

Section - D

30) Frequency distribution.

(choice 1)

(4)

Class	Frequency	x_i	$f x_i$
11-13	3	12	$3 \times 12 = 36$
13-15	6	14	$6 \times 14 = 84$
15-17	9	16	$9 \times 16 = 144$
17-19	13	18	$13 \times 18 = 234$
19-21	f	20	$f \times 20 = 20f$
21-23	5	22	$5 \times 22 = 110$
23-25	4	24	$4 \times 24 = 96$
Total: $\rightarrow$	$40 + f$		$704 + 20f$

Given, mean = 18. To find: Value of f .

We know,

$$\text{mean } (\bar{x}) = \frac{\sum f x_i}{\sum f_i}$$

$$\rightarrow 18 = \frac{704 + 20f}{40 + f}$$

$$16 = 2f$$

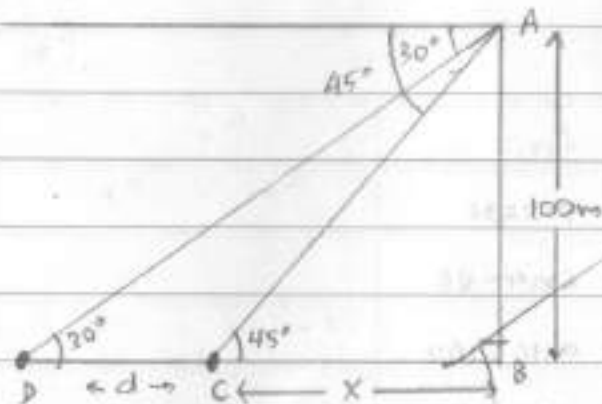
$$\Rightarrow f = 8.$$

$$720 + 18f = 704 + 20f.$$

The value of f is 8.

29) Diagram:

(4)



AB $\rightarrow$ lighthouse = 100m high

C $\rightarrow$ boat 1

D $\rightarrow$ boat 2.

To find: $\overline{CD}$ or d .
(distance b/w ships)

We know,

$$\tan \angle ACB = \frac{\text{Opp.}}{\text{adj.}} = \frac{AB}{BC}$$

$$\tan \angle ADB = \frac{\text{Opp.}}{\text{adj.}} = \frac{AB}{BD}$$

$$\rightarrow \tan 45^\circ = \frac{100}{x}$$

$$1 = \frac{100}{x}$$

$$\Rightarrow x = 100 \text{ m.}$$

$$\rightarrow \tan 30^\circ = \frac{100}{x+d}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{d+100} \quad [x=100]$$

$$100+d = 100\sqrt{3}$$

$$\rightarrow d = 100\sqrt{3} - 100 = 100(\sqrt{3} - 1)$$

Given, $\sqrt{3} \approx 1.732$,

$$\Rightarrow d = 100(1.732 - 1)$$

$$= 100 \times 0.732 = 73.2 \text{ m.}$$

$\rightarrow$ The distance between the boats is 73.2m.

27) To prove: $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

U

Simplifying LHS:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A (1 - 2\sin^2 A)}{\cos A (2\cos^2 A - 1)}$$

$$= \frac{\sin A [1 - (2\sin^2 A)]}{\cos A [2\cos^2 A - 1]}$$

$$= \frac{\sin A [\sin^2 A + \cos^2 A - 2\sin^2 A]}{\cos A [2\cos^2 A - (\sin^2 A + \cos^2 A)]}$$

$$= \frac{\sin A [\cos^2 A - \sin^2 A]}{\cos A [\cos^2 A - \sin^2 A]}$$

$$= \frac{\sin A}{\cos A} \times 1$$

$$= \tan A$$

$$\text{LHS} = \text{RHS}$$

hence proved.

$$\left[\because \frac{\sin A}{\cos A} = \tan A \right]$$

28) Given, metal bucket shaped like frustum of cone.

Dimensions: height = 24 cm. (h)

Lower diameter = 10 cm $\rightarrow$ radius = $\frac{10}{2} = 5$ cm (r_1)

Upper diameter = 30 cm $\rightarrow$ radius = $\frac{30}{2} = 15$ cm (r_2)

To find: Metal sheet needed to make bucket.

$\rightarrow$ Area of metal sheet = Curved surface area +
Area of base.

We know,

Curved surface area of frustum = $\pi \times (R+r) \times l$ sq. units.

Where $l = \sqrt{h^2 + (R-r)^2}$ units.

To find l : $l = \sqrt{h^2 + (R-r)^2}$ units.

$$= \sqrt{24^2 + (15-5)^2} \text{ cm}$$

$$= \sqrt{576 + 10^2}$$

$$= \sqrt{676}$$

$$l = 26 \text{ cm.}$$

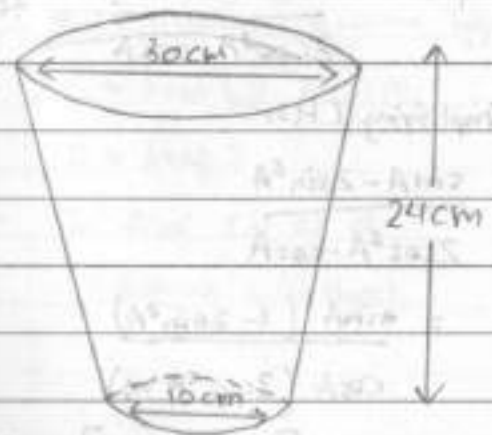
$$\Rightarrow \text{CSA} = \pi (R+r)l \text{ sq. units}$$

$$= 3.14 \times (15+5) \times 26 \text{ sq. cm.}$$

$$= 3.14 \times 20 \times 26$$

$$\text{CSA} = 1632.8 \text{ cm}^2$$

Bucket



We know, Area of circular base = πr^2 sq. units.

