of workers getting Rs. 65 or more but less than Rs. 85 as weekly wages = 72-35 = 37

[Check the cumulative frequency column and class interval column.]

Exercise 21.4

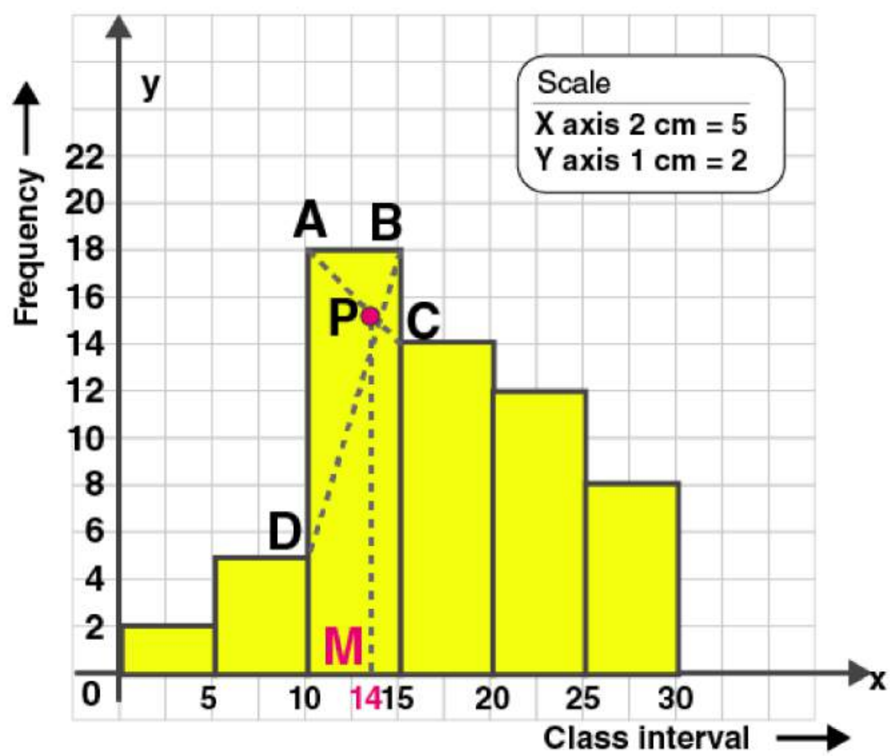
1. Draw a histogram for the following frequency distribution and find the mode from the graph :

Class	**0-5**	**5-10**	**10-15**	**15-20**	**20-25**	**25-30**
Frequency	**2**	**5**	**18**	**14**	**8**	**5**

Solution:
Construct histogram using given data.

Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	2	5	18	14	8	5

Represent class on X-axis and frequency on Y-axis.



In the highest rectangle, draw two straight lines AC and BD.
P is the point of intersection.
Draw a vertical line through P to meet the X-axis at M.
The abscissa of M is 14.
Hence the mode is 14.

2. Find the modal height of the following distribution by drawing a histogram :

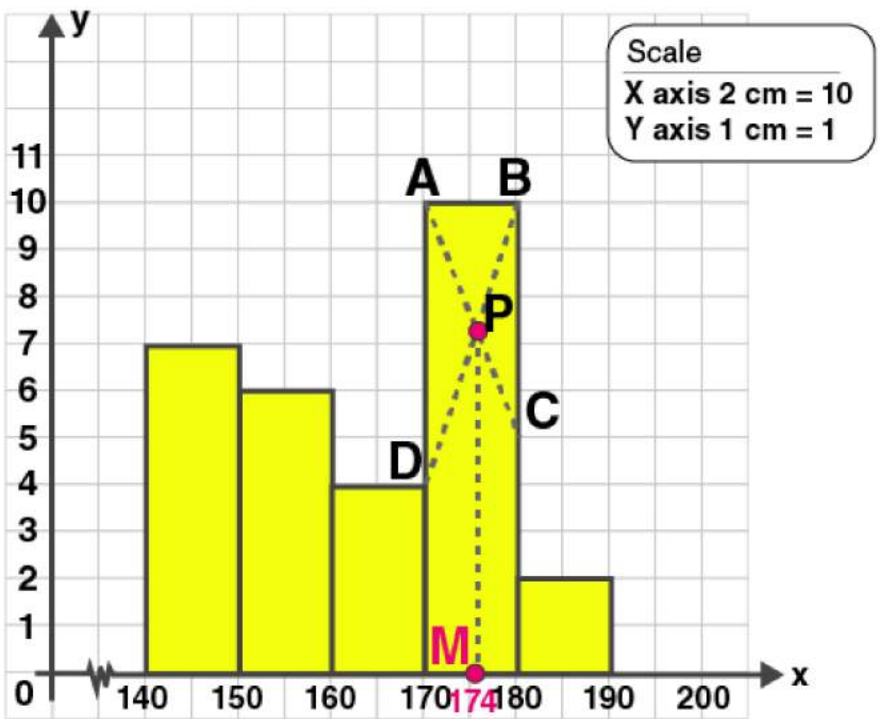
Height (in cm)	**140-150**	**150-160**	**160-170**	**170-180**	**180-190**
No. of students	**7**	**6**	**4**	**10**	**2**

Solution:

Construct histogram using given data.

Height (in cm)	140-150	150-160	160-170	170-180	180-190
No. of students	7	6	4	10	2

Represent height on X-axis and number of students on Y-axis.
Take scale: X axis : 2 cm = 10 (class interval)
Y axis : 1 cm = 1 (frequency)



In the highest rectangle, draw two straight lines AC and BD.
P is the point of intersection.
Draw a vertical line through P to meet the X-axis at M.
The abscissa of M is 174.
Hence the mode is 174.

3. A Mathematics aptitude test of 50 students was recorded as follows :

Marks	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of	**4**	**8**	**14**	**19**	**5**

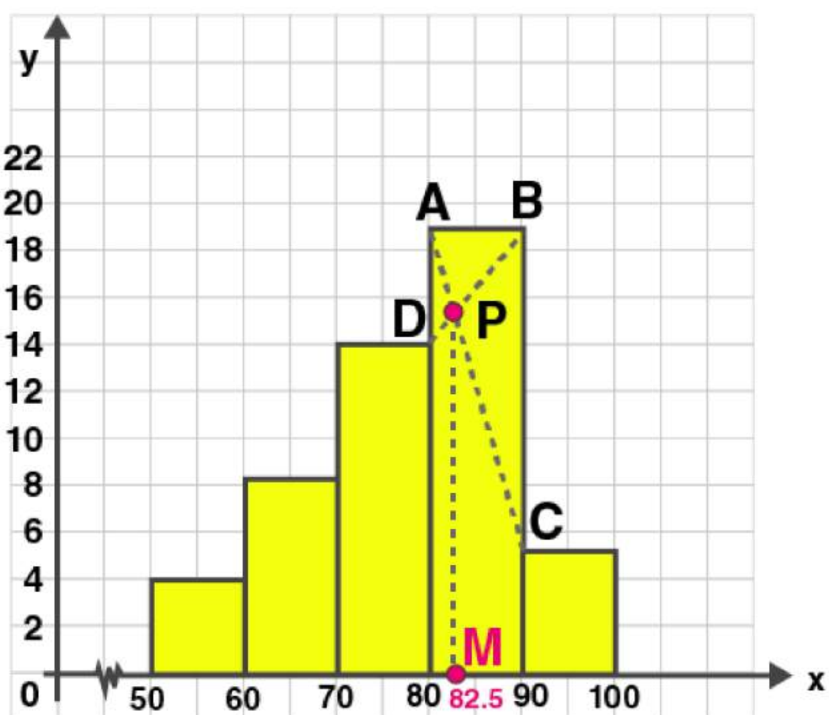
students					

Draw a histogram for the above data using a graph paper and locate the mode. (2011)

Solution:
Construct histogram using given data.

Marks	50-60	60-70	70-80	80-90	90-100
No. of students	4	8	14	19	5

Represent marks on X-axis and number of students on Y-axis.
Take scale: X axis : 2 cm = 10 (class interval)
Y axis : 1 cm = 1 (frequency)



In the highest rectangle, draw two straight lines AC and BD.
P is the point of intersection.
Draw a vertical line through P to meet the X-axis at M.
The abscissa of M is 82.5.
Hence the mode is 82.5.

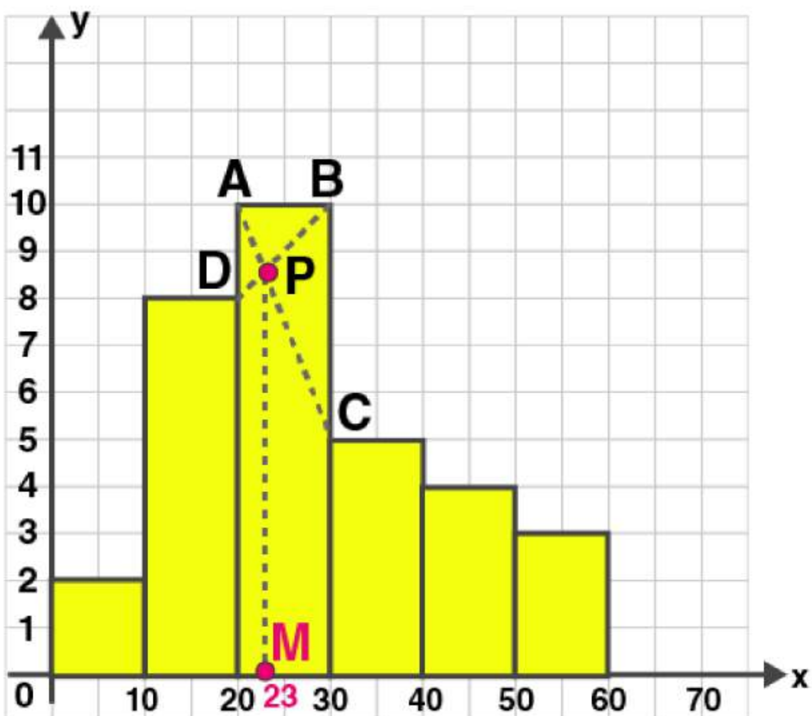
4. Draw a histogram and estimate the mode for the following frequency distribution :

Classes	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
Frequency	**2**	**8**	**10**	**5**	**4**	**3**

Solution:
Construct histogram using given data.

Classes	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	2	8	10	5	4	3

Represent classes on X-axis and frequency on Y-axis.
Take scale: X axis : 2 cm = 10 (class interval)
Y axis : 1 cm = 1 (frequency)



In the highest rectangle, draw two straight lines AC and BD.
P is the point of intersection.
Draw a vertical line through P to meet the X-axis at M.
The abscissa of M is 23.
Hence the mode is 23.

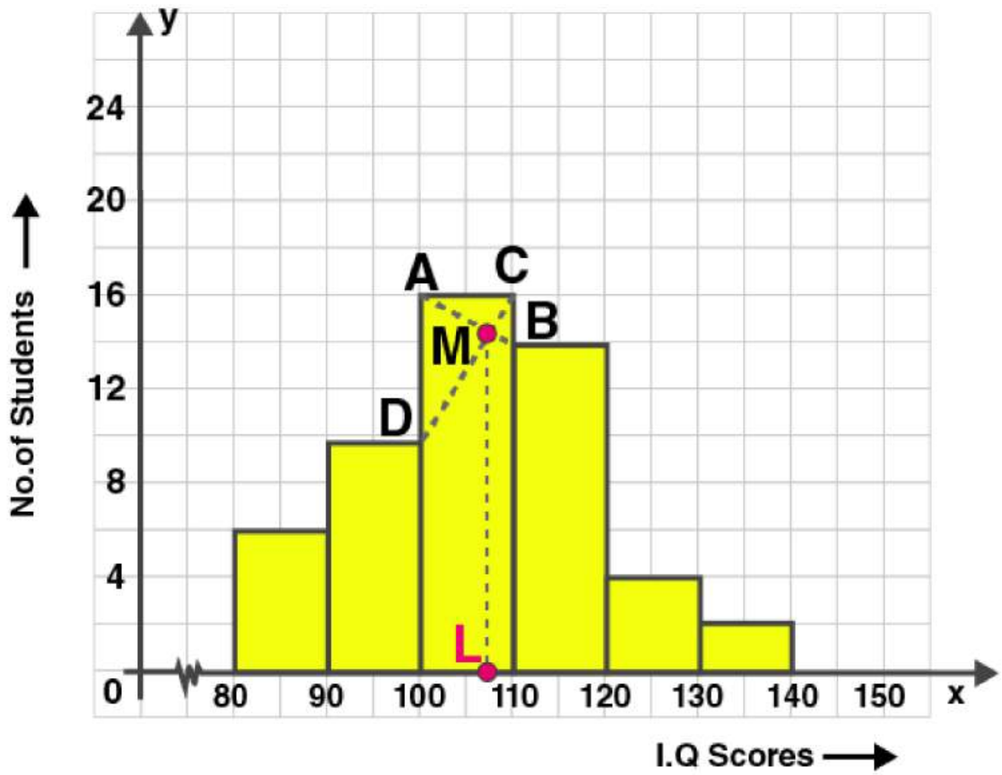
5. IQ of 50 students was recorded as follows.

IQ score	**80-90**	**90-100**	**100-110**	**110-120**	**120-130**	**130-140**
No. of students	**6**	**9**	**16**	**13**	**4**	**2**

Draw a histogram for the above data and estimate the mode.

Solution:
Construct histogram using given data.

Represent classes on X-axis and frequency on Y-axis.
Take scale: X axis : 1 cm = 10 (class interval)
Y axis : 1 cm = 1 (frequency)



In the highest rectangle, draw two straight lines AB and CD.
M is the point of intersection.
Draw a vertical line through M to meet the X-axis at L.
The abscissa of L is 107.
Hence the mode is 107.

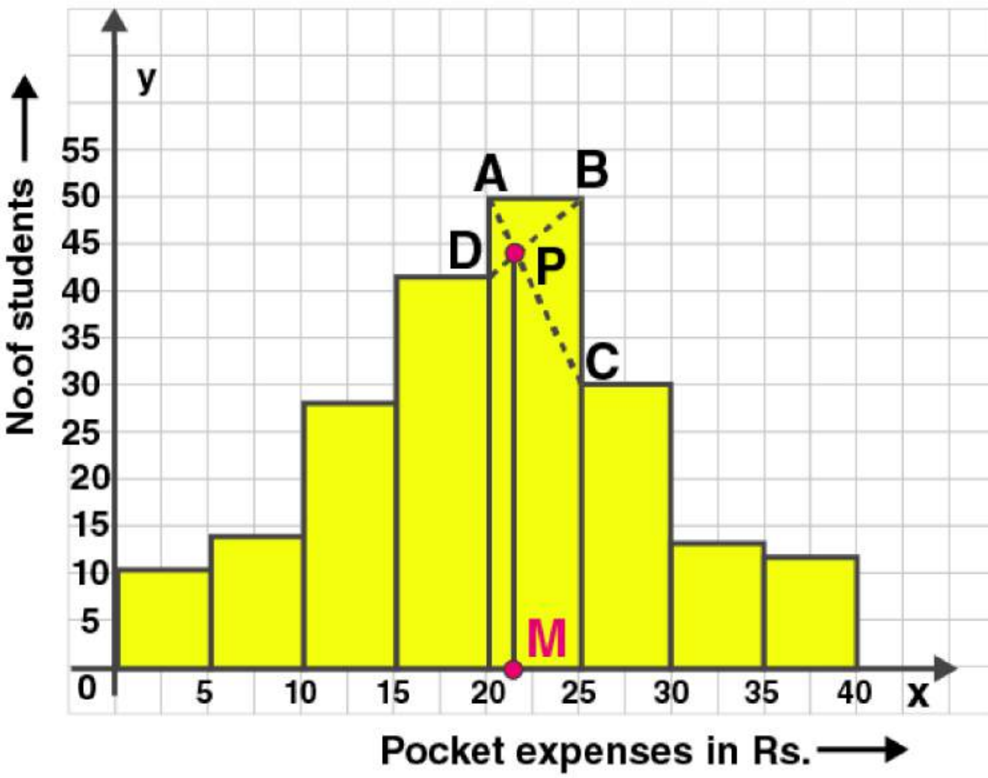
6. Use a graph paper for this question. The daily pocket expenses of 200 students in a school are given below:

Pocket expenses (in Rs)	Number of students (Frequency)
0-5	10
5-10	14
10-15	28
15-20	42
20-25	50
25-30	30
30-35	14

35-40	**12**

Draw a histogram representing the above distribution and estimate the mode from the graph.

Solution:
Construct histogram using given data.
Represent classes on X-axis and frequency on Y-axis.
Take scale: X axis : 2 cm = 5 (class interval)
Y axis : 1 cm = 5 (frequency)



In the highest rectangle, draw two straight lines AC and BD.
P is the point of intersection.
Draw a vertical line through P to meet the X-axis at M.
The abscissa of M is 21.
Hence the mode is 21.

