2.7 TRANSPORT PHENOMENA

2.220 (a) The fraction of gas molecules which traverses distances exceeding the mean free path without collision is just the probability to traverse the distance $s = \lambda$ without collision.

Thus $$P = e^{-1} = \frac{1}{e} = 0{\cdot}37$$

(b) This probability is

$$P = e^{-1} - e^{-2} = 0{\cdot}23$$

2.221 From the formula

$$\frac{1}{\eta} = e^{-\Delta l/\lambda} \quad \text{or} \quad \lambda = \frac{\Delta l}{\ln \eta}$$

2.222 (a) Let $P(t)$ = probability of no collision in the interval $(0, t)$. Then

$$P(t + dt) = P(t)(1 - \alpha\, dt)$$

or $$\frac{dP}{dt} = -\alpha P(t) \quad \text{or} \quad P(t) = e^{-\alpha t}$$

where we have used $P(0) = 1$

(b) The mean interval between collision is also the mean interval of no collision. Then

$$\langle t \rangle = \frac{\int_0^\infty t e^{-\alpha t} dt}{\int_0^\infty e^{-\alpha t} dt} = \frac{1}{\alpha} \frac{\Gamma(2)}{\Gamma(1)} = \frac{1}{\alpha}$$

2.223 (a) $$\lambda = \frac{1}{\sqrt{2}\, \pi d^2 n} = \frac{kT}{\sqrt{2}\, \pi d^2 p}$$

$$= \frac{1{\cdot}38 \times 10^{-23} \times 273}{\sqrt{2}\, \pi (0{\cdot}37 \times 10^{-9})^2 \times 10^5} = 6{\cdot}2 \times 10^{-8}\ \text{m}$$

$$\tau = \frac{\lambda}{\langle v \rangle} = \frac{6{\cdot}2 \times 10^{-8}}{454} s = {\cdot}136\ \text{n s}$$

$\lambda = 6.2 \times 10^6$ m

(b) $\eta = 1{\cdot}36 \times 10^4$ s $= 3{\cdot}8$ hours

2.224 The mean distance between molecules is of the order

$$\left(\frac{22{\cdot}4 \times 10^{-3}}{6{\cdot}0 \times 10^{23}}\right)^{1/3} = \left(\frac{224}{6}\right)^{1/3} \times 10^{-9}\ \text{meters} \approx 3{\cdot}34 \times 10^{-9}\ \text{meters}$$

This is about 18.5 times smaller than the mean free path calculated in 2.223 (a) above.

2.225 We know that the Vander Waal's constant b is four times the molecular volume. Thus

$$b = 4 N_A \frac{\pi}{6} d^3 \quad \text{or} \quad d = \left(\frac{3b}{2\pi N_A}\right)^{1/3}$$

Hence $$\lambda = \left(\frac{kT_0}{\sqrt{2}\, \pi p_0}\right)\left(\frac{2\pi N_A}{3b}\right)^{2/3}$$

2.226 The volocity of sound in N_2 is

$$\sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

so, $$\frac{1}{\nu} = \sqrt{\frac{\gamma RT_0}{M}} = \frac{RT_0}{\sqrt{2}\,\pi d^2 p_0 N_A}$$

or, $$\nu = \pi d^2 p_0 N_A \sqrt{\frac{2\gamma}{MRT_0}}$$

2.227 (a) $\lambda > l$ if $p < \dfrac{kT}{\sqrt{2}\,\pi d^2 l}$

Now $$\frac{kT}{\sqrt{2}\,\pi d^2 l} \text{ for } O_2 \text{ of } O \text{ is } 0{\cdot}7\text{ Pa.}$$

(b) The corresponding n is obtained by dividing by kT and is $1{\cdot}84 \times 10^{20}$ per $m^3 = 1{\cdot}84^{14}$per c.c. and the corresponding mean distance is $\dfrac{l}{n^{1/3}}$.