$$\rightarrow \text{Area} = 3.14 \times 5 \times 5 \text{ sq. cm}$$

$$= 3.14 \times 100$$

$$= \frac{314}{1}$$

$$\text{Area} = 78.5 \text{ cm}^2.$$

Total area of sheet = Curved area + Circular base area

$$= 1632.8 + 78.5 \text{ cm}^2$$

$$\text{Area} = 1711.3 \text{ cm}^2$$

The area of sheet needed is 1711.3 cm^2 .

- ii) Plastic buckets are less preferable to metal buckets because plastic buckets are more harmful to the environment. They may also leak harmful chemicals into the water being stored.

23) Representative diagram:

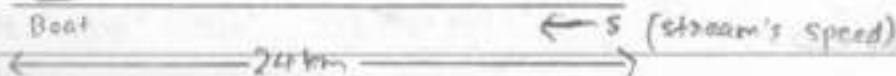
(choice 1)

4



Boat

$\rightarrow 18 \text{ km/hr}$



Given that:

Speed of boat = 18 km/hr in still water.

Speed of stream = s (variable, must find)

Distance upstream + back = 24 km

Time upstream = 1 hr more than time downstream.

We know, $\text{speed} = \frac{\text{Distance}}{\text{Time}} \rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$$\text{Time upstream} = \frac{24}{18-s}, \quad \text{Time downstream} = \frac{24}{18+s}$$

$$\rightarrow \frac{24}{18-s} = 1 + \frac{24}{18+s}$$

$$\frac{24}{18-s} = \frac{18+s+24}{18+s}$$

(Cross-multiplying)

$$24(18+s) = (42+s)(18-s)$$

$$432 + 24s = 756 + 18s - 42s - s^2$$

$$\rightarrow s^2 + 24s + 24s + 432 - 756 = 0$$

$$s^2 + 48s + (-324) = 0$$

$$s^2 + 54s - 6s - 324 = 0$$

$$s(s+54) - 6(s+54) = 0$$

$$(s-6)(s+54) = 0$$

Now, either $s - 6 = 0$ or $s + 54 = 0$.

$$\rightarrow s = 6$$

$$\rightarrow s = -54.$$

So speed = 6 or -54 km/hr.

But speed cannot be negative.

$\Rightarrow$ Speed of the stream is 6 km/hr.

25) Given: $\triangle ABC$ is equilateral.

$$\rightarrow AB = BC = CA, \angle A = \angle B = \angle C = 60^\circ$$

D is a point on BC such that $BD = \frac{1}{3} BC$.

To prove: $9(AD)^2 = 7(AB)^2$.

Construction: Draw $AE \perp BC$.

Proof: Let $BD = x$.

$$\Rightarrow BC = 3x = AB = AC \quad [\because \triangle ABC \text{ is equilateral}] \quad [\text{Given } BD = \frac{1}{3} BC].$$

Also, we know that $BE = \frac{1}{2} BC$ [Altitude in equilateral $\triangle$ bisects base].

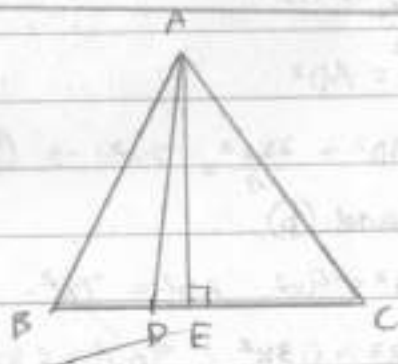
$$\text{As } \angle AEB = 90^\circ,$$

In $\triangle ABE$, by Pythagoras Theorem,

$$BE^2 + AE^2 = AB^2 \rightarrow AB^2 = 9x^2 \rightarrow \textcircled{a}.$$

$$\left(\frac{3x}{2}\right)^2 + AE^2 = (3x)^2.$$

$$\frac{9x^2}{4} + AE^2 = 9x^2 \rightarrow AE^2 = \frac{3 \times 9x^2}{4} \rightarrow \textcircled{b}$$



Now, in $\triangle ADE$, $\angle E = 90^\circ$.

$$DE = BE - BD$$

By Pythagoras Theorem,

$$DE = \frac{3x}{2} - x.$$

$$DE^2 + AE^2 = AD^2$$

$$\left(\frac{3x}{2} - x\right)^2 + \frac{27x^2}{4} = AD^2 \quad [\text{From ①}].$$

$$\left(\frac{x}{2}\right)^2 + \frac{27x^2}{4} = AD^2.$$

$$\frac{x^2 + 27x^2}{4} = AD^2$$

$$\Rightarrow AD^2 = \frac{28x^2}{4} = 7x^2 \rightarrow \text{⑥}.$$

From ④ and ⑥,

$$AB^2 = 9x^2, AD^2 = 7x^2.$$

$$7AB^2 = 63x^2, 9AD^2 = 63x^2.$$

$$\Rightarrow 7AB^2 = 9AD^2.$$

hence proved.

