Chapter 1

Error Analysis and R, L, C Measurements

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Electrical measuring instruments
- Indicating instruments
- Error analysis
- Sources of error
- Power of a factor
- Resistance measurement
- Kelvin's double bridge method
- Potentiometer method
- Measurement of medium resistance
- Substitution method
- Wheatstone bridge method

Electrical Measuring Instruments

Classification

There are many ways in which instruments are classified. Broadly instruments are classified into two.

1. Absolute Instruments
2. Secondary instruments

Absolute Instruments

These instruments give the magnitude of the quantity under measurement in terms of physical constants of the instrument and their deflection.

- No previous calibration or comparison is required.

Secondary Instruments

They are those in which the electrical quantity to be measured can be determined from the deflection of instrument only when they have been pre calibrated against an absolute instrument.

Classification of Secondary Instruments

Indicating Instruments

They indicate the instantaneous value of electrical quantity at the time at which it is being measured. The indications can be by pointers moving over calibrated dial.

Example: Ordinary ammeters, Voltmeters, Wattmeters, etc.

Recording Instruments

They give a continuous record of variations of a quantity over a selected period of time.

- The system carries an ink pen which rests lightly on a chart or graph, which is moved at a uniform—low speed, perpendicular to the deflection of the pen.
- The path traced out by the pen presents a continuous record of the variations in the deflection of the instrument.

Integrating Instruments

These instruments measure and register either the total quantity of electricity (Amp hr) or the total amount of electrical energy in (W-h (or) kWh) supplied to a circuit in a given time, by a set of dials and pointers.

Electrical Principles of Operations

Measuring Instruments are generally classified according to the physical effects of an electric current or potential, utilized in their operation.

The effects generally utilized are

1. Magnetic Effect (Ammeters and Voltmeters)
2. Electrostatic Effect (Voltmeters only)
3. Electrodynamic Effect (Ammeters, Voltmeters and Wattmeter)
4. Electromagnetic Effect (Ammeters, Voltmeters and watt hour meters)

5. Thermal Effect (Ammeters and Voltmeters)
6. Chemical Effect (Amp-hour meter)

Error Analysis

Error is defined as the deviation of the true value from the desired value. It is expressed either as limiting error or as a percentage of error.

1. **Limiting Error (Guarantee Error)**
   Limiting error can be defined as the difference between the true value of the variable and measured value of the variable.
   $$\pm\delta A = A_n - A_a$$
   $A_a \rightarrow$ Actual value (true value)
   $A_n \rightarrow$ Nominal value (measured value)
   $\delta A \rightarrow$ Limiting error (absolute error)
2. **Relative (%) Limiting Error**
   $$\%\text{ Error} = \frac{\text{Limiting error value}}{\text{Nominal value}} \times 100$$
   $$\%\text{ Error} = \frac{\delta A}{A_n} \times 100$$
   $$\%\text{ Error} = \left[\frac{A_n - A_a}{A_n}\right] \times 100$$

Types of Errors

Static Error: It is the numerical difference between the actual value of a quantity and its value as obtained by measurement. They are subdivided into the following types.

1. Gross error
2. Systematic error
3. Random error

(a) **Gross Errors:** It is due to human mistakes in reading or in using instruments or errors in recording observations.
(b) **Systematic Errors:** These errors occur due to short comings of the instrument such as a defective or worn parts or ageing or effects of the environment on the instrument.

1. **Instrumental Errors:** These are inherent in measuring instruments because of their mechanical structure. These can be avoided by
   - Selecting a suitable instrument for the particular measurement application.
   - By applying correction factors after determining the amount of instrumental error of calibrating the instrument against a standard.
2. **Environment Errors:** These are due to conditions external to the measuring device including conditions in the area surrounding the instrument such as effects of changes in temperature, humidity, barometric pressure or effect of magnetic or electrostatic fields.
3. **Observational Errors:** These are errors introduced by the observer. The most common is the parallax error introduced in reading a meter scale and the error of estimation.
4. **Random Errors:** These are due to unknown causes and are normally small and follow the law of probability and are mathematically treated.

Sources of Error

1. Insufficient knowledge of process parameters and design conditions.
2. Poor maintenance.
3. Change in process parameters irregularities, upsets, etc.
4. Design limitations and poor design.
5. Due to person operating the instrument.

Rules

When two or more quantities, each of which are subjected to error are combined.

Sum

Suppose y is the final result of sum of measured quantities p, q, and r each of which are subjected to possible systematic errors $\pm d_p$, $\pm d_q$, $\pm d_r$

$$y = p + q + r$$

Relative limiting error

$$\frac{dy}{y} = \pm\left[\frac{p}{y}\frac{dp}{p} + \frac{q}{y}\frac{dq}{v} + \frac{r}{y}\frac{dr}{r}\right]$$

Resultant systematic error is equal to the sum of the products formed by multiplying the individual systematic errors by the ratio of each terms to the function.

Difference

$$y = p - q$$

Taking log and differentiating with respect to y and simplify, we get

Relative limiting error

$$\frac{\delta y}{y} = \pm\left(\frac{p}{y}\frac{\delta u}{U} - \frac{q}{y}\frac{\delta q}{q}\right)$$

Product

$$y = p.q.r$$

Taking log and differentiating with respect to y and simplify, we get

Relative limiting error

$$\frac{\delta y}{y} = \pm\left(\frac{\delta p}{P} + \frac{\delta q}{q} + \frac{\delta r}{r}\right)$$

Quotient

$$y = p/q$$

Taking logarithm and differentiating with respect to y, we get

Relative limiting error

$$\frac{\delta y}{y} = \pm\left(\frac{\delta p}{p} - \frac{\delta q}{q}\right)$$

Maximum possible error occurs when $\frac{\delta p}{P}$ is positive and $\frac{\delta q}{q}$ is negative or vice versa.

Power of a Factor

$$y = U^n$$

n may be + or _, integral or fraction. Taking logarithm and differentiating w.r.t. y.

Relative Limiting Error is

$$\frac{\delta y}{y} = \pm n\frac{\delta u}{U}$$

if $\quad y = U^n V^m$

$$\frac{\delta y}{y} = \pm\left(\frac{n\delta u}{U} + \frac{m\delta v}{V}\right)$$

Statistical Analysis

1. **Arithmetic mean ($\bar{X}$):** The most probable value of the measured variable

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \cdots + X_n}{n} = \frac{\Sigma X}{n}$$

$\bar{X}$ = arithmetic mean

$X_1, X_2, X_3, \ldots, X_n$ = readings

n = number of readings

2. **Deviation (d):** Departure of the observed reading from arithmetic mean of the group of readings

Deviation $X_1 = d_1 = X_1 - \bar{X}$

Deviation of $X_2 = d_2 = X_2 - \bar{X}$

Algebraic sum of all deviations for a given arithmetic mean = 0.

3. **Average Deviation (D):** Sum of absolute values of deviation divided by number of readings.

$$\bar{D} = \frac{|-d_1| + |-d_2| + 1.....|-d_n|}{n} = \frac{\Sigma|d|}{n}$$

4. **Standard Deviation (S.D.):** The square root of sum of the individual deviation squared divided by number of readings.

$$\text{S.D.} = \sigma = \sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n}} = \sqrt{\frac{\Sigma d^2}{n}}$$

(for $n > 20$)

$$\text{S.D.} = S = \sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n-1}} = \sqrt{\frac{\Sigma d^2}{n-1}}$$

(for $n < 20$)

5. **Variance (V)**

V = (Standard deviation)^2

$$V = \text{S.D.}^2 = \sigma^2 = \frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n}$$

$$V = a^2 = \frac{\Sigma d^2}{n} (\text{for } n > 20)$$

6. **Probable error of one reading**

$$r_1 = 0.6745\ \frac{\sqrt{\Sigma|d|^2}}{n-1}$$

7. **Probable error of mean**

$$r_m = \frac{r_1}{\sqrt{n-1}}$$

Solved Examples

Example 1: The limiting errors of measurement of power consumed and the current passing through a resistance are ± 2.5% and ± 0.5%, respectively, the limiting error of measurement of resistance will then be

(A) ±3.5% (B) ±5%
(C) ±0.5% (D) ±1.0%

Solution: (A)

(Limiting error in the measurement of resistance R) = (Limiting error in measurement of power + 2 × Limiting error in measurement of current)

= ± (2.5 + 2 × 0.5)
= ± 3.5%.

Example 2: A 0–160 V voltmeter has an accuracy of 1% full scale reading. The voltage is 80 V. The limiting error is

(A) 1% (B) 2%
(C) 2.5% (D) 3%

Solution: (B)

Magnitude of error $\partial A = \varepsilon_r A_s$

$= 0.01 \times 160 = 1.6$ V

$$\text{Relative error} = \varepsilon_r = \frac{1.6}{80} = 0.02 \text{ V}$$

Voltage between limits

$$80(1 \pm 0.02) = 80 \pm 1.6 \text{ V}$$

$$\%\text{ limiting error} = \frac{1.6}{80} \times 100 = 2\%.$$

Example 3: A 0–200 V voltmeter has an error of ±2% of full-scale deflection. The range of readings if true voltage is 20 V would be

(A) 16–24 V (B) 19.6–20.4 V
(C) 10–30 V (D) None of the above

Solution: (A)

$$2\% \text{ of F.S.d} = 200 \times \frac{2}{100} = 4$$

Range would be $20 \pm 4 = 16–24$ V

Example 4: If a wattmeter reads 350 W on its 700 W full scale, the range of reading, if it has an error of ± 2% of true value, is

(A) 343–357 (B) 336–364
(C) 300–400 (D) None of the above

Solution: (A)

Basic Terminologies

1. **Accuracy:** It is the closeness with which an instrument reading approaches the true value of the quantity being measured.
   - It is specified in terms of limits of errors and is expressed as either percentage of scale or range of percentage of true value.
2. **Precision:** It is a measure of the reproducibility of the measurements, i.e. for a fixed value of a quantity precision is a measure of the degree of agreement within a group of measurements.
   - An indication of the measurement is obtained from the number of significant figures in which it is expressed.
   - The more the significant figure the greater the precision of measurement.
3. **Linearity:** When the output quantity of a measurement system is directly proportional to the input

   or

   Whenever the deflection θ (output) of the instrument is proportional to the current I (input). The system is said to be linear.
4. **Sensitivity:** It is the ratio of magnitude of the output signal or response to the magnitude of input signal or the quantity being measured.

$$\text{Sensitivity} = \frac{\text{Magnitude output response}}{\text{Magnitude of input}}$$

5. **Deflection Factor (or) Scale Factor (or) Inverse Sensitivity**

$$\text{S.F.} = \frac{\text{Magnitude of input}}{\text{Magnitude of output response}}$$

6. **Dead Time:** Dead time is defined as the time required by a measurement system to begin to respond after the measured quantity has been changed.
7. **Dead Zone:** It is defined as the largest change of input quantity for which there is no output of the instrument. It is also called THRESHOLD. It defines the smallest measurable input.
8. **Resolution:** The smallest increment in input (the quantity being measured).Which can be detected with certainly by an instrument is called resolution.