$$= \frac{10^{-2}}{(0{\cdot}184)^{1/3} \times 10^5} = 1{\cdot}8 \times 10^{-}\ m \approx 0{\cdot}18\ \mu\text{m}.$$

2.228 (a) $\nu = \dfrac{1}{\tau} = \dfrac{1}{\lambda/\langle v\rangle} = \dfrac{\langle v\rangle}{\lambda}$

$$= \sqrt{2}\,\pi d^2 n \langle v\rangle = {\cdot}74 \times 10^{10}\,\text{s}^{-1} \text{ (see 2.223)}$$

(b) Total number of collisions is

$$\frac{1}{2} n\nu \approx 1{\cdot}0 \times 10^{29}\,\text{s cm}^{-3}$$

Note, the factor $\frac{1}{2}\cdot$ When two molecules collide we must not count it twice.

2.229 (a) $\lambda = \dfrac{1}{\sqrt{2}\,\pi d^2 n}$

d is a constant and n is a constant for an isochoric process so λ is constant for an isochoric process.

$$\nu = \frac{\langle v\rangle}{\lambda} = \frac{\sqrt{\frac{8RT}{M\pi}}}{\lambda} \alpha \sqrt{T}$$

(b) $\lambda = \dfrac{1}{\sqrt{2}\,\pi d^2} \dfrac{kT}{p} \alpha\, T$ for an isobaric process.

$$\nu = \frac{\langle v\rangle}{\lambda} \alpha \frac{\sqrt{T}}{T} = \frac{1}{\sqrt{T}} \text{ for an isobaric process.}$$

2.230 (a) In an isochoric process λ is constant and

$$\nu\, \alpha \sqrt{T}\, \alpha \sqrt{pV}\, \alpha \sqrt{p}\, \alpha \sqrt{n}$$

(b) $\lambda = \dfrac{kT}{\sqrt{2}\,\pi d^2 p}$ must decrease n times in an isothermal process and ν must increase n times because $\langle v\rangle$ is constant in an isothermal process.

2.231 (a) $\lambda \alpha \frac{1}{n} \Rightarrow \frac{1}{N/V} = \frac{V}{N}$

Thus $$\lambda \alpha V \text{ and } \nu \alpha \frac{T^{1/2}}{V}$$

But in an adiabatic process $\left(\gamma = \frac{7}{5} \text{ here}\right)$

$$TV^{\gamma - 1} = \text{constant so } TV^{2/5} = \text{ constant}$$

or $$T^{1/2} \alpha V^{-1/5} \text{ Thus } \nu \alpha V^{-6/5}$$

(b) $\lambda \alpha \frac{T}{p}$

But $p\left(\frac{T}{p}\right)^{\gamma} = \text{constant}$ or $\frac{T}{p} \alpha p^{-1/\gamma}$ or $T \alpha p^{1 - 1/\gamma}$

Thus $$\lambda \alpha p^{-1/\gamma} = p^{-5/7}$$

$$\nu = \frac{<v>}{\lambda} \alpha \frac{p}{\sqrt{T}} \alpha p^{1/2 + \frac{1}{2\gamma}} = p^{\frac{\gamma + 1}{2\gamma}} = p^{6/7}$$

(c) $\lambda \alpha V$

But $$TV^{2/5} = \text{constant or } V \alpha T^{-5/2}$$

Thus $$\lambda \alpha T^{-5/2}$$

$$\nu \alpha \frac{T^{1/2}}{V} \alpha T^{3}$$

2.232 In the polytropic process of index n

$$pV^{n} = \text{constant, } TV^{n-1} = \text{constant and } p^{1-n} T^{n} = \text{constant}$$

