ICSE 2025 EXAMINATION

Sample Question Paper - 14

Mathematics

Time: 2 ½ Hours. **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) If $A = \begin{bmatrix} 3 & 0 \\ x & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 0 \\ 16 & -y \end{bmatrix}$, find x and y when $A^2 = B$.

(a) $x = 4$ and $y = -1$
(b) $x = -4$ and $y = 1$
(c) $x = 4$ and $y = 1$
(d) $x = -4$ and $y = -1$

ii) Which of the following is a solution of the quadratic equation $x^2 - 2x - 15 = 0$?

(a) 2
(b) 3
(c) 4
(d) 5

iii) A dealer in Delhi buys some goods worth Rs. 16,000. If the rate of GST is 12%, find the amount the dealer pays as CGST.

(a) Rs. 960
(b) Rs. 1920
(c) Rs. 17,920
(d) Rs. 16,960

iv) The roots of a quadratic equation $x^2 - 4x + 4 = 0$ are

(a) imaginary
(b) not real
(c) real and equal
(d) real and unequal

v) If the sum of first ‘n’ terms of an Arithmetic Progression 24, 21, 18, … is 78, then the value of ‘n’ is
(a) 4
(b) 12
(c) 14
(d) 16

vi) If a : b :: b : c, then $a^2 : b^2 =$
(a) c : a
(b) a : c
(c) a : b
(d) b : a

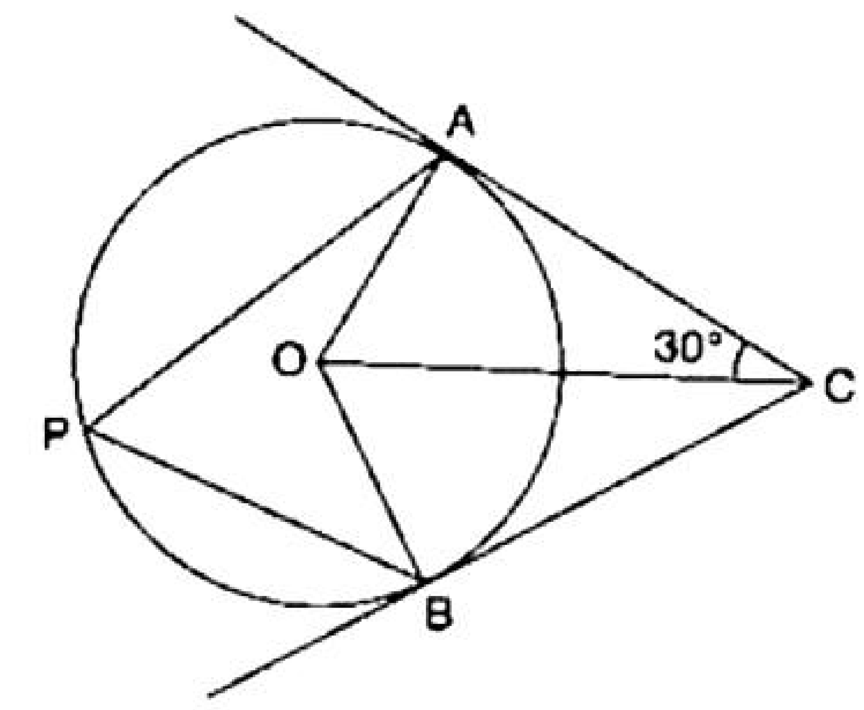
vii) Read the following statements and state which is valid?.
Statement 1: AA is the test of similarity.
Statement 2: RHS is the test of congruency.
(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

viii) If a cone has volume 154 cm^3 and the perpendicular height 12 cm, then the radius will be
(a) 3.5 cm
(b) 7 cm
(c) 3 cm
(d) 12 cm

ix) A card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting an ace.
(a) $\frac{1}{13}$
(b) $\frac{2}{13}$
(c) $\frac{3}{13}$
(d) $\frac{4}{13}$

x) Every point on the perpendicular bisector of PQ is
(a) Equidistant from either P or Q.
(b) Equidistant from Q only
(c) Equidistant from P only
(d) Equidistant from P and Q.

xi) In the given figure, O is the centre of the circle. Tangents AC and BC meet at point C. If $\angle ACO = 30^o$, find $\angle BCO$.



(a) 90^o
(b) 60^o
(c) 45^o
(d) 30^o

xii) The length of the shadow of a vertical tower is $\frac{1}{\sqrt{3}}$ times its height. Find the angle of elevation of the Sun.

(a) 30°
(b) 45°
(c) 60°
(d) 90°

xiii) The money required to buy 500 Rs. 30 shares at a premium of Rs. 10 is

(a) Rs. 20,000
(b) Rs. 15,000
(c) Rs. 10,000
(d) Rs. 5,000

xiv) The equation of the line parallel to the line $3x - 4y = 9$ and passing through the point (3, 5) is

(a) $3x + 4y + 11 = 0$
(b) $3x - 4y - 11 = 0$
(c) $3x + 4y - 11 = 0$
(d) $3x - 4y + 11 = 0$

xv) x is a positive odd integer.

Assertion (A): The solution set of $2x - 3 \leq \frac{x}{3} + 7$ is {1, 2, 3, 4, 5, 6}.

Reason (R): If each term of an inequation be multiplied or divided by the same positive number, the sign of inequality remains the same.

(a) A is true, R is false
(b) A is false, R is true
(c) Both A and R are true, and R is the correct reason for A.
(d) Both A and R are true, and R is the incorrect reason for A.

Question 2

i) The area of the base of a right circular cone is 28.26 sq. cm. If its height is 4 cm, find its volume and the curved surface area. (use $\pi = 3.14$) [4]

ii) Ahmed has a recurring deposit account in a bank. He deposits Rs. 2,500 per month for 2 years. If he gets Rs. 66,250 at the time of maturity, find the

a. Interest paid by the bank

b. Rate of interest [4]

iii) Prove that: [4]

$$\frac{\tan A}{1-\cot A}+\frac{1}{\tan A(1-\tan A)}=1+\operatorname{cosec}A\sec A$$

Question 3

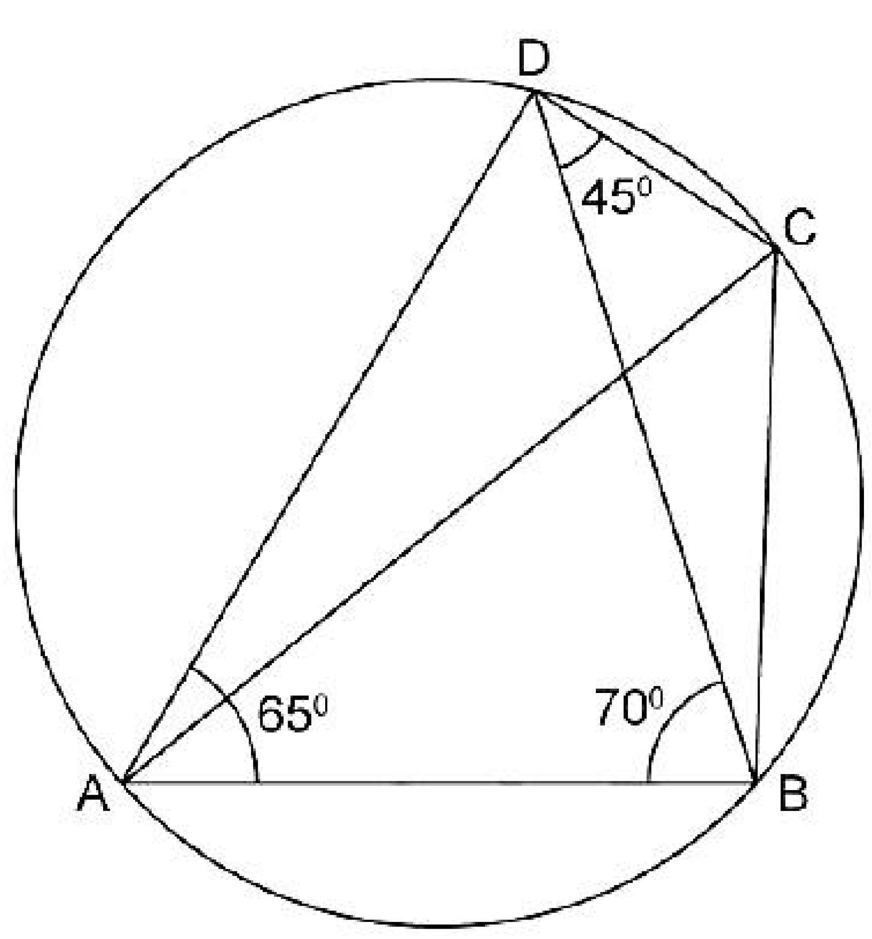
i) Using the properties of proportion, solve for x: $\frac{x^4+1}{2x^4}=\frac{313}{25}$ [4]

ii) In the given figure, $\angle BAD = 65°$, $\angle ABD = 70°$, $\angle BDC = 45°$. [4]

a) Prove that AC is a diameter of the circle.

b) Find $\angle ACB$.

c) Find $\angle DBC$.



iii) Draw a histogram of the following frequency distribution and use it to calculate the mode. [5]

C.I.	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	15	10	5	12	8

Section B

(Attempt any four questions from this Section.)

