Chapter 1

Number Systems

CHAPTER HIGHLIGHTS

- *Digital Circuits*
- *Number Systems with Different Base*
- *Conversion of Number Systems*
- *Subtraction with Complements*
- *Signed Binary Numbers - (four bit)*
- *Complements*
- *Signed Binary Numbers*
- *Subtraction with Complements*
- *Signed Binary Numbers - (four bit)*
- *Numeric Codes*
- *BCD Addition*
- *BCD Subtraction*
- *Binary Codes*

Digital Circuits

Computers work with binary numbers that use only the digits '0' and '1'. Since all the digital components are based on binary operation, it is convenient to use binary numbers when analysing or designing digital circuits.

Number Systems with Different Base

Decimal Number System

Decimal numbers are usual numbers that we use in our day-to-day life. The base of the decimal number system is 10. There are 10 numbers (0 to 9).

The value of the n^{th} digit of the number from the right side = n^{th} digit × $(\text{base})^{n-1}$

Solved Examples

Example 1: $(99)_{10} \rightarrow 9 \times 10^1 + 9 \times 10^0$
$= 90 + 9 = 99$

Example 2: $(332)_{10} \rightarrow 3 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$
$= 300 + 30 + 2$

Example 3: $(1{,}024)_{10} \rightarrow 1 \times 10^3 + 0 \times 10^2 + 2 \times 10^1 + \times 10^0$
$= 1{,}000 + 0 + 20 + 4$
$= 1{,}024$

Binary Number System

In binary number system, there are only two digits '0' and '1'. Since there are only two numbers, its base is 2.

Example 4: $(1{,}101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 8 + 4 + 1$
$= (13)_{10}$

Octal Number System

Octal number system has eight numbers 0 to 7. The base of the number system is 8. The number $(8)_{10}$ is represented by $(10)_8$.

Example 5: $(658)_8 = 6 \times 8^2 + 5 \times 8^1 + 8 \times 8^0$
$= 384 + 40 + 8$
$= (432)_{10}$

Hexadecimal Number System

Totally, in hexadecimal number system, there are 16 numbers 0 to 9 and digits from 10 to 15, which are represented by A to F, respectively. The base of hexadecimal number system is 16.

Example 6: $(1A5C)_{16} = 1 \times 16^3 + A \times 16^2 + 5 \times 16^1 + C \times 16^0$
$= 1 \times 4{,}096 + 10 \times 256 + 5 \times 16 + 12 \times 1$
$= 4{,}096 + 2{,}560 + 80 + 12$
$= (6{,}748)_{10}$.

Different Number Systems

Decimal	Binary	Octal	Hexadecimal
0	000	0	0
1	001	1	1
2	010	2	2
3	011	3	3
4	100	4	4
5	101	5	5
6	110	6	6

(Continued)

Decimal	Binary	Octal	Hexadecimal
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14

1. For a number system with base n, the number of different symbols in the number system will be n. For example, octal number system will have total of 8 numbers from 0 to 7.
2. The number 'n' in the number system with base 'n' is represented as $(10)_n$.
3. The equivalent of number $(a_3a_2a_1a_0 \cdot a_{-1}a_{-2})_n$ in decimal is $a_3 \times n^3 + a_2 \times n^2 + a_1 \times n^1 + a_0 \times n^0 + a_{-1} \times n^{-1} + a_{-2} \times n^{-2}$.

Conversion of Number Systems

The conversion of decimal to any other number system involves successive division by the radix until the dividend reaches 0. At each division, the remainder gives a digit of converted number and the last one is the most significant digit, and the remainder of the first division is the least significant digit.

The conversion of other number system to decimal involves multiplying each digit of number system with the weight of the position (in the power of radix) and sum of the products is calculated; further, the total is the equivalent value in decimal.

Decimal to Binary Conversion

Example 7: $(66)_{10}$

$$\begin{array}{r|ll} 2 & 66 & \\ 2 & 33 & 0 \\ 2 & 16 & 1 \\ 2 & 8 & 0 \\ 2 & 4 & 0 \\ & 2 & 0 \\ & 1 & 0 \end{array}$$

Reading remainders from bottom to top

$= (1000010)_2$

Example 8: $(928)_{10}$

$$\begin{array}{r|ll} 2 & 928 & \\ 2 & 464 & 0 \\ 2 & 232 & 0 \\ 2 & 116 & 0 \\ 2 & 58 & 0 \\ 2 & 29 & 0 \\ 2 & 14 & 1 \\ 2 & 7 & 0 \\ 2 & 3 & 1 \\ & 1 & 1 \end{array}$$

$= (1110100000)_2$

Example 9: $(105.75)_{10}$

$$\begin{array}{r|ll} 2 & 105 & \\ 2 & 52 & 1 \\ 2 & 26 & 0 \\ 2 & 13 & 0 \\ 2 & 6 & 1 \\ 2 & 3 & 0 \\ & 1 & 1 \end{array}$$