24) Given, sum of 4 consecutive no. in AP is 32.

Also, ratio of $t_1 \times t_n$ (first $\times$ last) and two middle terms product = 7:15.

Let no. of terms be n . n^{th} term = $t_n = a + (n-1)d$.

$$\rightarrow t_1 = a + (1-1)d.$$

$$t_n = a + (n-1)d.$$

As there are two middle terms, n is even.

$\Rightarrow$ middle term 1 = $a + \left(\frac{n+2}{2} - 1\right)d$. Let this be α .

middle term 2 = $a + \left(\frac{n}{2} - 1\right)d$. Let this be β .

Given, $\frac{t_1 + t_n}{\alpha \cdot \beta} = \frac{7}{15}$.

$$\frac{a \times [a + (n-1)d]}{\left(a + \frac{n}{2}d\right) \left[a + \left(\frac{n}{2} - 1\right)d\right]} = \frac{7}{15}$$

$$\rightarrow \frac{a^2 + a(n-1)d}{a^2 + \frac{n}{2}d \cdot \left(\frac{n}{2} - 1\right)d + a\left[\frac{n}{2}d + \left(\frac{n}{2} - 1\right)d\right]} = \frac{7}{15}$$

$$\frac{a^2 + a(n-1)d}{a^2 + \frac{n^2}{4}d^2 - \frac{n}{2}d^2 + a(n-1)d} = \frac{7}{15}$$

Cross-multiplying,

$$15a^2 + 15a(n-1)d = 7a^2 + \left(\frac{7n^2}{4} - \frac{7n}{2}\right)d^2 + 7a(n-1)d$$

$$8a^2 + 8a(n-1)d = \left(\frac{7n^2}{4} - \frac{7n}{2}\right)d^2$$

Assuming we take only 4 terms, then n will be 4.

Substituting ~~n~~ $n=4$ in above equation,

$$8a^2 + 8a(4-1)d = \left(\frac{7 \cdot 4^2}{4}, \frac{7 \times 4}{2}\right)d^2$$

$$8a^2 + 24ad = (28-14)d^2$$

$$8a^2 + 24ad = 14d^2 \rightarrow \textcircled{1}$$

$$\text{Sum} = 32 \rightarrow S_n = \frac{n}{2} [2a + (n-1)d]$$

$$32 = \frac{4}{2} [2a + 3d]$$

$$2a + 3d = 16 \rightarrow \textcircled{2}$$

$$\text{Squaring } \textcircled{2}, \quad 4a^2 + 9d^2 = 256 \rightarrow \textcircled{2.1}$$

$$+ 12ad$$

Substituting $\textcircled{3}$ in $\textcircled{2.1}$,

$$-15d^2 + 9d^2 = 256$$

$$-6d^2 = 256$$

$$d^2 = \frac{256}{6}$$

Substituting $\textcircled{3}$ in $\textcircled{2.1}$, $7d^2 + 9d^2 = 256$

$$16d^2 = 256 \rightarrow d^2 = 16, d = 4.$$

$$\text{Now, } 2a + 3(4) = 16.$$

$$2a = 4$$

$$a = 2.$$

$\rightarrow$ Terms = 2, 6, 10, 14.

a and a+3d.

The numbers are 2, 6, 10 and 14.

when $n=4$,

terms $a+d, a, a+2d, a+3d$.

$$7a(a+3d) = (a+d)(a+2d) \times 15$$

$$7(a^2 + 3ad) = (a^2 + ad + 2a^2 + 2ad) \times 15$$

$$\therefore 2d^2 = 0.$$

$$d = 0.$$

$$7a^2 + 21ad = 15a^2 + 15ad + 30d^2$$

$$-30d^2 = 8a^2 + 24ad$$

$$-15d^2 = 4a^2 + 12ad \rightarrow \textcircled{3}$$

terms $a+d, a, a+2d, a+3d$.

$$\frac{a(a+3d)}{(a+d)(a+2d)} = \frac{7}{15} \quad (\text{Given})$$

$$15a^2 + 45ad = 7[a^2 + 3ad + 2a^2]$$

$$15a^2 + 45ad = 7a^2 + 21ad + 14ad^2$$

$$8a^2 + 24ad + 24ad = 0.$$

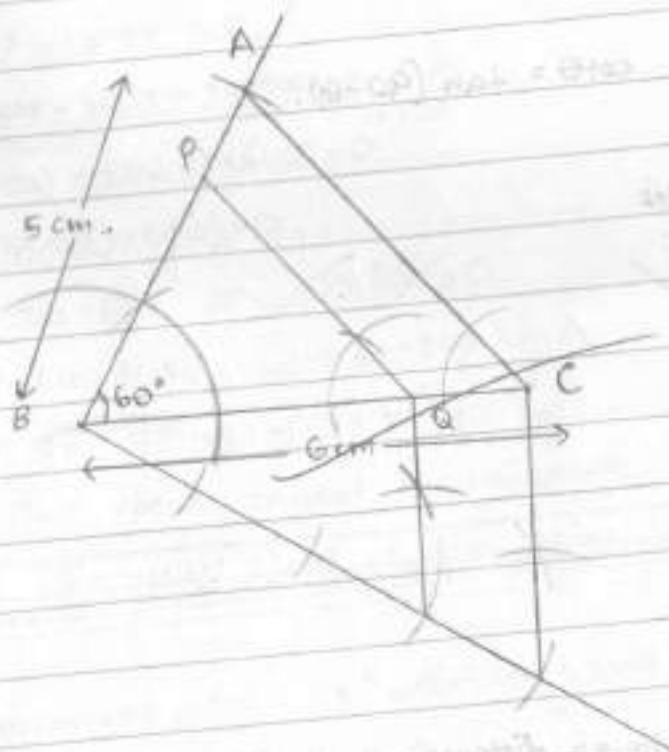
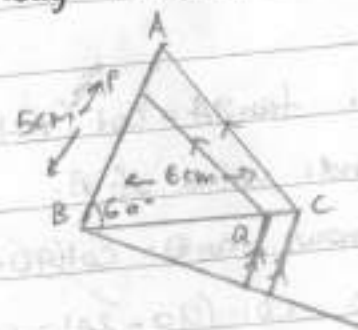
$$14d^2 = 8a^2 + 24ad.$$