Question 4

i) If matrix $X = \begin{bmatrix} 4 & -2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and X – 2Y is null matrix. Find the matrix X and Y. [3]

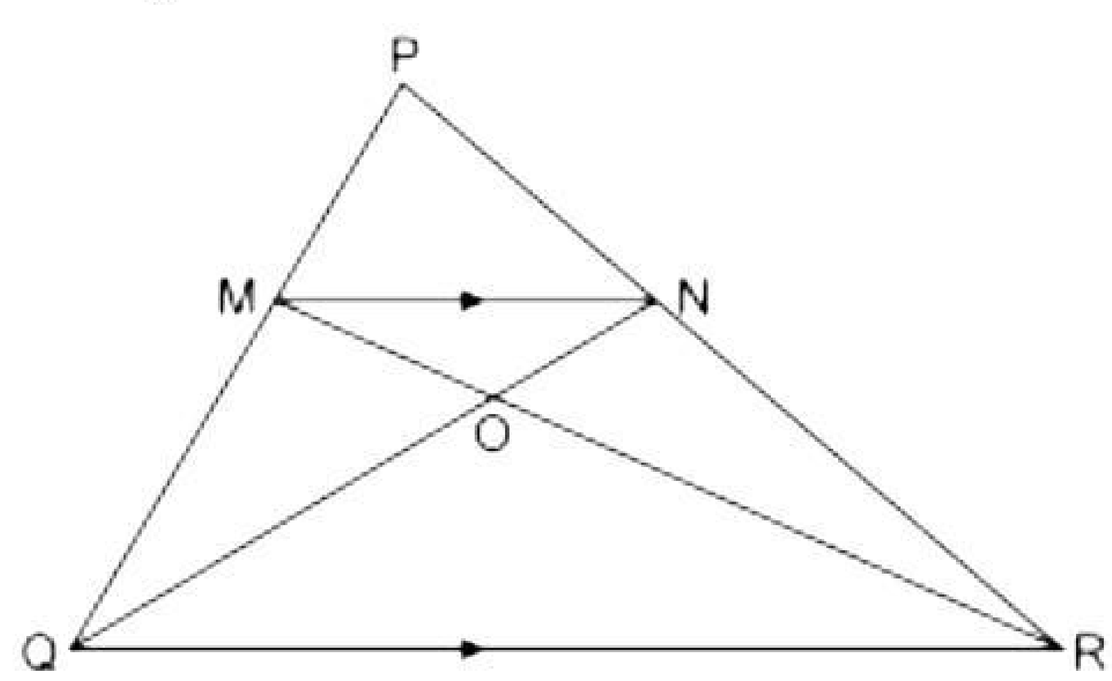
ii) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [3]

iii) In ΔPQR, MN is parallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$. [4]

(i) Find $\frac{MN}{QR}$

(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find, Area of ΔOMN : Area of ΔORQ



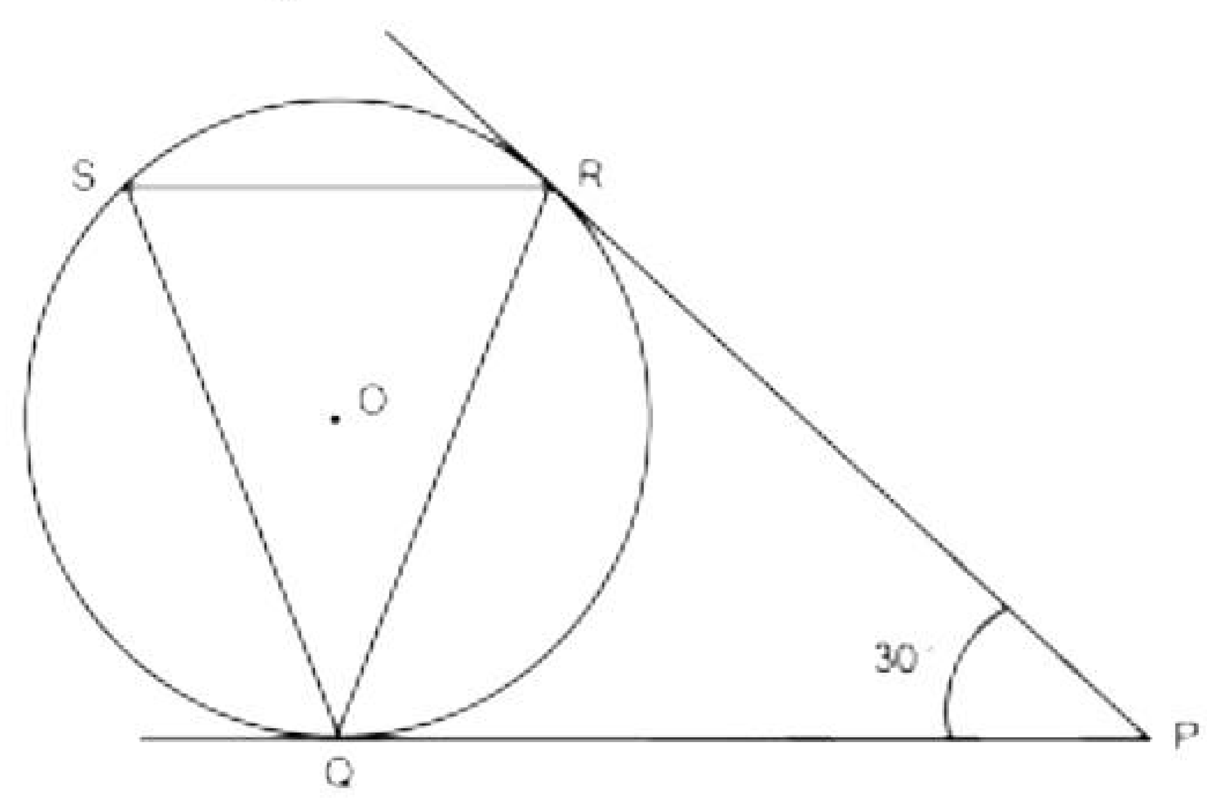
Question 5

i) Find the missing frequencies f_1 and f_2 in the table given below. It is given that the mean of the given frequency distribution is 50. [3]

Class	Frequency
0–20	17
20–40	f_1
40–60	32
60–80	f_2
80–100	19
Total	120

ii) A is a dealer in Banaras (U.P.). He supplies goods/services worth Rs. 8000 to a dealer B in Agra (U.P.). Dealer B, in turn, supplies the same goods/services to dealer C in Patna (Bihar) at a profit of Rs. 1200. Find the input and output taxes for the dealer C under GST system; if the rate of GST is 18% and C does not sell his goods/services further. [3]

iii) In the given figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$. [4]



Question 6

i) The 4th term of a G.P. is 16 and the 7th term is 128. Find the first term and common ratio of the series. [3]

ii) Draw a histogram and hence estimate the mode for the following distribution: [3]

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	4	13	7	9	10	5	2

iii) A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. This ice cream is to be filled into cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream. [4]

Question 7

i) A(−1, 3), B(4, 2) and C(3, −2) are the vertices of a triangle. [5]

(a) Find the coordinates of the centroid G of triangle ABC.

(b) Find the equation of the line through G and parallel to AC.

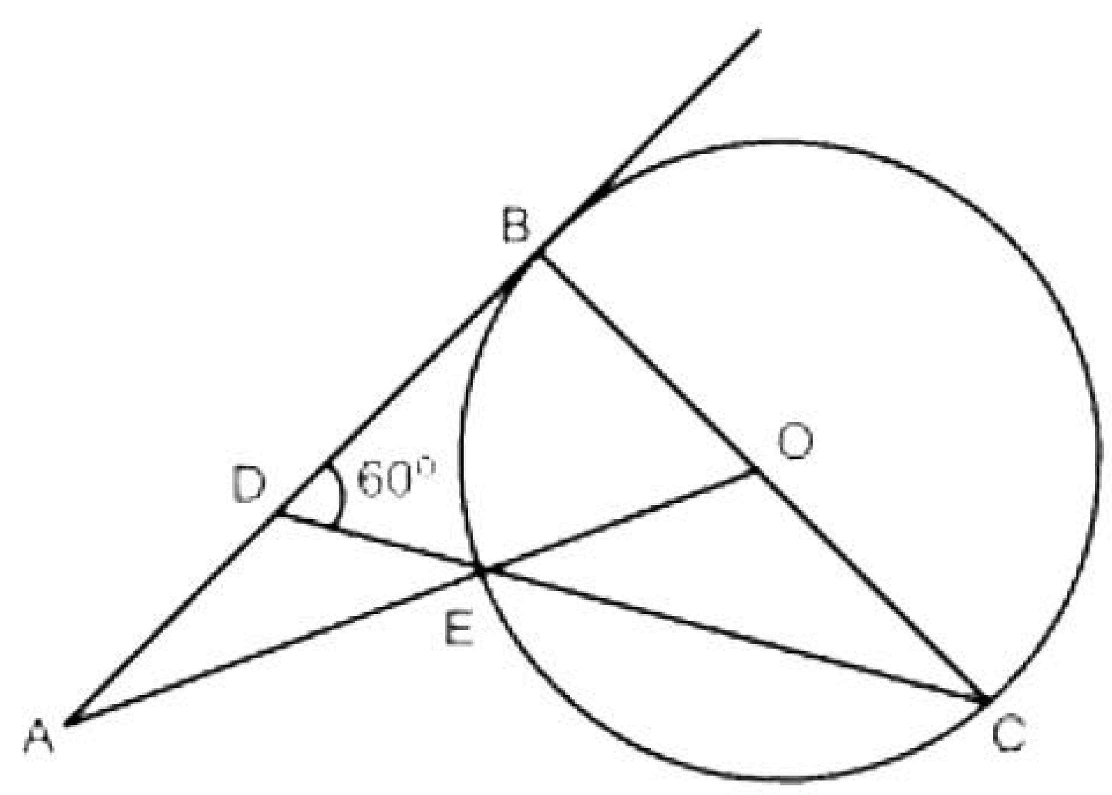
(c) Find the equation of the line through G and perpendicular to AC.