$(105)_{10} = (1101001)_2$

$(0.75)_{10}$

Multiply 0.75 by 2 = 1.50

Multiply 0.50 by 2 = 1.00

Reading integers from top to bottom 0.75 $= (0.11)_2$

Therefore, $(105.75)_{10} = (1101001.11)_2$

Binary to Decimal Conversion

Example 10: $(10100011)_2$

$= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 128 + 0 + 32 + 0 + 0 + 0 + 2 + 1$

$= (163)_{10}$

Example 11: $(11010011.101)_2$

$= 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$

$= 128 + 64 + 0 + 16 + 0 + 0 + 2 + 1 + 0.5 + 0 + 0.125$

$= (211.625)_{10}$

Decimal to Octal Conversion

Example 12: $(16)_{10}$

$$\begin{array}{r|ll} 8 & 16 & \\ & 2 & 0 \end{array}$$

Remainder from bottom to top $= (20)_8$

Example 13: $(347.93)_{10}$

$(.93)_{10}$

$0.93 \times 8 =$	7.44
$0.44 \times 8 =$	3.52
$0.52 \times 8 =$	4.16
$0.16 \times 8 =$	1.28

……..

Read the integers of octal point from top to bottom.

Therefore, $(0.93)_{10} = (0.7341)_8$

$(347)_{10}$

8	347	3
8	43	3
	5	

Therefore, $(347)_{10} = (533)_8$

Hence, the solution is $(533.7341)_8$

Octal to Decimal Conversion

Example 14: $(33)_8$

$3 \times 8^1 + 3 \times 8^0 = 24 + 3$

$(27)_{10}$.

Example 15: $(1,023.06)_8$

$1 \times 8^3 + 0 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$

$= 512 + 0 + 16 + 3 + 0 + 0.0937$

$= (2,095.0937)_{10}$.

Octal to Binary Conversion

To convert octal to binary, replace each octal digit with their equivalent 3-bit binary representation.

Example 16: $(7777)_8$

Convert each octal digit to binary

$= \frac{7}{111}\ \frac{7}{111}\ \frac{7}{111}\ \frac{7}{111}$

$= (111\ 111\ 111\ 111)_2$

Example 17: $(567.62)_8$

5	6	7	.	6	2
101	110	111	.	110	010

$= (101110111.110010)_2$

Binary to Octal Conversion

To convert a binary number to an octal number, we need to start from the binary point and make groups of 3 bits each on either side of the binary point; these 3-bit binary groups are replaced by the equivalent octal digits.

Example 18: $(010011101)_2$

010	011	101
2	3	5

$= (235)_8$

Example 19: $(10010111011.1011)_2$

010	010	111	011	.	101	100
2	2	7	3	.	5	4

$= (2273.54)_8$

Decimal to Hexadecimal Conversion

Example 20: $(527)_{10}$

16	527	
16	32	15
	2	0

Decimal		Hexa
2	→	2
0	→	0
15	→	F

$= (20F)_{16}$.

Example 21: $(18.675)_{10}$

$(18)_{10}$

16	18	
	1	2

Decimal		Hexa
1	–	1
2	–	2

$(18)_{10} = (12)_{16}$

$(0.675)_{10}$

0.675×16	10.8
0.800×16	12.8
0.800×16	12.8
0.800×16	12.8

Decimal	Hexa
10	A
12	C
12	C
12	C

$= (0.ACCC)_{16}$

Therefore, hexadecimal equivalent is

$= (12.AC\ CC)_{16}$

Hexadecimal To Decimal Conversion

Example 22: $(A3F)_{16}$

Decimal		Hexa
A	–	10
3	–	3
F	–	15

$\rightarrow 10 \times 16^2 + 3 \times 16^1 + 15 \times 16^0$

$\rightarrow 2,560 + 48 + 15$

$\rightarrow (2,623)_{10}$

Example 23: $(1F63.0EB)_{16}$

1	1
F	15
6	6
3	3
0	0
E	14
B	11

$\rightarrow 1 \times 16^3 + 15 \times 16^2 + 6 \times 16^1 + 3 \times 16^0 \times (0 \times 16^{-1}) + (14 \times 16^{-2}) + (11 \times 16^{-3})$

$\rightarrow 4{,}096 + 3840 + 96 + 3 + 0 + 0.0546 + 0.0026$

$\rightarrow (8{,}035.0572)_{10}$

Hexadecimal to Binary Number System

To represent hexadecimal in binary, represent each HEX number with its 4-bit binary equivalent.