Resistance Measurement

Classification of Resistance

1. **Low Resistances:** Resistance of the order 1 Ω and below.
2. **Medium Resistances:** Resistance of the range above 1 Ω to 0.1 MΩ.
3. **High Resistances:** Resistance of the order 0.1 MΩ and upwards.

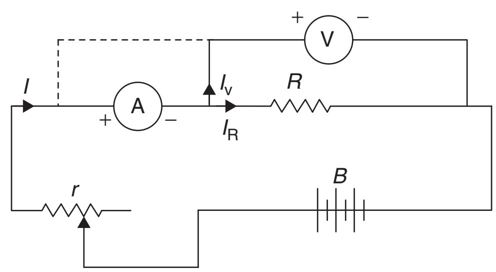
Measurement of Low Resistance

Ammeter–Voltmeter Method

- Voltmeter can be connected in 2 ways.
- Current through ammeter = current through unknown resistance + current through voltmeter.

$$I = I_R + I_V$$

[Steps of operation as explained in section. (III) (1)-measurement of medium resistance]



True value of unknown resistance $R = \dfrac{V}{I_R}$

$$= \frac{V}{I - I_v} = \frac{V}{I - \dfrac{V}{R_v}}$$

$$= \frac{V/I}{\left(1 - \dfrac{V}{IR_v}\right)} = \frac{R'}{\left(1 - \dfrac{R'}{R_v}\right)}$$

$$R = \frac{R'}{\left(1 - \dfrac{R'}{R_v}\right)}$$

where V = Voltage reading voltmeter
I = Current reading in ammeter
R_v = Resistance of voltmeter
R' = Circuit resistance

Accuracy of this method of measurement of low resistance is limited, since it depends on accuracy of voltmeter and ammeter.

Example 5: Given below are the test results for A–V method of measurement of resistance

$V = 200$ V (% error ± 2%)
$I = 16.67$ A (% error ± 2%)

The most approximate value of resistance measured (in Ω) would range between

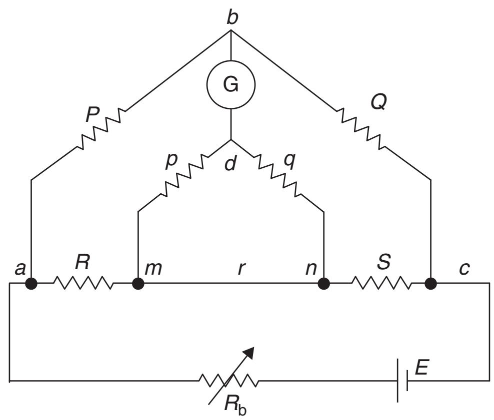
(A) 12 ± 4% (B) 12 ± 2%
(C) 12 exactly (D) Cannot be determined

Solution: (A)
In the A–V method of measurement of resistance. The % errors in ammeter and voltmeter readings would get added up

i.e. % Error = (±2%) + (±2%) = ±4%

$$R = \frac{V}{I} = \left(\frac{200}{16.67}\right) = 12 \pm 4\,\%$$

Kelvin's Double Bridge Method



- Kelvin's bridge is a modification of the wheat stone's bridge and provides increased accuracy.
- It incorporates the idea of second set of ratio arms, hence the name double bridge and the use of 4 terminal resistances for low arms.
- The first of ratio arms is P and Q-Second set of ratio arms p and q is used to connect galvanometer to a point d at the approximate potential between points m and n to eliminate the effect of connecting r between R and S.

Step 1: The ratio P/Q is made equal to p/q under balanced condition $I_G = 0$, which means $E_{AB} = E_{amd}$

$$E_{AB} = \frac{P}{P+Q} E_{AC}$$

$$E_{AB} = \left[\frac{P}{p+q}\right] I \left[R + S + \frac{(p+q)r}{p+q+r}\right] \quad (1)$$

$$E_{amd} = E_{am} + E_{md}$$

$$= IR + V_{mn}\left[\frac{p}{p+q}\right]$$

$$= IR + I(R_{eq})_{mn}\left[\frac{p}{p+q}\right]$$

$$= IR + I\left[\frac{(p+q)r}{p+q+r}\right]\left[\frac{p}{p+q}\right]$$

$$E_{amd} = I\left[R + \frac{p}{p+q}\left[\frac{(p+q)r}{p+q+r}\right]\right]$$

$$= I\left[R + \frac{Pr}{p+q+r}\right] \quad (2)$$

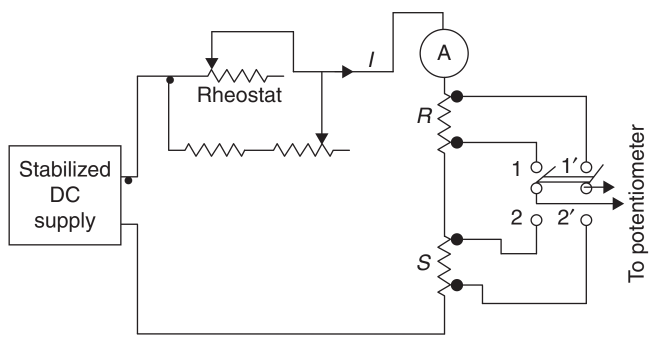
$$\left[\frac{P}{P+Q}\right] I \left[R + S + \frac{(p+q)r}{p+q+r}\right] = I\left[R + \frac{pr}{p+q+r}\right]$$

$$R = \frac{P}{Q}S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]$$

If $P/Q = p/q$ then

$$R = \frac{P}{Q} .\ S$$

Potentiometer Method



- The unknown resistor R is connected in series with standard resistors.
- The current through the circuit is controlled with the help of a rheostat.
- A two-pole double throw switch is used, when switch is in position 1–1′ it connects unknown resistance to potentiometer and let the reading of potentiometer is V_R.

$$V_R = IR \quad (3)$$

- When switch is in position 2–2′ this connects the standard resistor to potentiometer suppose the reading of potentiometer is V_s

$$V_s = IS \quad (4)$$

From (3) and (4) $\frac{R}{S} = \frac{V_R}{V_s}$ $R = \frac{V_R}{V_s}.S$

Factors For Accuracy

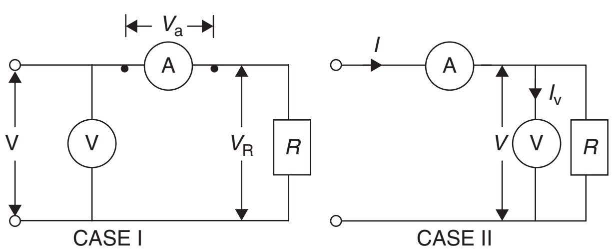
1. It mainly depends upon the assumption that there is no change in the value of current when two different measurements are taken. A stable DC supply is absolutely necessary.
2. S and R should be of same order or range.
3. The ammeter is within the capacity of resistors.

4. It is desirable that the current flowing through circuit be so adjusted that the value of voltage drop across each resistor is of the order of 1 V.

Measurement of Medium Resistance

Ammeter–Voltmeter Method

Two types of connections employed for ammeter voltmeter method are shown.



Case I:

- Ammeter measures true value of the current through resistance.
- Voltmeter does not measure true value across resistance but it indicates the sum of voltage across ammeter and measured resistance.
- Voltage across ammeter $V_a = IR_a$ (where R_a is resistance of ammeter)

Measured value of resistance $R_{m1} = \dfrac{V}{I}$

$$R_{m1} = \frac{V_R + V_a}{I} = \frac{IR + IR_a}{I} = R + R_a$$

True value of resistance $R = R_{m1} - R_a$

$$R = R_{m1}\left(1 - \frac{R_a}{R_{m1}}\right)$$

It is clear that $R = Rm$ only when $R_a = 0$.

$$\text{Relative Error } E_r = \frac{R_{m1} - R}{R} = \frac{R_a}{R}$$

- The error in measurement would be small when $R >>> R_a$.

Note: Case 1 to be used when measuring High resistance values.

Case II:

- Voltmeter measures the true value of voltage but ammeter measures the sum of currents through the resistance and voltmeter.
- Current through voltmeter $I_V = \dfrac{V}{R_V}$

 (R_v is the resistance of voltmeter)
 Measured value of resistance $R_{m2} = \dfrac{V}{I}$

$$= \frac{V}{I_R + I_V} = \frac{V}{\left(\frac{V}{R}\right) + \left(\frac{V}{R_v}\right)} = \frac{R}{1 + \left(\frac{R}{R_v}\right)}$$

True value of resistance $R = \dfrac{R_{m2} R_v}{R_v + R_{m2}}$

$$R = R_{m2}\left(\frac{1}{1 + \frac{R_{m2}}{R_v}}\right)$$

- It is clear that $R = R_{m2}$ only when $R_v = \mu$.

 Since $R_v >>> R_{m2}$ and $\dfrac{R_{m2}}{R_v}$ is very small

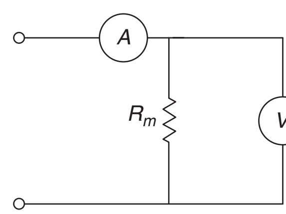
 We can write $R = R_{m2}\,(1 + R_{m2}/R_v)$

 Relative error $E_r = \dfrac{R_{m2} - R}{R} = \dfrac{R_{m2}^2}{R R_v}$

 $E_r = \dfrac{-R}{R_v}$ Error measurement would be small when $R <<< R_v$.