7. Draw a histogram for the following distribution :

Wt. in kg	**40-44**	**45-49**	**50-54**	**55-59**	**60-64**	**65-69**
No. of students	**2**	**8**	**12**	**10**	**6**	**4**

Hence estimate the modal weight.

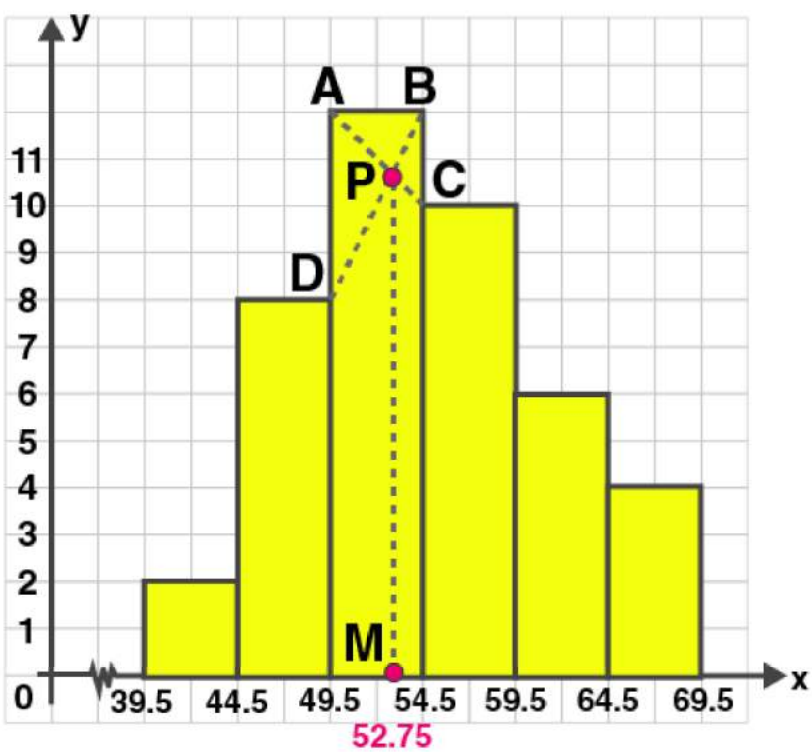
Solution:
The given distribution is not continuous.
Adjustment factor = (45-44)/2 = ½ = 0.5

We subtract 0.5 from lower limit of the class interval and add 0.5 to upper limit.

So the new table in continuous form is given below.

Weight in kg	Number of students (Frequency)
39.5-44.5	2
44.5-49.5	8
49.5-54.5	12
54.5-59.5	10
59.5-64.5	6
64.5-69.5	4

Construct histogram using given data.
Represent weight on X-axis and no. of students on Y-axis.
Take scale: X axis : 2 cm = 5 (class interval)
Y axis : 1 cm = 1 (frequency)



In the highest rectangle, draw two straight lines AC and BD.
P is the point of intersection.
Draw a vertical line through P to meet the X-axis at M.
The abscissa of M is 52.75.
Hence the mode is 52.75.

8. Find the mode of the following distribution by drawing a histogram

Mid value	**12**	**18**	**24**	**30**	**36**	**42**	**48**
Frequency	**20**	**12**	**8**	**24**	**16**	**8**	**12**

Also state the modal class.

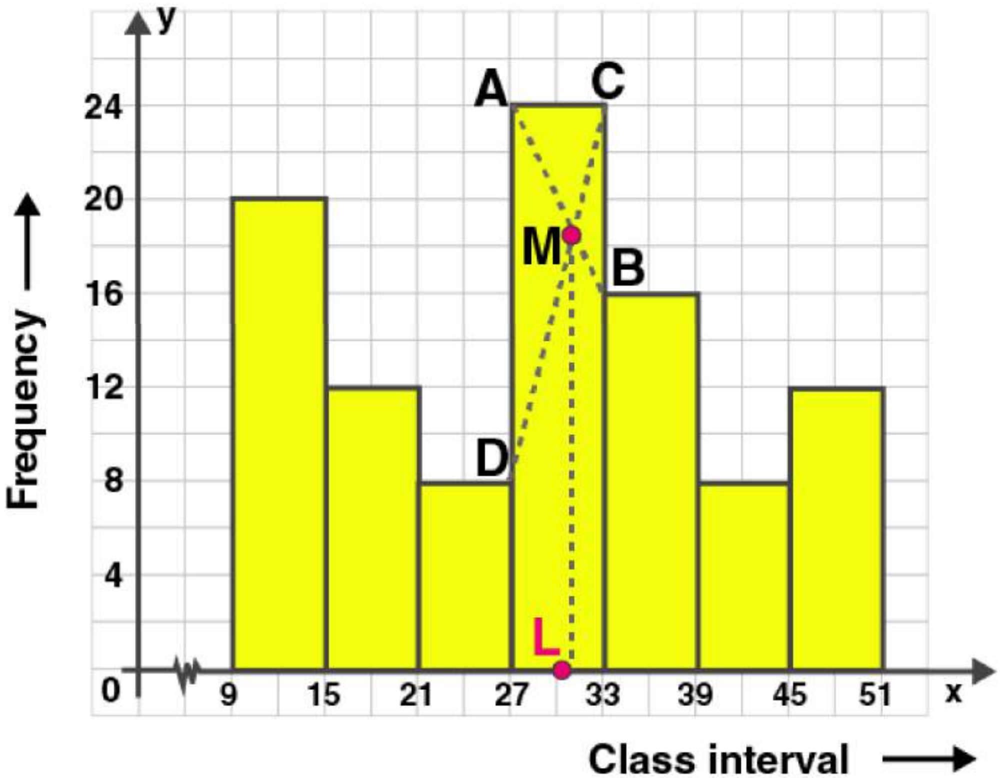
Solution:

Mid value	Frequency
12	20
18	12
24	8
30	24
36	16
42	8
48	12

Here mid value and frequency is given.
We can find the class size, h by subtracting second mid value from first mid value.
$\therefore h = 18-12 = 6$
So to find the lower limit of class interval, we subtract h/2 to the mid value.
To find the upper limit of class interval, we add h/2 to the mid value.
Here $h/2 = 6/2 = 3$
So lower limit = 12-3 = 9
Upper limit = 12+3 = 15
So the class interval is 9-15
Likewise we find the class interval of other values.

Mid value	Class interval	Frequency
12	9-15	20
18	15-21	12
24	21-27	8
30	27-33	24
36	33-39	16
42	39-45	8
48	45-51	12

Construct histogram using given data.
Take scale: X axis : 2 cm = 6 (class interval)
Y axis : 1 cm = 2 (frequency)



In the highest rectangle, draw two straight lines AB and CD.
M is the point of intersection.
Draw a vertical line through M to meet the X-axis at L.
The abscissa of L is 30.5.
Hence the mode is 30.5.

Modal class is the class with highest frequency.
Hence the modal class is 27-33.

Exercise 21.5

1.Draw an ogive for the following frequency distribution:

Height (in cm)	150-160	160-170	170-180	180-190	190-200
No. of students	8	3	4	10	2

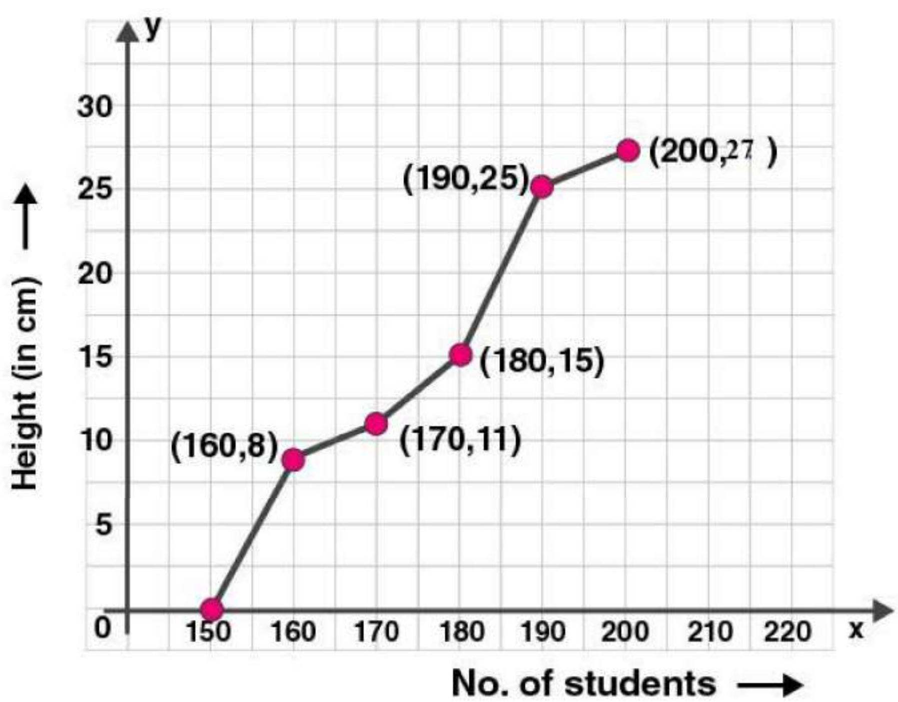
Solution:

We write the given data in cumulative frequency table.

Height in cm	No of students	Cumulative frequency
150-160	8	8
160-170	3	11
170-180	4	15
180-190	10	25
190-200	2	27

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis).

Plot the points (160, 8), (170, 11), (180, 15), (190, 25) and (200, 27) on the graph. Join the points with the free hand. We get an ogive as shown:



2. Draw an ogive for the following data:

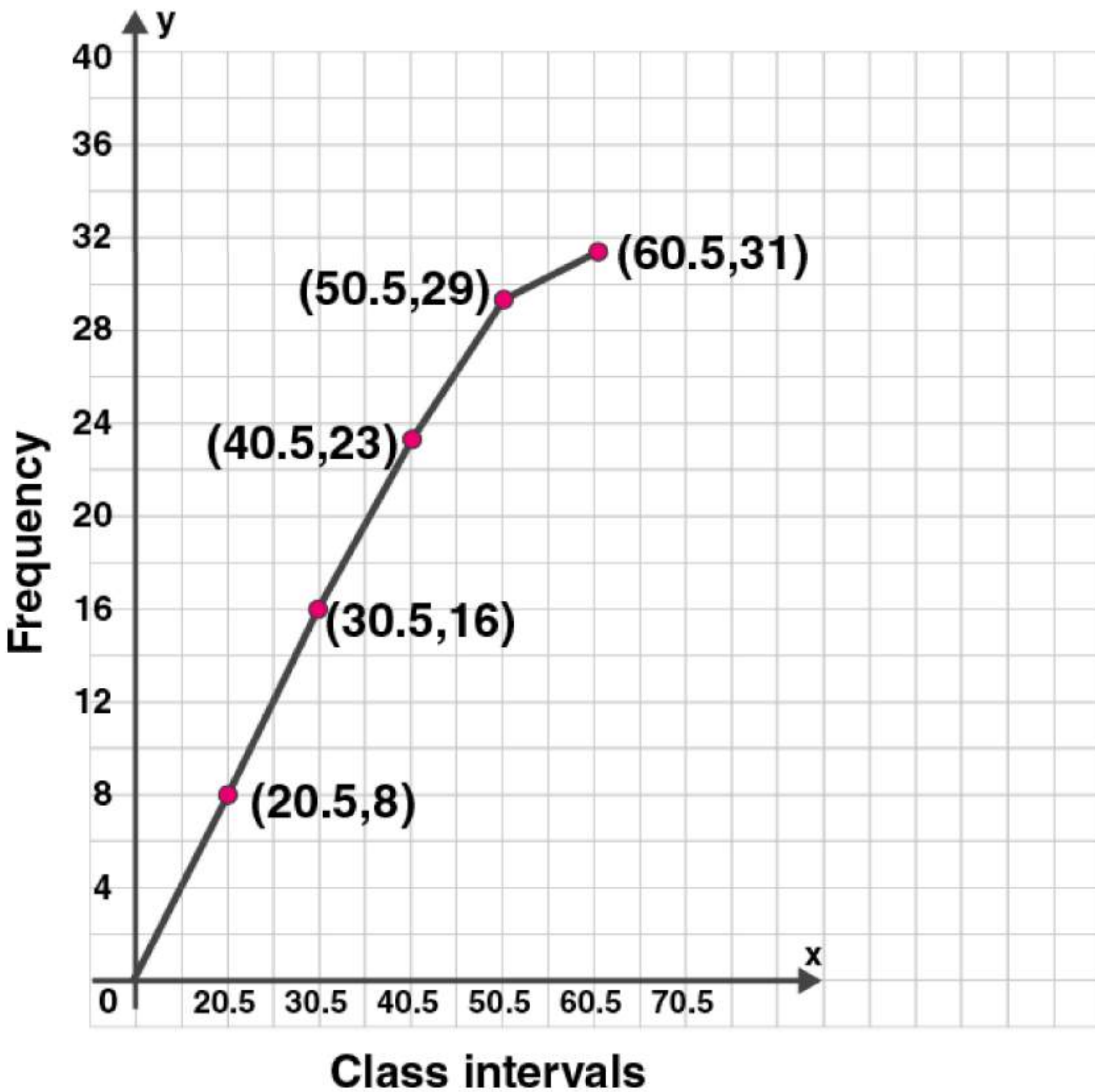
Class intervals	**1-10**	**11-20**	**21-30**	**31-40**	**41-50**	**51-60**
Frequency	**3**	**5**	**8**	**7**	**6**	**2**

Solution:
The given distribution is not continuous.
Adjustment factor = (11-10)/2 = ½ = 0.5
We subtract 0.5 from lower limit of the class interval and add 0.5 to upper limit.
So the new table in continuous form is given below.
We write the given data in cumulative frequency table.

Class intervals	frequency	Cumulative frequency
0.5-10.5	3	3
10.5-20.5	5	8
20.5-30.5	8	16
30.5-40.5	7	23
40.5-50.5	6	29
50.5-60.5	2	31

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis).

Plot the points (10.5, 3), (20.5, 8), (30.5, 16), (40.5, 23) , (50.5, 29) and (60.5, 31) on the graph.
Join the points with the free hand. We get an ogive as shown:



3. Draw a cumulative frequency curve for the following data:

Marks obtained	24-29	29-34	34-39	39-44	44-49	49-54	54-59
No. of students	1	2	5	6	4	3	2

Solution:

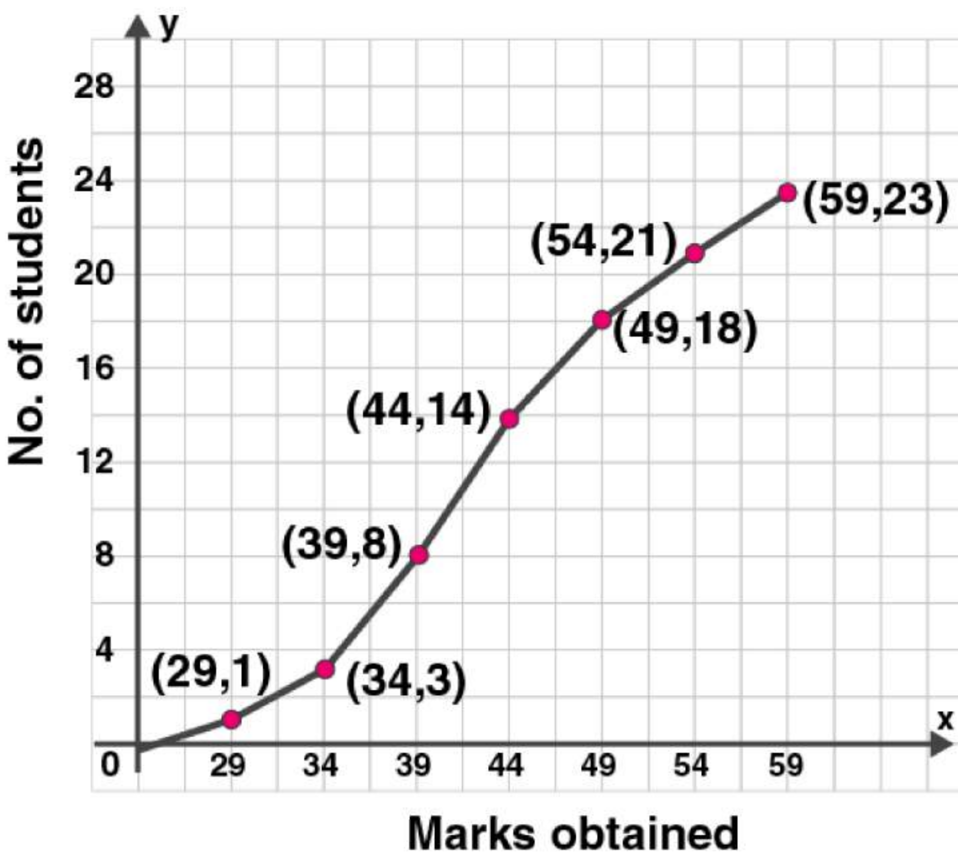
We write the given data in cumulative frequency table.