Example 24: $(34F)_{16}$

Hexa	Decimal	Binary
3	3	0011
4	4	0100
F	15	1111

$= (001101001111)_2$

Example 25: $(AFBC.BED)_{16}$

Hex	Decimal	Binary
A	10	1010
F	15	1111
B	11	1011
C	12	1100
B	11	1011
E	14	1110
D	13	1101

$= (1010111110111100.101111101101)_2$

Binary to Hexadecimal Number System

To convert binary number to a hexadecimal number, we need to start from the binary point and make groups of 4 bits each on either side of the binary point; these 4-bit groups are replaced by the equivalent hexadecimal digit.

Example 26: $(11001001)_2$

$\rightarrow \dfrac{1100}{12}\ \dfrac{1001}{9}$

$\rightarrow (C9)_{16}$

Example 27: $(1011011011.01111)_2$

$\dfrac{0010}{2}\ \dfrac{1101}{D}\ \dfrac{1011}{B} . \dfrac{0111}{7}\ \dfrac{1000}{8}$

$= (2DB.78)_{16}$

Hexadecimal to Octal Number System

The simplest way to convert hexadecimal to octal is to first convert the given hexadecimal number to binary and the binary number to octal.

Example 28: $(C3AF)_{16}$

$\rightarrow 001100001110101111$

$\rightarrow (141{,}657)_8$

Example 29: $(C6.AE)_{16}$

$\rightarrow 0011000110.10101110$

$\rightarrow (306.534)_8$

Octal to Hexadecimal Number System

The simplest way to convert octal to hexadecimal is to first convert the given octal number to binary and then the binary number to hexadecimal.

Example 30: $(775)_8$

$\rightarrow (000111111101)_2$

$\rightarrow (1FD)_{16}$

Example 31: $(34.7)_8$

$\rightarrow (00011100.1110)_2$

$\rightarrow (1C.E)_{16}$

Complements

Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.

There are two types of complements for each 'base-r' system.

1. Radix complement (or) r's complement:
   the r's complement of an m-digit number N in base r is $r^m - N$ for $N \neq 0$.
   For $N = 0$, r's complement is 0.
2. Diminished radix complement: (or) $(r - 1)$'s complement:
   Given a number N in base r having m digits, then $(r - 1)$'s complement is $(r^m - 1) - N$.
   For example, decimal number system will have 10's complement and 9's complement.
   Similarly, binary number system will have 2's complement and 1's complement.

Example 32: 10's complement of $(2{,}657)_{10}$ is $(10)^4 - 2{,}657$

$$\begin{array}{r} 10{,}000 \\ \underline{2{,}657} \\ \underline{7{,}343} \end{array}$$

Example 33: 9's complement of $(2{,}657)_{10}$ is $(10^4 - 1) - 2{,}657$

$$\begin{array}{r} 10{,}000 \\ \underline{-\quad 1} \\ 9{,}999 \\ \underline{2{,}657} \\ \underline{7{,}342} \end{array}$$

Further, r's complement can be obtained by adding 1 to $(r - 1)$'s complement.

$$r^m - N = \{(r^m - 1) - N\} + 1$$

Example 34: 2's complement of $(101101)_2$ is

$= (2)^6 - 101101$

$(2^6)_{10} = (100000)_2$

2's complement is 100000
−101101
010011

Example 35: 1's complement of $(101101)_2$ is

$2^6 - 1 =$ 1000000
− 1
111111
101101

1's complement −010010

The 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's, The 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged, and replacing 1's with 0's and 0's with 1's in all other bits.

If the number M contains radix point, the point should be removed temporarily in order to form the r's/$(r - 1)$'s complement.

The radix point is then restored to the complemented number in the same relative position.

Example 36: Find the 1's complement of $(1001.011)_2$.

Let us consider the number without radix point, that is, 1001011.

Therefore, 1's complement is 0110100.

By adding the radix point again, the solution will be $(0110.100)_2$

Example 37: Find the 2's complement of $(1001.011)_2$.

Let us consider the number without radix point, that is, 1001011.

Therefore, 2's complement is 0110101.

By adding the radix point again, the solution will be $(0110.101)_2$

Complement of a complement is equal to the original number $r^m - (r^m - M) = M$

Subtraction with Complements

The subtraction of two n-digit unsigned numbers $A - B$ in base r can be done by r's complement method.

Add A to the r's complement of B.

Mathematically, $A + (r^n - B) = A - B + r^n$

If $A \geq B$, the sum will produce an end carry r^n, which can be discarded (discarding carry is equivalent to subtracting r^n from the result). Then, the result is $A - B$.

$A = 1100$	→	1100
$B = 1010$ (2's complement)	+	0110
	Sum:	10010
discard carry $(-r^n)$	−	10000
	$A - B$:	0010

If $A < B$, the sum does not produce an end carry and result is $r^n - (B - A)$. Then, take r's complement of the sum and place a negative sign in front.