$$7d^2 = 4a^2 + 12ad \rightarrow \textcircled{3}$$

26) Given: $\triangle ABC$, $BC = 6\text{ cm}$, $AB = 5\text{ cm}$, $\angle ABC = 60^\circ$.

To draw: $\triangle$ w/ $\frac{3}{4}$ sides of $\triangle ABC$.

Rough Diagram:



$$AB = 5\text{ cm}$$

$$BC = 6\text{ cm}$$

$$\angle ABC = 60^\circ$$

$\triangle PBQ$ is required triangle.

$$PB = 3.75\text{ cm}$$

$$BQ = 4.5\text{ cm}$$

$$PQ = 4.25\text{ cm}$$

Section-c

19) Given: $\tan 2A = \cot (A-18)$, $0 \leq 2A < 90^\circ$. ($2A$ is acute)

(Choice 2) To find: value of A .

We know, $\tan \theta = \cot (90-\theta)$ and $\cot \theta = \tan (90-\theta)$.

$$\rightarrow \cot (90-2A) = \cot (A-18)$$

Applying $\cot^{-1}$, on both sides

$$90-2A = A-18.$$

$$108 = 3A.$$

$$\rightarrow A = 36^\circ.$$

The value of A is 36° .

16) Given: distance is 1500 km.

Usual speed = s .

We know, $\text{speed} = \frac{\text{distance}}{\text{time}} \rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}.$

$\rightarrow$ From question, $\frac{1500}{100+s} + \frac{1}{2} = \frac{1500}{s}$ [half an hour late].
(20 min = 0.5 hr).

$$\frac{1500}{100+s} = \frac{1500}{s} - \frac{1}{2}.$$

$$\frac{1500}{100+s} = \frac{3000-s}{2s}$$

Cross multiplying,

$$3000s = 300000 - 100s + 3000s - s^2$$

$$s^2 + 100s - 300000 = 0$$

$$s^2 + 600s - 500s - 300000 = 0$$

$$s(s+600) - 500(s+600) = 0$$

$$(s-500)(s+600) = 0$$

$$\Rightarrow s-500=0 \quad \text{or} \quad s+600=0$$

$$\Rightarrow s=500 \text{ km/h} \quad \rightarrow s=-600 \text{ km/h}$$

$$\Rightarrow s=500 \text{ or } -600 \text{ km/h}$$

But speed cannot be negative.

$$\Rightarrow \boxed{\text{The usual speed of the plane is } 500 \text{ km/h.}}$$

4) Given, polynomial $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$.

Two zeroes $\rightarrow 2+\sqrt{3}$ and $2-\sqrt{3}$.

$\Rightarrow$ Product of two zeroes is also a zero.

$$\Rightarrow (2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1 \quad [(a+b)(a-b) = a^2 - b^2]$$

As 1 is a zero, $\Rightarrow x-1$ is a factor.

Dividing,

$$x-1 \mid 2x^4 - 9x^3 + 5x^2 + 3x - 1 \quad (2x^3 - 7x^2 - 2x + 1)$$

$$\underline{2x^4 - 2x^3}$$

$$-7x^3 + 5x^2$$

$$\underline{-7x^3 + 7x^2}$$

$$-2x^2 + 3x$$

$$\underline{-2x^2 + 2x}$$

$$x - 1$$

$$\underline{x - 1}$$

$$0$$

$\Rightarrow$ By division algorithm,

$$p(x) = (x-1)(2x^3 - 7x^2 - 2x + 1) \rightarrow g(x).$$

Now, in a cubic polynomial, we know:

$$\text{sum of roots} = -\frac{\text{coeff. of } x^2}{\text{coeff. of } x^3}.$$

The roots of $g(x)$ are $2+\sqrt{3}$, $2-\sqrt{3}$ and α .

$$\rightarrow \alpha + 2 + \sqrt{3} + 2 - \sqrt{3} = -\frac{(-7)}{2}$$

$$\alpha + 4 = \frac{7}{2}$$

$$\alpha = -\frac{1}{2} \text{ which is hence a zero of } p(x)$$

$\Rightarrow$ All zeros are $-\frac{1}{2}$, 1 , $2+\sqrt{3}$ and $2-\sqrt{3}$.