Marks obtained	No of students	Cumulative frequency
24-29	1	1

29-34	2	3
34-39	5	8
39-44	6	14
44-49	4	18
49-54	3	21
54-59	2	23

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis).

Plot the points (29, 1), (34, 3), (39, 8), (44, 14), (49, 18) , (54, 21) and (59, 23) on the graph.
Join the points with the free hand. We get an ogive as shown:



Exercise 21.6

1. The following table shows the distribution of the heights of a group of a factory workers.

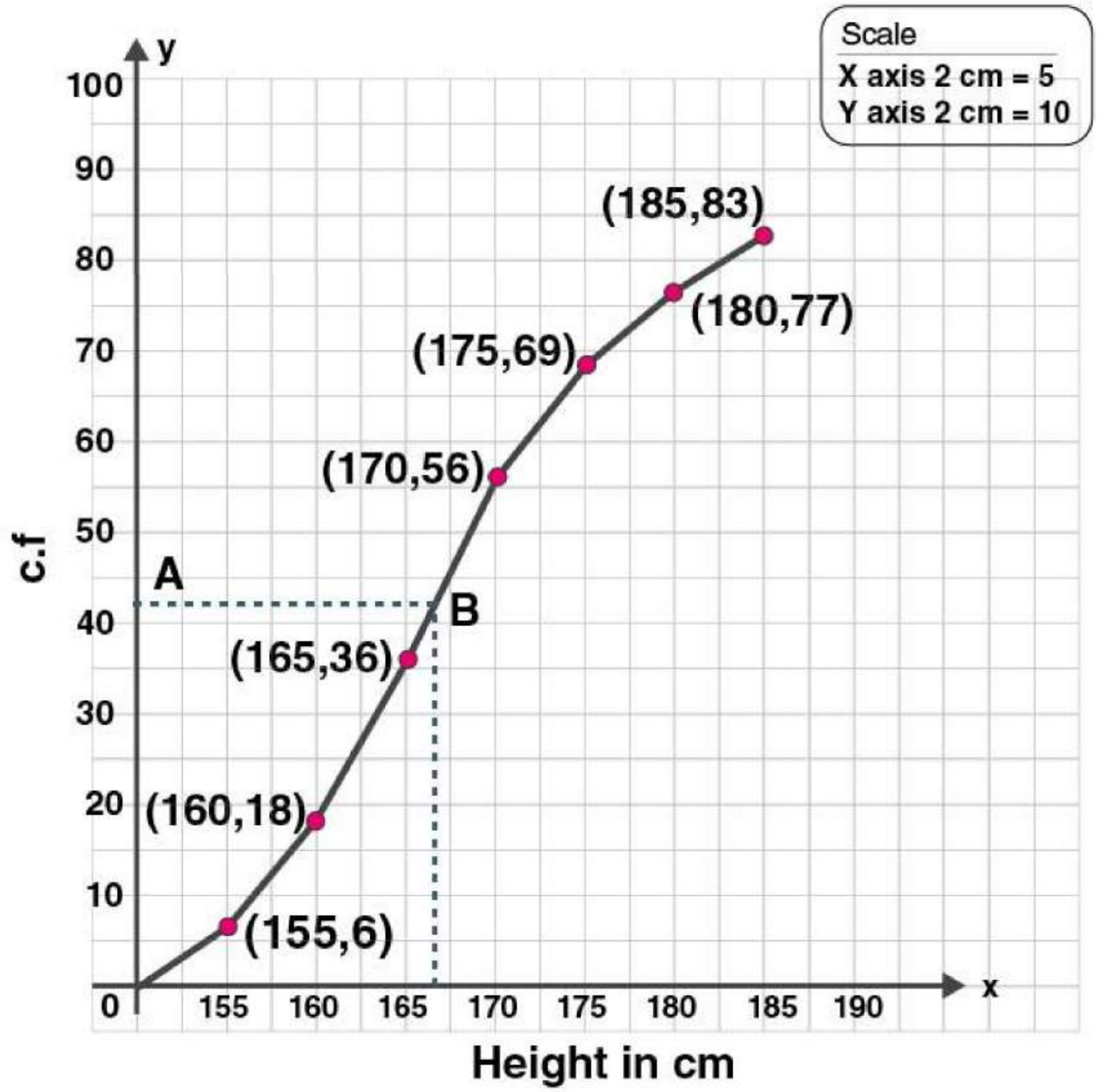
Height (in cm)	**150-155**	**155-160**	**160-165**	**165-170**	**170-175**	**175-180**	**180-185**
No. of workers	**6**	**12**	**18**	**20**	**13**	**8**	**6**

(i) Determine the cumulative frequencies.
(ii) Draw the cumulative frequency curve on a graph paper. Use 2 cm = 5 cm height on one axis and 2 cm = 10 workers on the other.
(iii) From your graph, write down the median height in cm.

Solution:
(i) We write the given data in cumulative frequency table.

Height in cm	No of workers f	Cumulative frequency
150-155	6	6
155-160	12	18
160-165	18	36
165-170	20	56
170-175	13	69
175-180	8	77
180-185	6	83

(ii)Plot the points (155, 6), (160, 18), (165, 36), (170, 56), (175, 69), (180, 77) and (185, 83) on the graph. Join the points with the free hand. We get an ogive as shown:



(iii)Here n = 83 , which is odd.
So median = $((n+1)/2)^{th}$ observation
= $((83+1)/2)^{th}$ observation
= $(84/2)^{th}$ observation
= 42^{th} observation

Take a point A(42) on Y-axis. From A, draw a horizontal line parallel to X-axis meeting the curve at B. From B draw a line perpendicular on the x-axis which meets it at C.
∴C is the median which is 166.5 cm.

2. Using the data given below construct the cumulative frequency table and draw the-Ogive. From the ogive determine the median.

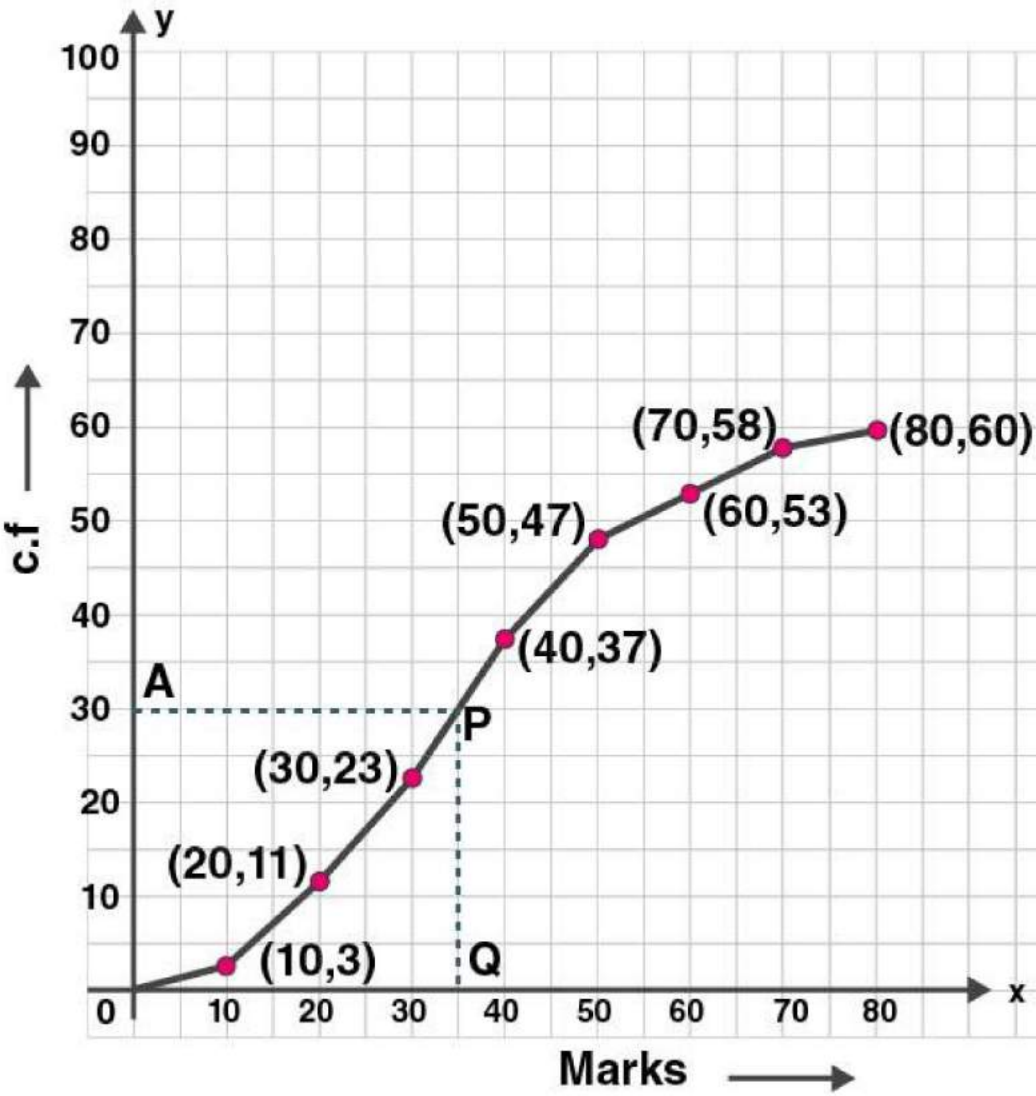
Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**
No. of students	**3**	**8**	**12**	**14**	**10**	**6**	**5**	**2**

Solution:
We write the given data in cumulative frequency table.

Marks	No of students f	Cumulative frequency c.f
0-10	3	3
10-20	8	11
20-30	12	23
30-40	14	37
40-50	10	47
50-60	6	53
60-70	5	58
70-80	2	60

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),

Plot the points (10, 3), (20, 11), (30, 23), (40, 37), (50, 47), (60, 53), (70, 58) and (80, 60) on the graph. Join the points with the free hand. We get an ogive as shown:



Here number of observations, n = 60 which is even.
So median = (n/2) th term
= (60/2 th term
= 30 th term

Mark a point A(30) on Y-axis. From A, draw a horizontal line parallel to X-axis meeting the curve at P. From P draw a line perpendicular on the x-axis which meets it at Q.
$\therefore$Q is the median .
Q = 35
Hence the median is 35 .

3. Use graph paper for this question. The following table shows the weights in gm of a sample of 100

potatoes taken from a large consignment:

Weight (gm)	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**	**100-110**	**110-120**	**120-130**
Frequency	**8**	**10**	**12**	**16**	**18**	**14**	**12**	**10**

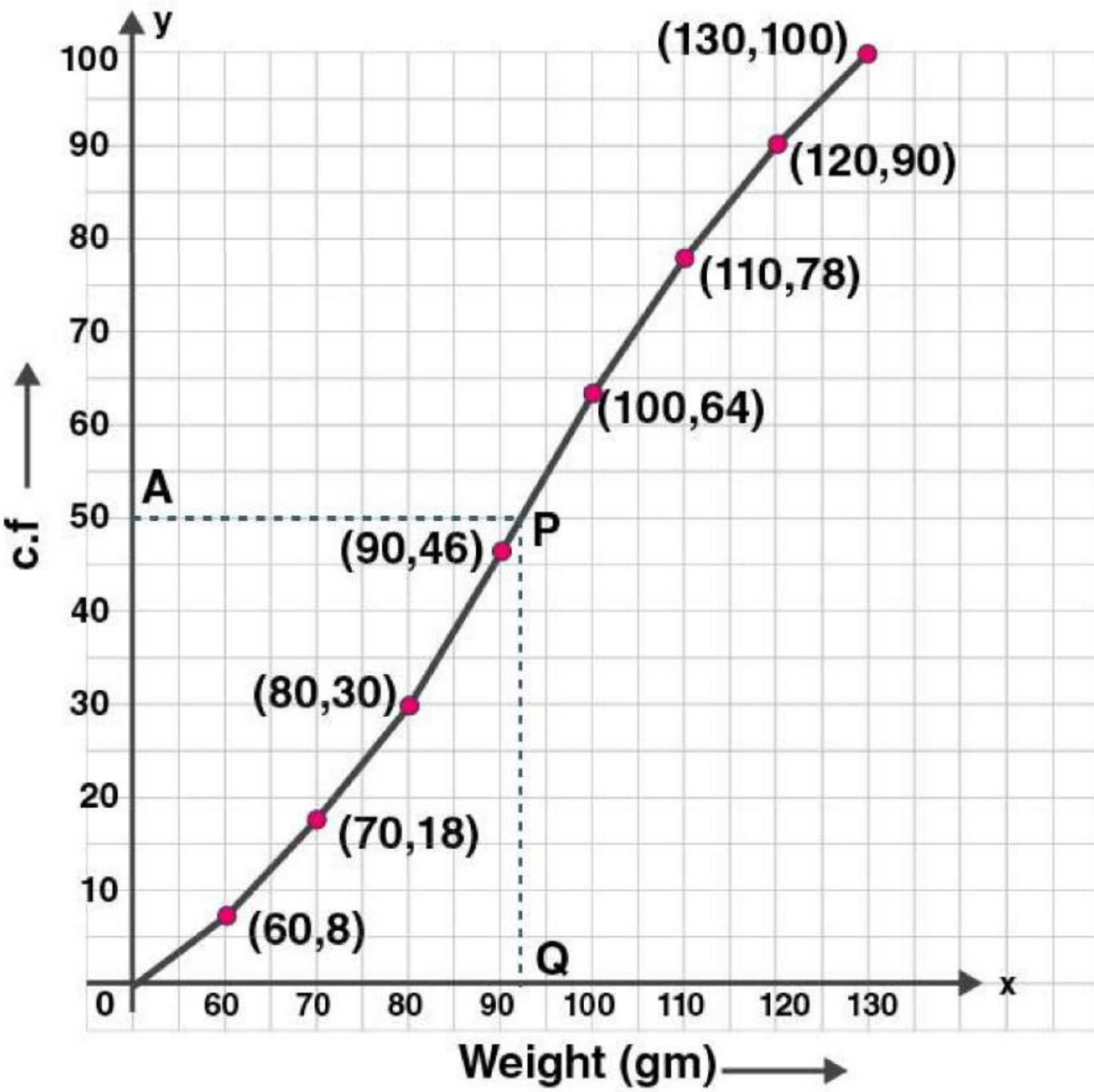
(i) Calculate the cumulative frequencies.
(ii) Draw the cumulative frequency curve and from it determine the median weight of the potatoes. (1996)

Solution:
(i)We write the given data in cumulative frequency table.

Marks	frequency f	Cumulative frequency c.f
50-60	8	8
60-70	10	18
70-80	12	30
80-90	16	46
90-100	18	64
100-110	14	78
110-120	12	90
120-130	10	100

(ii)To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),

Plot the points (60, 8), (70, 18), (80, 30), (90, 46), (100, 64), (110, 78), (120, 90) and (130, 100) on the graph. Join the points with the free hand. We get an ogive as shown:



Here n = 100 which is even.
So median = (n/2 th term)
= (100/2 th term)
= (50 th term)
Now mark a point A (50) on the Y-axis and from A draw a line parallel to X-axis meeting the curve at P. From P, draw a perpendicular on x-axis meeting it at Q.
Q is the median.
Q = 93 gm.
Hence the median is 93.