ii) A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff. At a point on the plane, 30 metres away from the tower, an observer notices that the angles of elevation of the top and bottom of the flagstaff are $60°$ and $45°$, respectively. Find the height of the flagstaff and that of the tower. [5]

Question 8

i) If P is the solution set of $-3x + 4 > 2x - 3$, $x \in W$ and Q is the solution set of $4x - 5 < 12$, $x \in W$. Find Q – P. [3]

ii) In the given figure, O is the centre of the circle. AB is a tangent to it at point B. $\angle BDC = 60°$. Find $\angle BAO$. [3]



iii) Calculate the ratio in which the line joining the points $(-4, -5)$ and $(1, 4)$ is divided by the line $y = -2$. Also, find the coordinates of the point of intersection. [4]

Question 9

i) If $\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \dfrac{5}{2}$, find the value of $\dfrac{a^2+b^2}{a^2-b^2}$. [3]

ii) The work done by $(x - 3)$ men in $(2x + 1)$ days and the work done by $(2x + 1)$ men in $(x + 4)$ days are in the ratio 3 : 10. Find the value of x. [3]

iii) Using ruler and compass only, construct ΔABC such that BC = 5 cm, AB = 6.5 cm and $\angle ABC = 120°$ [4]

(i) Construct a circumcircle of ΔABC

(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Question 10

i) When divided by $(x - 2)$, the polynomials $x^3 - (p + 5)x^2 + 17x + 28$ and $x^3 + px^2 + 2x - 8$ leaves the remainder whose difference is 6. Find the value of p. [3]

ii) Twenty identical cards are numbered from 1 to 20. A card is drawn randomly from those 20 cards. Find the probability that the number on the card drawn is [3]

a) Divisible by both 2 and 5.

b) Greater than 20.

iii) Use a graph paper for this question. [4]

The points A(4, 2), B(5, 1) and C(3, 1) are the vertices of triangle ABC.

a) Write the co-ordinates of A', B', C' if triangle A'B'C' is the image of triangle ABC, when reflected in the origin.

b) Write the co-ordinates of A", B", C" if triangle A"B"C" is the image of triangle ABC, when reflected in the x-axis.

c) Mention the special name of the quadrilateral ABB"A" and find its area.

Solution

Section A

Solution 1

i) Correct option: (a)

Explanation:

$$\text{Given}: A = \begin{bmatrix} 3 & 0 \\ x & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & 0 \\ 16 & -y \end{bmatrix}$$

As $A^2 = B$

$$\Rightarrow \begin{bmatrix} 3 & 0 \\ x & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ x & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 16 & -y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 \\ 4x & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 16 & -y \end{bmatrix}$$

$\Rightarrow 4x = 16$ and $1 = -y$

$\Rightarrow x = 4$ and $y = -1$

ii) Correct option: (d)

Explanation:

$x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x - 5) + 3(x - 5) = 0$

$\Rightarrow (x - 5)(x + 3) = 0$

$\Rightarrow x - 5 = 0$ or $x + 3 = 0$

$\Rightarrow x = 5$ or $x = -3$

iii) Correct option: (a)

Explanation:

It is a case of Intra-state transaction.

For the dealer, cost of goods = Rs. 16,000 and GST rate is 12%.

Then, CGST = 6% of Rs. 16,000 = $\frac{6}{100} \times 16{,}000$ = Rs. 960

iv) Correct option: (c)

Explanation:

Given equation is $x^2 - 4x + 4 = 0$.

Here $a = 1$, $b = -4$ and $c = 4$ s

Then, $b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$

Since the discriminant is 0, the roots are real and equal.

v) Correct option: (a)

Explanation:

In the A.P. 24, 21, 18,…,

First term, a = 24, common difference, d = 21 – 24 = –3

Let S_n be the sum of the first n terms of this A.P.

$$\Rightarrow S_n = \frac{n}{2}\left[2\times 24+(n-1)(-3)\right]$$

$$\Rightarrow 78 = \frac{n}{2}\left[48-3n+3\right]$$

$$\Rightarrow 156 = -3n^2+51n$$

$$\Rightarrow 52 = -n^2+17n$$

$$\Rightarrow n^2-17n+52=0$$

$$\Rightarrow (n-13)(n-4)=0$$

$$\Rightarrow n=13 \text{ or } n=4$$

vi) Correct option: (b)

Explanation:

a : b :: b : c

$$\Rightarrow \frac{a}{b}=\frac{b}{c}$$

$$\Rightarrow b^2=ac$$

$$\Rightarrow a^2:b^2=\frac{a^2}{b^2}=\frac{a^2}{ac}=\frac{a}{c}=a:c$$

vii) Correct option: (a)

Explanation:

Both statements are correct.

viii) Correct option: (a)

Explanation:

Volume of the cone = 154 cm^3

$$\Rightarrow \frac{1}{3}\times\pi r^2h=154$$

$$\Rightarrow \frac{1}{3}\times\frac{22}{7}\times r^2\times 12=154$$

$$\Rightarrow r^2=\frac{154\times 3\times 7}{12\times 22}=\frac{49}{4}$$

$$\Rightarrow r=\frac{7}{2}$$

$$\Rightarrow r=3.5 \text{ cm}$$

ix) Correct option: (a)

Explanation:

Let S denote the sample space of this experiment.

$\Rightarrow n(S) = 52$

Let A be an event of getting an ace card.

Number of ace cards in a deck of 52 cards = $4 \times 1 = 4$

$\Rightarrow$ Number of possible outcomes = $n(A) = 4$

$\Rightarrow$ Probability of getting an ace card = $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

x) Correct option: (d)

Explanation:

Every point on the perpendicular bisector of PQ is equidistant from P and Q.

xi) Correct option: (d)

Explanation:

In ΔAOC and ΔBOC,

AO = BO (radii)

AC = BC (tangents to a circle from an external point are equal in length)

OC = OC (Common)

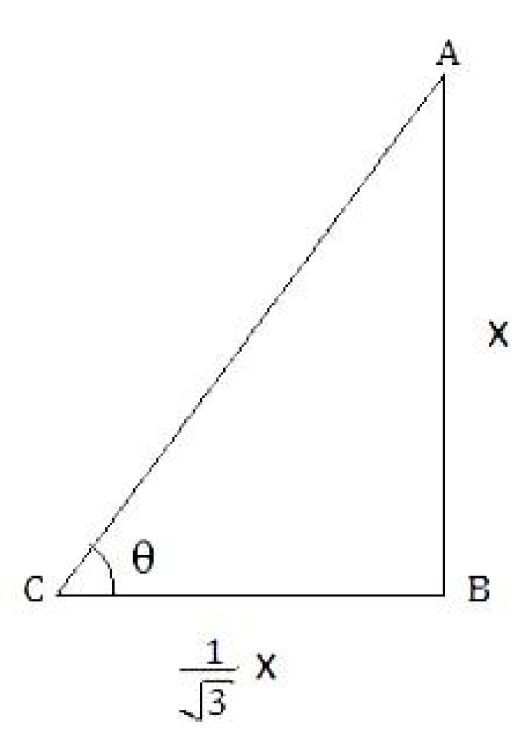
$\Delta AOC \cong \Delta BOC$ ... SSS congruency test

$\Rightarrow \angle BCO = \angle ACO = 30^{o}$ (cpct)

xii) Correct option: (c)

Explanation:

Let the height of the tower be 'x' m.



Therefore, the length of its shadow = $\dfrac{1}{\sqrt{3}}x$ m.