Note: Case 2 to be used to measure Low Resistance Value.

Example 6: The connection diagram for an ammeter voltmeter method of measuring resistance is as given bellow:



The readings of the voltmeter and ammeter are 80 V and 4 mA, respectively. If the voltmeter has a sensitivity of 1000 Ω/V and a f.s.d of 120 V then the percentage error due to relative loading will be.

(A) 16.66% (B) 13.33 %
(C) −16.66% (D) −13.33%

Solution: (C)

Total circuit resistance = $R_t = \dfrac{80}{4 \times 10^{-3}} = 20\ \text{k}\Omega$

Resistance of milli ammeter is very small and hence negligible.

Voltmeter resistance $R_v = 1000 \times 120 = 120\ \text{k}\Omega$
Total circuit resistance = R_v/R_m

$$R_t = \frac{R_v\, R_m}{R_v + R_m}$$

$$R_m = \frac{R_v\, R_t}{R_v - R_t} = \frac{20 \times 120}{120 - 20}$$

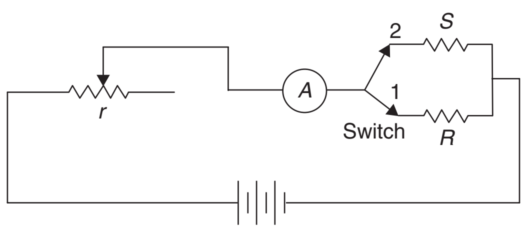
$$= 24\ \%\ \text{error} = \frac{\text{Measured value} - \text{True value}}{\text{True value}} \times 100$$

$$= \frac{20 - 24}{24} \times 100$$

$$= -16.66\%$$

Substitution Method

- In the connection diagram shown below, R is the unknown resistance while S is a standard variable resistance.
  A is an ammeter and r is regulating resistance. There is a switch for putting R and S into circuit alternately.

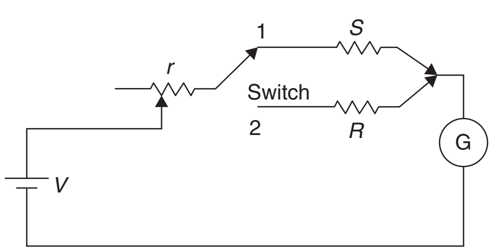


Step 1: Switch is put at position 1 and R is connected to the circuit. r is adjusted till the ammeter pointer is at a chosen scale mark.

Step 2: Switch is thrown to position 2 putting S in the circuit. The value of S is varied till the same deflection as was obtained with R in the circuit is obtained. The setting of the dials is read.

- Since substitution of one resistance for another has left the current unaltered, and provided that the emf of battery and position of r are unaltered the two resistances must be equal.
- Thus $R = S$.
- More accurate than ammeter–voltmeter Method.
- Accuracy is greatly affected if there is any change in battery emf during the time of readings on the two settings taken.
- Accuracy of measurement depends on the constancy of the battery emf and of the resistance of the circuit excluding R and S, upon sensitivity of the instrument, and upon the accuracy with which standard resistance S is known.

Example 7: The experiments set-up given below measures the value of unknown resistance in two steps.



- When $V = 24$ V, r is set at 400 Ω and $S = 80$ Ω, galvanometer shows a deflection of 30°.
- When switch is thrown to position 2 the supply voltage drops down to 22 V and the galvanometer still shows a deflection of 30°.

The value of unknown resistance R is

(A) 80 Ω (B) 60 Ω
(C) 40 Ω (D) None of the above

Solution: (C)

The galvanometer deflects in proportion to the current flowing through the circuit

$$\theta \propto I$$

Step 1: $I = \dfrac{24}{400 + 80} = \dfrac{24}{480} = 0.05$

Step 2: $I = \dfrac{22}{400 + R}$

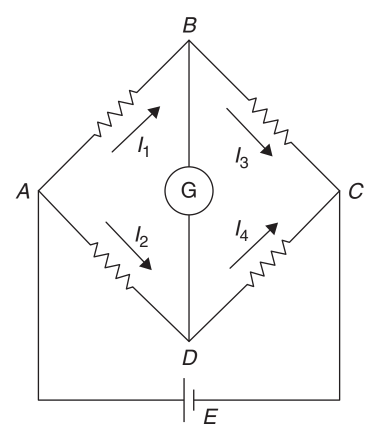
$$= \frac{22}{400 + R} = 0.05$$

$$400 + R = \frac{22}{0.05} = 440$$

$$R = 40\ \Omega.$$

Wheatstone Bridge Method

- Most widely used methods for measurement of medium resistance.
- Wheatstone bridge consists of known variable resistance P, Q and S. The unknown resistance is R.
- The resistance are arranged to form two parallel circuits and a sensitive galvanometer is bridged across two circuits between B and D.
- While balancing the bridge the values of resistance P, Q and S are so adjusted that



$$I_1 P = I_2 R \tag{5}$$

$$I_3 Q = I_4 S \tag{6}$$

But $$I_1 = I_3 = \frac{E}{P+Q}$$

$$I_2 = I_4 = \frac{E}{R+S}$$

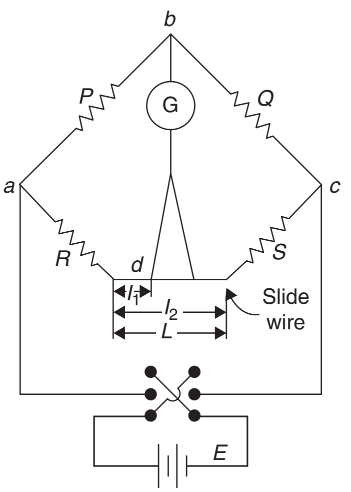
(5) ÷ (6) $$\frac{P}{Q} = \frac{R}{S}$$

$$R = S\,(P/Q)$$

Carey–Foster Bridge Method

- This bridge is especially suited for the comparison of two nearly equal resistances.
- The circuit consists of length L is included between R and S. Resistance P and Q are adjusted so that $\frac{P}{Q}$ is approximately equal to R/S.
- Exact balance is obtained by adjustment of the sliding contact on slide wire. Let l_1 be the distance of the sliding contact from the left hand end of the slide wire.
- R and S are interchanged and balance is again obtained. Let the distance now be l_2.

Let r be resistance/unit length of slide wire.



For first balance $\frac{P}{Q} = \frac{R + l_1 r}{S + (L - l_1) r}$ (7)

For second balance $\frac{P}{Q} = \frac{S + l_2 r}{R + (L - l_2) r}$ (8)

Adding (7) and (8) and add 1 on the both sides

$$\frac{R + l_1 r}{S + (L - l_1) r} + 1 = \frac{S + l_2 r}{R + (L - l_2) r} + 1$$

$$\frac{R + S + L_r}{S + (L - l_1) r} = \frac{S + R + L_r}{R + (L - l_2) r}$$

$$(S - R) = (l_1 - l_2)\ r$$

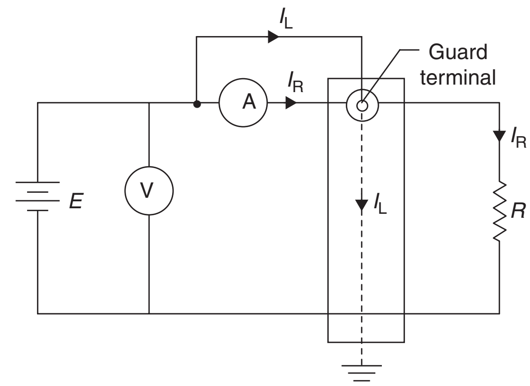
- Thus, S–R is obtained from resistance per unit length of the slide wire together with $(l_1 - l_2)$.
- The slide wire is then calibrated, i.e. r is obtained and again determining the difference in length $(l_1' - l_2')$.
- Suppose S is known as S' is its value when shunted by a known resistance.

$$\frac{(S - R)}{l_1 - l_2} = \frac{S' - R}{l_1' - l_2'}$$

$$\Rightarrow \qquad R = \frac{S(l_1' - l_2') - S'(l_1 - l_2)}{(l_1' - l_2' - l_1 + l_2)}$$

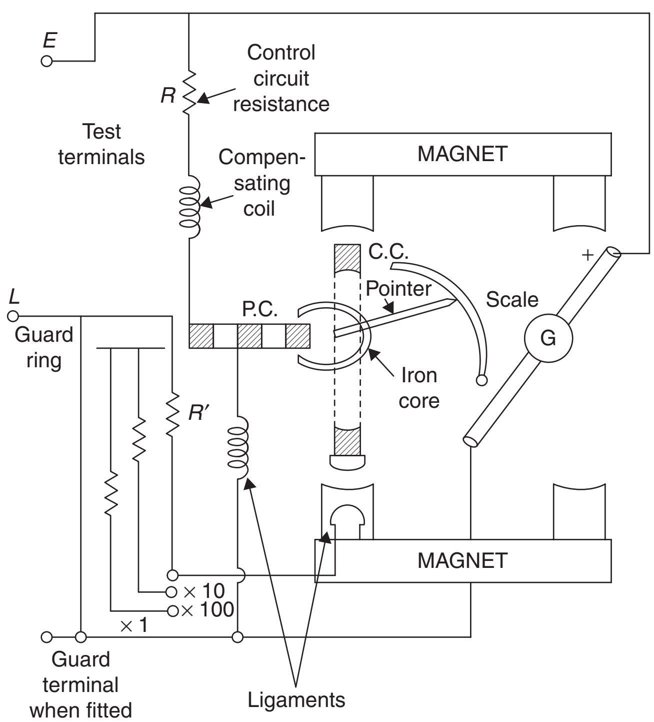
Measurement of High Resistance

Direct Deflection Method



- A guard terminal surrounding resistance terminal is connected to the battery side and micro ammeter.
- The guard terminal and resistance terminal are almost at the same potential and hence there is no flow of current between them.
- The leakage current I_L which would otherwise flow through the μ ammeter bypasses the μ ammeter.
- The μ ammeter indicates the current I_R only.
- The resistance values is determined by the readings of voltmeter and ammeter as $R = \frac{V}{I_R}\ \Omega$.

Megger Method



Construction

- It consists of a hand-driven DC generator and a direct reading ohmmeter.