4. Attempt this question on graph paper.

Age(yrs)	**5-15**	**15-25**	**25-35**	**35-45**	**45-55**	**55-65**	**65-75**
No. of casualities due to accidents	**6**	**10**	**15**	**13**	**24**	**8**	**7**

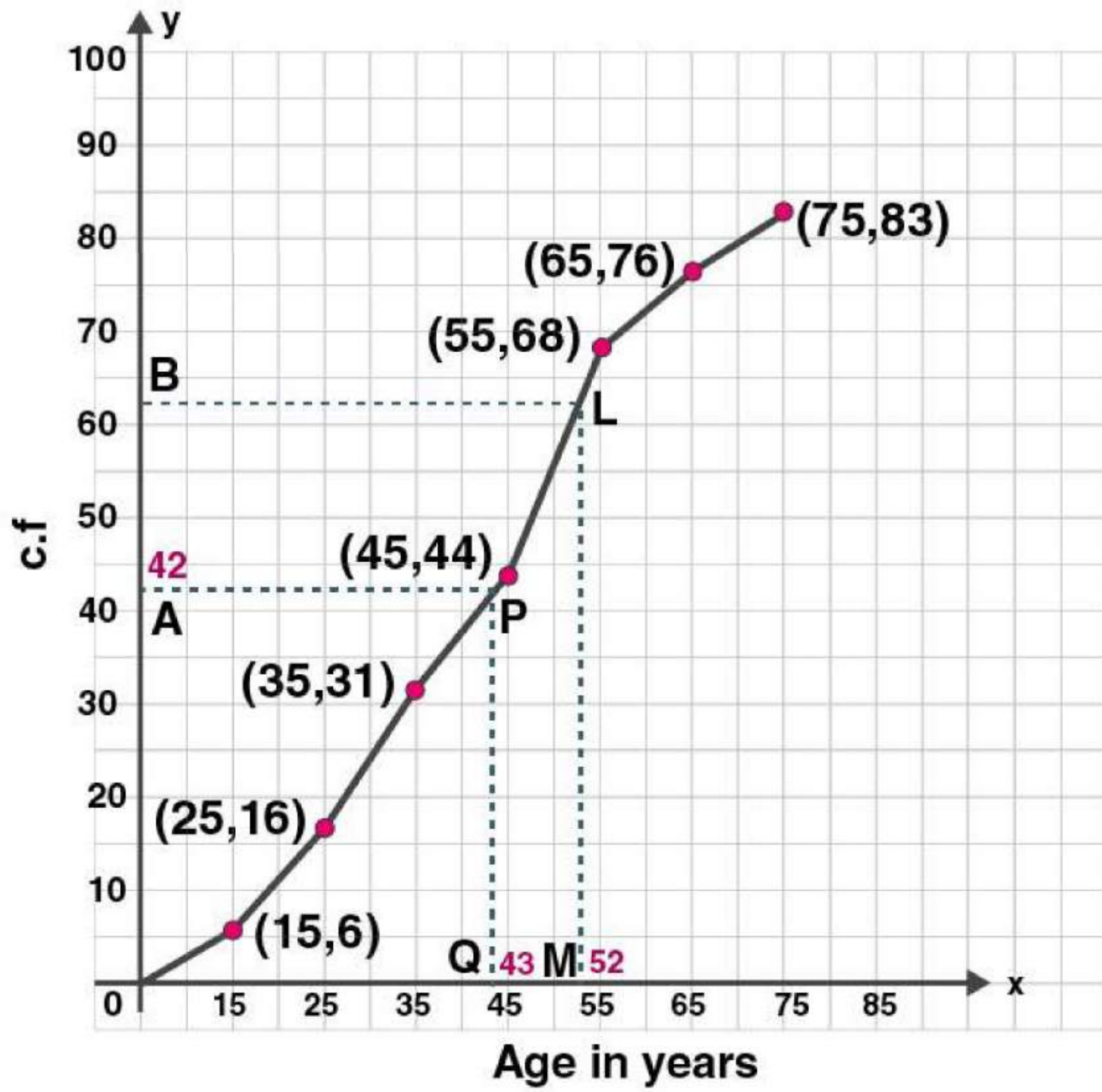
(i) Construct the 'less than' cumulative frequency curve for the above data, using 2 cm = 10 years, on one axis and 2 cm = 10 casualties on the other.
(ii) From your graph determine
(1) the median and (2) the upper quartile

Solution:
(i)We write the given data in cumulative frequency table.

Age (yrs)	No of casualities due to accidents f	Cumulative frequency c.f
5-15	6	6
15-25	10	16
25-35	15	31
35-45	13	44
45-55	24	68
55-65	8	76
65-75	7	83

(ii)To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),

Plot the points (15, 6), (25, 16), (35, 31), (45, 44), (55, 68), (65, 76) and (75, 83) on the graph.
Join the points with the free hand. We get an ogive as shown:



(ii)(1). Here n = 83, which is odd.
So median = $(n+1)/2)^{th}$ term
= ((83+1)/2)
= 84/2
= 42
Now mark a point A (42) on the Y-axis and from A draw a line parallel to X-axis meeting the curve at P. From P, draw a perpendicular on x-axis meeting it at Q.
Q is the median.
Q = 43
Hence the median is 43.

(ii)(2). Upper quartile = (3(n+1)/4)
= (3×(83+1)/4)
= (3×(84)/4)
= 63
Now mark a point B (63) on the Y-axis and from A draw a line parallel to X-axis meeting the curve at L. From L, draw a perpendicular on x-axis meeting it at M.
M = 52
Hence the upper quartile is 52.

5. The weight of 50 workers is given below:

Weight in kg	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**	**100-110**	**110-120**
No. of workers	**4**	**7**	**11**	**14**	**6**	**5**	**3**

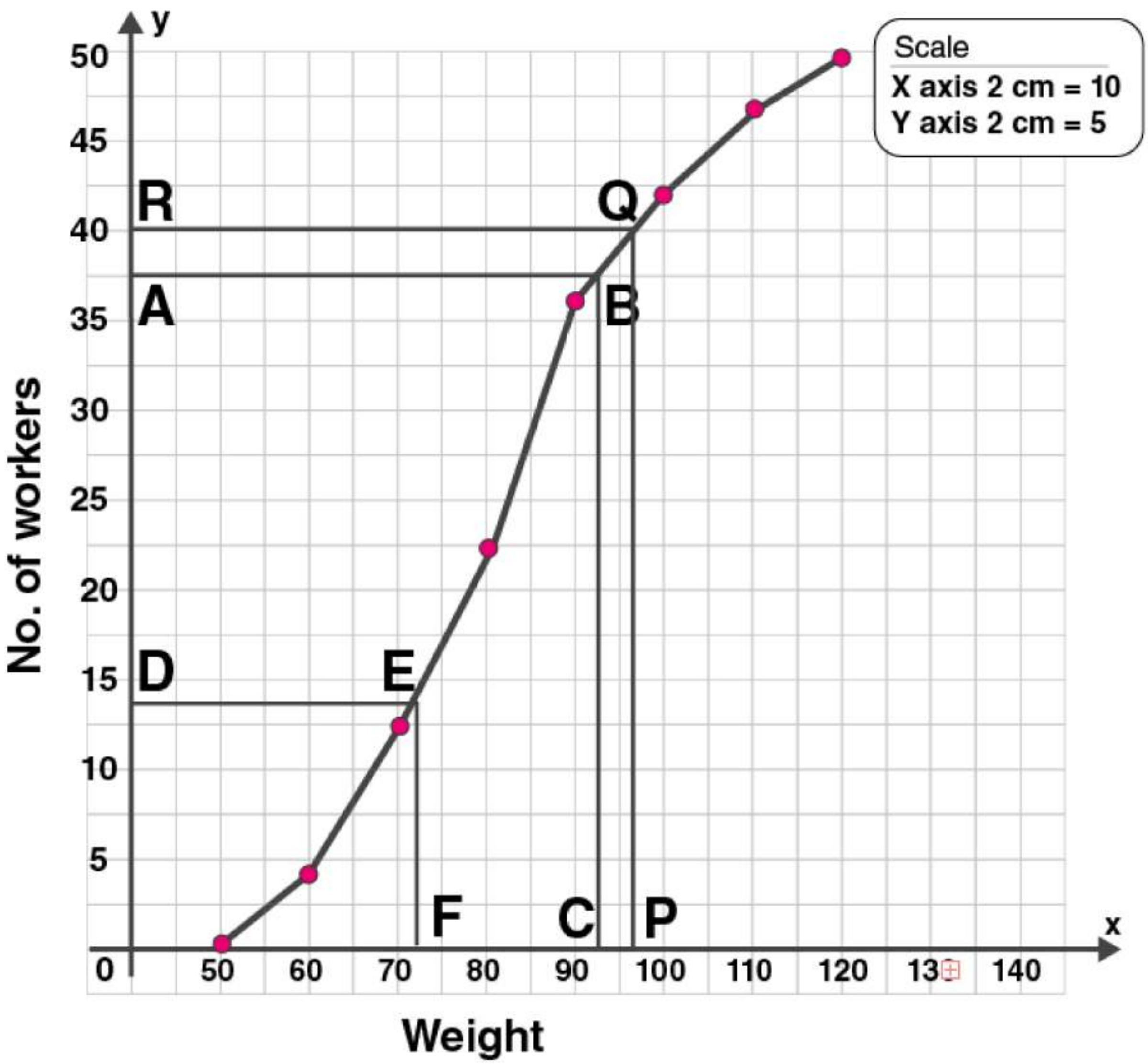
Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis , and 2 cm = 5 workers along the other axis.
Use a graph to estimate the following:
(i) the upper and lower quartiles.
(ii) if weighing 95 kg and above is considered overweight find the number of workers who are overweight. (2015)

Solution:
We write the given data in cumulative frequency table.

Weight in kg	No of workers f	Cumulative frequency c.f
50-60	4	4
60-70	7	11
70-80	11	22
80-90	14	36
90-100	6	42
100-110	5	47
110-120	3	50

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),

Plot the points (60, 4), (70, 11), (80, 22), (90, 36), (100, 42), (110, 47) and (120, 50) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)Here n = 50, which is even.
Upper quartile = 3n/4
= 3×50/4
= 150/4
= 37.5
Now mark a point A (37.5) on the Y-axis and from A draw a line parallel to X-axis meeting the curve at B. From B, draw a perpendicular on x-axis meeting it at C.
C = 92.5
Hence the upper quartile is 92.5 kg.

Lower quartile, $Q_1 = (n/4)^{th}$ term
= 50/4
= 12.5
Now mark a point D(12.5) on the Y-axis and from D draw a line parallel to X-axis meeting the curve at E. From

E, draw a perpendicular on x-axis meeting it at F.
F = 72
Hence the lower quartile is 72 kg.
(ii) Mark on the graph point P which is 95 kg on X axis.
Through P draw a vertical line to meet the ogive at Q. Through Q, draw a horizontal line to meet y-axis at R.
The ordinate of point R represents 40 workers on the y-axis .
∴ The number of workers who are 95 kg and above = Total number of workers – number of workers of weight less than 95 kg = 50-40 = 10

6. The table shows the distribution of scores obtained by 160 shooters in a shooting competition. Use a graph sheet and draw an ogive for the distribution. (Take 2 cm = 10 scores on the x-axis and 2 cm = 20 shooters on the y-axis)

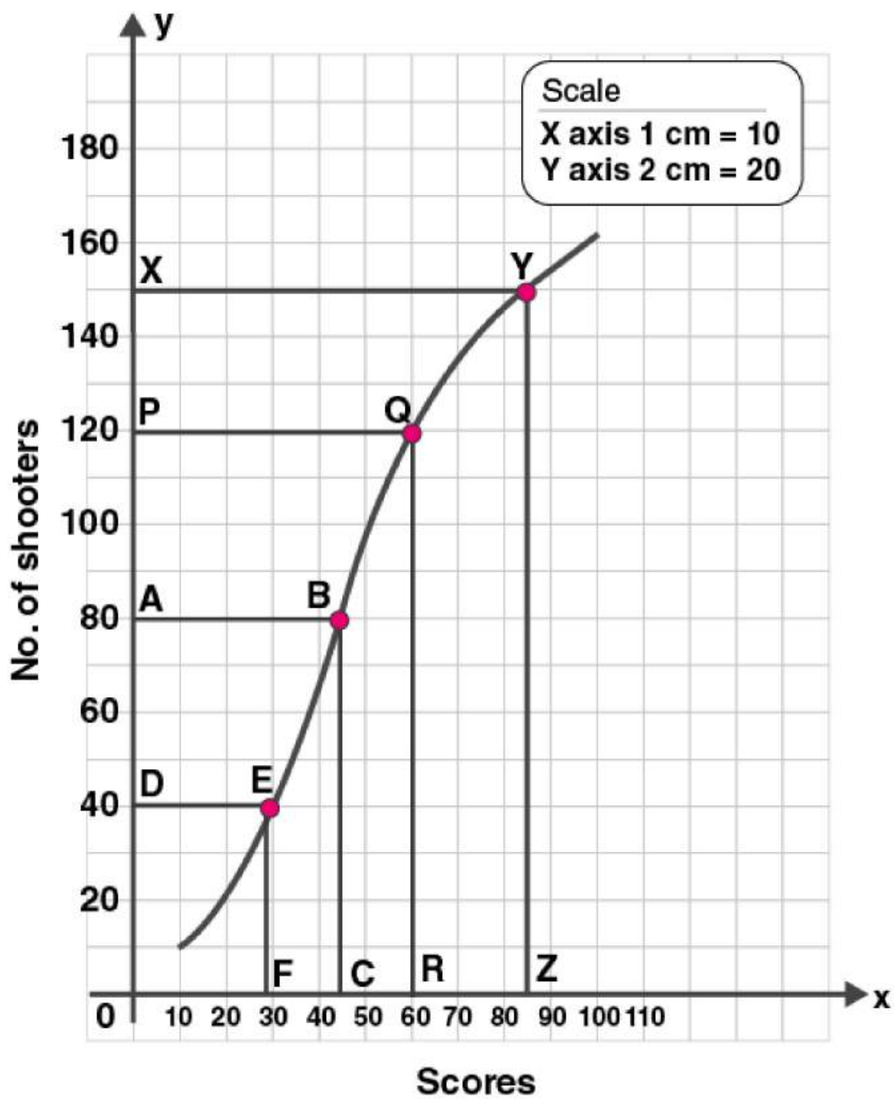
Scores	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of shooters	**9**	**13**	**20**	**26**	**30**	**22**	**15**	**10**	**8**	**7**

Use your graph to estimate the following:
(i) The median.
(ii) The interquartile range.
(iii) The number of shooters who obtained a score of more than 85%.

Solution:
We write the given data in cumulative frequency table.

Scores	No of shooters f	Cumulative frequency c.f
0-10	9	9
10-20	13	22
20-30	20	42
30-40	26	68
40-50	30	98
50-60	22	120
60-70	15	135
70-80	10	145
80-90	8	153
90-100	7	160

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),
Plot the points (10, 9), (20, 22), (30, 42), (40, 68), (50, 98), (60, 120), (70, 135), (80, 145), (90, 153) and (100, 160) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)Here n = 160, which is even.
So median = n/2 = 80
Now mark a point A(80) on the Y-axis and from A draw a line parallel to X-axis meeting the curve at B. From P, draw a perpendicular on x-axis meeting it at C.
C is the median.
C = 44

(ii) lower quartile, $Q_1 = (n/4)^{th}$ term
= 160/4
= 40
Now mark a point D(40) on the Y-axis and from that point draw a line parallel to X-axis meeting the curve at E. From E, draw a perpendicular on x-axis meeting it at F.
F = 29

So Q_1= 29
Upper quartile, $Q_3 = (3n/4)^{th}$ term
= 3×160/4
= 3×40
= 120
Mark a point P(120) on the Y-axis and from that point draw a line parallel to X-axis meeting the curve at Q. From Q, draw a perpendicular on x-axis meeting it at R.
R = 60
So Q_3 = 60
Inter quartile range = Q_3- Q_1
= 60-29
= 31
Hence the Inter quartile range is 31.

(iii) Mark a point Z(85) on the X axis.
From Z on X-axis, draw a perpendicular to it meeting the curve at Y. From Y, draw a line parallel to X-axis meeting Y-axis at X. X is the required point which is 150.
Number of shooters getting more than 85% scores = Total number of shooters - number of shooters who got till 85% = 160-150 = 10.
Hence the number of shooters getting more than 85% scores is 10.

7. The daily wages of 80 workers in a project are given below

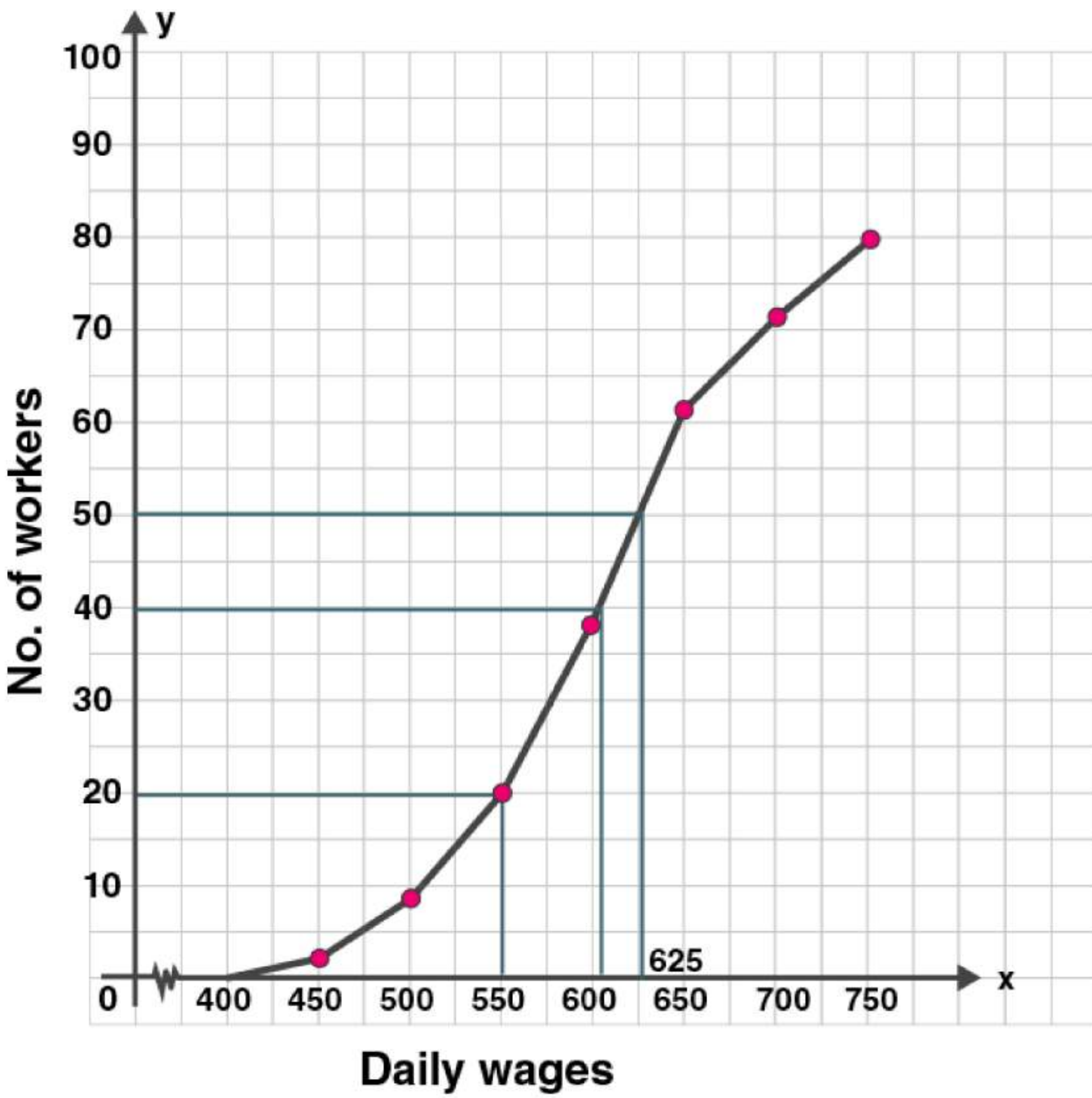
Wages in Rs	**400-450**	**450-500**	**500-550**	**550-600**	**600-650**	**650-700**	**700-750**
No. of workers	**2**	**6**	**12**	**18**	**24**	**13**	**5**

Use a graph paper to draw an ogive for the above distribution. (a scale of 2 cm = Rs 50 on x- axis and 2 cm = 10 workers on y-axis). your ogive to estimate
(i) the median wage of the workers.
(ii) the lower quartile wage of the workers.
(iii) the number of workers who earn more than Rs 625 daily. (2017)

Solution:
We write the given data in cumulative frequency table.