(a) $\lambda \alpha V$

$$\nu \alpha \frac{T^{1/2}}{V} = V^{\frac{1-n}{2}} V^{-1} = V^{\frac{-n+1}{2}}$$

(b) $\lambda \alpha \frac{T}{p}$, $T^{n} \alpha p^{n-1}$ or $T \alpha p^{1 - \frac{1}{n}}$

so $$\lambda \alpha p^{-1/n}$$

$$\nu = \frac{<v>}{\lambda} \alpha \frac{p}{\sqrt{T}} \alpha p^{1 - \frac{1}{2} + \frac{1}{2n}} = p^{\frac{n+1}{2n}}$$

(c) $\lambda \alpha \frac{T}{p}$, $p \alpha T^{\frac{n}{n-1}}$

$$\lambda \alpha T^{1 - \frac{n}{n1}} = T^{-\frac{1}{n-1}} = T^{\frac{1}{1-n}}$$

$$\nu \alpha \frac{p}{\sqrt{T}} \alpha T^{\frac{n}{n-1} - \frac{1}{2}} = T^{\frac{n+1}{2(n-1)}}$$

2.233 (a) The number of collisions between the molecules in a unit volume is

$$\frac{1}{2} n\, \nu = \frac{1}{\sqrt{2}} \pi d^2 n^2 <v> \alpha \frac{\sqrt{T}}{V^2}$$

This remains constant in the poly process pV^{-3} = constant

Using (2.122) the molar specific heat for the polytropic process

$$pV^{\alpha} = \text{constant},$$

is
$$C = R\left(\frac{1}{\gamma - 1} - \frac{1}{\alpha - 1}\right)$$

Thus
$$C = R\left(\frac{1}{\gamma - 1} + \frac{1}{4}\right) = R\left(\frac{5}{2} + \frac{1}{4}\right) = \frac{11}{4} R$$

It can also be written as $\frac{1}{4} R\,(1 + 2\,i)$ where $i = 5$

(b) In this case $\frac{\sqrt{T}}{V}$ = constant and so pV^{-1} = constant

so
$$C = R\left(\frac{1}{\gamma - 1} + \frac{1}{2}\right) = R\left(\frac{5}{2} + \frac{1}{2}\right) = 3R$$

It can also be written as $\frac{R}{2}(i + 1)$

2.234 We can assume that all molecules, incident on the hole, leak out. Then,

$$-dN = -d\,(nV) = \frac{1}{4} n <v> S\, dt$$

or
$$dn = -n \frac{dt}{4v/S <v>} = -n \frac{dt}{\tau}$$

Integrating $n = n_0\, e^{-t/\tau}$. Hence $<v> = \sqrt{\frac{8RT}{\pi M}}$

2.235 If the temperature of the compartment 2 is η times more than that of compartment 1, it must contain $\frac{1}{\eta}$ times less number of molecules since pressure must be the same when the big hole is open. If M = mass of the gas in 1 than the mass of the gas in 2 must be $\frac{M}{\eta}$. So immediately after the big hole is closed.

$$n_1^0 = \frac{M}{mV},\ n_2^0 = \frac{M}{mV\eta}$$

where m = mass of each molecule and n_1^0, n_2^0 are concentrations in 1 and 2. After the big hole is closed the pressures will differ and concentration will become n_1 and n_2 where

$$n_1 + n_2 = \frac{M}{m\,V\eta}(1 + \eta)$$

On the other hand

$$n_1 <v_1> = n_2 <v_2> \quad \text{i.e. } n_1 = \sqrt{\eta}\, n_2$$

Thus $$n_2'(1+\sqrt{\eta}) = \frac{m}{mV\eta}(1+\eta) = n_2^0(1+\eta)$$

So $$n_2 = n_2^0 \frac{1+\eta}{1+\sqrt{\eta}}$$

2.236 We know

$$\eta = \frac{1}{3}<v>\lambda\rho = \frac{1}{3}<v>\frac{1}{\sqrt{2}\,\pi d^2} m \alpha \sqrt{T}$$

Thus η changing α times implies T changing α^2times.
On the other hand

$$D = \frac{1}{3}<v>\lambda = \frac{1}{3}\sqrt{\frac{8kT}{\pi m}}\frac{kT}{\sqrt{2}\,\pi d^2 p}$$