If $A = 1010$

$B = 1100$

$A - B$ can be done as

$A \rightarrow$	1010
$B \rightarrow$ 2's complement	+ 0100
Sum:	1110

Here, no carry generated, and therefore, result is a negative number.

2's complement of the result → 0010 = 2
= −2

The subtraction of unsigned numbers by using $(r - 1)$'s complement can be done in similar way. However, $(r - 1)$'s complement is one less than the r's complement. Because of this, the sum produced is one less than the correct difference when an end carry occurs. Therefore, end carry will be added to the sum. Removing the end carry and adding 1 to the sum is referred to as an end-around carry.

Consider $A = 1100$ and $B = 1010$

For $A - B$

$A \rightarrow$	1100
$B \rightarrow$(1's complement) +	0101
Sum:	10001
End-around carry +	1
$A - B =$	0010

For $B - A$

$B \rightarrow$	1010
$A \rightarrow$ (1's complement)+	0011
Sum:	1101

There is no end carry, and therefore, the result is

$-(B - A) = -$ (1's complement of 1101)

$= -0010 = -2$

Signed Binary Numbers

Positive integers can be represented as unsigned numbers; however, to represent negative integer, we need a notation for negative values in binary.

It is customary to represent the sign with a bit placed in the left-most position of the number. The convention is to make the sign bit 0 for positive and 1 for negative. This representation of signed numbers is referred to as signed-magnitude convention.

S Magnitude

+24 is 0 11000
sign Magnitude

−24 is 1 11000
sign Magnitude

Another notation for the representation of signed numbers is signed-complement system. This is convenient to use in a computer for arithmetic operations. This system negates a number by taking its complement (i.e., complement of corresponding positive number), whereas the signed-magnitude system negates a number by changing its sign bit. Positive numbers use same notation in signed-magnitude as well as in signed-complement systems.

The signed-complement system can be used either as the 1's complement or the 2's complement. However, 2's complement is the most common.

1. +24 in 1's or 2's complement representation is 011000.
2. –24 in 1's complement representation is 100111.
3. –24 in 2's complement representation is 101000.

Signed Binary Numbers (four bit)

Decimal	Signed Magnitude	Signed 1's Complement	Signed 2's Complement
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
–0	1000	1111	-
–1	1001	1110	1111
–2	1010	1101	1110
–3	1011	1100	1101
–4	1100	1011	1100
–5	1101	1010	1011
–6	1110	1001	1010
–7	1111	1000	1001
–8	–	–	1000

The ranges of signed binary numbers with n bits.

Signed-magnitude: $-2^{n-1} + 1$ to $+2^{n-1} - 1$
1's complement representation: $-2^{n-1} + 1$ to $+2^{n-1} - 1$
2's complement representation: -2^{n-1} to $+2^{n-1} - 1$

Signed 2's complement representation can be directly used for arithmetic operations. The carry out of the sign bit position is discarded.

In order to obtain a correct answer, we must ensure that the result has a sufficient number of bits to accommodate the sum or product.

Example 38

X = 00110, Y = 11100 are represented in 5-bit signed 2's complement system

Find their sum $X + Y$ in 6-bit signed 2's complemented representation.

Solution

X = 00110 and Y = 11100 are 5-bit numbers.
However, result needs to be in 6-bit format.
Operands X and Y should also be in 6-bit format

$$\begin{array}{r} X = 000110 \\ Y = \underline{111100} \\ 000010 \end{array} \quad X + Y = (1)$$

The carry out of sign bit position is discarded and the result is 000010.

Example 39

$(36x70)_{10}$ is 10's complement of $(yzyz0)_{10}$
Find the values of x, y, and z?
(A) 4, 5, 2 (B) 4, 6, 3 (C) 3, 6, 3 (D) 3, 5, 4

Solution

$(36x70)_{10}$ is 10's complement of $(yzyz0)_{10}$.
10's complement of $(yzyz0)_{10}$ is
$10^5 - yzyz0 = 36x70$
Therefore, $36x70 + yzyz0 = 100000$

$$\begin{array}{l} \ 36x70 \\ +\underline{yzyz0} \\ 100000 \end{array}$$

Then, $7 + z = 10$,
$1 + x + y = 10$; $z = 3$
$1 + 6 + z = 10$; $y = 6$
$1 + 3 + y = 10$,
$\rightarrow x = 3$

Example 40

The 10's complement of $(843)_{11}$ is
(A) $(157)_{11}$ (B) $(267)_{11}$ (C) $(156)_{11}$ (D) $(268)_{11}$

Solution

Given $(843)_{11}$ is base-11 number system; the number in the number system ranges from 0 to 9 and A (A = 10)
10's complement for $(843)_{11}$ means $(r - 1)$'s complement
Therefore, $(r^n - 1) - N = [(11)^n - 1] - N$

$$(11)^n - 1 \Rightarrow \begin{array}{r} 1000 \\ \underline{-\quad 1} \\ AAA \\ \underline{-843} \\ 267 \end{array}$$

10's complement is $(267)_{11}$

Example 41

Consider the signed binary number 10111011. What is the decimal equivalent of this number if it is in signed-magnitude form or in 1's complement form or in 2's complement form?