Wages in Rs.	No of workers f	Cumulative frequency c.f
400-450	2	2
450-500	6	8
500-550	12	20
550-600	18	38
600-650	24	62
650-700	13	75
700-750	5	80

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis), Plot the points (450, 9), (500, 22), (550, 42), (600, 68), (650, 98), (700, 120) and (750, 135) on the graph. Join the points with the free hand. We get an ogive as shown:



(i)Here n = 80.
Median = $(n/2)^{th}$ term
= 80/2
= 40^{th} term
Mark a point (40) on Y axis. Draw a line from that point parallel to X axis. Let it meet the curve at A.
Draw a perpendicular from A to meet X axis at B.
The value of B is 604.
Hence the median is 604.

(ii)Lower quartile, Q1 = $(n/4)^{th}$ term

= 80/4
= 20^{th} term
= 550 [from graph]

(iii) Draw a vertical line through the point 625 on X axis. which meets the graph at point C. From C, draw a horizontal line which meets the y-axis at the mark of 50.
Thus, number of workers that earn more Rs 625 daily = Total no. of workers - no. of workers who earn upto 625
= 80-50 = 30

8. Marks obtained by 200 students in an examination are given below

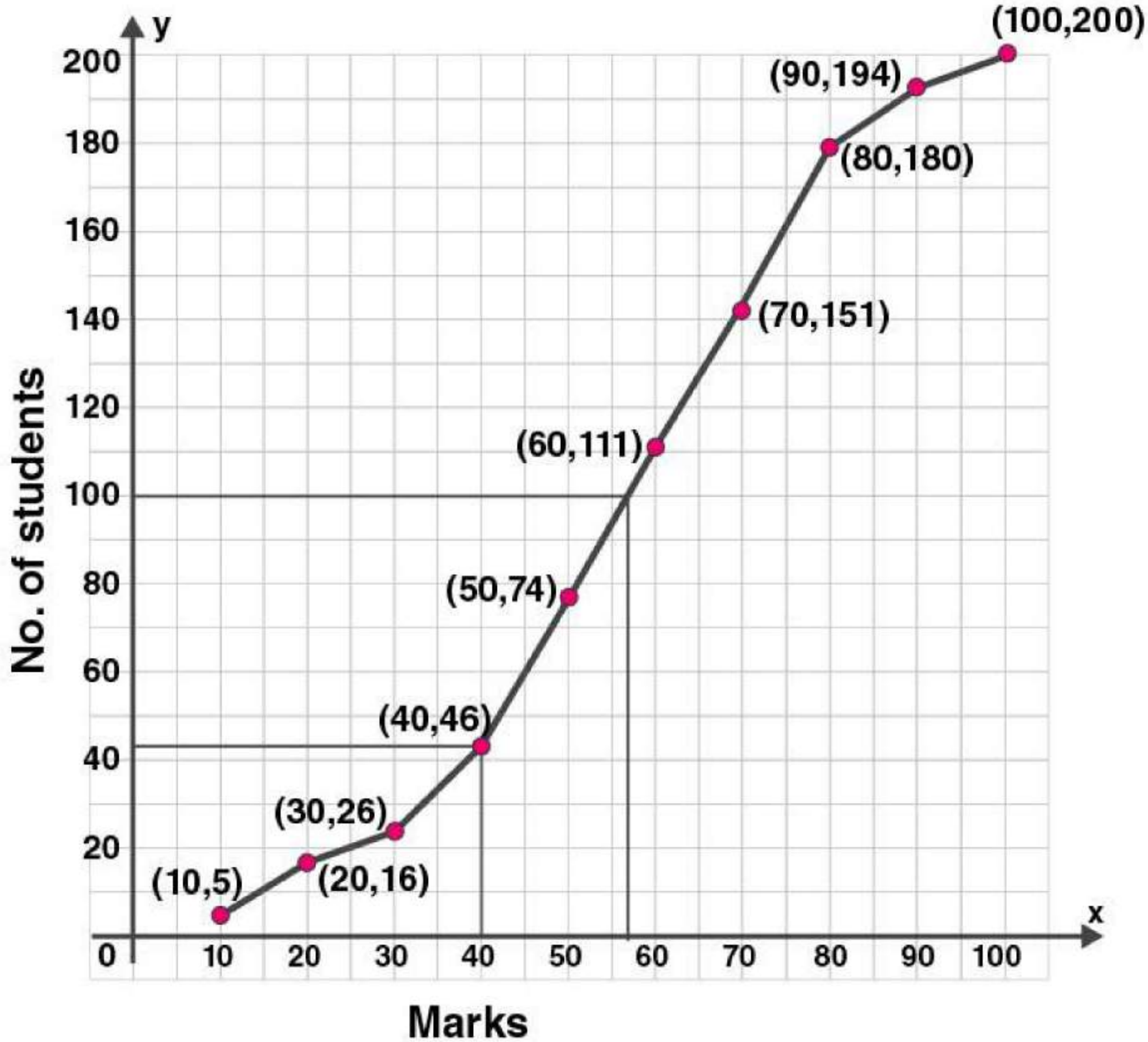
marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of students	**5**	**11**	**10**	**20**	**28**	**37**	**40**	**29**	**14**	**6**

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.
Using the graph, determine
(i) The median marks.
(ii) The number of students who failed if minimum marks required to pass is 40.
(iii) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination.

Solution:
We write the given data in cumulative frequency table.

Marks	No of students f	Cumulative frequency c.f
0-10	5	5
10-20	11	16
20-30	10	26
30-40	20	46
40-50	28	74
50-60	37	111
60-70	40	151
70-80	29	180
80-90	14	194
90-100	6	200

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),
Plot the points (10, 5), (20, 16), (30, 26), (40, 46), (50, 74), (60, 111) , (70, 151) (80, 180) (90, 194) and (100, 200) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)Here n= 200
Median = $(n/2)^{th}$ term
= 200/2
= 100^{th} term
= 57 [from graph]
(ii)number of students failed if minimum marks required to pass is 40 = 44 [from graph]
(iii)Number of students who got grade 1 = number of students who scored 85 and more
= 200-188
= 12[From graph]

9. The monthly income of a group of 320 employees in a company is given below

Monthly income	**No. of employees**
6000-7000	**20**

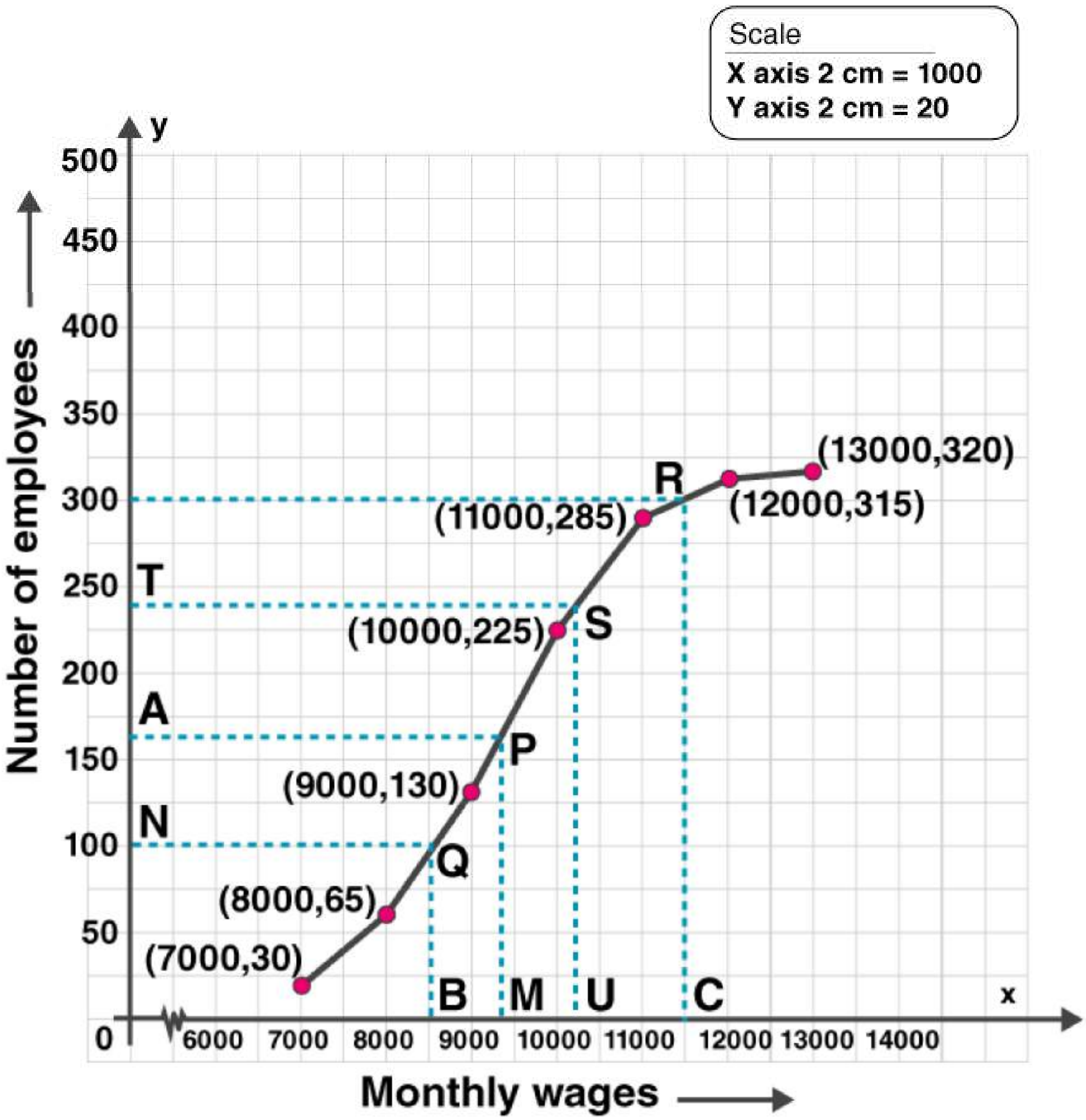
7000-8000	**45**
8000-9000	**65**
9000-10000	**95**
10000-11000	**60**
11000-12000	**30**
12000-13000	**5**

Draw an ogive of the given distribution on a graph sheet taking 2 cm = Rs. 1000 on one axis and 2 cm = 50 employees on the other axis.
From the graph determine
(i) the median wage.
(ii) the number of employees whose income is below Rs. 8500.
(iii) If the salary of a senior employee is above Rs. 11500, find the number of senior employees in the company.
(iv) the upper quartile.

Solution:
We write the given data in cumulative frequency table.

Monthly income	No. of employees	Cumulative frequency c.f
6000-7000	20	20
7000-8000	45	65
8000-9000	65	130
9000-10000	95	225
10000-11000	60	285
11000-12000	30	315
12000-13000	5	320

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),
Plot the points (7000, 20), (8000, 65), (9000, 130), (10000, 225), (11000, 285), (12000, 315) and (13000, 320) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)here n = 320
Median = (n/2)th term
= 320/2
= 160th term
Mark the point A(160) on Y axis.
Draw a line parallel to x axis from that point.
Let it meet the curve at P.
Draw a perpendicular from P to X axis which meets at M.
M is the median.
Here median is 9300.   [from graph]

(ii)Mark the point B(8500) on X axis.
Draw a line parallel to Y axis which meets curve at Q.
From Q draw a line parallel to X axis which meets Y axis at N.
N = 98
number of employees whose income is below 8500 = 98

(iii) Mark the point C(11500) on the x-axis.
Draw a line perpendicular to x-axis meeting the curve at R.
From R, draw a line parallel to x-axis meeting y-axis at L which is 300
No. of employees getting more than Rs. 11500 = 320-300 = 20

(iv)upper quartile = 3n/4
= 3×320/4
= 240
Mark the point T(240) on Y axis.
From that point on y-axis, draw a line perpendicular on the x-axis which meets the curve at S.
From S, draw a perpendicular on x-axis meeting it at U, which is 10250.
Hence upper quartile is 10250.

10. Using a graph paper, draw an ogive for the following distribution which shows a record of the weight in kilograms of 200 students

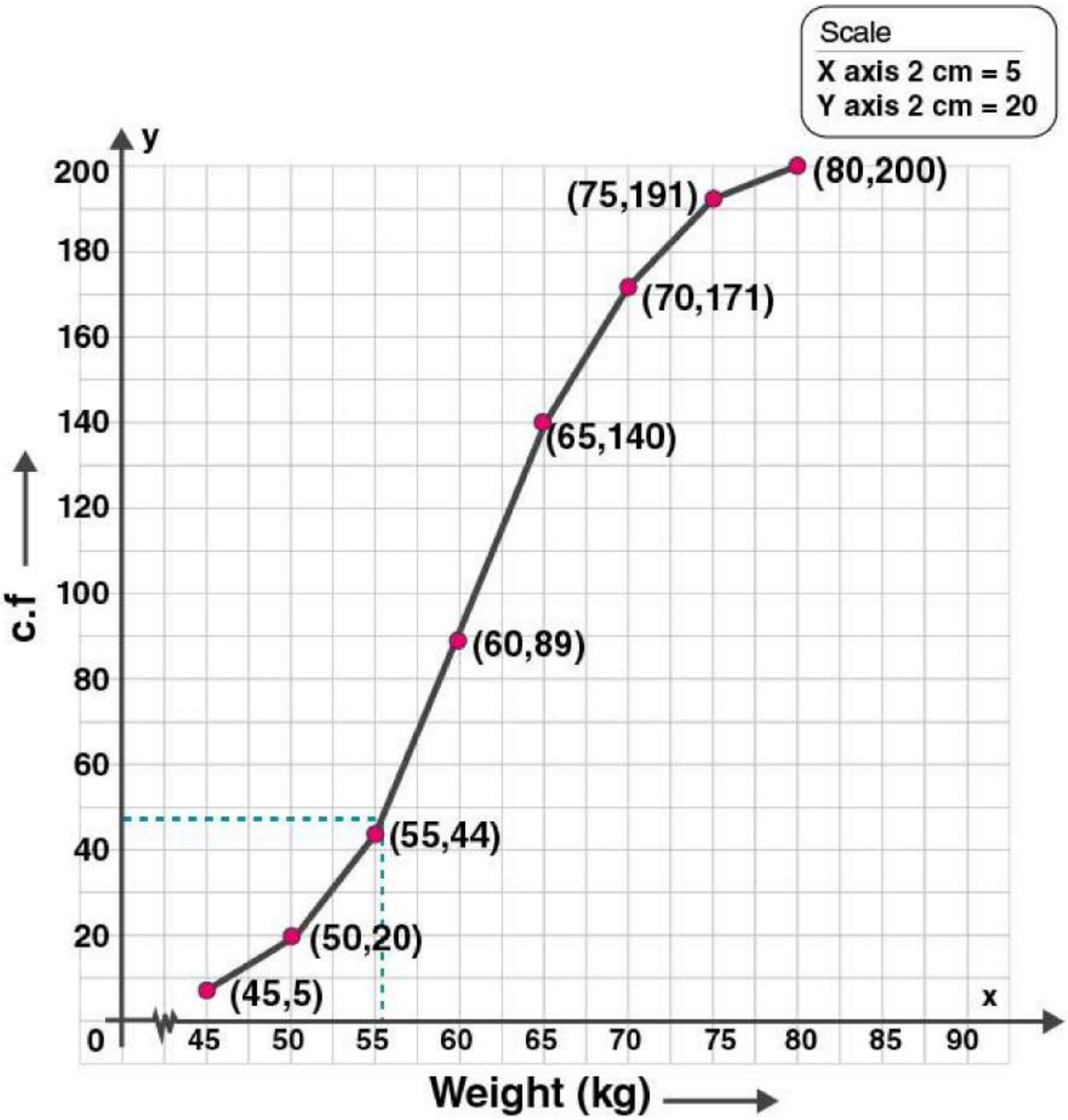
Weight	**40-45**	**45-50**	**50-55**	**55-60**	**60-65**	**65-70**	**70-75**	**75-80**
Frequency	**5**	**17**	**22**	**45**	**51**	**31**	**20**	**9**

Use your ogive to estimate the following:
(i) The percentage of students weighing 55 kg or more.
(ii) The weight above which the heaviest 30% of the students fall,
(iii) The number of students who are :
1. under-weight and
2. over-weight, if 55.70 kg is considered as standard weight.