Thus D changing β times means $\frac{T^{3/2}}{p}$ changing β times

So p must change $\frac{\alpha^3}{\beta}$ times

2.237 $D \alpha \frac{\sqrt{T}}{n} \propto V\sqrt{T}$, $\eta \propto \sqrt{T}$

(a) D will increase n times
η will remain constant if T is constant

(b) $D \alpha \frac{T^{3/2}}{p} \alpha \frac{(pV)^{3/2}}{p} = p^{1/2} V^{3/2}$

$$\eta \alpha \sqrt{pV}$$

Thus D will increase $n^{3/2}$ times, η will increase $n^{1/2}$ times, if p is constant

2.238 $D \alpha V\sqrt{T}$, $\eta \alpha \sqrt{T}$

In an adiabatic process

$$TV^{\gamma-1} = \text{constant, or } T \propto V^{1-\gamma}$$

Now V is decreased $\frac{1}{n}$ times. Thus

$$D \alpha V^{\frac{3-\gamma}{2}} = \left(\frac{1}{n}\right)^{\frac{3-\gamma}{2}} = \left(\frac{1}{n}\right)^{4/5}$$

$$\eta \alpha \text{ of } V^{\frac{1-\gamma}{2}} = \left(\frac{1}{n}\right)^{-1/5} = n^{1/5}$$

So D decreases $n^{4/5}$times and η increase $n^{1/5}$times.

2.239 (a) $D \alpha V\sqrt{T} \alpha \sqrt{pV^3}$

Thus D remains constant in the process pV^3 = constant
So polytropic index $n = 3$

(b) $\eta \alpha \sqrt{T} \alpha \sqrt{pV}$

So η remains constant in the isothermal process

$$pV = \text{constant}, \; n = 1, \text{ here}$$

(c) Heat conductivity $\kappa = \eta\, C_V$

and C_V is a constant for the ideal gas

Thus $$n = 1 \text{ here also,}$$

2.240 $$\eta = \frac{1}{3}\sqrt{\frac{8\,kT}{\pi\, m}}\,\frac{m}{\sqrt{2}\,\pi\, d^2} = \frac{2}{3}\sqrt{\frac{m\,kT}{\pi^3}}\,\frac{1}{d^2}$$

or $$d = \left(\frac{2}{3\,\eta}\right)^{1/2}\left(\frac{m\,kT}{\pi^3}\right)^{1/4} = \left(\frac{2}{3\times 18{\cdot}9}\times 10^6\right)^{1/2}\left(\frac{4\times 8{\cdot}31\times 273\times 10^{-3}}{\pi^3\times 36\times 10^{46}}\right)^{1/4}$$

$$= 10^{-10}\left(\frac{2}{3\times 18{\cdot}9}\right)^{1/2}\left(\frac{4\times 83{\cdot}1\times 273}{\pi^3\times{\cdot}36}\right)^{1/4} \approx 0{\cdot}178\text{ nm}$$

2.241 $$\kappa = \frac{1}{3}<v>\lambda\,\rho\, c_V$$

$$= \frac{1}{3}\sqrt{\frac{8\,kT}{m\,\pi}}\,\frac{1}{\sqrt{2}\,\pi\, d^2\, n}\, mn\,\frac{C_V}{M}$$

$\left(C_V \text{ is the specific heat capacity which is } \frac{C_V}{M}\right)$. Now C_V is the same for all monoatomic gases such as He and A. Thus

$$\kappa\,\alpha\,\frac{1}{\sqrt{M}\, d^2}$$

or $$\frac{\kappa_{He}}{\kappa_A} = 8{\cdot}7 = \frac{\sqrt{M_A}\, d_A^2}{\sqrt{M_{H_e}}\, d_{H_e}^2} = \sqrt{10}\,\frac{d_A^2}{d_{H_e}^2}$$

$$\frac{d_A}{d_{H_e}} = \sqrt{\frac{8{\cdot}7}{\sqrt{10}}} = 1{\cdot}658 \approx 1{\cdot}7$$

