

Introduction to Trigonometry

Olympiad
Comprehensive
Book

QUESTIONS

1. The value of $\sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$ is equal to _____

(a) $\frac{\sqrt{3} + 1}{2}$ (b) $\frac{\sqrt{6} - \sqrt{2}}{4}$

(c) $\frac{\sqrt{2}}{4}(\sqrt{3} + \sqrt{2})$ (d) $\frac{\sqrt{6} - \sqrt{2}}{2\sqrt{2}}$

(e) None of these

2. The value of $\tan 10^\circ \cdot \tan 20^\circ \cdot \tan 30^\circ \cdot \tan 45^\circ \cdot \tan 60^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ$ is:

(a) 0 (b) -1

(c) 1 (d) 2

(e) None of these

3. If $\sin \theta = \frac{p}{q}$ then $\cos \theta$ equals to _____

(a) $\sqrt{q^2 - p^2}$ (b) $\frac{\sqrt{q^2 - p^2}}{q}$

(c) $\sqrt{p^2 - q^2}$ (d) $\frac{\sqrt{p^2 + q^2}}{q}$

(e) None of these

4. The value of $\cos \theta$ in terms of $\tan \theta$ is

(a) $\frac{1}{\sqrt{1 - \tan^2 \theta}}$ (b) $\frac{1}{\sqrt{1 + \tan^2 \theta}}$

(c) $\frac{\sqrt{1 - \tan^2 \theta}}{\sqrt{1 + \tan^2 \theta}}$ (d) $\frac{1}{\sqrt{3 - 3 \tan^2 \theta}}$

(e) None of these

5. $\sin^4 \theta + \cos^4 \theta$ equals to _____

(a) $1 + 2 \sin^2 \theta \cos^2 \theta$ (b) $2 \sin^2 \theta \cos^2 \theta - 1$

(c) $1 - 2 \sin^2 \theta \cos^2 \theta$ (d) $3 + 2 \sin^2 \theta \cos^2 \theta$

(e) None of these

- 6.** Simplify $\frac{\sin^2 30^\circ \cdot \cos^2 30^\circ - \tan^2 30^\circ \cdot \tan^2 60^\circ}{(\sin 60^\circ + \cos 60^\circ)^2}$
- (a) $\frac{-1}{2\sqrt{3} - 4}$ (b) $\frac{-13(\sqrt{3} + 2)}{4}$
 (c) $\frac{1}{4 + 2\sqrt{3}}$ (d) $\frac{13}{8}(\sqrt{3} - 2)$
 (e) None of these
- 7.** If $4 \tan A = 3$, then the value of $\frac{3\cos^3 A - 5\sin^3 A}{2\cos A + 5}$ is _____ (where A is an acute angle)
- (a) $\frac{18}{275}$ (b) $\frac{19}{175}$
 (c) $\frac{20}{105}$ (d) $\frac{19}{275}$
 (e) None of these
- 8.** Which among the following is different?
- $2\cos^2 30^\circ - 1, 1 - 2\sin^2 30^\circ$
 $\cos^2 30^\circ - \sin^2 30^\circ, \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$
- (a) $2\cos^2 30^\circ - 1$ (b) $1 - 2\sin^2 30^\circ$
 (c) $\cos^2 30^\circ - \sin^2 30^\circ$ (d) $\frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$
 (e) None of these
- 9.** Which among the following is not similar?
- $\sin 60^\circ, 2\sin 30^\circ \cdot \cos 30^\circ, \frac{2\tan 30^\circ}{1 + \tan^2 30^\circ}, \frac{\sqrt{3}}{2}$
- (a) $\sin 60^\circ$ (b) $2\sin 30^\circ \cdot \cos 30^\circ$
 (c) $\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ}$ (d) $\frac{\sqrt{3}}{2}$
 (e) None of these
- 10.** If $\sin \theta - \cos \theta = 0$, then find the value of $(\sin^4 \theta + \cos^4 \theta)$, where θ is a acute angle.
- (a) $\frac{1}{3}$ (b) $\frac{\sqrt{2}}{4}$