Solution:
We write the given data in cumulative frequency table.

Weight	Frequency	Cumulative frequency c.f
40-45	5	5
45-50	17	22
50-55	22	44
55-60	45	89
60-65	51	140
65-70	31	171
70-75	20	191
75-80	9	200

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis),
Plot the points (45, 5), (50, 22), (55, 44), (60, 89), (65, 140), (70, 171), (75, 191) and (80, 200) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)Total number of students = 200
The number of students weighing 55 kg or more = 200-44 = 156 [From the graph]
Percentage = (156/200)×100
= 156/2
= 78%.

(ii)30% of 200 = (30/100)×200
= 30×2

= 60
No of heaviest students = 31+20+9 = 60
60 students fall above 65 kg.

(iii)If 55.70 kg is the standard weight,
No. of students who are under weight = 47 [from graph]
No. of students who are overweight = 200-47 = 153

11. The marks obtained by 100 students in a Mathematics test are given below

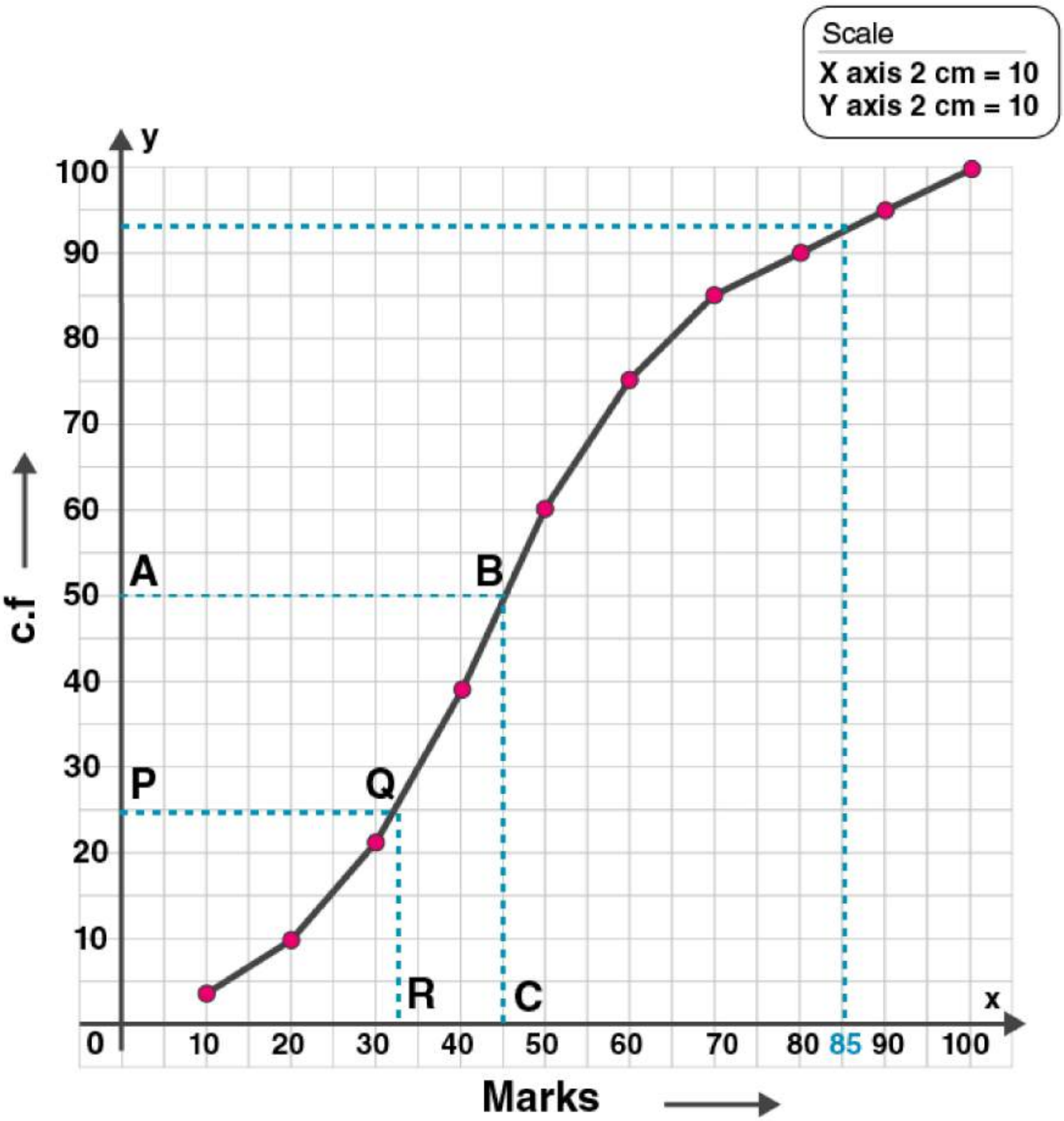
Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of students	**3**	**7**	**12**	**17**	**23**	**14**	**9**	**6**	**5**	**4**

Draw an ogive on a graph sheet and from it determine the :
(i) median
(ii) lower quartile
(iii) number of students who obtained more than 85% marks in the test.
(iv) number of students who did not pass in the test if the pass percentage was 35.

Solution:
We write the given data in cumulative frequency table.

Marks	No. of students	Cumulative frequency c.f
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

Plot the points (10, 3), (20, 10), (30, 22), (40, 39), (50, 62), (60, 76), (70, 85), (80, 91), (90, 96) and (100, 100) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)Here n = 100
Median = n/2
= 50 th term
Mark a point A(50) on Y axis. Draw a line parallel to X axis from A.
Let it meet the curve at B. From B draw a perpendicular which meets X axis at C.
The point C is 45.
Hence median is 45.

(ii)Lower quartile = n/4

= 100/4
= 25th term
Mark a point P (25) on Y axis. Draw a line parallel to X axis from that point.
Let it meet the curve at Q. From that point draw a perpendicular which meets X axis at R.
The point R is 32.
Hence lower quartile is 32.

(iii)no. of students who obtained more than 85% = 100-94 = 6 [from graph]

(iv)No of students who failed if 35% is the pass percentage = 25 [from graph]

12. The marks obtained by 120 students in a Mathematics test are-given below

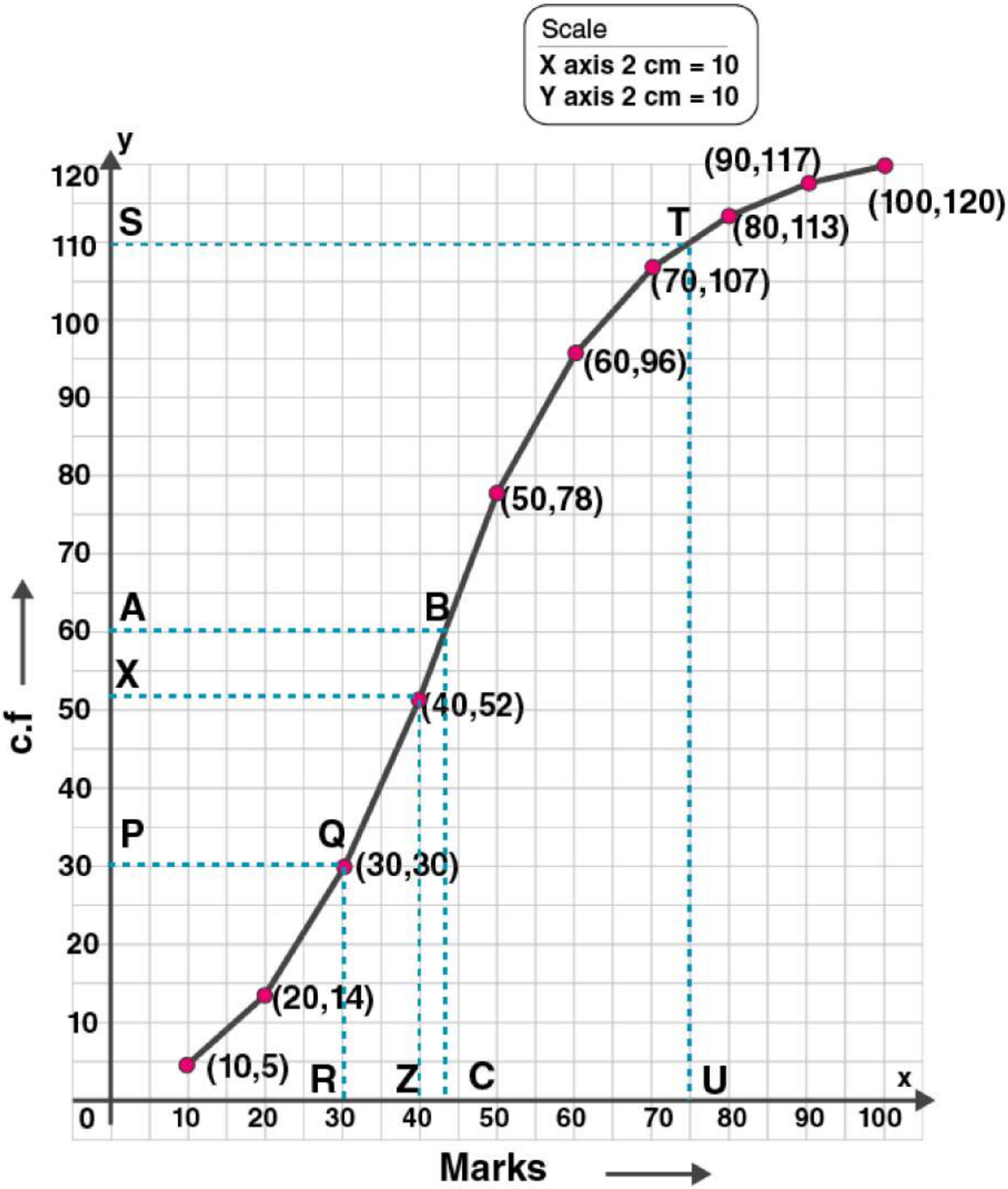
Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
No. of students	**5**	**9**	**16**	**22**	**26**	**18**	**11**	**6**	**4**	**3**

Draw an ogive for the given distribution on a graph sheet. Use a suitable scale for ogive to estimate the following:
(i) the median
(ii) the lower quartile
(iii) the number of students who obtained more than 75% marks in the test.
(iv) the number of students who did not pass in the test if the pass percentage was 40. (2002)

Solution:
We write the given data in cumulative frequency table.

Marks	No. of students	Cumulative frequency c.f
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

Plot the points (10, 5), (20, 14), (30, 30), (40, 52), (50, 78), (60, 96), (70, 107), (80, 113), (90, 117) and (100, 120) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)Here n = 120

Median = $(n/2)^{th}$ term

= 120/2

= 60^{th} term

Mark point A(60) on Y axis. Draw a line parallel to X axis from A.

Let it meet the curve at B. Draw a straight line from B to X axis which meets at C.
C = 50
Hence median is 50.

(ii)Lower quartile = $(n/4)^{th}$ term
= 120/4
= 30^{th} term
Mark a point P (30) on Y axis. Draw a line parallel to X axis from that point.
Let it meet the curve at Q. From that point draw a perpendicular which meets X axis at R.
The point R is 30.
Hence lower quartile is 30.

(iii)Mark a point U(75) on X axis.
Draw a line parallel to Y axis which meets curve at T.
From T, draw a line parallel to X axis to meet Y axis at S.
S = 110
No. of students who obtained more than 75% = 120-110 = 10

(iv) Mark a point Z(40) on X axis.
Draw a line parallel to Y axis which meets curve at Y.
From Y, draw a line parallel to X axis to meet Y axis at X.
X = 52
No of students who failed if 40% is the pass percentage is 52.

13. The following distribution represents the height of 160 students of a school.

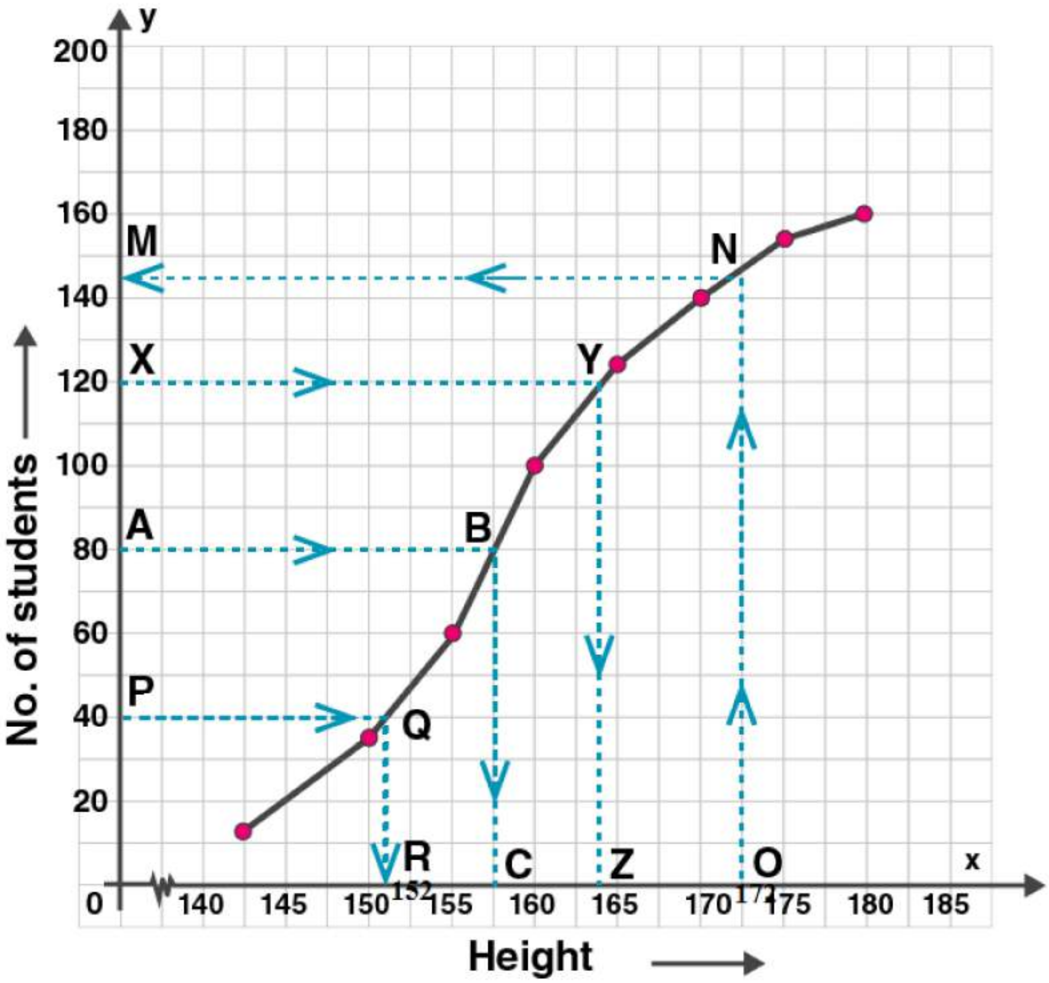
Height	**140-145**	**145-150**	**150-155**	**155-160**	**160-165**	**165-170**	**170-175**	**175-180**
No. of students	**12**	**20**	**30**	**38**	**24**	**16**	**12**	**8**

Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis.
Using the graph, determine :
(i)The median height.
(ii)The inter quartile range.
(iii) The number of students whose height is above 172 cm.

Solution:
We write the given data in cumulative frequency table.

Height	No. of students	Cumulative frequency c.f
140-145	12	12
145-150	20	32
150-155	30	62
155-160	38	100
160-165	24	124
165-170	16	140
170-175	12	152
175-180	8	160

Plot the points (145, 12), (150, 32), (155, 62), (160, 100), (165, 124), (170, 140), (175, 152), and (180, 160) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i)Here n = 160
Median = $(n/2)^{th}$ term
= 160/2
= 80^{th} term
Mark point A(80) on Y axis. Draw a line parallel to X axis from A.
Let it meet the curve at B. Draw a straight line from B to X axis which meets at C.
C = 157.5
Hence median is 157.5.