2.242 In this case

$$N_1\,\frac{r_2^2 - r_1^2}{r_1^2\, r_2^2} = 4\,\pi\,\eta\,\omega$$

or $$N_1\,\frac{2\,R\,\Delta R}{R^4} \approx 4\,\pi\,\eta\,\omega \quad\text{or}\quad N_1 = \frac{2\,\pi\,\eta\,\omega\, R^3}{\Delta R}$$

To decrease N_1, n times η must be decreased n times. Now η does not depend on pressure until the pressure is so low that the mean free path equals, say, $\frac{1}{2}\Delta R$. Then the mean free path is fixed and η decreases with pressure. The mean free path equals $\frac{1}{2}\Delta R$ when

$$\frac{1}{\sqrt{2}\,\pi\, d^2\, n_0} = \Delta R \;\; (n_0 = \text{concentration})$$

Corresponding pressure is $p_0 = \dfrac{\sqrt{2}\,k\,T}{\pi\,d^2\,\Delta R}$

The sought pressure is n times less

$$p = \frac{\sqrt{2}\,k\,T}{\pi\,d^2\,n\,\Delta R} = 70{\cdot}7 \times \frac{10^{-23}}{10^{-18} \times 10^{-3}} = 0{\cdot}71 \text{ Pa}$$

The answer is qualitative and depends on the choice $\dfrac{1}{2}\Delta R$ for the mean free path.

2.243 We neglect the moment of inertia of the gas in a shell. Then the moment of friction forces on a unit length of the cylinder must be a constant as a function of r.

So, $$2\,\pi\,r^3\,\eta\,\frac{d\omega}{dr} = N_1 \quad \text{or} \quad \omega(r) = \frac{N_1}{4\,\pi\,\eta}\left(\frac{1}{r_1^2} - \frac{1}{r^2}\right)$$

and $$\omega = \frac{N_1}{4\,\pi\,\eta}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \quad \text{or} \quad \eta = \frac{N_1}{4\,\pi\,\omega}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

2.244 We consider two adjoining layers. The angular velocity gradient is $\dfrac{\omega}{h}$. So the moment of the frictional force is

$$N = \int_0^a r \cdot 2\,\pi\,r\,dr \cdot \eta\,r\,\frac{\omega}{h} = \frac{\pi\,\eta\,a^4\,\omega}{2h}$$

2.245 In the ultrararefied gas we must determine η by taking $\lambda = \dfrac{1}{2}h$. Then

$$\eta = \frac{1}{3}\sqrt{\frac{8kT}{m\pi}} \times \frac{1}{2}h \times \frac{mp}{kT} = \frac{1}{3}\sqrt{\frac{2M}{\pi\,RT}}\,hp$$

so, $$N = \frac{1}{3}\,\omega\,a^4\,p\sqrt{\frac{\pi\,M}{2RT}}$$

2.246 Take an infinitesimal section of length dx and apply Poiseuilles equation to this. Then

$$\frac{dV}{dt} = \frac{-\pi\,a^4}{8\eta}\,\frac{\partial p}{\partial x}$$

From the formula $$pV = RT \cdot \frac{m}{M}$$

$$pdV = \frac{RT}{M}\,dm$$

or $$\frac{dm}{dt} = \mu = -\frac{\pi\,a^4\,M}{8\,\eta\,RT}\,\frac{pdp}{dx}$$

This equation implies that if the flow is isothermal then $p\dfrac{dp}{dx}$ must be a constant and so equals $\dfrac{\left|p_2^2 - p_1^2\right|}{2\,l}$ in magnitude.