If θ is the angle of elevation of the Sun, then

$$\tan\theta = \frac{x}{\frac{1}{\sqrt{3}}x} = \sqrt{3} = \tan 60$$

$\Rightarrow \theta = 60$

Hence, the angle of elevation of the Sun is 60^{o}.

xiii) Correct option: (a)

Explanation:

Number of shares to be bought = 500

Rs. 30 at a premium of Rs. 10,

⇒ Nominal value of the share = Rs. 30

And, market value = 30 + 10 = Rs. 40

Money required to buy 1 share = Rs. 40

Money required to buy 500 shares = 500 × 40 = Rs. 20000

xiv) Correct option: (d)

Explanation:

Given line is 3x – 4y = 9

Converting it into the form y = mx + c, we get

$$y = \frac{3}{4}x - \frac{9}{4}$$

⇒ Slope of the given line = $\frac{3}{4}$ = slope of the required parallel line

Hence, for the required parallel line, m = $\frac{3}{4}$ and $(x_1, y_1) = (3, 5)$

Therefore, the equation is given by

$$y - 5 = \frac{3}{4}(x - 3)$$

$$4y - 20 = 3x - 9$$

$$3x - 4y + 11 = 0$$

xv) Correct option: (b)

Explanation:

$$2x - 3 \leq \frac{x}{3} + 7$$

$$\Rightarrow 2x - \frac{x}{3} \leq 7 + 3$$

$$\Rightarrow \frac{5x}{3} \leq 10$$

$$\Rightarrow 5x \leq 30$$

$$\Rightarrow x \leq 6$$

Since, x is a positive odd integer, the solution set = {1, 3, 5}.

Hence, the assertion is false.

The statement given in reason is true.

Hence, the reason is true.

Solution 2

i) Area of the base of cone = 28.26 cm^2

$$\Rightarrow \pi r^2 = 28.26$$

$$\Rightarrow r^2 = \frac{28.26}{3.14} = 9$$

$$\Rightarrow r = 3 \text{ cm}$$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times 28.26 \times 4$$

$$= 37.68 \text{ cm}^3$$

$$\text{Curved surface area of the cone} = \pi r l = \pi r\sqrt{r^2 + h^2}$$

$$= 3.14 \times 3 \times \sqrt{3^2 + 4^2}$$

$$= 3.14 \times 3 \times 5$$

$$= 47.10 \text{ cm}^2$$

ii) Given, P = Rs. 2500, n = 2 years = 24 months, maturity value = Rs. 66,250

i. Total deposit = Rs. 2500 × 24 = Rs. 60,000
   Amount paid at maturity = Rs. 66,250
   Therefore, interest paid by the bank = Rs. (66250 – 60000) = Rs. 6250

ii. Interest = Rs. 6250

$$P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100} = 6250$$

$$\Rightarrow 2500 \times \frac{24 \times 25}{2 \times 12} \times \frac{r}{100} = 6250$$

$$\Rightarrow 25 \times 25 \times r = 6250$$

$$\Rightarrow 625r = 6250$$

$$\Rightarrow r = 10\% \text{ p.a.}$$

iii) $$\text{L.H.S.} = \frac{\tan A}{1 - \cot A} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan^2 A}{(\tan A - 1)} - \frac{1}{\tan A(\tan A - 1)}$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

$$= \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A(\tan A - 1)} \qquad [a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$= \tan A + 1 + \cot A$$

$$= 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= 1 + \frac{1}{\sin A \cos A}$$

$$= 1 + \operatorname{cosec} A \sec A$$

$$= \text{R.H.S}$$

Solution 3

i)

$$\frac{x^4 + 1}{2x^4} = \frac{313}{25}$$

$$\Rightarrow \frac{x^4 + 1 + 2x^4}{x^4 + 1 - 2x^4} = \frac{313 + 25}{313 - 25} \quad \text{(Using componendo and dividendo)}$$

$$\Rightarrow \frac{(x^2 + 1)^2}{(x^2 - 1)^2} = \frac{338}{288}$$

$$\Rightarrow \frac{(x^2 + 1)^2}{(x^2 - 1)^2} = \frac{169}{144}$$

$$\Rightarrow \frac{x^2 + 1}{x^2 - 1} = \frac{13}{12}$$

$$\Rightarrow \frac{x^2 + 1 + x^2 - 1}{x^2 + 1 - (x^2 - 1)} = \frac{13 + 12}{13 - 12} \quad \text{(Using componendo and dividendo)}$$

$$\Rightarrow \frac{2x^2}{2} = \frac{25}{1}$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = 5$$

ii)

a) In ΔABD,

$$\angle DAB + \angle ABD + \angle ADB = 180^\circ$$

$$\Rightarrow 65^\circ + 70^\circ + \angle ADB = 180^\circ$$

$$\Rightarrow 135^\circ + \angle ADB = 180^\circ$$

$$\Rightarrow \angle ADB = 180^\circ - 135^\circ = 45^\circ$$

Now, $\angle ADC = \angle ADB + \angle BDC = 45^\circ + 45^\circ = 90^\circ$

Since $\angle ADC$ is the angle in a semicircle, so AC is a diameter of the circle.

b) $\angle ACB = \angle ADB$ ....(angles in the same segment of a circle)

$\Rightarrow \angle ACB = 45°$

c) $\angle ABC = \angle ABD + \angle DBC$

$\angle ABC = 90°$ (Angle in a semicircle)

$\Rightarrow \angle ABD + \angle DBC = 90°$

$\Rightarrow 70° + \angle DBC = 90°$

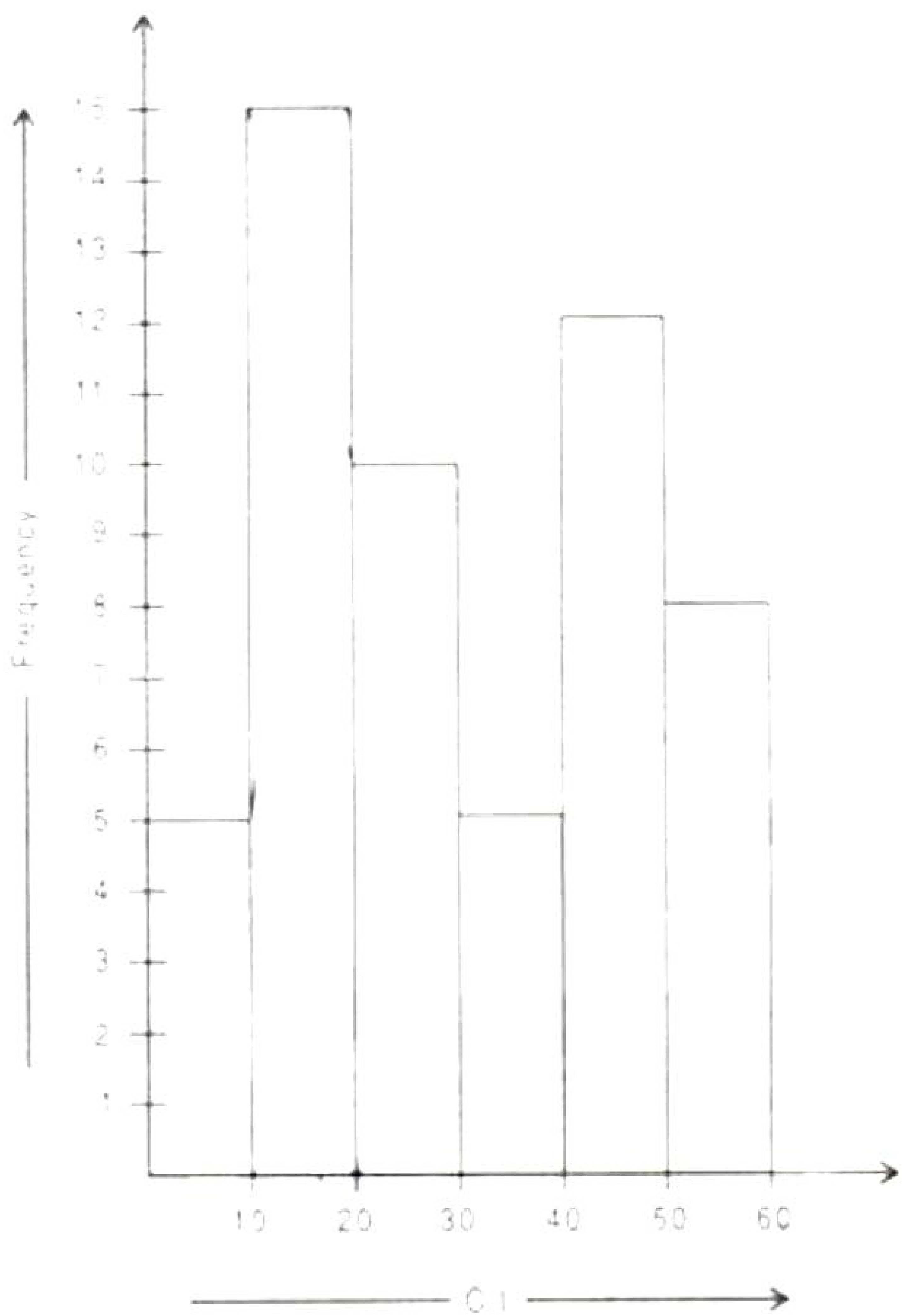
$\Rightarrow \angle DBC = 20°$

iii)

Steps of construction for histogram:

i. Represent the class intervals on the horizontal axis by choosing the scale as 1 cm = 10 units and frequency on the vertical axis. Since the maximum frequency is 15, choose the scale as 1 cm = 1 unit.