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3) Numbers: 404, 96. To find: HCF and LCM.

$$\begin{array}{l} 2 \overline{) 404, 96} \\ 2 \overline{) 202, 48} \\ 101, 24 \end{array} \Rightarrow \text{Then HCF is 4.}$$

$$\begin{array}{c} 404 \\ 2 \overline{) 404} \\ 2 \overline{) 202} \\ 2 \overline{) 101} \\ 7 \overline{) 101} \\ 7 \overline{) 101} \\ 13 \end{array}$$

$$\begin{array}{c} 96 \\ 2 \overline{) 96} \\ 2 \overline{) 48} \\ 2 \overline{) 24} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \end{array}$$

$$404 = 2^2 \times 7 \times 13$$

$$96 = 2^5 \times 3$$

$$\text{HCF} = \text{greatest common factor} = 2^2 = 4.$$

$$\text{LCM} = \text{all factors (least power)} = 2^5 \times 3 \times 7 \times 13$$

$$= 96 \times 101$$

$$= 9696$$

$$\text{Product of two numbers} = 96 \times 404$$

$$= 38784$$

$$\text{Product of HCF + LCM} = 9696 \times 4$$

$$= 38784$$

Hence, $\text{HCF} \times \text{LCM} = \text{product of two numbers.}$

15) Vertices of quadrilateral ABCD:

(choice 2) A (-5, 7), B (-4, -5), C (-1, -6), D (4, 5)

3

Area of quad ABCD.

= area $\triangle ABD$ + area $\triangle BCD$.

area $\triangle ABD \rightarrow$

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \text{ sq. units.}$$

$$= \frac{1}{2} [-5(-5 - 5) + (-4)(5 - 7) + 4(7 + 5)]$$

$$= \frac{1}{2} [50 + 8 + 48]$$

$$= \frac{1}{2} [58 + 48]$$

$$= \frac{1}{2} \times 106 = 53 \text{ sq. units}^2.$$

$$\text{area } \triangle BCD = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [-4(-6 - 5) + (-1)(5 + 5) + (4)(-5 + 6)]$$

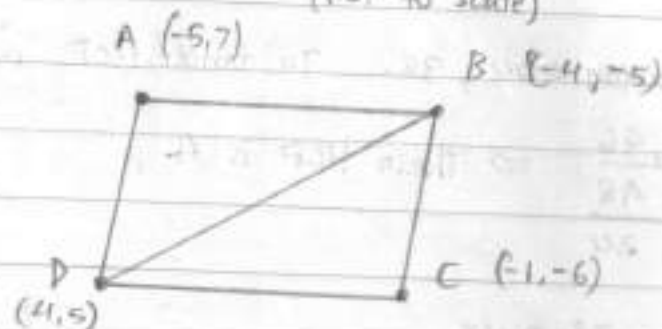
$$= \frac{1}{2} [44 - 10 + 4]$$

$$= \frac{1}{2} \times 38 = 19 \text{ units}^2.$$

$$\Rightarrow \text{Area of quadrilateral} = \text{Area of two triangles} = 53 + 19 = 72 \text{ sq. units.}$$

Area of quadrilateral ABCD is 72 sq. units

(not to scale)



18) Given: Circle (O, r) . AP and PB
are tangents drawn to the circle.

3

To prove: $PA = PB$.

Construction: Join OA, OB and OP.

Proof: $OA = OB$ [radius]. (side).

$\angle OAP = \angle OBP = 90^\circ$ (right angle).

[$\because$ radius is perpendicular to
tangent at point of contact].

$OP = OP$ (hypotenuse).

So in $\triangle OAP$ and $\triangle OBP$,

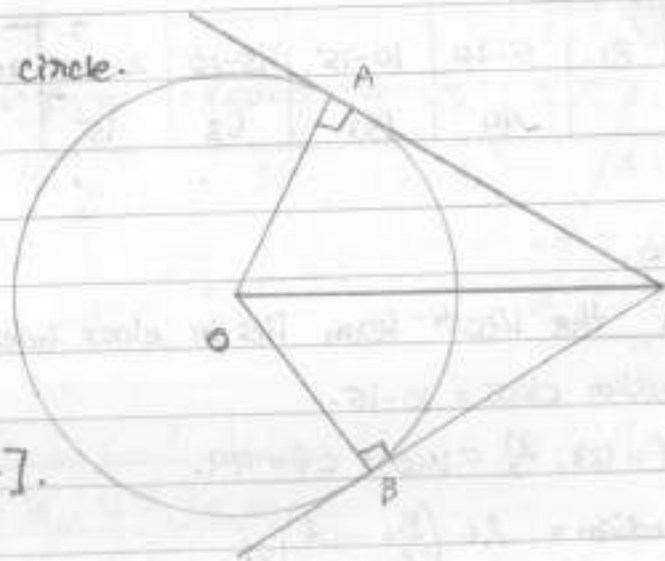
by RHS congruency,

$\rightarrow \triangle OAP \cong \triangle OBP$.

by CPCT,

$\Rightarrow AP = BP$.

hence proved.



22) Distribution of frequencies:

Salary in thousand Rs.	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of persons	49	133	68	15	6	7	4	2	1

To find: median.

no. of people = 280.

$\Rightarrow \frac{n}{2} = 140$, the 140th term lies in class interval 10-15.

$\Rightarrow$ median class = 10-15.

$l = 10$, $h = 5$, $f = 133$, $\frac{n}{2} = 140$, $cf = 49$.