Solution

Given binary number is 10111011. As sign bit is 1, it is a negative number. If it is in sign magnitude format, then MSB is sign bit and remaining bits represent the magnitude.

$(0111011)_2 = 32 + 16 + 8 + 2 + 1 = 59$.
Therefore, if the given number is in signed-magnitude format, the number is –59.
If it is in 1's complement or in 2's complement form, then the magnitude of negative number can be obtained by taking 1's complement or 2's complement for the number, respectively.
$10111011 \Rightarrow$ 1's complement $\Rightarrow 01000100 = (68)_{10}$.
In 1's complement format, the number is –68.
$10111011 \Rightarrow$ 2's complement $\Rightarrow 01000101 = (69)_{10}$.
In 2's complement format, the number is –69.

Ex: Find $(–9.625)_{10}$ in signed 2's complement representation.
Signed-binary fraction can be represented in the same way of signed integer.

$$\begin{array}{r|l} 2 & 9 \\ \hline 2 & 4 - 1 \\ \hline 2 & 2 - 0 \\ \hline & 1 - 0 \end{array}$$

$0.625 \times 2 = 1.25$
$0.25 \times 2 = 0.5$
$0.5 \times 2 = 1.0$
$= 0.101$
$+(9.625) = 01001.101$
$–9.625 = 10110.011$ (by taking 2's complement)

Binary Multipliers

The multiplication of binary number is done in the same way as multiplication of decimal.

The multiplicand (m) is multiplied by each bit of the multiplier (N), starting from the LSB.

Let
$M = B_3\, B_2\, B_1\, B_0$
$N = A_3\, A_2\, A_1\, A_0$
If $M \times N = P$

			A_0B_3	A_0B_2	A_0B_1	A_0B_0	
		A_1B_3	A_1B_2	A_1B_1	A_1B_0		
	A_2B_3	A_2B_2	A_2B_1	A_2B_0			
A_3B_3	A_3B_2	A_3B_1	A_3B_0				
	P_5	P_4	P_3	P_2	P_1	P_0	$=P$

Ex: Let $M = 1\,0\,1\,1$
$N = 1\,1\,0\,0$
$M \times N = P$

$$\begin{array}{r} 1\,0\,1\,1 \\ \times\; 1\,1\,0\,0 \\ \hline 0\,0\,0\,0 \\ 0\,0\,0\,0\;\; \\ 1\,0\,1\,1\;\;\;\; \\ 1\,0\,1\,1\;\;\;\;\;\; \\ 1\,1\,1\;\;\;\;\;\;\;\; \\ \hline 1\,0\,0\,0\,0\,1\,0\,0 = P \end{array}$$

Binary Codes

Binary codes can be classified as numeric codes and alphanumeric codes. Figure 1 shows the classification of codes.

Numeric Codes

Numeric codes are the codes that represent numericals in binary, that is, only numbers as a series of 0's and 1's.

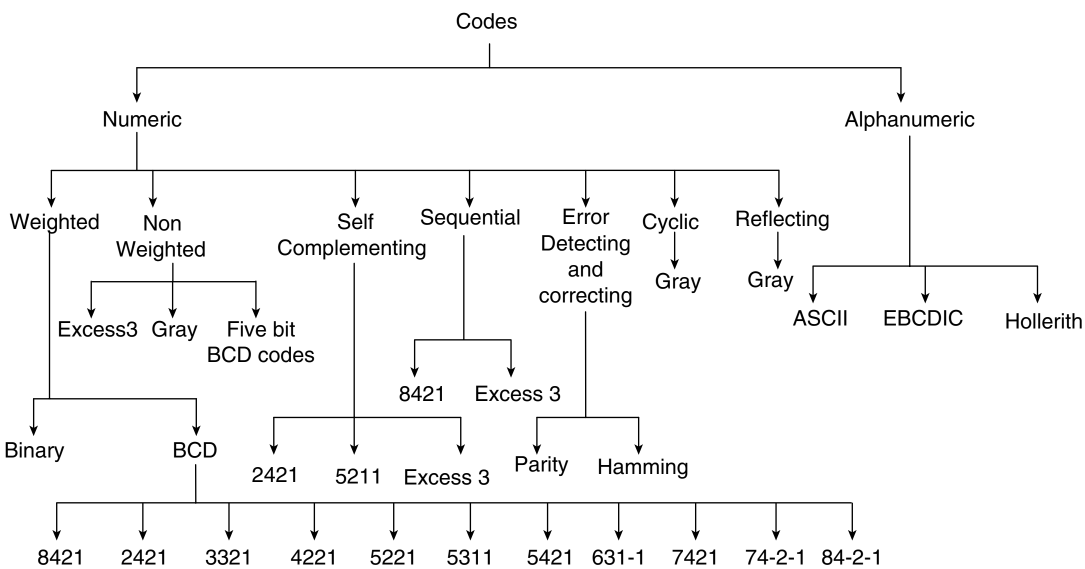

Figure 1 Classification of binary codes.