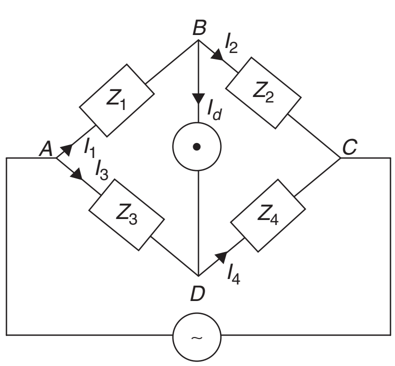
- Permanent element consist of three coils, 1 current coil, potential or pressure coil and compensating coil.
- The coils are connected to the circuit by flexible leads or ligaments.
- *E* and *L* are test terminals.
- Current coil is connected in series with terminal *L*.
- *R′* protects the current coil in case the test terminals are short circuited and controls the range of the instrument.
- Pressure coil (PC) in series with a compensating coil and protection resistance *R* is connected across generator terminals.
- Compensating coil is provided for better scale proportions and to make instrument astatic.

Working

- When, current from the generator flows through the pressure coil, the coil tends to align itself at right angles to the permanent magnet field.
- When test terminals are open, i.e. infinite resistance no current flows through current coil and pressure coil governs the motion of moving elements causing it to move to its extreme counter clockwise position, i.e. infinite resistance. Current coil produces clockwise torque on moving element.
- When terminals *L* and *E* are short circuited, the current flowing through the current coil (if external resistance = 0) is large enough to move the pointer to its extreme clockwise position zero.
- For any resistance connected between L and E, the opposing torque of the coils balances each other so that pointer comes to rest at some intermediate point on the scale.

Bridges

- Measurement of inductance and capacitance may be made conveniently and accurately by employing AC bridge networks.
- The AC bridge is a modified form of the Wheatstone bridge. It consists of four arms, a source of excitation and a balance detector which is sensible to small alternating potential differences.



- If *B* and *D* are at same potential $V_{AB} = V_{AD}$

$$I_1Z_1 = I_3Z_3 \tag{9}$$

- In this condition $V_{BC} = V_{DC}$

$$I_2Z_2 = I_4Z_4 \tag{10}$$

Divide (9) by (10)

$$\frac{I_1Z_1}{I_2Z_2} = \frac{I_3Z_3}{I_4Z_4}$$

But in balance condition $I_D = 0$.

$$I_1 = I_2 \text{ and } I_3 = I_4.$$

Also

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$Z_1Z_4 = Z_2Z_3$$

In polar form, $Z_1Z_4(\angle\theta_1 + \theta_4) = Z_2Z_3(\angle\theta_3 + \theta_2)$

Two conditions for balancing bridge

1. $Z_1Z_4 = Z_2Z_3$
2. $\angle\theta_1 + \theta_4 = \angle\theta_2 + \theta_3$

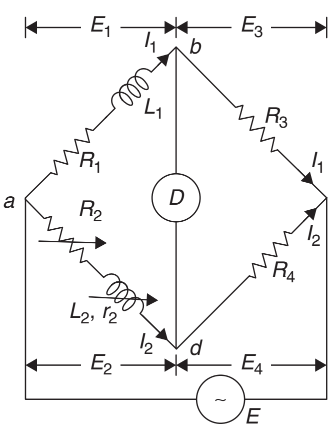
Measurement of Inductance

Measurement of Self-inductance

Maxwell's Inductance Bridge

- This bridge circuit measures an inductance by comparing with a variable standard self-inductance.

Let L_1 = Unknown inductance of resistance R_1
L_2 = Unknown inductance of fixed resistance r_2
R_2 = Variable resistance connected in series with inductor L_2.
R_3, R_4 = Known non-inductive resistances



From concept of bridges (1.3)

$$Z_1Z_4 = Z_2Z_3$$

$$(R_1 + j\omega L_1)\, R_4 = (R_2 + j\omega L_2)\, R_3$$

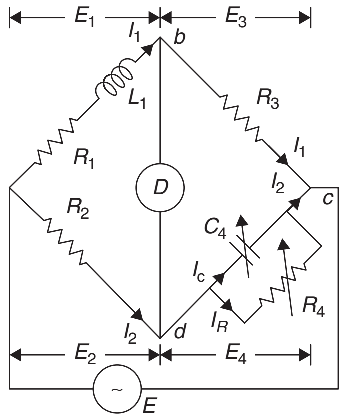
Equating real and imaginary parts separately

$$R_1R_4 = R_2R_3 \text{ (or) } R_1 = \left(\frac{R_3}{R_4}\right).(R_2 + r_2)$$

$$j\omega L_1\, R_4 = j\omega L_2\, R_3 \text{ (or) } L_1 = \left(\frac{R_3}{R_4}\right)L_2$$

Maxwell's Wien Bridge

- In this bridge, an inductance is measured by comparison with a standard variable capacitance.
- The connection diagram is as given below.



$$Z_1 = R_1 + j\omega L_1;\ Z_3 = R_3$$

$$Z_2 = R_2\ Z_4 = \cfrac{1}{\cfrac{1}{R_4} + \cfrac{1}{jxc}}$$

$$Z_4 = \cfrac{1}{\cfrac{1}{R_4} + j\omega C_4} = \frac{R_4}{1 + j\omega C_4 R_4}$$

At balance

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_2\ R_3$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3\ C_4 R_4.$$

Equating real and imaginary parts

$$R_1 = \frac{R_2\ R_3}{R_4}$$

$$L_1 = C_4 R_2 R_3$$

Advantages

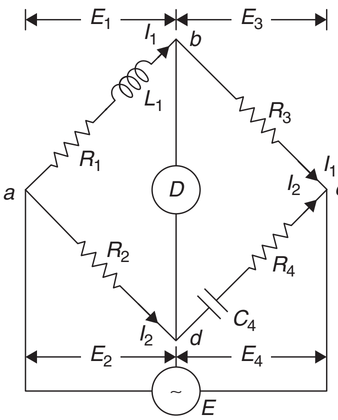
1. If we choose R_4 and C_4 as variable elements, the two balance equations are independent.
2. Frequency does not appear in any of the equations.
3. This bridge yields simple expressions for L_1 and R_1 in terms of known bridge elements.
4. This bridge is useful for measurement of a wide range of inductance at power and audio frequencies.

Disadvantages

1. Bridge requires a variable standard capacitor which may be very expensive if calibrated to a high degree of accuracy.

Hay's Bridge

- It is modification of Maxwell's Bridge.
- It uses a resistance in series with a standard capacitor.



L_1 = Unknown inductance having a resistance

R_2, R_3, R_4 = Known non-inductive resistance

C_4 = Standard capacitor

$$Z_1 = R_1 + j\omega L_1,\ Z_2 = R_2,\ Z_3 = R_3$$

$$Z_4 = (R_4 - j/\omega C_4)$$

At balance

$$(R_1 + j\omega L_1)\ (R_4 - j/\omega C_4) = R_2 R_3$$

$$(R_1 R_4) + \frac{L_1}{C_4} + j\omega L_1 R_4 - jR_1/\omega C_4 = R_2\ R_3$$

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \tag{11}$$

$$\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0 \tag{12}$$

Solving (11) and (12), we get

$$L_1 = \frac{R_2\ R_3\ C_4}{1 + \omega^2\ R_4^2\ C_4^2}$$

$$R_1 = \omega^2\ R_4\ C_4 \times \frac{R_2\ R_3\ C_4}{1 + \omega^2\ R_4^2\ C_4^2}$$

$$R_1 = \frac{\omega^2 R_2\ R_3\ R_4\ C_4^2}{1 + \omega^2\ R_4^2\ C_4^2}$$

Q-factor of the coil $Q = \omega L_1/R_1 = \dfrac{1}{\omega C_4 R_4}$

$$\therefore \quad L_1 = \frac{R_2\ R_3\ C_4}{1 + (1/Q)^2}$$

Advantages

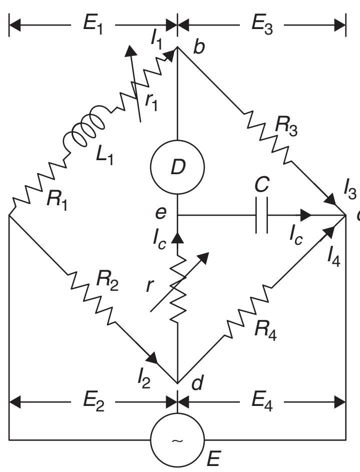
1. This bridge gives very simple expression for unknown inductance for high Q coils and is suitable for coils with $Q > 10$.
2. The expression for Q-factor is simple small when compared to Maxwell's bridge which requires a parallel R_4 with a very high value.

Disadvantages

1. This bridge is suitable for measurement of high Q inductors ($Q > 10$).
2. For inductor with $Q < 10$ this bridge is not suited and Maxwell's bridge is preferred.

Anderson's Bridge

- It is a modification of Maxwell's Wien bridge.
- In this bridge self-inductance is measured in terms of a standard capacitor.