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{2}}{2}$

(e) None of these

11. If $\tan \theta = \sqrt{3}$ then find the value of $\sin^2 \theta - \cos^2 \theta$, where θ is acute angle.

(a) $\frac{1}{2}$

(b) $\frac{-1}{3}$

(c) $\frac{-1}{2}$

(d) $\frac{1}{4}$

(e) None of these

12. Simplified form of $(1 + \cot^2 \theta)(1 - \cos \theta)(1 + \cos \theta)$ is _____

(a) 1

(b) 0

(c) Cosec θ

(d) Sin θ

(e) None of these

13. If $m = \sin \theta + \cos \theta$ and $n = \sin \theta - \cos \theta$, then which of the following is true?

(a) $m^2 + n^2 = 2$

(b) $m^2 + n^2 = 1$

(c) $m^2 - n^2 = 1$

(d) $m^2 + n^2 = 3$

(e) None of these

14. The value of $\cot 5^\circ \cdot \cot 15^\circ \cdot \cot 25^\circ \cdot \cot 35^\circ \cdot \cot 45^\circ \cdot \cot 55^\circ \cdot \cot 65^\circ \cdot \cot 75^\circ \cdot \cot 85^\circ$ is _____.

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) 1

(d) $2\sqrt{3}$

(e) None of these

15.
$$\frac{\sin 75^\circ \cdot \cos 25^\circ \cdot \tan 50^\circ}{\sin 65^\circ \cdot \cos 15^\circ \cdot \cot 40^\circ}$$

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

(e) None of these

16. If $2 \sin \alpha = \sqrt{3}$ then $\frac{\tan^2 \alpha + 3 \cdot \cot \alpha}{\cot^2 \alpha + 2 \tan \alpha}$ is equal to _____ (where α is an acute angle)

(a) $\frac{3(\sqrt{3}+1)}{1+6\sqrt{3}}$

(b) $\frac{-45-51\sqrt{3}}{-107}$

(c) $\frac{50\sqrt{3}+45}{-106}$

(d) $\frac{-51\sqrt{3}}{107}$

(e) None of these

17. In a $\triangle PQR$, right-angled at Q, and if $PR - QR = 2$ cm and $PQ = 10$ cm then _____

(a) $\operatorname{cosec} R = \frac{13}{12}$

(b) $\sec R = \frac{13}{5}$

(c) $\tan P = \frac{12}{5}$

(d) All the above

(e) None of these

18. If $(p^2 - q^2) \sin \theta + 2pq \cos \theta = p^2 - q^2$, then the value of $\tan \theta$ is _____

(a) $\frac{p+q}{2pq}$

(b) $\frac{p-q}{2pq}$

(c) $\frac{p^2 - q^2}{2pq}$

(d) $\frac{p^2 + q^2}{2pq}$

(e) None of these

19. If θ is an acute angle and $4 \cos \theta + 4\sqrt{3} \sin \theta = 8 \sin \theta \cos \theta + 2 - \sqrt{3}$, then which one among the following is correct?

(a) $\sin \theta = \frac{\sqrt{3}}{2}$

(b) $\cos \theta = \frac{1}{2}$

(c) $\theta = 30^\circ$

(d) $\tan \theta = \sqrt{3}$

(e) None of these

20. For $0^\circ < \theta < 90^\circ$, if $\sqrt{2}(\sin \theta + \cos \theta) = 1 + 2 \sin \theta \cos \theta$ then which among the following is not correct?

(a) $\sin \theta - \cos \theta = 0$

(b) $\sin \theta + \cos \theta = 0$

(c) $\theta = 45^\circ$

(d) $\sin 2\theta = 2 \cos(\theta + 15)$

(e) None of these

21. Simplify: $\frac{3 \sin 67^\circ}{5 \cos 23^\circ} + \frac{5 \sec 39^\circ \cdot \cos 51^\circ}{2(1 + \cot^2 51)} + \frac{8}{9} (\cosec^2 42^\circ - \tan^2 48^\circ)$

- (a) $3\frac{89}{90}$
 (b) $2\frac{81}{90}$
 (c) $\cos 67^\circ + \sec 39^\circ$
 (d) $\frac{3}{5} \cos 67^\circ + \frac{5}{2} \sec 51^\circ + \frac{8}{9} \cot^2 48^\circ$