(ii) Lower quartile , $Q_1 = (n/4)^{th}$ term
= 160/4
= 40^{th} term
Proceeding in the same way mentioned in (i),
we get lower quartile = 152 [Point R]

Upper quartile, Q_3 = 3n/4
= 3×160/4
= 3×40
= 120^{th} term
Proceeding in the same way mentioned in (i),
we get upper quartile = 164 [Point Z]
Interquartile range = Q_3-Q_1
= 164-152
= 12

(iii)Mark a point O(172) on X axis. Draw a line parallel to Y axis from O.
Let it meet the curve at N. Draw a straight line from N to Y axis which meets at M.
M = 144
Hence number of students whose height is more than 172 cm is 160-144 = 16

14. 100 pupils in a school have heights as tabulated below

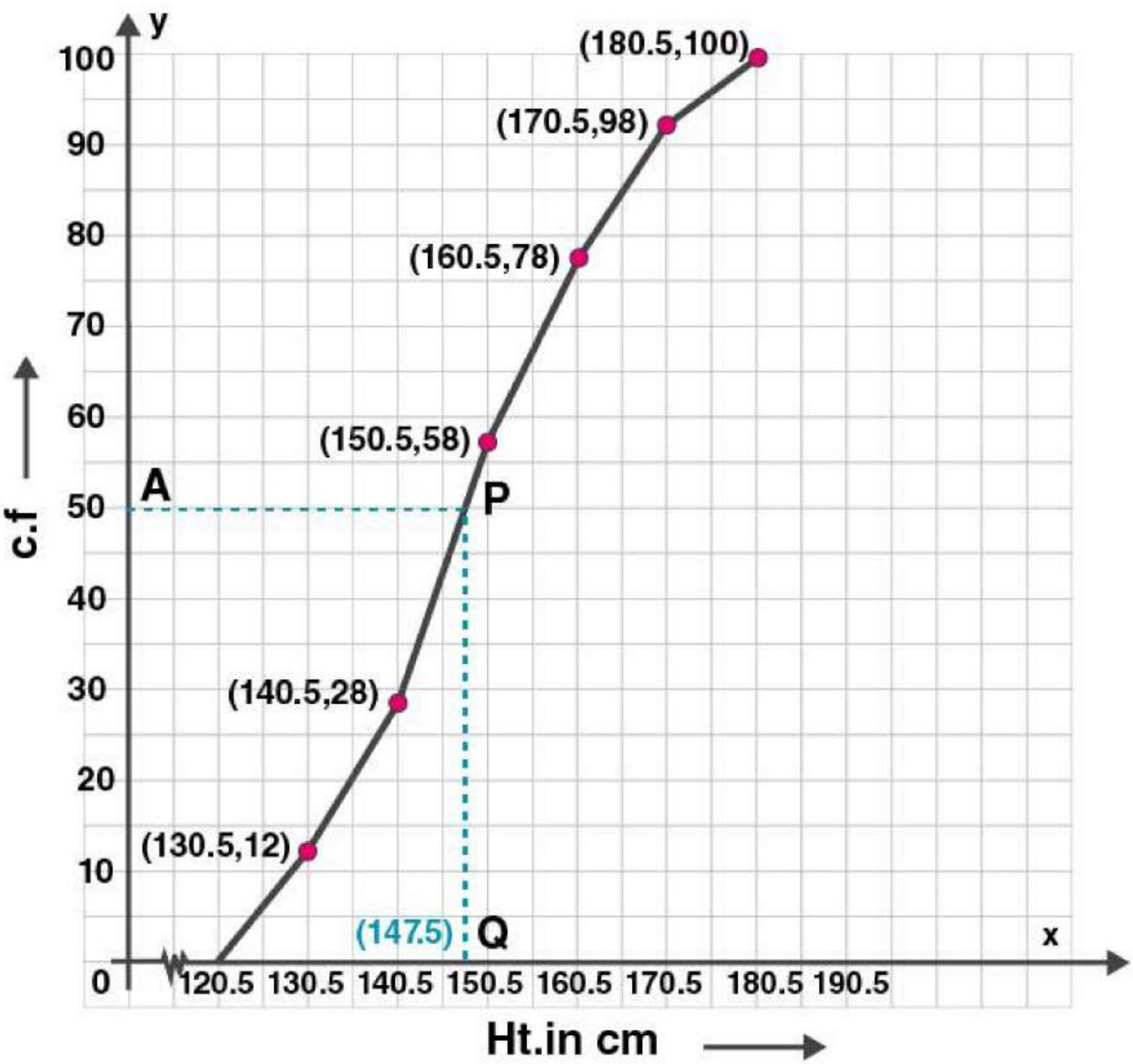
Height in cm	**121-130**	**131-140**	**141-150**	**151-160**	**161-170**	**171-180**
No. of pupils	**12**	**16**	**30**	**20**	**14**	**8**

Draw the ogive for the above data and from it determine the median (use graph paper).

Solution:
We write the given data in cumulative frequency table (in continuous distribution):

Height	No. of students	Cumulative frequency c.f
120.5-130.5	12	12
130.5-140.5	16	28
140.5-150.5	30	58
150.5-160.5	20	78
160.5-170.5	14	92
170.5-180.5	8	100

Plot the points (130.5, 12), (140.5, 28), (150.5, 58), (160.5, 78), (170.5, 92) and (180.5, 100) on the graph.
Join the points with the free hand. We get an ogive as shown:



Here n = 100
Median = $(n/2)^{th}$ term
= 100/2
= 50^{th} term
Mark point A(50) on Y axis. Draw a line parallel to X axis from A.
Let it meet the curve at P. Draw a straight line from P to X axis which meets at Q.
Q = 147.5
Hence median is 147.5.

Chapter Test

1. Arun scored 36 marks in English, 44 marks in Civics, 75 marks in Mathematics and x marks in Science. If he has scored an average of 50 marks, find x.

Solution:
Marks scored in English = 36
Marks scored in Civics = 44
Marks scored in Mathematics = 75
Marks scored in Science = x
No. of subjects = 4
Average marks = sum of marks / No. of subjects = 50 [Given]
$\therefore (36+44+75+x)/4 = 50$
$\therefore 155+x = 4\times50$
$\therefore 155+x = 200$
$\therefore x = 200-155$
$\therefore x = 45$
Hence the value of x is 45.

2. The mean of 20 numbers is 18. If 3 is added to each of the first ten numbers, find the mean of new set of 20 numbers.

Solution:
Given the mean of 20 numbers = 18
$\therefore$ Sum of numbers = $18\times20 = 360$
If 3 is added to each of first 10 numbers, then new sum = $(3\times10)+360$
= 30+360
= 390
$\therefore$ New mean = 390/20
= 19.5
Hence the mean of new set of 20 numbers is 19.5.

3. The average height of 30 students is 150 cm. It was detected later that one value of 165 cm was wrongly copied as 135 cm for computation of mean. Find the correct mean.

Solution:
Average height of 30 students = 150 cm
So sum of height = $150\times30 = 4500$
Difference between correct value and wrong value = 165-135 = 30
So actual sum = 4500+30 = 4530
So actual mean = 4530/30 = 31
Hence the correct mean 31.

4. There are 50 students in a class of which 40 are boys and the rest girls. The average weight of the students in the class is 44 kg and average weight of the girls is 40 kg. Find the average weight of boys.

Solution:
Total number of students = 50
No. of boys = 40

∴No. of girls = 50-40 = 10
Average weight of 50 students = 44 kg
So sum of weight = 44×50 = 2200 kg
Average weight of girls = 40 kg
So sum of weight of girls = 40×10 = 400 kg
∴Total weight of boys = 2200-400 = 1800 kg
∴Average weight of boys = 1800/40 = 45 kg
Hence the average weight of boys is 45 kg.

5. The contents of 50 boxes of matches were counted giving the following results.

No. of matches	**41**	**42**	**43**	**44**	**45**	**46**
No. of boxes	**5**	**8**	**13**	**12**	**7**	**5**

Calculate the mean number of matches per box.

Solution:

No. o f matches x	No. of boxes f	fx
41	5	205
42	8	336
43	13	559
44	12	528
45	7	315
46	5	230
Total	Σf = 50	Σfx =2173

Mean = Σfx/Σf
= 2173/50
= 43.46
Hence the mean is 43.46.

6. The heights of 50 children were measured (correct to the nearest cm) giving the following results

Height (in cm)	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**
No. of children	**1**	**4**	**5**	**7**	**11**	**10**	**6**	**4**	**2**

Calculate the mean height for this distribution correct to one place of decimal.

Solution:

Height x	No. of children f	fx
65	1	65
66	4	264
67	5	335
68	7	476
69	11	759

70	10	700
71	6	426
72	4	288
73	2	146
Total	Σf = 50	Σfx = 3459

Mean = Σfx/Σf
= 3459/50
= 69.18
= 69.2 [corrected to one decimal place]
Hence the mean is 69.2.

7. Find the value of p for the following distribution whose mean is 20.6.

Variate (x_i)	**10**	**15**	**20**	**25**	**35**
Frequency (f_i)	**3**	**10**	**p**	**7**	**5**

Solution:

Variate (x_i)	Frequency (f_i)	fx
10	3	30
15	10	150
20	p	20p
25	7	175
35	5	175
Total	Σf_i = 25+p	$\Sigma f_i x_i$ = 530+20p

Mean = Σfx/Σf
∴ 20.6 = (530+20p)/(25+p) [Given mean = 20.6]
∴ 20.6(25+p) = (530+20p)
∴ 515 + 20.6p = 530+20p
20.6p-20p = 530-515
0.6p = 15
p = 15/0.6
p = 25
Hence the value of p is 25.

8. Find the value of p if the mean of the following distribution is 18.

Variate (x_i)	**13**	**15**	**17**	**19**	**20+p**	**23**
Frequency (f_i)	**8**	**2**	**3**	**4**	**5p**	**6**

Solution:

Variate (x_i)	Frequency (f_i)	f_i x_i

13	8	104
15	2	30
17	3	51
19	4	76
20+p	5p	$5p^2+100p$
23	6	138
Total	$\Sigma f_i = 23+5p$	$\Sigma f_i x_i = 399+5p^2+100p$

Mean = $\Sigma f_i x_i / \Sigma f_i$

$\therefore 18 = (399+5p^2+100p)/(23+5p)$ [Given mean = 18]

$\therefore 18(23+5p) = 399+5p^2+100p$

$\therefore 414 + 90p = 399+5p^2+100p$

$\therefore 5p^2+100p-90p+399-414 = 0$

$\therefore 5p^2+10p-15 = 0$

Dividing by 5, we get

$\therefore p^2+2p-3 = 0$

$\therefore (p-1)(p+3) = 0$

$\therefore p-1 = 0$ or $p+3 = 0$

$p = 1$ or $p = -3$

p cannot be negative.

So $p = 1$

Hence the value of p is 1.

9. Find the mean age in years from the frequency distribution given below:

Age in years	**25-29**	**30-34**	**35-39**	**40-44**	**45-49**	**50-54**	**55-59**
No. of persons	**4**	**14**	**22**	**16**	**6**	**5**	**3**

Solution:

The given distribution is not continuous.

Adjustment factor = (30-29)/2 = ½ = 0.5

We subtract 0.5 from lower limit of the class interval and add 0.5 to upper limit.

So the new table in continuous form is given below.

Age	Mid value x_i	No. of persons f_i	$f_i x_i$
24.5-29.5	27	4	108
29.5-34.5	32	14	448
34.5-39.5	37	22	814
39.5-44.5	42	16	672
44.5-49.5	47	6	282
49.5-54.5	52	5	260
54.5-59.5	57	3	171
Total		$\Sigma f_i = 70$	$\Sigma f_i x_i = 2755$

Mean = $\Sigma f_i x_i / \Sigma f_i$

= 2755/70
= 39.357
= 39.36
Hence the mean age is 39.36 years.

10. Calculate the Arithmetic mean, correct to one decimal place, for the following frequency

Marks	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
Students	**2**	**4**	**5**	**16**	**20**	**10**	**6**	**8**	**4**

Solution:
Class mark, x_i = (upper class limit + lower class limit)/2

Marks	Students f_i	Class mark x_i	$f_i x_i$
10-20	2	15	30
20-30	4	25	100
30-40	5	35	175
40-50	16	45	720
50-60	20	55	1100
60-70	10	65	650
70-80	6	75	450
80-90	8	85	680
90-100	4	95	380
Total	$\Sigma f_i = 75$		$\Sigma f_i x_i = 4285$

Mean = $\Sigma f_i x_i / \Sigma f_i$
= 4285/75
= 57.133
= 57.1
Hence the mean is 57.1.

11. The mean of the following frequency distribution is 62.8. Find the value of p.

Class	**0-20**	**20-40**	**40-60**	**60-80**	**80-100**	**100-120**
Frequency	**5**	**8**	**p**	**12**	**7**	**8**

Solution:

Class	Frequency f_i	Class mark x_i	$f_i x_i$
0-20	5	10	50
20-40	8	30	240
40-60	p	50	50p
60-80	12	70	840
80-100	7	90	630
100-120	8	110	880
Total	$\Sigma f_i = 40+p$		$\Sigma f_i x_i = 2640+50p$

Mean = $\Sigma f_i x_i / \Sigma f_i$
62.8 = (2640+50p)/(40+p)   [Given mean = 62.8]

62.8(40+p) = 2640+50p
2512+62.8p = 2640+50p
62.8p-50p = 2640-2512
12.8p = 128
p = 128/12.8
∴p = 10
Hence the value of p is 10.

12. The daily expenditure of 100 families are given below. Calculate f1, and f2, if the mean daily expenditure is Rs 188.

Expenditure in Rs	**140-160**	**160-180**	**180-200**	**200-220**	**220-240**
No. of families	**5**	**25**	**f_1**	**f_2**	**5**

Solution:
Given mean = 188

Class	Frequency f_i	Class mark x_i	f_ix_i
140-160	5	150	750
160-180	25	170	4250
180-200	f_1	190	$190f_1$
200-220	f_2	210	$210f_2$
220-240	5	230	1150
Total	$\Sigma f_i = 35+f_1+f_2 = 100$		$\Sigma f_ix_i = 6150+190f_1+210f_2$

Given no. of families = 100
So $35+f_1+f_2 = 100$
∴ $f_1+f_2 = 100-35 = 65$
∴ $f_1 = 65-f_2$ ..(i)

Mean = $\Sigma f_i x_i / \Sigma f_i$
$188 = (6150+190f_1+210f_2)/100$ [Given mean = 188]
$188(100) = 6150+190f_1+210f_2$
$18800 = 6150+190f_1+210f_2$
$18800-6150 = 190f_1+210f_2$
$12650 = 190f_1+210f_2$ ..(ii)
Substitute (i) in (ii)
$12650 = 190(65-f_2)+210f_2$
$12650 = 12350-190f_2+210f_2$
$12650-12350 = -190f_2+210f_2$
$300 = 20f_2$
∴ $f_2 = 300/20 = 15$
Put f_2 in (i)
∴ $f_1 = 65-15$
∴ $f_1 = 50$
Hence the value of f_1 and f_2 is 50 and 15 respectively.

13. The measures of the diameter of the heads of 150 screw is given in the following table. If the mean

diameter of the heads of the screws is 51.2 mm, find the values of p and q .

Diameter in mm	**32-36**	**37-41**	**42-46**	**47-51**	**52-56**	**57-61**	**62-66**
No. of screws	**15**	**17**	**p**	**25**	**q**	**20**	**30**

Solution:

Given mean = 51.2 mm

The given distribution is not continuous.

Adjustment factor = (37-36)/2 = ½ = 0.5

We subtract 0.5 from lower limit of the class interval and add 0.5 to upper limit.

So the new table in continuous form is given below.

Diameter in mm	Mid value x_i	No. of screws f_i	f_ix_i
31.5-36.5	34	15	510
36.5-41.5	39	17	663
41.5-46.5	44	p	44p
46.5-51.5	49	25	1225
51.5-56.5	54	q	54q
56.5-61.5	59	20	1180
61.5-66.5	64	30	1920
Total		$\Sigma f_i = 107+p+q$	$\Sigma f_ix_i = 5498+44p+54q$

Given No. of screws = 150

$\therefore 107+p+q = 150$

$p = 150-107-q$

$p = 43-q$ ..(i)

Mean = $\Sigma f_i x_i / \Sigma f_i$

$\therefore 51.2 = (5498+44p+54q)/150$

$51.2\times150 = 5498+44p+54q$

$7680 = 5498+44(43-q)+54q$

$7680 = 5498+1892-44q+54q$

$7680 -5498-1892 = -44q+54q$

$290 = 10q$

$\therefore q = 290/10 = 29$

Put q in (i)

$p = 43-29$

$\therefore p = 14$

Hence the value of p and q is 14 and 29 respectively.