Let
L_1 = Inductance to be measured
R_1 = Resistance of L_1
r_1 = Resistance connected in series with L_1
r, R_2, R_3, R_4 = Known non-inductive resistor
C = Fixed standard capacitor
$I_1 = I_3$ and $I_2 = I_c + I_4$

At balance,

$$I_1 R_3 = I_c \times \frac{1}{j\omega c}$$

$$\therefore \quad I_c = I_1 j\omega C R_3$$

Writing the other balance equations

$$I_1 (r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_c r \tag{13}$$

and
$$I_c \left(r + \frac{1}{j\omega c} \right) = (I_2 - I_c) R_4 \tag{14}$$

Sub value of I_c in (14) and (13),we get

$$I_1 (r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_1 j\omega C R_3 r$$

$$I_1 (r_1 + R_1 + j\omega L_1 - j\omega C R_3 r) = I_2 R_2 \tag{15}$$

$$J\omega C R_3 I_1 \left(r + \frac{1}{j\omega c} \right) = (I_2 - I_1 j\omega C R_3) R_4$$

$$I_1 (j\omega C R_3 r + j\omega C R_3 R_4 + R_3) = I_2 R_4 \tag{16}$$

From (15) and (16), we can write

$$I_1 (r_1 + R_1 + j\omega L_1 - j\omega C R_3 r)$$

$$= I_1 \left(\frac{R_2 R_3}{R_4} + \frac{j\omega C R_2 R_3 r}{R_4} + j\omega C R_2 R_3 \right)$$

Equating real and imaginary parts,

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

$$L_1 = C \frac{R_3}{R_4} [\, r (R_4 + R_2) + R_2 R_4 \,]$$

Advantages

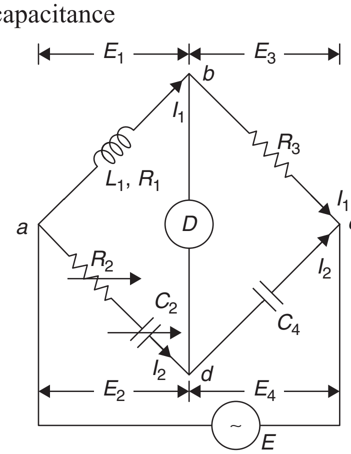
1. In case adjustments are made by manipulating control over r_1 and r_2, they become independent of each other. This is a marked superiority over sliding conditions met with low Q coils when measuring with Maxwell's bridge.
2. For low Q coils, it is much easier to attain balance in case of Anderson's Bridge than in Maxwell's bridge.
3. A fixed capacitor can be used instead of a variable capacitor.
4. Bridge may also be used to determine capacitance in terms of inductance.

Disadvantages

1. It is more complex than its prototype Maxwell's bridge. Anderson's bridge has more parts and is more complicated to set up and manipulate.
2. Balance equations are not simple and in fact are much more tedious.
3. An additional junction point increases the difficulty of shielding the bridge.

Owen's Bridge

- This bridge may be used for measuring inductance in terms of capacitance



L_1 = Unknown self-inductance of resistance R_1
R_2 = Variable non-inductive resistor
R_3 = Fixed non-inductive resistor
C_2 = Variable standard capacitor
C_4 = Fixed standard capacitor

At balance

$$(R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = \left(R_2 + \frac{1}{j\omega C_2}\right)R_3$$

Equating real and imaginary terms,

$$L_1 = R_2 R_3 C_4$$

$$R_1 = R_3 \frac{C_4}{C_2}$$

Advantages

1. Examining the equations for balance we obtain two independent equations in case C_2 and R_2 are made variable. Since R_2 and C_2 are in the same arm, convergence to balance condition is much easier.
2. The balance equations are quite simple and do not contain any frequency component.
3. A wide range of inductance can be measured.

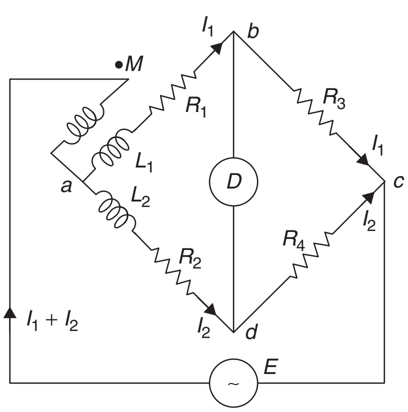
Disadvantages

1. The variable capacitor is expensive and its accuracy is about 1%.
2. For high Q-coils, the value of C_2 tends to become target.

Measurement of Mutual Inductance

Heaviside's Bridge

- This bridge measures mutual inductance in terms of known self-inductance.



M = Unknown mutual inductance
L_1 = Self-inductance of secondary of M
L_2 = Known self-inductance
R_1, R_2, R_3, R_4 = Non-inductive resistors

At balance $\quad V_{BC} = V_{DC}$

$$I_1R_3 = I_2R_4$$

$$(I_1 + I_2)\, j\omega M + I_1(R_1 + R_3 + j\omega L_1)$$
$$= I_2(R_2 + R_4 + j\omega L_2)$$

$$I_2\left(\frac{R_4}{R_3}+1\right)j\omega M + I_2\left(\frac{R_4}{R_3}\right)(R_1 + R_3 + j\omega L_1)$$
$$= I_2(R_2 + R_4 + j\omega L_2)$$

$$j\omega M\left(\frac{R_4}{R_3}+1\right) + \frac{R_4}{R_3}.R_1 + R_4 + j\omega L_1\frac{R_4}{R_3}$$
$$= R_2 + R_4 + j\omega.L_2$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

$$M = \frac{L_2 - L_1\dfrac{R_4}{R_3}}{\dfrac{R_4}{R_3}+1} = \frac{R_3 L_2 - R_4 L_1}{R_3 + R_4}$$

For the value of M to be measured, the value of L must be known.

If $R_3 = R_4$, we get $M = \dfrac{L_2 - L_1}{2}$ and $R_1 = R_2$

This method can be used for measuring self-inductance.

$$L_2 = \frac{M(R_3 + R_4) + R_4 L_1}{R_3}$$

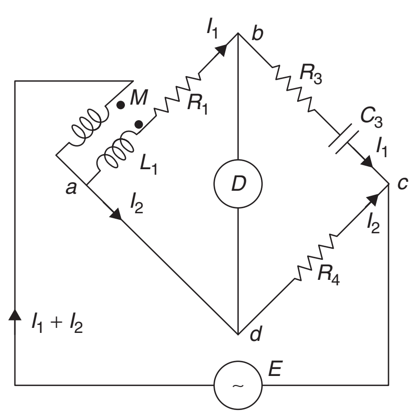
$$= M\left(1+\frac{R_4}{R_3}\right)+\left(\frac{R_4}{R_3}\right)L_1$$

$$R_2 = R_1R_4/R_3$$

If $R_4 = R_3 L_2 = 2M + L_1$ and $R_2 = R_1$

Carey Foster Bridge

- The bridge is used for the measurement of standard capacitance in terms of mutual inductance and mutual inductance in terms of standard capacitance.



At balance $V_{AB} = V_{AD} = 0$

$$I_1\,[R_1 + j\omega L_1] - j\omega M\,[I_1 + I_2] = 0$$

$$I_1\,[R_1 + j\omega L_1] = j\omega M\,(I_1 + I_2) \tag{17}$$

Also
$$V_{BC} = V_{CD}$$

$$I_1\left(R_3 - \frac{j}{\omega C_3}\right) = I_2 R_4$$

$$I_2 = \frac{I_1\left(R_3 - \frac{j}{\omega C_3}\right)}{R_4} \tag{18}$$

Sub (18) from (17)

$$I_1\,[R_1 + j\omega L_1] = j\omega M\left(I_1 + \frac{I_1\left(R_3 - \frac{j}{\omega C_3}\right)}{R_4}\right)$$

$$R_1 + j\omega L_1 = j\omega M\left(\frac{R_3 - \frac{j}{\omega C_3}}{R_4} + 1\right)$$

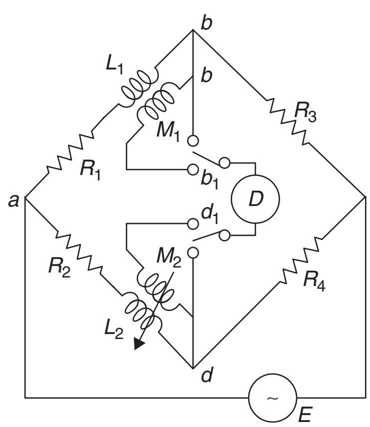
Equating real and imaginary parts, we get

$$R_1 = \frac{M}{C_3\,R_4} \quad \text{or} \quad C_3 = \frac{M}{R_1\,R_4}$$

$$L_1 = M\left(\frac{R_3}{R_4} + 1\right) \quad \text{or} \quad M = \frac{L_1}{(1 + R_3/R_4)}$$

Campbell's Bridge

- This bridge measures an unknown mutual inductance in terms of a standard mutual inductance.



$M_1 \rightarrow$ Unknown mutual inductance
$L_1 \rightarrow$ Self-inductance of secondary of M_1
$M_2 \rightarrow$ Variable standard mutual inductance
$L_2 \rightarrow$ Self-inductance of secondary M_2
$R_1, R_2, R_3, R_4 \rightarrow$ Non-inductive resistances.

There are two steps in balancing process.

1. Detector is connected between b and d. The circuit now becomes a simple self-inductance comparison bridge. The requirement for balance is

$$\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

   Bridge may be balanced by R_3 (or R_4) and R_1 (or R_2).

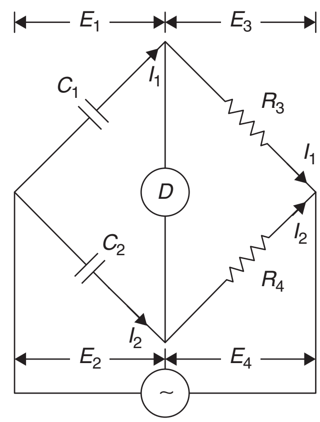
2. Detector is connected between b' and d' keeping adjustments as in step 1 above, the variable mutual inductance M_2 is varied to get balance $\frac{M_1}{M_2} = \frac{R_3}{R_4}$

$$M_1 = M_2\left(\frac{R_3}{R_4}\right)$$

Measurement of Capacitance

De Sauty's Bridge

- This bridge is the simplest method of comparing two capacitances.