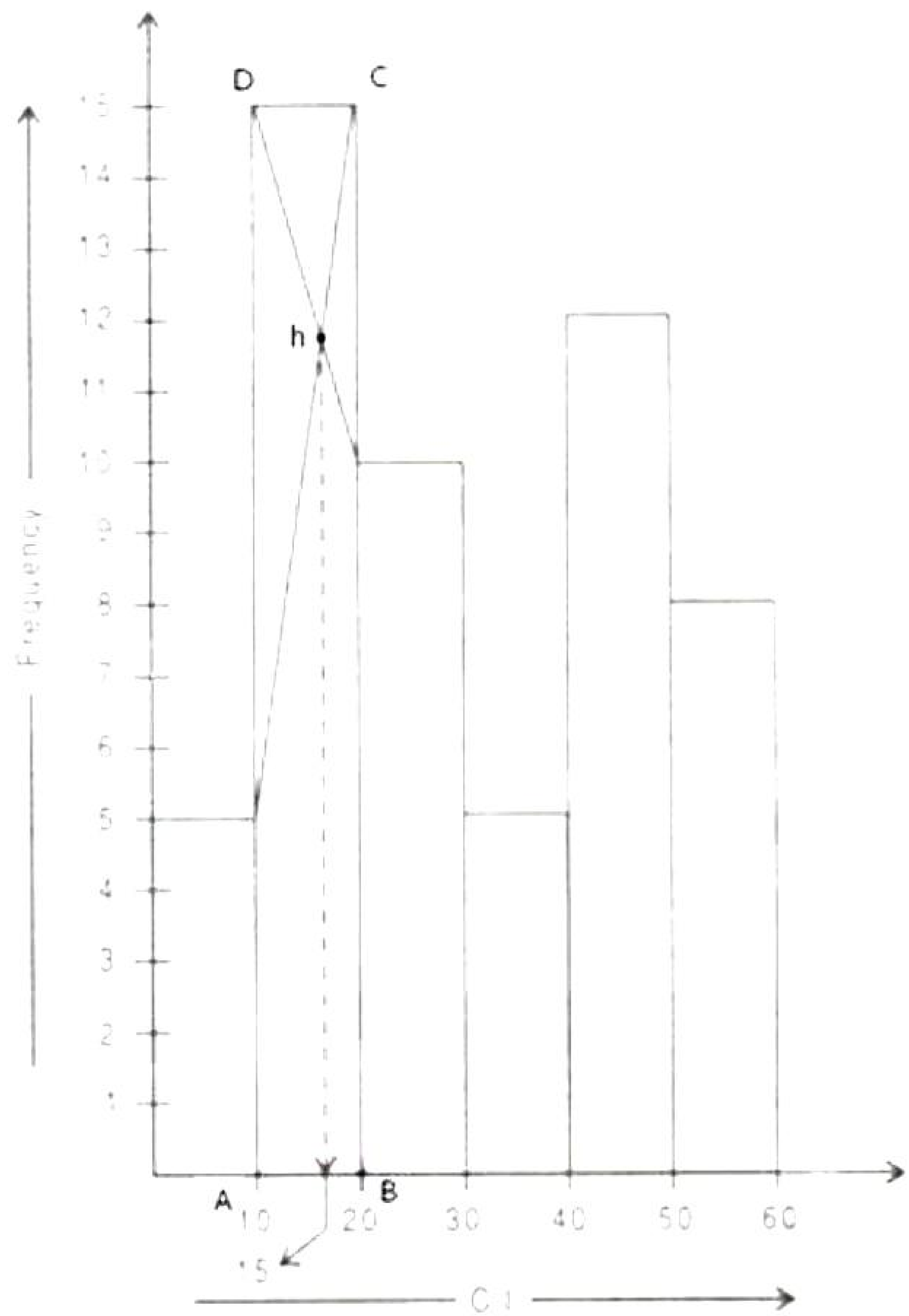
ii. Draw rectangles (or rectangular bars) of width equal to the class-size and lengths according to the frequencies of the corresponding class intervals.



To locate the mode from the histogram, we proceed as follows:

1. Find the modal class. Rectangle ABCD is the largest rectangle. It represents the modal class, that is, the mode lies in this rectangle. The modal class is 10–20.

2. Draw two lines diagonally from the vertices C and D to the upper corners of the two adjacent rectangles. Let these rectangles intersect at point h.
3. The x-value of the point 'h' is the mode. Thus, mode of the given data is approximately 15.



Section B

Solution 4

i)

$$X = \begin{bmatrix} 4 & -2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 12+2 \\ 15-3 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \end{bmatrix}$$

Now, $X - 2Y = 0$

Let $Y = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 14 \\ 12 \end{bmatrix} - 2\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 14 \\ 12 \end{bmatrix} - \begin{bmatrix} 2a \\ 2b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 14-2a \\ 12-2b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Comparing both sides, we get

$\Rightarrow$ 14 – 2a = 0 and 12 – 2b = 0

$\Rightarrow$ a = 7 and b = 6

$$\Rightarrow Y = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

ii)

Given quadratic equation is $x^2 + 7x = 7$

$\Rightarrow x^2 + 7x - 7 = 0$

Comparing with $ax^2 + bx + c = 0$, we have $a = 1$, $b = 7$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{77}}{2}$$

$$\Rightarrow x = \frac{-7 \pm 8.77}{2}$$

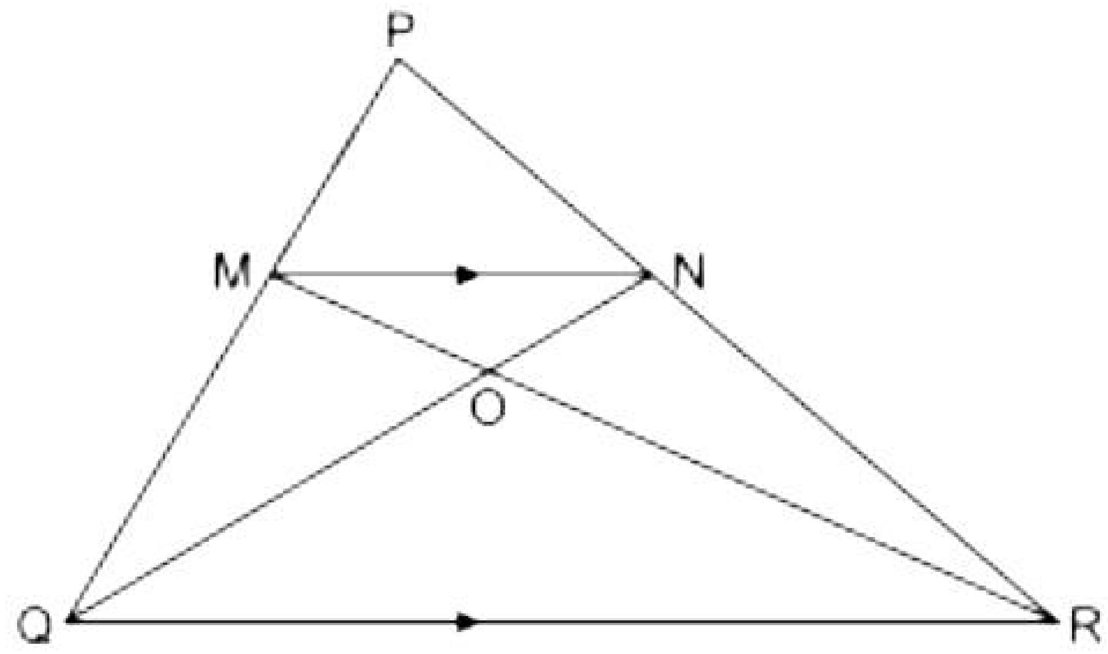
$$\Rightarrow x = \frac{-7 + 8.77}{2} \text{ and } x = \frac{-7 - 8.77}{2}$$

$$\Rightarrow x = \frac{1.77}{2} \text{ and } x = \frac{-15.77}{2}$$

$\Rightarrow x = 0.885$ and $x = -7.885$

$\Rightarrow x = 0.89$ and $x = -7.89$ (correct to two decimal places)

iii)



(i) In ΔPMN and ΔPQR, MN is parallel to QR

$\Rightarrow \angle PMN = \angle PQR$ (corresponding angles)

$\Rightarrow \angle PNM = \angle PRQ$ (corresponding angles)