We know, median = $l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$.

$$\Rightarrow \text{median} = 10 + \frac{140 - 49}{133} \times 5.$$

$$= 10 + \frac{91}{133} \times 5.$$

$$= 10 + \frac{65}{19}$$

$$= 10 + 3.421$$

$$= 13.421.$$

The median salary is 13.421 thousand rupees.

2) Conical heap of rice:

(choice 2) Dimensions: diameter = 24m, height 3.5m. $\rightarrow$ radius = 12m.

Volume of cone = $\frac{1}{3} \times \pi r^2 h$ cu. units.

$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \text{ cu. m.}$$

$$= 132 \times 4$$

$$= 528 \text{ cu. m.}$$

The volume of the rice heap is 528 cu. m.

Area of cloth required = Curved surface area.

CSA of cone = $\pi r l$ sq. units where $l = \sqrt{h^2 + r^2}$ units.

Finding l : $l = \sqrt{h^2 + r^2}$ units

$$= \sqrt{3.5^2 + 12^2} \text{ m}$$

$$= \sqrt{12.25 + 144}$$

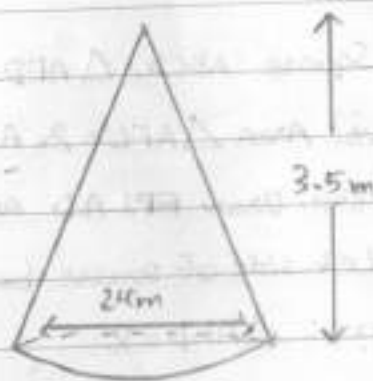
$$= \sqrt{156.25}$$

$$= 12.5 \text{ m.}$$

$\Rightarrow$ CSA = $\pi r l$ sq. units

$$= \frac{22}{7} \times 12.5 \times 12$$

$$= \frac{22 \times 150}{7} = \frac{3300}{7} = 471.428571 \text{ m}^2.$$



The area of canvas cloth required is 471.428571 m^2 .

17) Given: Square ABCD. $\triangle AED$ and $\triangle AFC$ are equilateral.

(choice 1) To prove: Area $\triangle AFC = 2 \times$ Area $\triangle AED$.

Construction: Draw $EP \perp AD$ and $FQ \perp AC$.

Proof: Let side of square be x .

$\Rightarrow$ sides of $\triangle AED = x$.

In $\triangle ABC$, $\angle B = 90^\circ$.

$\Rightarrow$ By Pythagoras Theorem,

$$AB^2 + BC^2 = AC^2$$

$$x^2 + x^2 = AC^2 \Rightarrow AC = \sqrt{2}x. \Rightarrow \text{sides of } \triangle AFC = \sqrt{2}x.$$

We know, altitude of equilateral $\triangle$ bisects the base.

$$\rightarrow PD = \frac{x}{2}, \quad AQ = \frac{x}{\sqrt{2}}.$$

In $\triangle AEP$, $\angle P = 90^\circ$.

By Pythagoras theorem, $AE^2 = EP^2 + AP^2$.

$$x^2 = EP^2 + \left(\frac{x}{2}\right)^2.$$

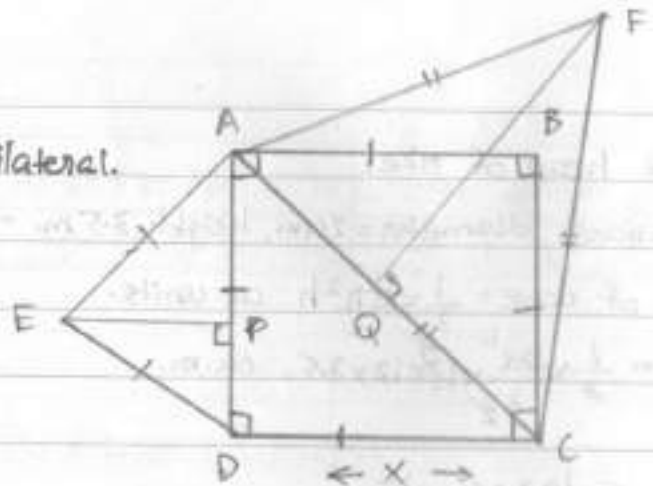
$$EP^2 = \frac{3x^2}{4} \rightarrow EP = \frac{\sqrt{3}}{2}x.$$

In $\triangle AFR$, $\angle Q = 90^\circ$.