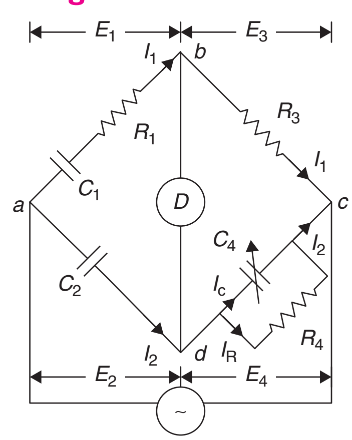
C_1 = Capacitor whose capacitance is to be measured
C_2 = A standard capacitor
R_3, R_4 = Non-inductive resistors

At balance

$$\left(\frac{1}{j\omega C_1}\right)R_4 = \left(\frac{1}{j\omega C_2}\right)R_3$$

$$C_1 = C_2\left(\frac{R_4}{R_3}\right)$$

Schering Bridge



C_1 = Capacitor whose capacitance is to be determined
r_1 = Series resistance representing loss in C_1
C_2 = Standard capacitor,
R_2 = Non-inductive resistance

C_4 = A variable capacitor
R_4 = Variable non-inductive resistance in parallel with variable capacitor C_4.

$$Z_1 = \left(r_1 + \frac{1}{j\omega C_1}\right) Z_2 = \frac{1}{j\omega C_2}$$

$$Z_3 = (R_3)\ Z_4 = \left(\frac{R_4}{1 + j\omega C_4 R_4}\right)$$

At balance

$$\left(r_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = \frac{1}{j\omega C_2} . R_3$$

$$\left(r_1 + \frac{1}{j\omega C_1}\right) R_4 = \frac{R_3}{j\omega C_2}(1 + j\omega C_4 R_4)$$

$$r_1 R_4 - \frac{jR_4}{\omega C_1} = -j\frac{R_3}{\omega C_2} = \frac{R_3 R_4 C_4}{C_2}$$

Equating real and imaginary terms

We obtain $r_1 = R_3 C_4 / C_2$

$$C_1 = C_2\left(\frac{R_4}{R_3}\right)$$

Two independent balance equations are obtained if C_4 and R_4 are chosen as variable elements.

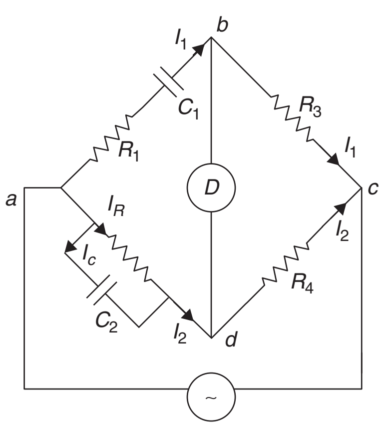
Dissipation factor $D_1 = \tan\delta = \omega C_1 r_1 = \omega C_2 \left(\frac{R_4}{R_3}\right)$

$D_1 = \omega C_4 R_4$

δ = Loss angle of the capacitor

$\sin\delta$ = Power factor of the capacitor

Wien's Bridge



- Wien's bridge is primarily used for the measurement of frequency.
- Its balance equation involves frequency even the performance of individual bridge elements is independent of frequency.
- It may be employed in harmonic distortion analyser, audio and HF oscillators.

$$Z_1 = R_1 - \frac{j}{\omega C_1} \quad Z_2 = \frac{R_2}{1 + j\omega C_2 R_2} S$$

$$Z_3 = R_3\ Z_4 = R_4$$

At balance

$$\left(R_1 - \frac{j}{\omega C_1}\right) R_4 = R_3 \frac{R_2}{(1 + j\omega C_2 R_2)}$$

$$R_4\left(R_1 - \frac{j}{\omega C_1}\right)(1 + j\omega C_2 R_2) = R_2 R_3$$

Equating real and imaginary terms,

$$R_1 R_4 + R_2 R_4 \frac{C_2}{C_1} = R_2 R_3$$

$$\frac{C_2}{C_1} = \frac{R_3}{R_4} - \frac{R_1}{R_2}$$

$$\frac{-R_4}{\omega C_1} + \omega C_2 R_1 R_2 R_4 = 0$$

$$\omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$$C_1 = \frac{1}{\omega^2 R_1 R_2 C_2}$$

For measurement of frequency

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad \text{and} \quad f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \text{Hz}$$

Example 8: In Wien's bridge method of measurement of frequency, among the following; the combination of bridge elements which could not result in frequency measurement of 2.5 kHz would be.

1. C_1 = 0.2 μF, C_2 = 0.8 μF
   R_1 = 100 Ω, R_2 = 253.3 Ω
2. C_1 = 0.1 μF, C_2 = 0.8 μF
   R_1 = 200 Ω, R_2 = 253.3 Ω
3. C_1 = 0.8 μF, C_2 = 3.16 × 10^{-4} F
   R_1 = 200 Ω, R_2 = 253.3 Ω

(A) 1 only (B) 2 only
(C) 3 only (D) 1, 2, and 3

Solution: (C)

Wien's Bridge method to calculate frequency

$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

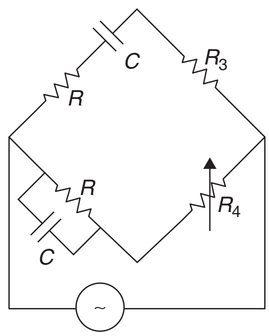
$$1.\ = \frac{1}{2\pi\sqrt{0.2\,\mu \times 0.8\,\mu \times 100 \times 253.3}} = 2.5 \text{ kHz}$$

$$2.\ = \frac{1}{2\pi\sqrt{0.8\,\mu \times 0.1\,\mu \times 200 \times 253.3}} = 2.5 \text{ kHz}$$

3. $= \dfrac{1}{2\pi\sqrt{0.8\mu\times 3.16\times 10.9\times 200\times 258.3}} = 44.47 \text{ kHz}$

1 and 2 would measure 2.5 kHz
(3) alone measures 44.47 Hz

Example 9: If R and C are variables connected as shown in the Wien bridge given which of the following is true, when bridge is balanced and $f = \dfrac{1}{2\pi CR}$



(A) $R_4 = R_3$
(B) $R_4 = 2R_3$
(C) $R_4 = R_3/2$
(D) $R_4 = 3R_3$

Solution: (C)
At balance

$$R_4\left(R + \frac{1}{j\omega C}\right) = R_3\left(\frac{R\cdot\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\right)$$

$$\frac{R_4(1 + j\omega RC)}{j\omega C} = \frac{R_3 R}{(1 + j\omega RC))}$$

$$R_4\left(-C^2\omega^2R^2 + 1 + 2j\omega\ RC\right) = jR_3RC_\omega$$

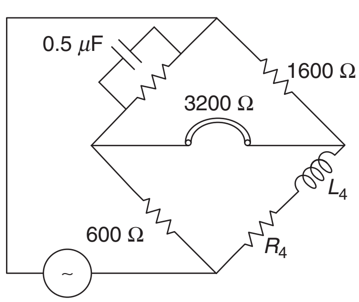
Equating real and imaginary terms

$$2\omega\ CR\ R_4 = \omega\ CR\ R_3$$

$$R_4 = \frac{R_3}{2}$$

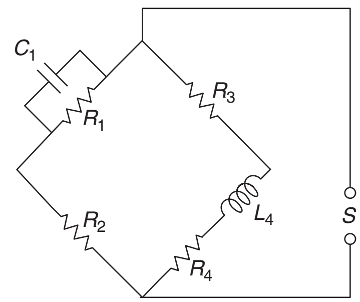
Example 10: In the Maxwell's bridge, the component values shown are when the bridge is balanced determines R_4 and L_4

(A) 300 Ω, 0.48 H
(B) 30 Ω, 4.8H
(C) 37.5 Ω, 75 mH
(D) 75Ω, 7.5 mH



Solution: (A)
At balance

$$\left(\frac{R_1\cdot\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}\right)(R_4 + j\omega L_4) = R_2\ R_3$$



$$R_1R_4 + j\omega L_4R_1 = R_2R_3 + j\ \ R_2R_3R_1C_1\omega$$

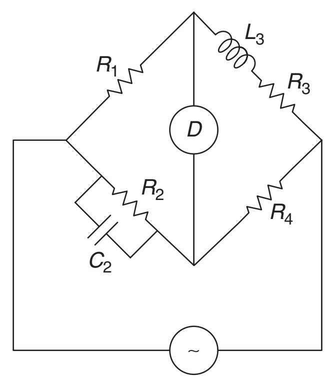
$$R_4 = \frac{R_2R_3}{R_1}\quad L_4 = R_2R_3C_1$$

for the given problem

$$R_1 = 1600\times\frac{600}{3200} = 300\ \Omega$$

$$L_1 = 1600\times 600\times 0.5\times 10^{-6}$$
$$= 0.48 \text{ H}$$

Example 11: For a balanced bridge shown below what are the expressions for R_3, L_3, Q_3, respectively.



(A) $\dfrac{R_4}{R_1}R_2, \dfrac{R_4\ R_1}{C_2}, \omega C_2 R_2$

(B) $\dfrac{R_2}{R_4}.R_1, \dfrac{R_1\ R_4}{C_2}, \omega C_2 R_2$

(C) $\dfrac{R_1\ R_4}{R_2}, R_1\ R_4\ C_2, \omega\, R_2C_2$

(D) $\dfrac{R_2\ R_1}{R_4}, R_1\ R_4\ , C_2, \omega\, R_2C_2$

Solution: (C)

At balance $R_1R_4 = (R_3 + j\omega L_3)\left(\dfrac{R_2}{1 + j\omega R_2 C_2}\right)$

$$(1 + j\omega R_2C_2)(R_1 R_4) = R_2 (R_3 + j\omega L_3)$$

Equating real and imaginary parts

$$R_1R_4 = R_2R_3$$

$$R_2 = \left(\frac{R_1 R_4}{R_3}\right)$$

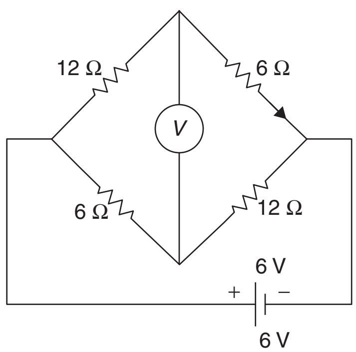
$$\omega C_2R_2R_1R_4 = \omega R_2L_2$$

$$L_2 = R_1 R_4 C_2$$

$$Q_2 = \omega C_2R_2$$

Example 12: The reading of high impedance voltmeter in the given bridge is

(A) Zero (B) 4 V
(C) 6.66 V (D) 2 V



Solution: (D)

Current in either branch is $I = \dfrac{6}{18} = \left(\dfrac{1}{3}\right)$ A

$$V_{BC} = 12\times\frac{1}{3} - 6\times\frac{1}{3} = 2 \text{ V}.$$

Example 13: A slide potentiometer has 5 wires 2 m each. With the help of a standard voltage source of 1.234 V, it is standardized by keeping the jockey at 123.4 cm. If resistance of potentiometer wire is 1000 Ω, then the value of working current will be

(A) 1 mA (B) 0.5 mA
(C) 0.1 mA (D) 10 mA

Solution: (D)

Total length of slide wire = 5 × 2

= 10 m = 1000 cm

Total resistance of slide wire = 1000 Ω

Resistance per cm = 1 Ω

Resistance of 123.4 cm wire = 123.4 Ω

Which corresponds to a voltage of 1.234 V

$$\text{Current} = \frac{1.234}{123.4} = 10 \text{ mA}.$$

Exercises

Practice Problems I

1. The Maxwell's inductance capacitance bridge is used to measure inductance of
   (A) Low Q coils
   (B) High Q coils
   (C) Both (A) and (B)
   (D) None

2. Match list I (terms) with List II definitions and select the correct answer using the codes given below

List I	List II
A. Resolution	1. Closeness with which the instrument reading approaches the true value
B. Sensitivity	2. Reproducibility of measurements
C. Accuracy	3. Smallest change in measured value to which the instrument can respond
D. Precision	4. Ratio of response of the instrument to the input variable.