$\Rightarrow \Delta PMN \sim \Delta PQR$ (AA postulate)

$$\Rightarrow \frac{PM}{PQ} = \frac{MN}{QR}$$

$$\Rightarrow \frac{2}{5} = \frac{MN}{QR} \quad \left[\frac{PM}{MQ} = \frac{2}{3} \Rightarrow \frac{PM}{PQ} = \frac{2}{5}\right]$$

(ii) In ΔOMN and ΔORQ,

$\angle OMN = \angle ORQ$ (alternate angles)

$\angle MNO = \angle OQR$ (alternate angles)

$\Rightarrow \Delta OMN \sim \Delta ORQ$ (AA postulate)

(iii) $\dfrac{\text{Area of } \Delta OMN}{\text{Area of } \Delta ORQ} = \dfrac{MN^2}{RQ^2} = \left(\dfrac{2}{5}\right)^2 = \dfrac{4}{25}$

Solution 5

i)

We have, $17 + f_1 + 32 + f_2 + 19 = 120 \Rightarrow f_2 = 52 - f_1$

Class	Frequency f_i	Mid Value x_i	$f_i x_i$
0–20	17	10	170
20–40	f_1	30	$30f_1$
40–60	32	50	1600
60–80	$52 - f_1$	70	$3640 - 70f_1$
80–100	19	90	1710
	$\sum f_i = 120$		$\sum f_i x_i = 7120 - 40f_1$

Then, Mean, $\bar{x} = \dfrac{\sum(f_i \times x_i)}{\sum f_i}$

$\Rightarrow 50 = \dfrac{7120 - 40f_1}{120}$

$\Rightarrow 7120 - 40f_1 = 6000$

$\Rightarrow 40f_1 = 1120$

$\Rightarrow f_1 = 28$

Thus, $f_1 = 28$ and $f_2 = (52 - 28) = 24$

ii)

For the dealer A (intra-state transaction)

SP = Rs. 8,000

For the dealer B (intra-state transaction)

CP = Rs. 8,000

CGST = 9% of 8,000 = Rs. 720

SGST = 9% of 8,000 = Rs. 720

Profit = Rs. 1,200

SP = Rs. 9,200

For the dealer C (inter-state transaction)

CP = Rs. 9,200

IGST = 18% of Rs. 9,200 = $\frac{18}{100} \times 9{,}200 = \text{Rs. } 1656$

Input Tax = Rs. 1,656

Since, the dealer in Patna does not sell the product.

Output Tax (tax on sale) = Rs. 0

iii)

Since RS is drawn parallel to the tangent PQ,

$\angle SRQ = \angle PQR$ (alternate angles)

Also, PQ = PR

$\Rightarrow \angle PQR = \angle PRQ$

In ΔPQR,

$\angle PQR + \angle PRQ + \angle QPR = 180°$

$\Rightarrow \angle PQR + \angle PQR + 30° = 180°$

$\Rightarrow 2\angle PQR = 150°$

$\Rightarrow \angle PQR = 75°$

$\angle SRQ = \angle PQR = 75°$ (alternate angles)

The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.