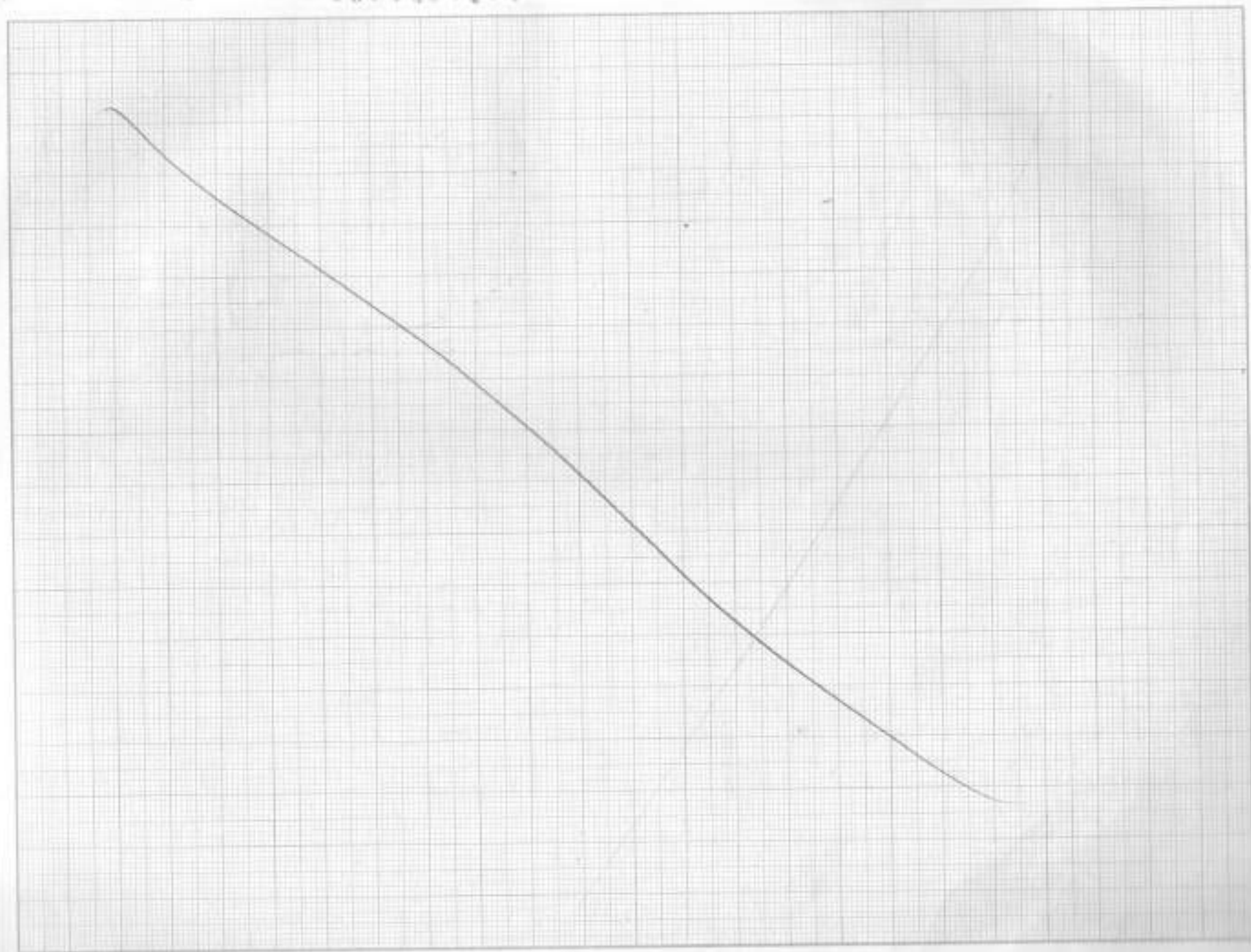
By Pythagoras theorem, $AF^2 = FR^2 + AQ^2$

$$2x^2 = FR^2 + \frac{x^2}{2}.$$

$$FR^2 = \frac{3x^2}{2} \rightarrow FR = \frac{\sqrt{3}}{\sqrt{2}}x.$$



CBS 2000



We know, Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{height}$ sq. units.

$$\begin{aligned} \Rightarrow \text{Area of } \triangle AFC &= \frac{1}{2} \times \sqrt{2}x \times FP \\ &= \frac{1}{2} \times \sqrt{2}x \times \frac{\sqrt{3}}{\sqrt{2}}x \\ &= \frac{\sqrt{3}}{2}x^2. \end{aligned}$$

$$\text{Area of } \triangle AED = \frac{1}{2} \times x \times EP.$$

$$\begin{aligned} &= \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x \\ &= \frac{\sqrt{3}}{4}x^2. \end{aligned}$$

$$2 \cdot \text{Area of } \triangle AED = \frac{\sqrt{3}}{2}x^2 = \text{Area of } \triangle AFC.$$

hence proved.

20) Given: side of square ABCD = 12 cm.

To find: shaded area.

Shaded area = Area of 4 quadrants = Area of square.

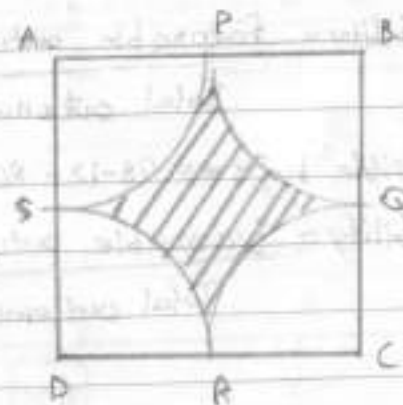
Area of square = s^2 sq. units

$$= 12^2 = 144 \text{ cm}^2.$$

Area of quadrant = $\frac{1}{4} \times \pi r^2$ sq. units

$$= \frac{1}{4} \times 3.14 \times \frac{12^2}{2} \times \frac{12^2}{2}$$

$$= 9 \times 3.14 = 28.26 \text{ cm}^2.$$



$\Rightarrow$ Shaded area = Area of square - $4 \times$ (Area of quadrant) sq. units

(c) $= 144 - 4(28.26)$ sq. cm

$$= 144 - 113.04$$

$$= 30.96 \text{ cm}^2.$$

The area of the shaded region is 30.96 cm^2 .