Codes

(A) A B C D
4 3 1 2

(B) A B C D
3 4 2 1

(C) A B C D
4 3 2 1

(D) A B C D
3 4 1 2

3. Which of the following statements are correct? The application of instrument in wrong manner in the procedure of measuring result in an AC
   (A) Systematic error
   (B) Instrument error
   (C) Random error
   (D) Gross error

4. Commonly used detectors for AC bridges are
   (A) Headphones
   (B) Vibration galvanometer
   (C) Tunable amplifier detector
   (D) All of the above

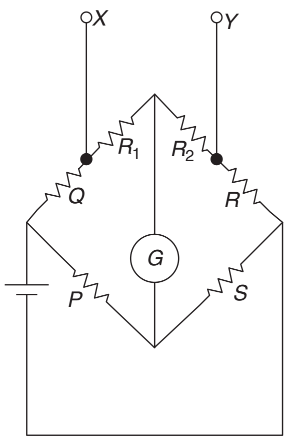
5. Slide wire in a potentiometer is made of
   (A) Platinum–silver alloy
   (B) Silver alloy
   (C) Chromium alloy
   (D) Lead alloy

6. Accuracy of an instrument is
(A) The measure of consistency and reproducibility of measurements
(B) The smallest change in measurableinput
(C) The ratio of change in output signal to the change in input signal
(D) The closeness with which an instrument reading approaches the true value of the quantity being measured

7. A symmetrical square-wave voltage is read on an average response electronic voltmeter whose scale is calibrated in terms of RMS value of a sinusoidal wave. The error in the reading is
(A) −3.1% (B) +6.4%
(C) −11% (D) +11%

8. The current coil of a wattmeter has a resistance of 0.05 Ω and its pressure coil has a resistance of 10,000 Ω. The wattmeter is used to measure the power of a load taking 40 A at 230 V and 0.7 p.f. The percentage error in the reading if the current coil is on the load side is
(A) 1.24% (B) 2.48%
(C) 0.62% (D) 1.96%

9. A Wheatstone bridge is modified by taking leads as shown in the figure



If an external resistance R_3 is connected across the leads such that $R_1 = 600\ \Omega$, $R_1 = 800R_2$, $P = 400R_2$, $P = 1.5Q$, $S = 0.6667R_2$

The value of unknown resistance R would be
(A) 0.25 Ω (B) 0.5 Ω
(C) 1.33 Ω (D) 1.5 Ω

10. In a Carey Fosters bridge, a resistance of 2.0250 Ω is compared with a known standard resistance of 2 Ω. The slide wire is 2 meters long and has a resistance of 0.1 Ω in 100 divisions. The ratio arms are actually 20.01 Ω and 19.9 Ω, respectively, the balance positions on the slide wire are obtained at
(A) 25 and 175 cm
(B) 15 and 185 cm
(C) 54 and 146 cm
(D) None of the above

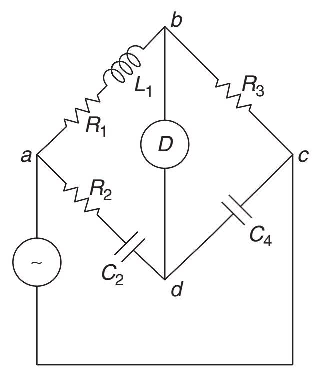
11. Kelvins double bridge was employed to measure an unknown resistance. The bridge had the following components Outer ratio arms 50.12 Ω and 100 Ω Inner ratio arms 50.155 Ω and 100 Ω known standard resistance of 50.015 μΩ. Resistance link connecting the standard resistance and unknown resistance is 350 μΩ

The value of unknown resistance will be
(A) 15 μΩ (B) 25 μΩ
(C) 30 Ω (D) 25 Ω

12. The drawback of Maxwell's inductance capacitance bridge over Hay's bridge is because
(A) Its balance equation do not contain frequency term
(B) It cannot be used for low-quality factor coil
(C) It cannot be used to measure coils with high Q-factor
(D) None of the above

13. Frequency measurement can be performed using a
(A) Wien's bridge
(B) Heaviside's Bridge
(C) Schering Bridge
(D) Maxwell's Bridge

14.



The properties of a sample sheet metal are measured using an Owen's bridge at 1 kHz. At balance, arm ab is test specimen $R_3 = 50\ \Omega$, $C_4 = 0.05\ \mu F$, $R_2 = 417\ \Omega$ in series with $C_2 = 0.062\ \mu F$. The effective impedance of the specimen under test condition would be
(A) 40.85 mΩ
(B) 40.85 μΩ
(C) 40.85 Ω
(D) 4.085 Ω

15. For a measuring instrument when a set of measurements were noted down it was observed that the readings had a wide range, the instrument has
(A) High accuracy
(B) Low accuracy
(C) High precision
(D) Low precision

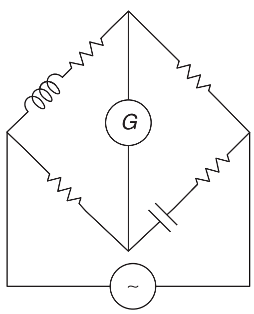
Practice Problems 2

1. Kelvin's double bridge is used to
(A) Measure capacitance
(B) Measure frequency
(C) Measure high resistance
(D) Measure low resistance

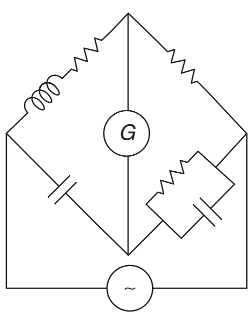
2. Carey Foster slide wire bridge is used to
(A) Measure low resistance
(B) Measure high resistance
(C) Measure the deviation of a unknown resistance from a standard one
(D) Measure the deviation of an unknown capacitance from a standard one

3. Hay's bridge from the following is

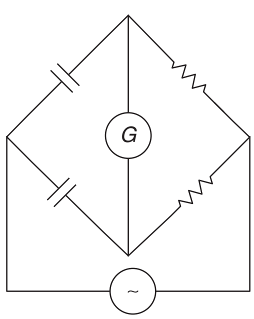
(A)



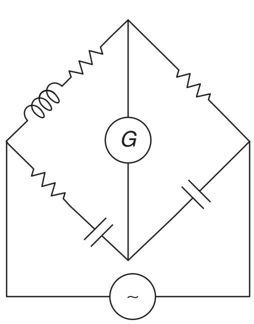
(B)



(C)



(D)



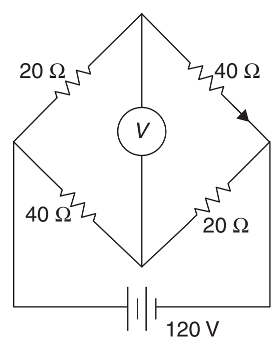
4. Group I represents the bridges available for measurement of certain parameters with reasonable accuracy. Group II represents the parameters that are to be measured. Select the correct choice of item in group I for the corresponding item in group II.

Group I	Group II
P. Wheatstone Bridge	1. Resistance in milli-ohm range
Q. Kelvin double	2. Low values of bridge capacitance
R. Schering bridge	3. Comparison of inductance with standard capacitor
S. Hay's bridge	4. Mutual inductance of a coil
T. Carey Foster bridge	
U. Wien's bridge	

(A) $1 = Q$; $2 = R$; $3 = T$; $4 = S$
(B) $1 = Q$; $2 = T$; $3 = V$; $4 = S$
(C) $1 = Q$; $2 = R$; $3 = S$; $4 = T$
(D) $1 = P$; $2 = R$; $3 = S$; $4 = U$

5. A slide wire potentiometer has 20 wires at 50 cm each. The bridge is standardized with a voltage of 1.232 V and keeping the jockey at 123.2 cm. The total resistance of the potentiometer is 2 kΩ, value of working current is
(A) 2 mA
(B) 1 mA
(C) 5 mA
(D) 20 mA

6. The reading shown by high impedance voltmeter is

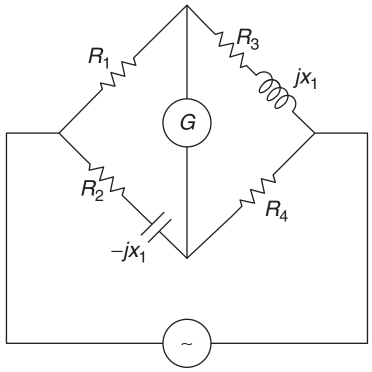


(A) 10 V
(B) 120 V
(C) 30 V
(D) 40 V

7. A DC potentiometer is used to measure up to 3 V with a slide wire of 900mm. A standard cell of 1.53 V attains balance at 660 mm. A test cell obtains balance at 720 mm. The emf of test cell is
(A) 1.83 V
(B) 2 V
(C) 1.67 V
(D) 1.52 V

8. The potentiometer wire should have
(A) Low resistivity and high temperature coefficient
(B) High resistivity and high temperature coefficient
(C) High resistivity and low temperature coefficient
(D) Low resistivity and low temperature coefficient

9. In the bridge circuit shown below, the sequence that is most suitable for balancing the bridge is



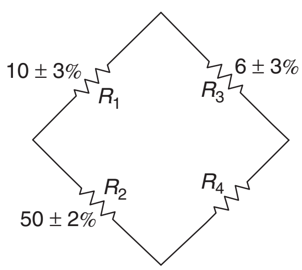
(A) First adjust R_1 and then adjust R_4
(B) First adjust R_3 and then adjust R_2
(C) First adjust R_1 and then adjust R_2
(D) First adjust R_3 and then adjust R_4

10. The current through a resistance R is measured as

$$I = 6\text{ A} \pm 0.25\%.$$

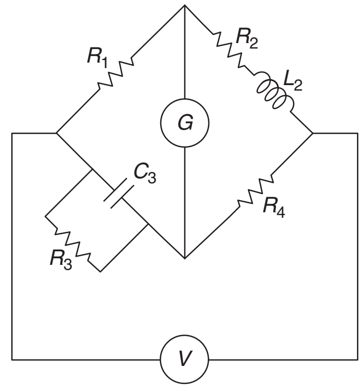
If the resistance $R = 100 \pm 0.5\%$, the uncertainty in the measurement of power is
(A) 3600 W ± 0.45%
(B) 3600 W ± 1%
(C) 3600 W ± 0.05%
(D) 3600 W ± 1.5%

11.