Weighted and Non-weighted Codes

1. The weighted codes are those that obey the position-weighting principle. Each position of a number represents a specific weight. For example, BCD, binary, 84-2-1, and 2421.
2. Non-weighted codes are codes that are not assigned fixed values. For example, Excess-3 and Gray code.

2421, 5211, 84 – 2 – 1 are examples of weighted codes, in which weight is assigned to each position in the number.

$(27)_{10}$ in 2421 code → 0010 1101
$(45)_{10}$ in 5211 code → 0111 1000
$(36)_{10}$ in 84 – 2 – 1 code → 0101 1010

Any digit in decimal will be represented by the weights represented by the code.

Error Detecting and Correcting Codes

Codes that allow only error detection are error detecting codes. For example, parity.

Codes that allow error detection as well as correction are called error-correcting codes. For example, Hamming codes.

Sequential Codes

A sequential code is one in which each succeeding code word is one binary number greater than the preceding code word. For example, XS–3 and BCD

Cyclic Codes (Unit Distance Codes)

Cyclic codes are those in which each successive code word differs from the preceding one in only one bit position. For example, Gray Code.

Reflective Codes

Binary code in which the n least significant bits for code words 2^n through $2^{n+1} - 1$ are the mirror images for 0 through $2^n - 1$ and it is called reflective codes. For example, Gray Code

Self-complementing Codes

A code is said to be self complementing, if the code word of the 9's complement of number 'N'; this means '9-N' can be obtained from the code word of 'N' by interchanging all the 0's and 1's, that is, by taking 1's complement. In other words, logical complement of number code is equivalent to the representation of its arithmetic complement. For example, 84-2-1, 2421, and XS -3.

All weighted BCD codes are self-complementing codes.

Binary-coded Decimal (BCD)

In BCD, each decimal digit 0 to 9 is coded by a 4-bit binary number. BCD codes are convenient to convert to/or from decimal number system.

Decimal	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Example 42: $(628)_{10} = (0110\ \ 0010\ \ 1000)_{BCD}$

BCD addition

1. BCD addition is performed by individually adding the corresponding digits of the decimal number expressed in 4-bit binary groups starting from the LSB.
2. If there is no carry and the sum term is not an illegal code, no correction is needed.
3. If there is a carry out of one group to the next group or if the sum term is an illegal code, $(6)_{10}$ is added to the sum term of that group and the resulting carry is added to the next group.

Example 43: 44 + 12

```
  0100   0100  (44 in BCD)
  0001   0010  (12 in BCD)
  ------------------------
  0101   0110  (56 in BCD)
```

Example 44: 76.9 + 56.6

```
0111  0110 . 1001
0101  0110 . 0110     (all are
-----------------
1100  1100 . 1111      illegal codes)
0110  0110 . 0110
-----------------
0010  0010 . 0101
  +1     +1     +1    (propagate carry)
-----------------
0001  0011   0011 .  0101
1   3       3   .   5
```

BCD subtraction BCD subtraction is performed by subtracting the digits of each 4-bit group of the subtrahend from the corresponding 4-bit group of the minuend in the binary starting from the LSB.

Example 45:

```
  42     0100    0010    (42 in BCD)
 –12    –0001    0010    (12 in BCD)
  30     0011    0000    (no borrow, and therefore, this is the correct difference)
```

Example 46:

```
  247.7  0010  0100  0111 . 0111
 –156.9  0001  0101  0110 . 1001
   90.8  0000  0111  0000 . 1110    (Borrow are present, and therefore, subtract 0110)
              –0110          –0110
               1001  0000  . 1000    Corrected difference (90.8)
```

Excess-3 (XS-3) Code

Excess-3 code is a non-weighted BCD code, where each digit binary code word is the combination of corresponding 8421 code word and 0011.

Find the XS-3 code in the following examples:

Example 47: $(3)_{10} \rightarrow (0011)_{BCD} = (0110)_{XS\text{-}3}$

Example 48: $(16)_{10} \rightarrow (0001\ 0110)_{BCD}$
$\rightarrow (0100\ 1001)_{XS\text{-}3}$

Gray Code

Each Gray code number differs from the preceding number by a single bit.

Decimal	Gray code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111

Binary to Gray conversion

Step 1:
Shift the binary number one position to the right, LSB of the shifted number is discarded.

Step 2:
Exclusive OR the bits of the binary number with those of the binary number shifted.