Thus, $$\mu = \frac{\pi\,a^4\,M}{16\,\eta\,R\,T}\,\frac{\left|p_2^2 - p_1^2\right|}{l}$$

2.247 Let $T =$ temperature of the interface.

Then heat flowing from left = heat flowing into right in equilibrium.

Thus, $$\kappa_1 \frac{T_1 - T}{l_1} = \kappa_2 \frac{T - T_2}{l_2} \quad \text{or} \quad T = \frac{\left(\frac{\kappa_1 T_1}{l_1} + \frac{\kappa_2 T_2}{l_2}\right)}{\left(\frac{\kappa_1}{l_1} + \frac{\kappa_2}{l_2}\right)}$$

2.248 We have

$$\kappa_1 \frac{T_1 - T}{l_1} = \kappa_2 \frac{T - T_2}{l_2} = \kappa \frac{T_1 - T_2}{l_1 + l_2}$$

or using the previous result

$$\frac{\kappa_1}{l_1}\left(T_1 - \frac{\frac{\kappa_1 T_1}{l_1} + \frac{\kappa_2 T_2}{l_2}}{\frac{\kappa_1}{l_1} + \frac{\kappa_1}{l_2}}\right) = \kappa \frac{T_1 - T_2}{l_1 + l_2}$$

or $$\frac{\kappa_1}{l_1} \frac{\frac{\kappa_2}{l_2}(T_1 - T_2)}{\frac{\kappa_1}{l_1} + \frac{\kappa_2}{l_2}} = \kappa \frac{T_1 - T_2}{l_1 + l_2} \quad \text{or} \quad \kappa = \frac{l_1 + l_2}{\frac{l_1}{\kappa_1} + \frac{l_2}{\kappa_2}}$$

2.249 By definition the heat flux (per unit area) is

$$\dot{Q} = -K \frac{dT}{dx} = -\alpha \frac{d}{dx} \ln T = \text{constant} = +\alpha \frac{\ln T_1/T_2}{l}$$

Integrating $$\ln T = \frac{x}{l} \ln \frac{T_2}{T_1} + \ln T_1$$

where $T_1 =$ temperature at the end $x = 0$

So $$T = T_1 \left(\frac{T_2}{T_1}\right)^{x/l} \quad \text{and} \quad \dot{Q} = \frac{\alpha \ln T_1/T_2}{l}$$

2.250 Suppose the chunks have temperatures T_1, T_2 at time t and $T_1 - dT_1$, $T_2 + dT_2$ at time $dt + t$.

Then $$C_1\, dT_1 = C_2\, dT_2 = \frac{\kappa S}{l}(T_1 - T_2)\, dt$$

Thus $$d\,\Delta T = -\frac{\kappa S}{l}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \Delta T\, dt \text{ where } \Delta T = T_1 - T_2$$

Hence $$\Delta T = (\Delta T)_0\, e^{-t/\tau} \text{ where } \frac{1}{\tau} = \frac{\kappa s}{l}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

2.251 $\dot{Q} = \kappa \frac{\partial T}{\partial x} = -A\sqrt{T}\,\frac{\partial T}{\partial x}$

$$= -\frac{2}{3}A\,\frac{\partial T^{3/2}}{\partial x},\ (A = \text{constant})$$

$$= \frac{2}{3}A\,\frac{(T_1^{3/2} - T_2^{3/2})}{l}$$

Thus $\qquad T^{3/2} = \text{constant} - \frac{x}{l}\left(T_1^{3/2} - T_2^{3/2}\right)$

or using $\qquad T = T_1$ at $x = 0$

$$T^{3/2} = T_1^{3/2} + \frac{x}{l}\left(T_2^{3/2} - T_1^{3/2}\right) \text{ or } \left(\frac{T}{T_1}\right)^{3/2} = 1 + \frac{x}{l}\left(\left(\frac{T_2}{T}\right)^{3/2} - 1\right)$$

$$T = T_1\left[1 + \frac{x}{l}\left\{\left(\frac{T_2}{T_1}\right)^{3/2} - 1\right\}\right]^{2/3}$$