Section-B

(2) Integers, 1 to 100. (between)

$\Rightarrow$ total = 98 possible outcomes.

i) divisible by 8 $\rightarrow$ 12 numbers. (8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96).

$$\Rightarrow \text{Probability} = \frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{12}{98} = \frac{6}{49}.$$

ii) not divisible by 8 $\Rightarrow 98 - 12 = 86$ numbers.

$$\Rightarrow \text{Probability} = \frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{86}{98} = \frac{43}{49}.$$

$$\Rightarrow \text{Shaded area} = \text{Area of square} - 4(\text{Area of quadrant}) \text{ sq. units}$$

$$= 144 - 4(28.26) \text{ sq. cm.}$$

$$= 144 -$$

11) Two dice tossed together.

$$\Rightarrow \text{Total outcome} = 36.$$

2) i) Doublet: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) $\rightarrow$ 6 possibilities.

$$\text{Probability} = \frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{6}{36} = \boxed{\frac{1}{6}}.$$

ii) Sum of 10: (4,6), (6,4), (5,5) $\rightarrow$ 3 possibilities.

$$\text{Probability} = \frac{\text{Favorable outcome}}{\text{Total outcome}} = \frac{3}{36} = \boxed{\frac{1}{12}}.$$

12) Sum of first 8 multiples of 3:

Forms an AP, $a=3$, $d=3$, $n=8$.

$$\text{Sum} = S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_8 = \frac{8}{2} [2 \times 3 + (8-1)3] = 4 [6 + 21] = 4 \times 27 = 108.$$

The sum of the first 8 multiples of 3 is 108.

8) Given, rectangle ABCD.

$\Rightarrow$ opposite sides are equal.

hence, $x+y=30 \rightarrow ①$

$x-y=14 \rightarrow ②$

$①+② \rightarrow 2x=44$

$x=22$.

Substituting in ①, $22+y=30$

$y=8$.

$\Rightarrow \boxed{x=22, y=8}$



9) Given, $\sqrt{2}$ is irrational

To prove: $5+3\sqrt{2}$ is irrational.

Proof: Let us assume $5+3\sqrt{2}$ is rational. So it is in form $\frac{a}{b}$.
 $\left[\begin{array}{l} a, b \in \mathbb{R}, b \neq 0 \\ a, b \in \mathbb{Z}, b \neq 0, \text{HCF}(a, b) = 1 \end{array} \right]$

$\Rightarrow 5+3\sqrt{2} = \frac{a}{b}$

$3\sqrt{2} = \frac{a}{b} - 5$

$3\sqrt{2} = \frac{a-5b}{b}$

$\sqrt{2} = \frac{a-5b}{3b}$

This shows that $\sqrt{2}$ is ^{rational} ~~irrational~~ ($a-5b$ and $3b$ are integers).

But we know that $\sqrt{2}$ is irrational.

This contradicts our assumption that $5+3\sqrt{2}$ is rational.

$\Rightarrow 5+3\sqrt{2}$ is irrational, hence proved.

Ex 19) Points $A(2,3)$, $B(6,-3)$ divided by $P(4,m)$.

Let the ratio be $k:1$.

② By seg. section formula,

$$P(4,m) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$(4,m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1} \right)$$

$$\Rightarrow \frac{6k+2}{k+1} = 4$$

$$6k+2 = 4k+4$$

$$2k = 2$$

$$k = 1$$

→ The ratio is $1:1$.

now, $m = \frac{-3k+3}{k+1}$

$$m = \frac{-3+3}{1+1}$$

$$m = 0$$

→ Value of m is 0 , the point is $P(4,0)$.

Section - A.

6) $\frac{AB}{PA} = \frac{1}{3}$. $\frac{\text{ar} \Delta ABC}{\text{ar} \Delta PQR} = \frac{AB^2}{PA^2} = \frac{1}{3^2} = \frac{1}{9}$.

Ratio of areas is $\frac{1}{9}$.

5) $\cos^2 67^\circ - \sin^2 23^\circ$
 $= \cos^2 67^\circ - \cos^2 67^\circ$

$(\cos 67^\circ + \sin 23^\circ)(\cos 67^\circ - \sin 23^\circ)$

$= (\cos 67^\circ + \cos 67^\circ)(\cos 67^\circ - \cos 67^\circ)$ $[\cos \theta = \sin(90^\circ - \theta)]$

$= 0$.

The value is 0.

4) $d = -4$, $a_7 = 4$.

The first term is 28.

$t_n = a + (n-1)d$

$t_7 = a + (7-1)(-4)$

$4 = a + 6(-4)$

$a = 24 + 4$

$a = 28$.

3) Distance between (x, y) and $(0, 0)$.

$$\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(x - 0)^2 + (y - 0)^2}$$

$$= \sqrt{x^2 + y^2}$$

The distance is $\sqrt{x^2 + y^2}$.

2) Smallest prime = 2

Smallest composite = 4

$$\text{HCF}(2, 4) = 2$$

The HCF of the smallest prime and smallest composite is 2.

1) $x^2 - 2kx - 6 = 0$. Let α be other root.

$$\text{Product} = \frac{c}{a} = \frac{-6}{1} = -6$$

$$3 \times \alpha = -6$$

$$\alpha = -2$$

$$\text{Sum} = \frac{-b}{a} = \frac{-(-2k)}{1} = 2k$$

$$\Rightarrow 3 + (-2) = 2k$$

$$1 = 2k, k = \frac{1}{2}$$

Value of k is $\frac{1}{2}$