Example 49: Convert $(1001)_2$ to Gray code

Binary	$\rightarrow$ 1010
Shifted Binary	$\rightarrow$ 101 ⊗
Gray	$\rightarrow$ 1111

Gray to binary conversion

1. Take the MSB of the binary number is same as MSB of Gray code number.
2. X-OR the MSB of the binary to the next significant bit of the Gray code.
3. X-OR the second bit of binary to the third bit of Gray code to get third bit binary and so on.
4. Continue this till all the Gray bits are exhausted.

Example 50: Convert Gray code 1010 to binary

Gray	1010	1	0	1	0
	1100	⇓	⊕↗ ||	⊕↗ ||	⊕↗ ||
$= (1100)_2$		1	1	0	0

Exercises

Practice Problems I

Direction for questions 1 to 15: Select the correct alternative from the given choices.

1. Assuming all the numbers are in 2's complement representation, which of the following is divisible by 11110110?
(A) 11101010 (B) 11100010
(C) 11111010 (D) 11100111

2. If $(84)_x$ (in base-x number system) is equal to $(64)_y$ (in base-y number system), then possible values of x and y are
(A) 12, 9 (B) 6, 8
(C) 9, 12 (D) 12, 18

3. Let A = 1111 1011 and B = 0000 1011 be two 8-bit signed 2's complement numbers. Their product in 2's complement representation is
(A) 11001001 (B) 10011100
(C) 11010101 (D) 10101101

4. Let r denotes number system's radix. The only value(s) of r that satisfy the equation $\sqrt[3]{(1331)_r} = (11)_r$ is/are
(A) 10 (B) 11
(C) 10 and 11 (D) any $r > 3$

5. X is 16-bit signed number. The 2's complement representation of X is $(F76A)_{16}$. The 2's complement representation of $8 \times X$ is
(A) $(1,460)_{16}$ (B) $(D643)_{16}$
(C) $(4,460)_{16}$ (D) $(BB50)_{16}$

6. The HEX number $(CD.\ EF)_{16}$ in octal number system is
(A) $(315.736)_8$ (B) $(513.637)_8$
(C) $(135.673)_8$ (D) $(531.367)_8$

7. 8-bit 2's complement representation a decimal number is 10000000. The number in decimal is
(A) +256 (B) 0 (C) −128 (D) −256

8. The range of signed decimal numbers that can be represented by 7-bit 1's complement representation is
(A) −64 to +63 (B) −63 to +63
(C) −127 to +128 (D) −128 to +127

9. Decimal 54 in hexadecimal and BCD number system is, respectively,
(A) 63, 10000111 (B) 36,01010100
(C) 66, 01010100 (D) 36, 00110110

10. A new binary-coded hexadecimal (BCH) number system is proposed in which every digit of a base-6 number system is represented by its corresponding 3-bit binary code. For example, the base-6 number 35 will be represented by its BCH code 011101.
In this numbering system, the BCH code 001001101011 corresponds to the following number in base-6 system.
(A) 4,651 (B) 4,562
(C) 1,153 (D) 1,353

11. The signed 2's complement representation of $(-589)_{10}$ in hexadecimal number system is
(A) $(F24D)_{16}$ (B) $(FDB3)_{16}$
(C) $(F42D)_{16}$ (D) $(F3BD)_{16}$

12. The base of the number system for which the following operation is to be correct $\frac{66}{5}=13$

(A) 6 (B) 7 (C) 8 (D) 9

13. The solution to the quadratic equation $x^2 - 11x + 13 = 0$ (in number system with radix r) are $x = 2$ and $x = 4$. Then, the base of the number system is (r =)

(A) 7 (B) 6 (C) 5 (D) 4

14. The 16's complement of BADA is

(A) 4,525 (B) 4,526
(C) ADAB (D) 2,141

15. In expression $(11A1B)_8 = (12CD)_{16}$, A and B represent positive digits in octal number system, and C and D have their original meaning in hexadecimal. Find the values of A and B?

(A) 2, 5 (B) 2, 3 (C) 3, 2 (D) 3, 5

Practice Problems 2

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. The hexadecimal representation of $(567)_8$ is
(A) 1 A F (B) D 7 7 (C) 1 7 7 (D) 1 3 3

2. $(2326)_8$ is equivalent to
(A) $(14D6)_{16}$ (B) $(103112)_4$
(C) $(1283)_{10}$ (D) $(09AC)_{16}$

3. $(0.46)_8$ equivalent in decimal is
(A) 0.59375 (B) 0.3534 (C) 0.57395 (D) 0.3435

4. The 15's complement of $(C\ A\ F\ A)_{16}$ is
(A) $(2{,}051)_{16}$ (B) $(2{,}050)_{16}$
(C) $(3{,}506)_{16}$ (D) $(3{,}505)_{16}$