At balanced condition, the value of R_4 is
(A) 30 ± 6% (B) 30 ± 4%
(C) 30 ± 8% (D) 30 ± 3%

Common Data for Questions 12 and 13:



The above bridge is used to measure the inductance L_2. Resistance R_3 is adjusted and the bridge is balanced $R_1 = 500\ \Omega$,

$$R_3 = 1000\ \Omega \text{ and } R_4 = 800\ \Omega,\ C_3 = 0.6\ \mu\text{F}$$

12. The value of L_2 and R_2 is
(A) 0.12 H, 200 Ω
(B) 0.24 H, 200 Ω
(C) 0.12 H, 400 Ω
(D) 0.24 H, 400 Ω

13. The Q-factor of the coil is
(A) 3.77 (B) 3.14
(C) 1.87 (D) 2.56

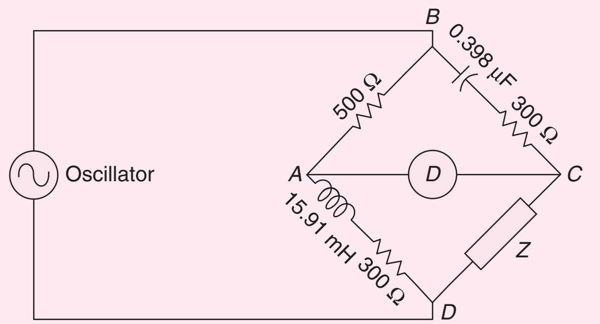
Common Data for Questions 14 and 15:
A voltmeter having a full-scale deflection of 100 V has a specified accuracy of ±1%

14. At full-scale deflection of 100 V, the error will be
(A) ± 0.01%of 100 V
(B) ± 0.1% of 100 V
(C) ±1 % of 100 V
(D) ± 99%of 100 V

15. When the voltmeter is reading 50 V, the error will be
(A) ±0.5% of measured voltage
(B) ±1% of measured voltage
(C) ±1.5% of measured voltage
(D) ±2% of measured voltage

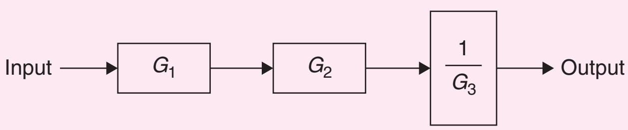
PREVIOUS YEARS' QUESTIONS

1. The AC bridge shown in the figure is used to measure the impedance Z. If the bridge is balanced for oscillator frequency $f = 2$ kHz, then the impedance Z will be **[2008]**



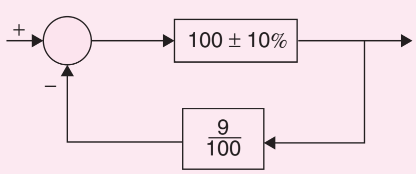
(A) $(260 + j0)\ \Omega$ (B) $(0 + j200)\ \Omega$
(C) $(260 - j200)\ \Omega$ (D) $(260 + j200)\ \Omega$

2. The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as G_1, G_2 and $\dfrac{1}{G_3}$. The relative small errors associated with each respective subsystems G_1, G_2 and G_3 are ε_1, ε_2 and ε_3. The error associated with the output is: **[2009]**



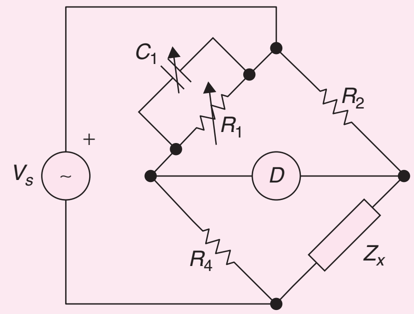
(A) $\varepsilon_1 + \varepsilon_2 + \dfrac{1}{\varepsilon_3}$ (B) $\dfrac{\varepsilon_1 \cdot \varepsilon_2}{\varepsilon_3}$
(C) $\varepsilon_1 + \varepsilon_2 - \varepsilon_3$ (D) $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$

3. As shown in the figure, a negative feedback system has an amplifier of gain 100 with ±10% tolerance in the forward path, and an attenuator of value 9/100 in the feedback path. The overall system gain is approximately: **[2010]**



(A) 10 ± 1% (B) 10 ± 2%
(C) 10 ± 5% (D) 10 ± 10%

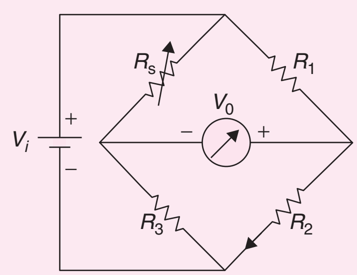
4. The bridge circuit shown in the figure below is used for the measurement of an unknown element Z_x. The bridge circuit is best suited when Z_x is a. **[2011]**



(A) Low resistance (B) High resistance
(C) Low Q inductor (D) Lossy capacitor

5. The bridge method commonly used for finding mutual inductance is **[2012]**
(A) Heaviside Campbell bridge
(B) Schering bridge
(C) De Sauty's bridge
(D) Wien bridge

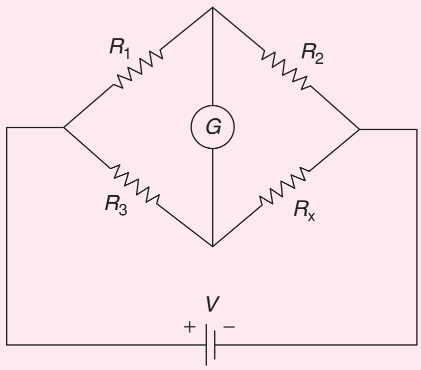
6. A strain gauge forms one arm of the bridge shown in the figure below and has a nominal resistance without any load as $R_s = 300\ \Omega$. Other bridge resistances are $R_1 = R_2 = R_3 = 300\ \Omega$. The maximum permissible current through the strain gauge is 20 mA. During certain measurement when the bridge is excited by maximum permissible voltage and the strain gauge resistance is increased by 1% over the nominal value, the output voltage V_0 in mV is **[2013]**



(A) 56.02 (B) 40.83
(C) 29.85 (D) 10.02

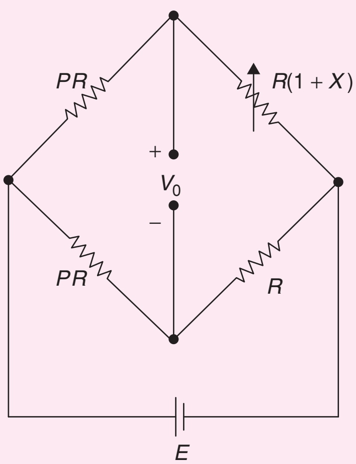
7. Suppose that resistors R_1 and R_2 and connected in parallel to given an equivalent resistor R. If resistors R_1 and R_2 have tolerance of 1% each, the equivalent resistor R for resistors $R_1 = 300\ \Omega$ and $R_2 = 200\ \Omega$ will have tolerance of **[2014]**
(A) 0.5% (B) 1%
(C) 1.2% (D) 2%

8. When the Wheatstone bridge shown in the figure is used to find the value of resistor R_x, the galvanometer G indicates zero current when $R_1 = 50\ \Omega$, $R_2 = 65\ \Omega$ and $R_3 = 100\ \Omega$. If R_3 is known with ±5% tolerance on its nominal value of 100 |, what is the range of R_x in Ohms? **[2015]**



(A) [123.50, 136.50] (B) [125.89, 134.12]
(C) [117.00, 143.00] (D) [120.25, 139.75]

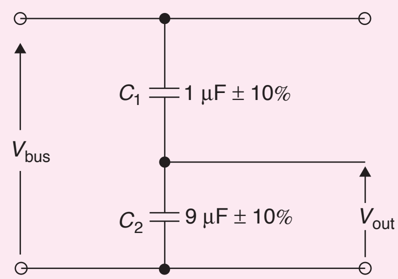
9.



An unbalanced DC Wheatstone bridge is shown in the figure. At what value of p will the magnitude of V_0 be maximum? **[2015]**

(A) $\sqrt{(1+x)}$ (B) $(1+x)$
(C) $\sqrt{(1+x)}$ (D) $\sqrt{(1-x)}$

10. A capacitive voltage divider is used to measure the bus voltage V_{bus} in a high-voltage 50 Hz AC system as shown in the figure. The measurement capacitors C_1 and C_2 have tolerances of ±10% on their nominal capacitance values. If the bus voltage V_{bus} is 100 kV rms, the maximum rms output voltage V_{out} (in kV), considering the capacitor tolerance, is _______. **[2015]**



Answer Keys

Exercises

Practice Problems 1

1. D **2.** D **3.** B **4.** D **5.** A **6.** D **7.** D **8.** A **9.** A **10.** C
11. B **12.** C **13.** A **14.** C **15.** D

Practice Problems 2

1. D **2.** C **3.** A **4.** C **5.** C **6.** D **7.** C **8.** C **9.** D **10.** B
11. C **12.** D **13.** A **14.** C **15.** D

Previous Years' Questions

1. A **2.** C **3.** A **4.** C **5.** A **6.** C **7.** B **8.** A **9.** A
10. 11.75 to 12.25