$\Rightarrow \angle RSQ = \angle RQP = 75°$

In ΔRSQ,

$\angle RSQ + \angle SRQ + \angle RQS = 180°$

$\Rightarrow 75° + 75° + \angle RQS = 180°$

$\Rightarrow \angle RQS = 30°$

Solution 6

4^{th} term of G.P. $= 16$

$\Rightarrow ar^3 = 16$

7^{th} term of G.P. $= 128$

$\Rightarrow ar^6 = 128$

so, $\frac{ar^3}{ar^6} = \frac{16}{128}$

$\Rightarrow \frac{1}{r^3} = \frac{1}{8} \Rightarrow r = 2$

$\therefore ar^3 = 16$

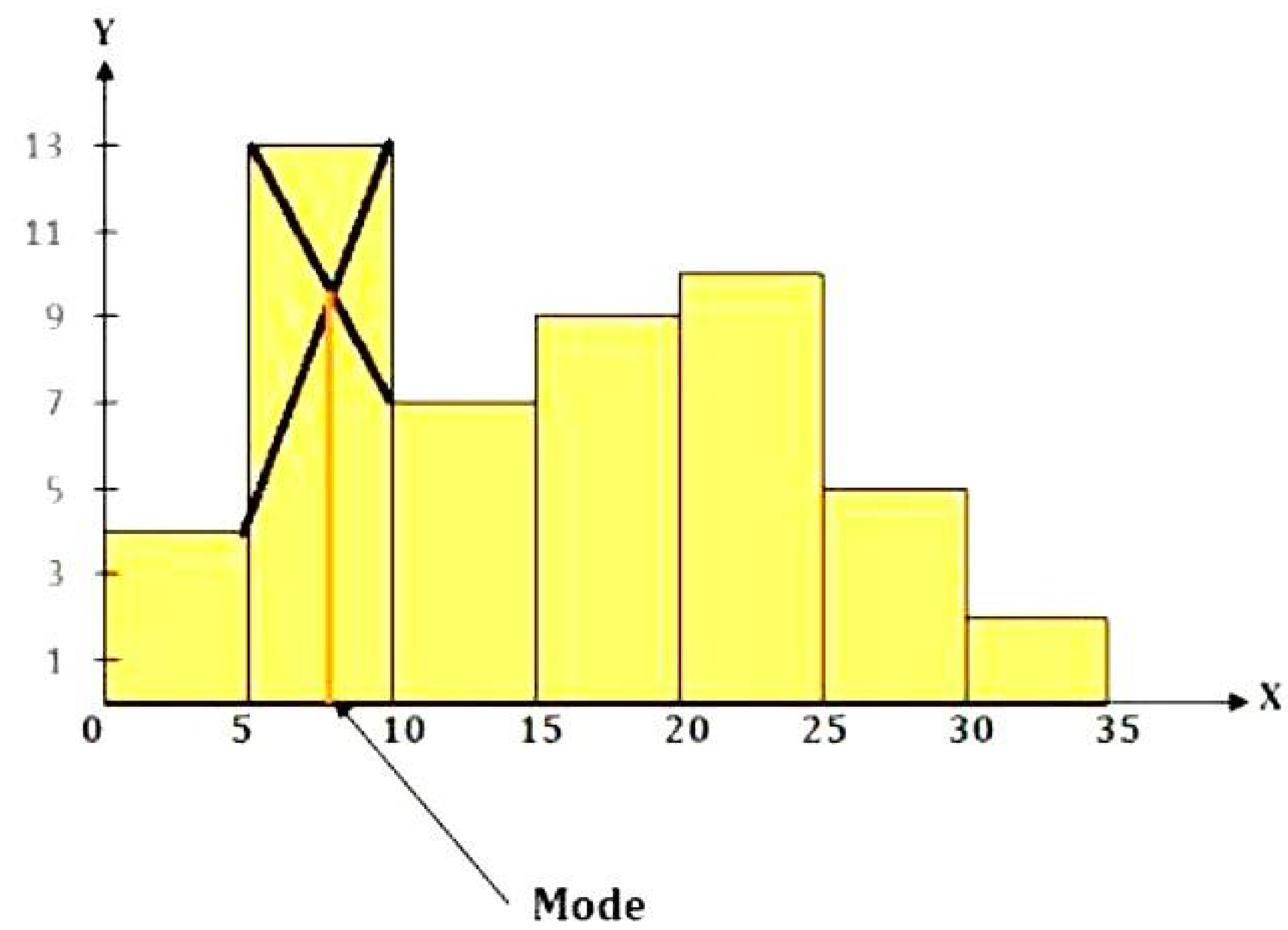
$\therefore a \times 2^3 = 16$

$\therefore a \times 8 = 16$

$\therefore a = 2$

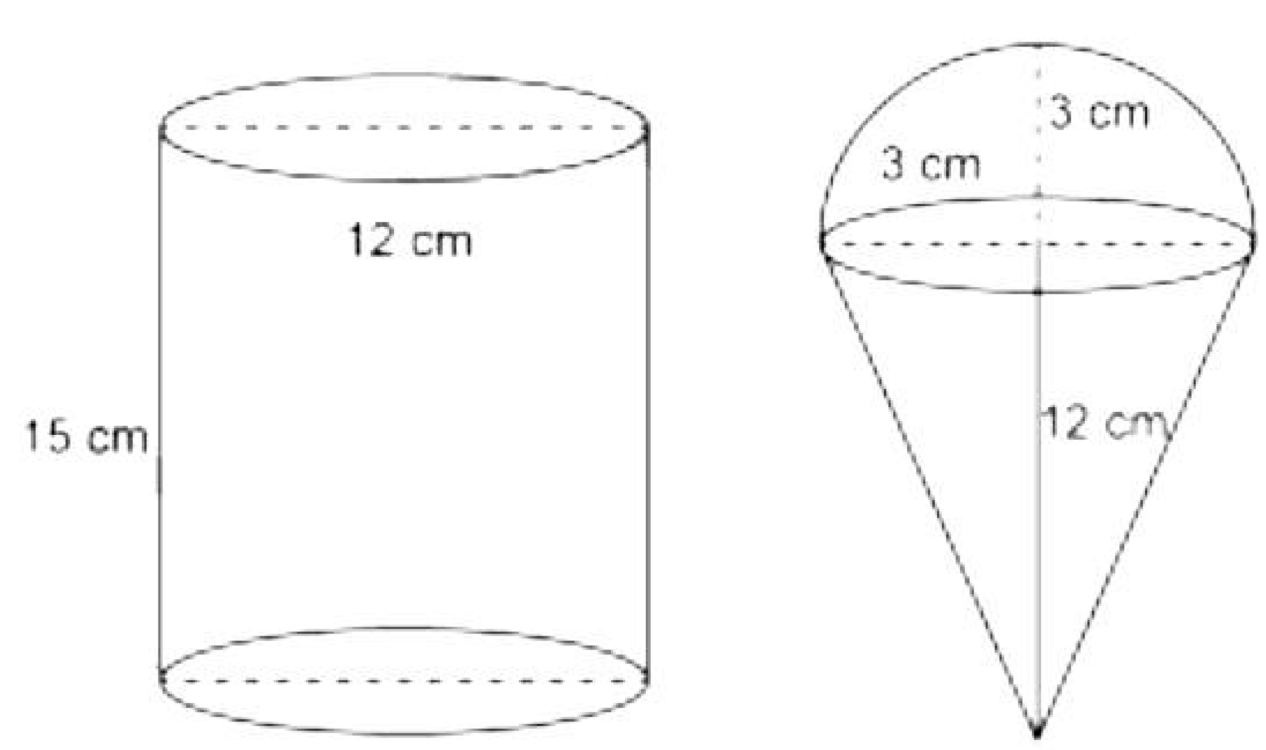
i)

Starting from 0, mark 5, 10, 15, 20, 25, 30 and 35 on the x-axis at equal distances.
Mark 1, 3, 5, 7, 9, 11 and 13 on the y-axis at equal distances.
Now, draw rectangles of width 0-5, 5-10, 10-15 and so on and of height 4, 13, 7, 9, 10, 5, 2, respectively.



Now, the height of the second rectangle is the largest among all the rectangles, i.e. 13.
Draw a line which connects the top left of the second rectangle to the top left of the third rectangle and draw another line which connects the right top of the second rectangle to the top right of the first rectangle.
The intersection point of these two lines will give the mode.
Thus, the mode = 8.

ii)



Height of the cylindrical container, H = 15 cm
Radius of the cylindrical container, R = 6 cm

Volume of the container = $\pi R^2 H = \pi \times 6 \times 6 \times 15 = 540\pi \text{ cm}^3$

Height of the cone, h_1 = 12 cm
Radius of the cone, r_1 = 3 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r_1^2 h_1$$

$$= \frac{1}{3}\pi \times 3 \times 3 \times 12$$

$$= 36\pi \text{ cm}^3$$

Radius of the hemisphere, $r_2 = 3$ cm

$$\text{Volume of the hemisphere} = \frac{2}{3}\pi r_2^3 = \frac{2}{3}\pi \times 3 \times 3 \times 3 = 18\pi \text{ cm}^3$$

Volume of the cone + volume of the hemisphere = $36\pi + 18\pi = 54\pi$ cm^3

Therefore, number of cones which can be filled with ice-cream.

$$= \frac{\text{Volume of the container}}{\text{Volume of cone} + \text{Volume of hemisphere}}$$

$$= \frac{540\pi}{54\pi}$$

$$= 10$$

Solution 7

i)

Given vertices: $A(-1, 3)$, $B(4, 2)$ and $C(3, -2)$

(a) Coordinates of the centroid G of ΔABC are given by

$$G = \left(\frac{-1+4+3}{3}, \frac{3+2-2}{3}\right) = \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$$

(b) Since the line through G is parallel to AC, the slope of the lines are the same.

$$\Rightarrow \text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{3-(-1)} = \frac{-5}{4}$$

So, equation of the line passing through G(2, 1) and with slope $\frac{-5}{4}$ is given by,

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 1 = \frac{-5}{4}(x - 2)$$

$$\Rightarrow 4y - 4 = -5x + 10$$

$\Rightarrow 5x + 4y = 14$ is the required equation.

(c) Since the line through G is perpendicular to AC,

$$\Rightarrow \text{Slope of the line} = m_1 = -\frac{1}{\text{Slope of AC}} = -\frac{1}{-5/4} = \frac{4}{5}$$

So, equation of the line passing through G(2, 1) and with slope $\frac{4}{5}$ is given by,

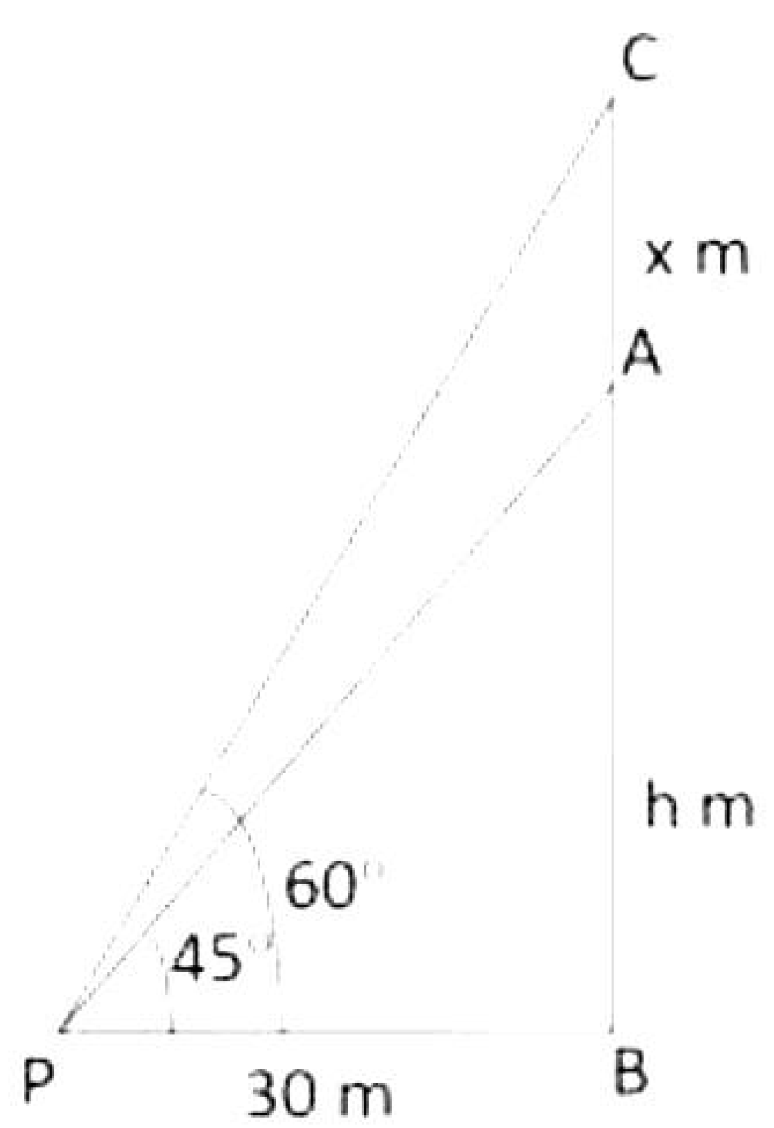
$$y - y_1 = m_1(x - x_1)$$

$$\Rightarrow y - 1 = \frac{4}{5}(x - 2)$$

$$\Rightarrow 5y - 5 = 4x - 8$$

$\Rightarrow 4x - 5y = 3$ is the required equation.

ii) Let AB be the tower with height = h metre
AC = flag staff = x m
PB = 30 m



In ΔPBC,
$\angle CPB = 60°$ and $\angle CBP = 90°$

$$\frac{BC}{PB} = \tan 60°$$

$$\frac{x+h}{30} = \sqrt{3}$$

$$\therefore x + h = 30\sqrt{3} \quad ....(1)$$

In ΔAPB,

$\angle APB = 45°, \angle ABP = 90°$

$$\frac{AB}{PB} = \tan 45°$$

$$\frac{h}{30} = 1$$

$$\therefore h = 30 \quad ....(2)$$

Putting value of h in (1), we get

$x + 30 = 30\sqrt{3}$

$\therefore x = 30\sqrt{3} - 30 = 30\left(\sqrt{3} - 1\right) = 21.96$

Height of the flag staff = 21.96 m
Height of the tower = 30 m

Solution 8

i)

$-3x + 4 > 2x - 3, x \in W$

$\Rightarrow 4 + 3 > 2x + 3x$

$\Rightarrow 7 > 5x$

$\Rightarrow \frac{7}{5} > x$

$\Rightarrow 1\frac{2}{5} > x$

The solution set is $P = \{x : x \in W,\ x < 1\frac{2}{5}\} = \{0, 1\}$.