5. 53 in 2's complement can be given as
(A) 1001011 (B) 001010
(C) 0110101 (D) 001011

6. Signed 2's complement representation of $(-15)_{10}$ is
(A) 11111 (B) 10001 (C) 01111 (D) 10000

7. $(0.25)_{10}$ in binary number system is
(A) (0.01) (B) (0.11) (C) 0.001 (D) 0.101

8. The equivalent of $(25)_6$ in number system with base 7 is
(A) 22 (B) 23 (C) 24 (D) 26

9. The operation 35 + 26 = 63 is true in number system with radix
(A) 7 (B) 8 (C) 9 (D) 11

10. The hexadecimal equivalent of largest binary number with 14 bits is
(A) 2FFF (B) 3FFFF (C) FFFF (D) 1FFFF

11. If x is radix of number system, $(211)_x = (152)_8$, then x is
(A) 6 (B) 7 (C) 9 (D) 5

12. The value of r, for which $\sqrt{(224)_r} = (13)_r$ is valid expression, in number system with radix r is
(A) 5 (B) 6 (C) 7 (D) 8

13. Which of the representation in binary arithmetic has a unique zero?
(A) signed magnitude (B) 1's compliment
(C) 2's complement (D) All of these

14. For the binary number 101101111, the equivalent hexadecimal number is
(A) 14E (B) 9E (C) B78 (D) 16F

15. Subtract 1001 from 1110
(A) 0010 (B) 0101 (C) 1011 (D) 1010

16. Which of the following is a positively weighted code?
(A) 8421 (B) 84-2-1 (C) EXS-3 (D) 74-2-1

17. Match the items correctly

Column 1	Column 2
(P) 8421	(1) cyclic code
(Q) 2421	(2) self-complementing
(R) 5212	(3) sequential code
(S) Gray code	(4) non-sequential code

(A) P-2, Q-4, R-3, S-1 (B) P-1, Q-4, R-3, S-2
(C) P-3, Q-2, R-4, S-1 (D) P-2, Q-4, R-1, S-2

18. Perform the subtraction in XS-3 code
57.6 – 27.8
(A) 0101 1100.1011 (B) 0010 1001.1100
(C) 00011101.1100 (D) 1010 1110.1011

19. The 2's complement representation of –17 is
(A) 101110 (B) 111110
(C) 101111 (D) 110001

20. The decimal 398 is represented in 2421 code by
(A) 110000001000 (B) 001110011000
(C) 001111111110 (D) 010110110010

Previous Years' Questions

1. The range is signed decimal numbers that can be represented by 6-bit 1's complement number is **[2004]**
(A) –31 to +31 (B) –63 to +64
(C) –64 to +63 (D) –32 to + 31

2. 11001, 1001, and 111001 correspond to the 2's complement representation of which one of the following sets of numbers? **[2004]**
(A) 25, 9, and 57 (B) –6, –6, and –6
(C) –7, –7, and –7 (D) –25, –9, and –57

3. Decimal 43 in hexadecimal and BCD number system is, respectively, **[2005]**
(A) B2, 0000 0011 (B) 2B, 0100 0011
(C) 2B, 0011 0100 (D) B2, 0100 0100

4. A new binary-coded pentary (BCP) number system is proposed in which every digit of a base-5 number is represented by its corresponding 3-bit binary code. For example, the base-5 number 24 will be represented by its BCP code 010100. In this numbering system, the BCP code 100010011001 corresponds to the following number in base-5 system **[2006]**
(A) 423 (B) 1324 (C) 2201 (D) 4231

5. $X = 01110$ and $Y = 11001$ are two 5-bit binary numbers represented in two's complement format. The sum of X and Y represented in 2's complement format using 6 bits is: **[2007]**
(A) 100111 (B) 001000
(C) 000111 (D) 101001

6. The two numbers represented in signed 2's complement form are $P = 11101101$ and $Q = 11100110$. If Q is subtracted from P, the value obtained in signed 2's complement form is **[2008]**
(A) 100000111 (B) 00000111
(C) 11111001 (D) 111111001

7. The number of bytes required to represent the decimal number 1,856,357 in packed BCD form is ________ **[2014]**

Answer Keys

Exercises

Practice Problems 1

1. B **2.** C **3.** A **4.** D **5.** D **6.** A **7.** C **8.** B **9.** B **10.** C
11. B **12.** D **13.** C **14.** B **15.** D

Practice Problems 2

1. C **2.** B **3.** A **4.** D **5.** D **6.** B **7.** A **8.** B **9.** B **10.** B
11. B **12.** A **13.** C **14.** D **15.** B **16.** A **17.** C **18.** A **19.** C **20.** C

Previous Years' Questions

1. A **2.** C **3.** B **4.** D **5.** C **6.** B **7.** 3.9 to 4.1