2.252 $\kappa = \frac{1}{3}\sqrt{\frac{8RT}{\pi M}}\,\frac{1}{\sqrt{2}\,\pi d^2 n}\,mn\,\frac{R\frac{i}{2}}{M} = \frac{R^{3/2}\,i\,T^{3/2}}{3\pi^{3/2}d^2\sqrt{M}\,N_A}$

Then from the previous problem

$$q = \frac{2i\,R^{3/2}\,(T_2^{3/2} - T_1^{3/2})}{9\pi^{3/2}d^2\sqrt{M}\,N_A\,l},\ i = 3 \text{ here.}$$

2.253 At this pressure and average temparature $= 27°C = 300\text{K} = T = \frac{(T_1 + T_2)}{2}$

$\lambda = \frac{1}{\sqrt{2}\,\pi d^2}\frac{\kappa T}{p} = 2330 \times 10^{-5}\text{m} = 23{\cdot}3\text{mm} >> 5{\cdot}0\text{mm} = l$

The gas is ultrathin and we write $\lambda = \frac{1}{2}l$ here

Then $\qquad q = \kappa\frac{dT}{dx} = \kappa\,\frac{T_2 - T_1}{l}$

where $\qquad \kappa = \frac{1}{3}<v> \times \frac{1}{2}l \times \frac{MP}{RT} \times \frac{R}{\gamma - 1} \times \frac{1}{M} = \frac{p<v>}{6T(\gamma-1)}\,l$

and $\qquad q = \frac{p<v>}{6T(\gamma - 1)}(T_2 - T_1)$

where $\quad <v> = \sqrt{\frac{8RT}{M\pi}}$. We have used $T_2 - T_1 << \frac{T_2 + T_1}{2}$ here.

2.254 In equilibrium $2\pi r \kappa \frac{dT}{dr} = -A =$ constant. So $T = B - \frac{A}{2\pi\kappa} \ln r$

But $T = T_1$ when $r = R_1$ and $T = T_2$. when $r = R_2$.

From this we find $T = T_1 + \frac{T_2 - T_1}{\ln \frac{R_2}{R_1}} \ln \frac{r}{r_1}$

2.255 In equilibrium $4\pi r^2 \kappa \frac{dT}{dr} = -A =$ constant

$$T = B + \frac{A}{4\pi\kappa} \frac{1}{r}$$

Using $T = T_1$ when $r = R_1$ and $T = T_2$ when $r = R_2$,

$$T = T_1 + \frac{T_2 - T_1}{\frac{1}{R_2} - \frac{1}{R_1}} \left(\frac{1}{r} - \frac{1}{R_1} \right)$$

2.256 The heat flux vector is $-\kappa$ grad T and its divergence equals w. Thus

$$\nabla^2 T = -\frac{w}{\kappa}$$

or $$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = -\frac{w}{\kappa} \text{ in cylindrical coordinates.}$$

or $$T = B + A \ln r - \frac{\omega}{2x} r^2$$

Since T is finite at $r = 0, A = 0$. Also $T = T_0$ at $r = R$

so $$B = T_0 + \frac{w}{4\kappa} R^2$$

Thus $$T = T_0 + \frac{w}{4\kappa} (R^2 - r^2)$$

r here is the distance from the axis of wire (axial radius).

2.257 Here again

$$\nabla^2 T = -\frac{w}{\kappa}$$

So in spherical polar coordinates,

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = -\frac{w}{\kappa} \text{ or } r^2 \frac{\partial T}{\partial r} = -\frac{w}{3\kappa} r^3 + A$$

or $$T = B - \frac{A}{r} - \frac{w}{6\kappa} r^2$$

Again $$A = 0 \text{ and } B = T_0 + \frac{w}{6\kappa} R^2$$

so finally $$T = T_0 + \frac{w}{6\kappa} (R^2 - r^2)$$