(e) None of these

22. In a $\triangle PQR$, $\angle Q$ is a right angle and QT is a perpendicular drawn on PR. If PR = 15 cm and QT = 5 cm, then the value of $\frac{\tan P + \tan R}{\tan R \cdot \tan P}$ is _____

- (a) $\frac{1}{3}$ (b) 2
 (c) $\frac{1}{2}$ (d) 3
 (e) None of these

23. Find the value of $\frac{k^2 - 1}{k^2 + 1}$ if $k = \sec \theta - \tan \theta$.

- (a) $\sin \theta$ (b) $\cos \theta$
 (c) $-\sin \theta$ (d) $-\cos \theta$
 (e) None of these

24. Which among the following statements is/ are true?

- (a) The value of $\sin \theta$ decreases as θ increases. (when $0^\circ < \theta < 90^\circ$)
 (b) The value of $\cos \theta$ increases (when $0^\circ < \theta < 90^\circ$)
 (c) $\sin(A + B) = \sin A \cdot \cos B$
 (d) The value of $\tan \theta$ is always greater than $\sin \theta$.
 (e) None of these

25. If $\text{cosec } \theta - \cot \theta = 3$ then find the positive value of $\cos \theta$.

- (a) $\frac{4}{5}$ (b) 1
(c) 0 (d) 2
(e) None of these

26. If $A = \frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta}$, then A equals to _____.

- (a) $\sec^2 \theta$ (b) $2\sec^2 \theta$
(c) $\cos^2 \theta$ (d) $2\cos^2 \theta$
(e) None of these

27. The value of $\log \cos 0^\circ + \log \cos 1^\circ + \log \cos 2^\circ + \dots + \log \cos 90^\circ$ is equal to _____

- (a) 1 (b) -1
(c) 0 (d) Undefined
(e) None of these

28. If $\sin \theta + \cos \theta = p$, then the value of $\sin^6 \theta + \cos^6 \theta$ equals to _____

- (a) $\frac{4 + 3(p^2 - 1)}{4}$ (b) $\frac{4 - 3(p^2 - 1)^2}{4}$
(c) $4 - 3(p^2 - 1)$ (d) $4 - 3(p^2 - 1)^2$
(e) None of these

29. If $\cos^2 \theta + \cos \theta = 1$, then find the value of $\sin^4 \theta + \sin^2 \theta$.

- (a) 0 (b) 1
(c) -1 (d) Cos9
(e) None of these

30. The value of $\log_{\sec \theta} \cos^3 \theta \left[(\cos \theta - \sin \theta)^2 + 2\sin \theta \cos \theta \right]$ is _____

- (a) 0 (b) -1
(c) 2 (d) -3
(e) None of these

37. If $\cos 3x = 4 \cos^3 30^\circ - 3 \cos^3 0^\circ$ then the value of x is _____

- (a) 0
 - (b) 30°
 - (c) 20°
 - (d) 15°
 - (e) None of these

38. If $\sin 2x = \frac{2\tan 30^\circ}{1 + \tan^2 30}$, then the value of $\frac{1 - \tan^2 x}{1 + \tan^2 x}$ is equal to ____ (where $0^\circ < x < 90^\circ$)

39. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$ then the value of $\sin^4\theta - \cos^4\theta$

- (a) $\frac{4}{3}$ (b) $\frac{3}{4}$
 (c) $\frac{5}{3}$ (d) $\frac{3}{5}$
 (e) None of these

40. If $\cos\alpha = \sin 14\alpha$ (where $0^\circ < 14\alpha < 90^\circ$), then find the value of $\sin 5\alpha + \cos(12\alpha - 27^\circ) - \operatorname{cosec}(3\alpha + 12^\circ)$.

ANSWER - KEY

1. (B)	2. (C)	3. (B)	4. (B)	5. (C)
6. (D)	7. (D)	8. (E)	9. (C)	10. (C)
11. (A)	12. (A)	13. (A)	14. (C)	15. (A)
16. (B)	17. (D)	18. (C)	19. (C)	20. (B)
21. (A)	22. (D)	23. (C)	24. (B)	25. (B)
26. (B)	27. (D)	28. (B)	29. (B)	30. (D)
31. (C)	32. (B)	33. (C)	34. (B)	35. (D)
36. (C)	37. (B)	38. (A)	39. (D)	40. (C)