14. The median of the following numbers, arranged in ascending order is 25. Find x.
11, 13, 15, 19, x + 2, x + 4, 30, 35, 39, 46

Solution:

Here n = 10, which is even

Median = 25

So Median = ½ ($n/2^{th}$ term + $((n/2)+1)^{th}$ term)

$\therefore 25 = ½ ((10/2)^{th}$ term + $(10/2)+1)^{th}$ term)

$\therefore 25 = ½ ((5)^{th}$ term + $(6)^{th}$ term)

$\therefore 25 = ½ (x+2 + x+4)$
$\therefore 25 = ½ (2x+6)$
$\therefore 2x+6 = 25\times2$
$\therefore 2x+6 = 50$
$\therefore 2x = 50-6 = 44$
$\Rightarrow x = 44/2 = 22$
Hence the value of x is 22.

15. If the median of 5, 9, 11, 3, 4, x, 8 is 6, find the value of x.

Solution:
Arranging numbers in ascending order
3,4,5,x,8,9,11
Here n = 7 which is odd
Given median = 6
So Median =((n+1)/2) th term
$\therefore 6 = ((7+1)/2)^{th}$ term
$\therefore 6 = ((8/2)^{th}$ term
$\therefore 6 = 4^{th}$ term
$\therefore 6 = x$
Hence the value of x is 6.

16. Find the median of: 17, 26, 60, 45, 33, 32, 29, 34, 56 .
If 26 is replaced by 62, find the new median.

Solution:
Arranging numbers in ascending order
17,26,29,32,33,34,45,56,60
Here n = 9 which is odd.
So Median =((n+1)/2) th term
$\therefore$ Median = $((9+1)/2)^{th}$ term
$\therefore$ Median = $(10/2)^{th}$ term
$\therefore$ Median = 5^{th} term
$\therefore$ Median = 33
Hence median is 33.

If 26 is replaced by 62, new set of numbers in ascending order is shown below.
17,29,32,33,34,45,56,60,62
Here n = 9 which is odd.
So Median =((n+1)/2) th term
$\therefore$ Median = $((9+1)/2)^{th}$ term
$\therefore$ Median = $(10/2)^{th}$ term
$\therefore$ Median = 5^{th} term
$\therefore$ Median = 34
Hence median is 34.

17. The marks scored by 16 students in a class test are : 3, 6, 8, 13, 15, 5, 21, 23, 17, 10, 9, 1, 20, 21, 18, 12
Find
(i) the median

(ii) lower quartile
(iii) upper quartile

Solution:
Arranging data in ascending order
1,3,5,6,8,9,10,12,13,15,17,18,20,21,21,23
Here n = 16 which is even
(i) So median $= \frac{1}{2}$ ($n/2$ th term + $((n/2)+1)^{th}$ term)
$= \frac{1}{2}$ ($16/2$ th term + $((16/2)+1)^{th}$ term)
$= \frac{1}{2}$ (8 th term + $(8+1)^{th}$ term)
$= \frac{1}{2}$ (8 th term + 9^{th} term)
$= \frac{1}{2}$ (12+13)
$= \frac{1}{2} \times 25$
= 12.5
Hence the median is 12.5.

(ii) Lower quartile, $Q_1 = (n/4)^{th}$ term
= (16)/4
= 4^{th} term
= 6
Hence the lower quartile is 6.

(iii)Upper quartile, $Q_3 = (3n/4)^{th}$ term
$= (3\times16/4)^{th}$ term
$= (3\times4)^{th}$ term
$= 12^{th}$ term
= 18
Hence the upper quartile is 18.

18. Find the median and mode for the set of numbers : 2, 2, 3, 5, 5, 5, 6, 8, 9

Solution:
Here n = 9 which is odd.
$\therefore$Median $= ((n+1)/2)^{th}$ term
$\therefore$Median $= ((9+1)/2)^{th}$ term
$\therefore$Median $= (10/2)^{th}$ term
$\therefore$Median $= 5^{th}$ term
$\therefore$Median = 5

Mode is the number which appears most often in a set of numbers.
Here 5 occurs maximum number of times.
So mode is 5.

19. Calculate the mean, the median and the mode of the following distribution.

Age in years	**12**	**13**	**14**	**15**	**16**	**17**	**18**
No. of students	**2**	**3**	**5**	**6**	**4**	**3**	**2**

Solution:

Age in years x_i	No. of students f_i	Cumulative frequency	f_ix_i
12	2	2	24
13	3	5	39
14	5	10	70
15	6	16	90
16	4	20	64
17	3	23	51
18	2	25	36
Total	$\Sigma f_i = 25$		$\Sigma f_ix_i = 374$

Mean = $\Sigma f_ix_i / \Sigma f_i$
= 374/25
= 14.96
Hence the mean is 14.96.

Here n = 25 which is odd.
∴ Median = $((n+1)/2)^{th}$ term
∴ Median = $((25+1)/2)^{th}$ term
∴ Median = $(26/2)^{th}$ term
∴ Median = 13^{th} term
∴ Median = 15
Hence the median is 15.

Here 15 occurs maximum number of times. i.e., 6 times.
Hence the mode is 15.

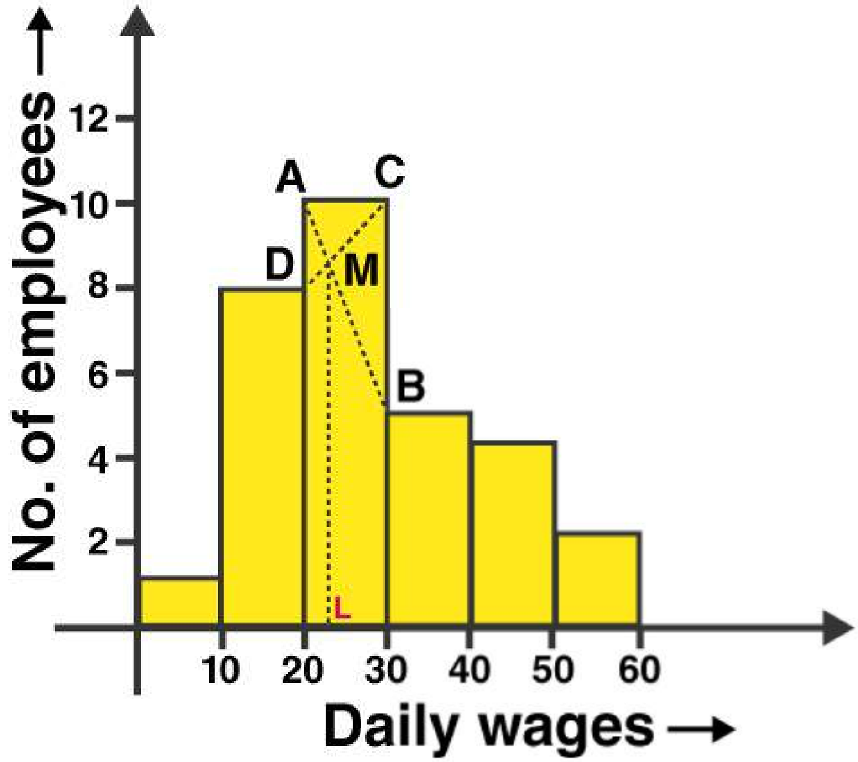
20. The daily wages of 30 employees in an establishment are distributed as follows:

Daily wages in Rs	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
No. of employees	**1**	**8**	**10**	**5**	**4**	**2**

Estimate the modal daily wages for this distribution by a graphical method.
Solution:

Daily wages in Rs.	No. of employees
0-10	1
10-20	8
20-30	10
30-40	5
40-50	4
50-60	2

Taking daily wages on x-axis and No. of employees on the y-axis and draw a histogram as shown below.



Join AB and CD intersecting each other at M. From M draw ML perpendicular to x-axis.
L is the mode
Here Mode = Rs 23
Hence the mode is Rs. 23.

21. Using the data given below, construct the cumulative frequency table and draw the ogive. From the ogive, estimate :
(i) the median
(ii) the inter quartile range.

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**
Frequency	**3**	**8**	**12**	**14**	**10**	**6**	**5**	**2**

Also state the median class

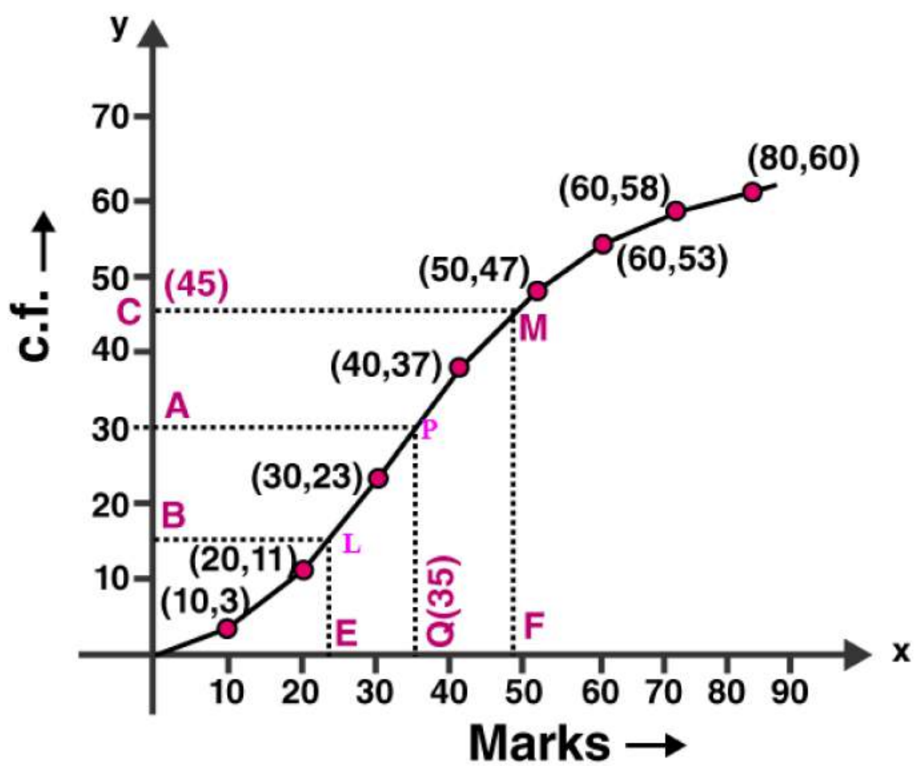
Solution:
Arranging the data in cumulative frequency table.

Marks	Frequency	Cumulative frequency
0-10	3	3
10-20	8	11
20-30	12	23
30-40	14	37
40-50	10	47

50-60	6	53
60-70	5	58
70-80	2	60

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis).

Plot the points (10, 3), (20, 11), (30, 23), (40, 37), (50, 47), (60, 53), (70, 58) and (80, 60) on the graph. Join the points with the free hand. We get an ogive as shown:



(i)Here number of observations, n = 60 which is even.
So median = (n/2) th term
= (60/2) th term
= 30 th term

Mark a point A(30) on Y-axis. From A, draw a horizontal line parallel to X-axis meeting the curve at P. From P draw a line perpendicular to the x-axis which meets it at Q.
∴Q is the median .
Q = 35
Hence the median is 35 .

(ii) Lower quartile, Q_1 = n/4 = 60/4 = 15th term
Upper quartile, Q_3 = 3n/4 = 3×60/4 = 45th term
Mark a point B(15) and C(45) on Y-axis. From B and C, draw a horizontal line parallel to X-axis meeting the curve at L and M respectively. From L and M, draw lines perpendicular to the x-axis which meets it at E and F

respectively.
E is the lower quartile .
E = 22.3
F is the upper quartile.
F = 47
Inter quartile range = Q_3-Q_1
= 47-22.3
= 24.7
Hence interquartile range is 24.7.

22. Draw a cumulative frequency curve for the following data :

Marks obtained	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
No. of students	**8**	**10**	**22**	**40**	**20**

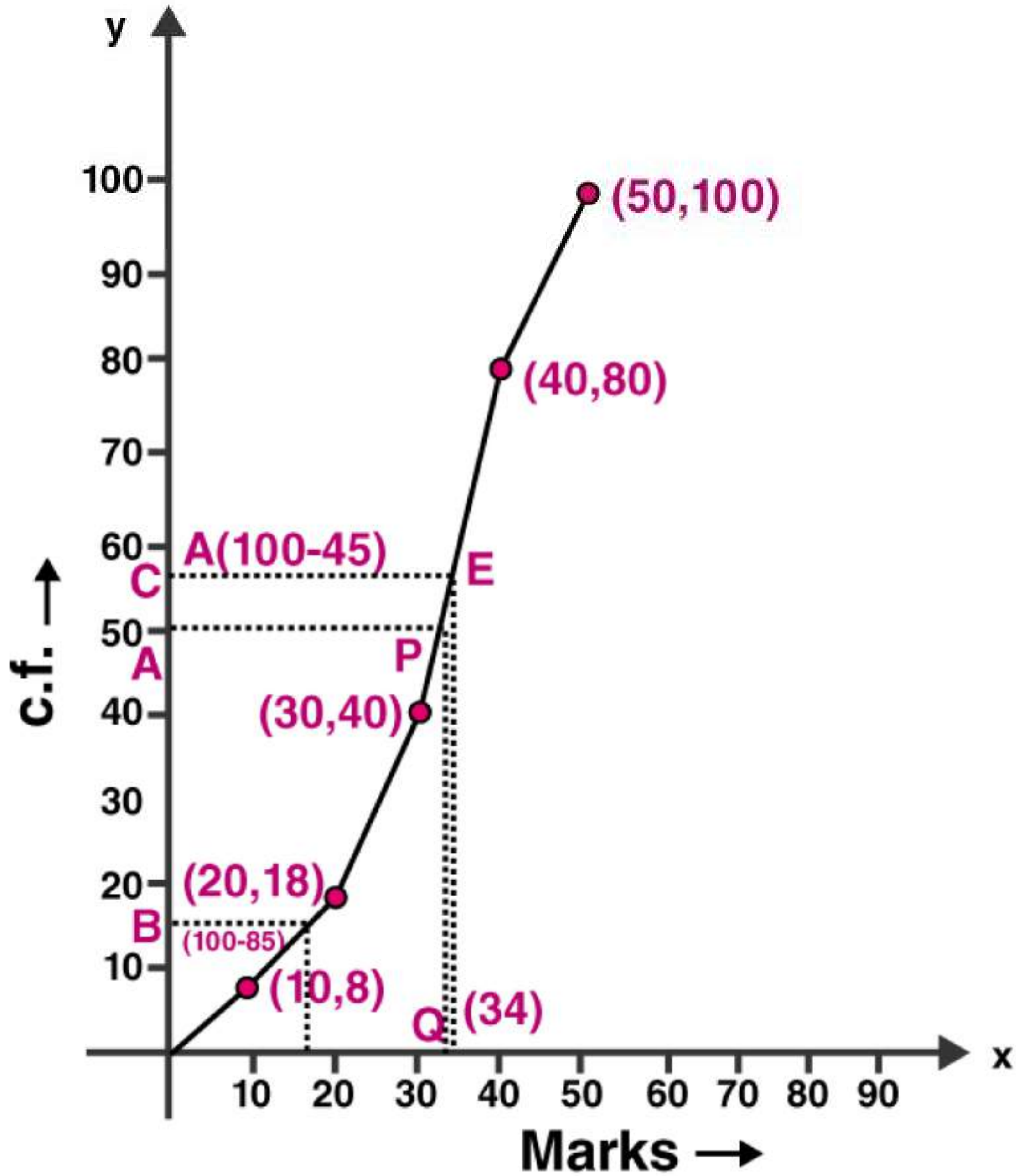
Hence determine:
(i) the median
(ii) the pass marks if 85% of the students pass.
(iii) the marks which 45% of the students exceed.

Solution:
Arranging the data in cumulative frequency table.

Marks obtained	No. of students f	Cumulative frequency
0-10	8	8
10-20	10	18
20-30	22	40
30-40	40	80
40-50	20	100

To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis).

Plot the points (10, 8), (20, 18), (30, 40), (40, 80), and(50,100) on the graph.
Join the points with the free hand. We get an ogive as shown:



(i) Here number of observations, n = 100 which is even.
So median = (n/2) th term
= (100/2) th term
= 50 th term
Mark a point A(50) on Y-axis. From A, draw a horizontal line parallel to X-axis meeting the curve at P. From P, draw a line perpendicular to the x-axis which meets it at Q.

$\therefore$ Q is the median .
Q = 32.5
Hence the median is 32.5 .

(ii)Total number of students = 100
85% of 100 = 85
Remaining number of students = 100-85 = 15
Mark a point B(15) on Y axis. From B, draw a horizontal line parallel to X-axis meeting the curve at L. From L, draw a line perpendicular to the x-axis which meets it at M.
Here M = 18
The pass marks will be 18 if 85% of students passed.

(iii) Total number of students = 100
45% of 100 = 45
Remaining number of students = 100-45 = 55
Mark a point C(55) on Y axis. From C, draw a horizontal line parallel to X-axis meeting the curve at E. From E, draw a line perpendicular to the x-axis which meets it at F.
Here F = 34
Hence marks which 45% of students exceeds is 34 marks.