$4x - 5 < 12, x \in W$

$\Rightarrow 4x < 17$

$\Rightarrow x < \frac{17}{4}$

$\Rightarrow x < 4\frac{1}{4}$

The solution set is $Q = \{x : x \in W,\ x < 4\frac{1}{4}\} = \{0, 1, 2, 3, 4\}$.

Hence, $Q - P = \{2, 3, 4\}$

ii)

In ΔBDC,

$\angle DBC = 90°$ (tangent $\perp$ radius)

$\angle BDC = 60°$ ...(1)(given)

Then, $\angle DCB + \angle DBC + \angle BDC = 180°$ (angle sum property in a triangle)

$\therefore \angle DCB + 90° + 60° = 180°$

$\therefore \angle DCB = 30°$ ...(2)

In ΔOEC,

$OE = OC$ (radii of the same circle)

$\therefore \angle OEC = \angle OCE$ (isosceles triangle property)

$\therefore \angle OEC = \angle BCD = 30°$

$\angle ADE + \angle BDE = 180°$ (straight line property)

$\therefore \angle ADE + 60° = 180°$ ($\angle BDE = \angle BDC$ and from (1)]

$\therefore \angle ADE = 120°$

Also, $\angle DEA = \angle OEC = 30°$ (vertically opposite angles)

In ΔAED,

∠DAE + ∠ADE + ∠DEA = 180° (angle sum property in a triangle)

∴ ∠DAE + 120° + 30° = 180°

∴ ∠DAE = 30°

∴ ∠BAO = ∠DAE = 30°

iii)

Let y = −2 divides the join of points (−4, −5) and (1, 4) in the ration $m_1 : m_2$.

$$\Rightarrow y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = -2$$

$$\Rightarrow m_1y_2 + m_2y_1 = -2(m_1 + m_2)$$

$$\Rightarrow m_1y_2 + m_2y_1 = -2m_1 - 2m_2$$

Here, $y_1 = -5$ and $y_2 = 4$

$$\Rightarrow m_1 \times 4 + m_2 \times (-5) = -2m_1 - 2m_2$$

$$\Rightarrow 4m_1 + 2m_1 = -2m_2 + 5m_2$$

$$\Rightarrow 6m_1 = 3m_2$$

$$\Rightarrow m_1 : m_2 = 1 : 2$$

∴ The required ratio is 1 : 2.

$$\Rightarrow x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times 1 + 2 \times (-4)}{2+1} = \frac{-7}{3}$$

∴ The required point of intersection is $\left(\frac{-7}{3}, -2\right)$.

Solution 9

i)

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{5}{2}$$

Using componendo-dividendo,

$$\Rightarrow \frac{\sqrt{a} + \sqrt{b} + \sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b} - \left(\sqrt{a} - \sqrt{b}\right)} = \frac{5+2}{5-2}$$

$$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{7}{3} \quad \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{7}{3}$$

$$\Rightarrow \frac{a}{b} = \frac{49}{9}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{2401}{81}$$

Using componendo-dividendo,

$$\Rightarrow \frac{a^2+b^2}{a^2-b^2}=\frac{2401+81}{2401-81}$$

$$\Rightarrow \frac{a^2+b^2}{a^2-b^2}=\frac{2482}{2320}$$

ii)

Let us assume that all men do the same amount of work in one day, and one-day work of each man is 1 unit.

Amount of work done by (x – 3) men in (2x + 1) days

= Amount of work done by (x – 3)(2x + 1) men in one day = (x – 3)(2x + 1) units

Similarly, amount of work done by (2x + 1) men in (x + 4) days

= Amount of work done by (2x + 1)(x + 4) men in one day = (2x + 1)(x + 4) units

According to the question,

$$\frac{(x-3)(2x+1)}{(2x+1)(x+4)}=\frac{3}{10}$$

$$\Rightarrow \frac{2x^2-5x-3}{2x^2+9x+4}=\frac{3}{10}$$

$$\Rightarrow 20x^2-50x-30=6x^2+27x+12$$

$$\Rightarrow 14x^2-77x-42=0$$

$$\Rightarrow 2x^2-11x-6=0$$

$$\Rightarrow (x-6)(2x+1)=0$$

$$\Rightarrow x = 6 \text{ or } x = \frac{-1}{2}$$

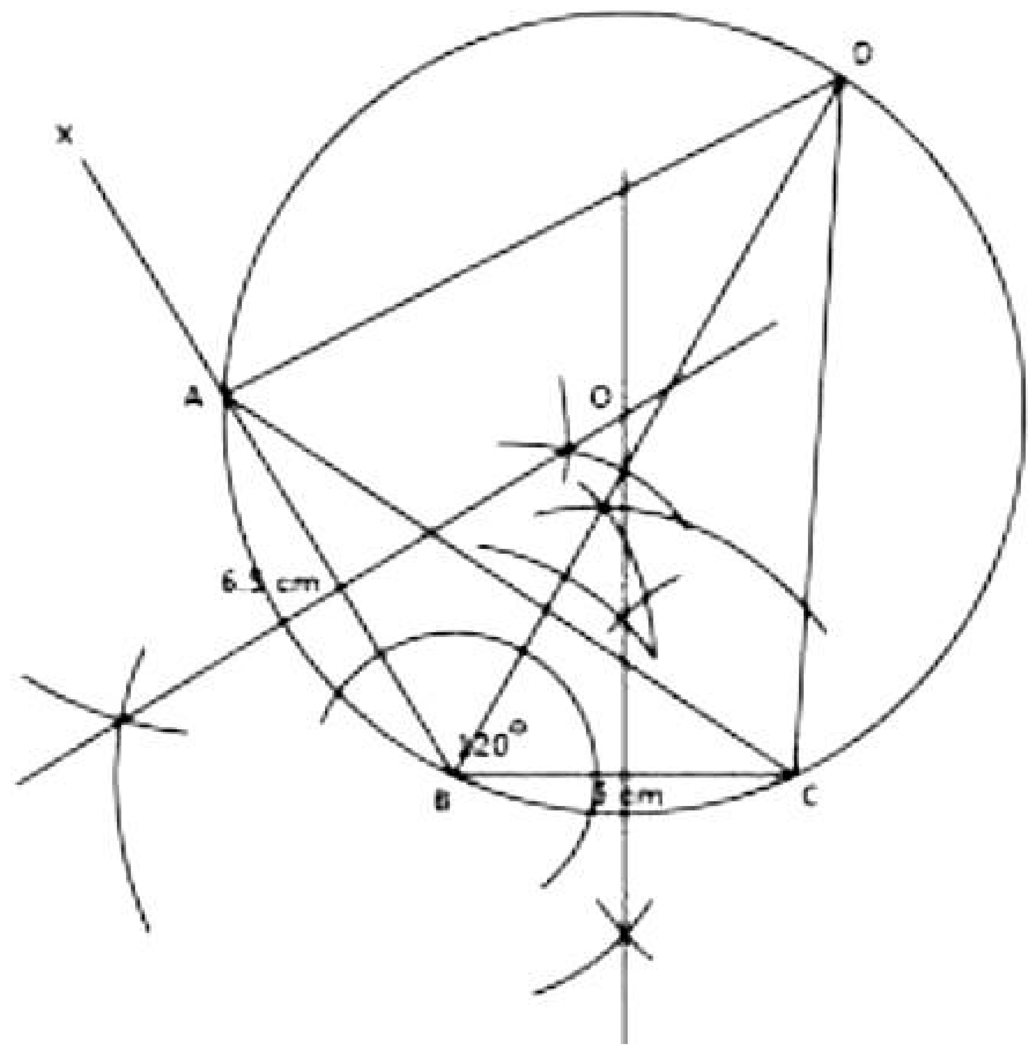
As $x = \frac{-1}{2}$ is not possible. Hence, x = 6.

iii) Steps of construction:

1) Draw a line segment BC of length 5 cm.
2) At B, draw a ray BX making an angle of 120° with BC.
3) With B as centre and radius 6.5 cm, draw an arc to cut the ray BX at A. Join AC. ΔABC will be obtained.

a) Draw the perpendicular bisectors of AB and BC to meet at point O.
With O as centre and radius OA, draw a circle. The circle will circumscribe ΔABC.

b) Draw the angle bisector of ∠ABC and let it meet circle at point D.
Join AD and DC to obtain the required cyclic quadrilateral ABCD such that point D is equidistant from AB and BC.



Solution 10

i)

Let $p_1(x) = x^3 - (p + 5)x^2 + 17x + 28$

$p_2(x) = x^3 + px^2 + 2x - 8$

By remainder theorem,

$p_1(2) = 2^3 - (p + 5) \times 2^2 + 17 \times 2 + 28 = -4p + 50$

$p_2(2) = 2^3 + p \times 2^2 + 2 \times 2 - 8 = 4p + 4$

According to the question,

$\Rightarrow (-4p + 50) - (4p + 4) = 6$

$\Rightarrow -4p + 50 - 4p - 4 = 6$

$\Rightarrow -8p = -40$

$\Rightarrow p = 5$

ii)

Let S denote the sample space of this experiment.

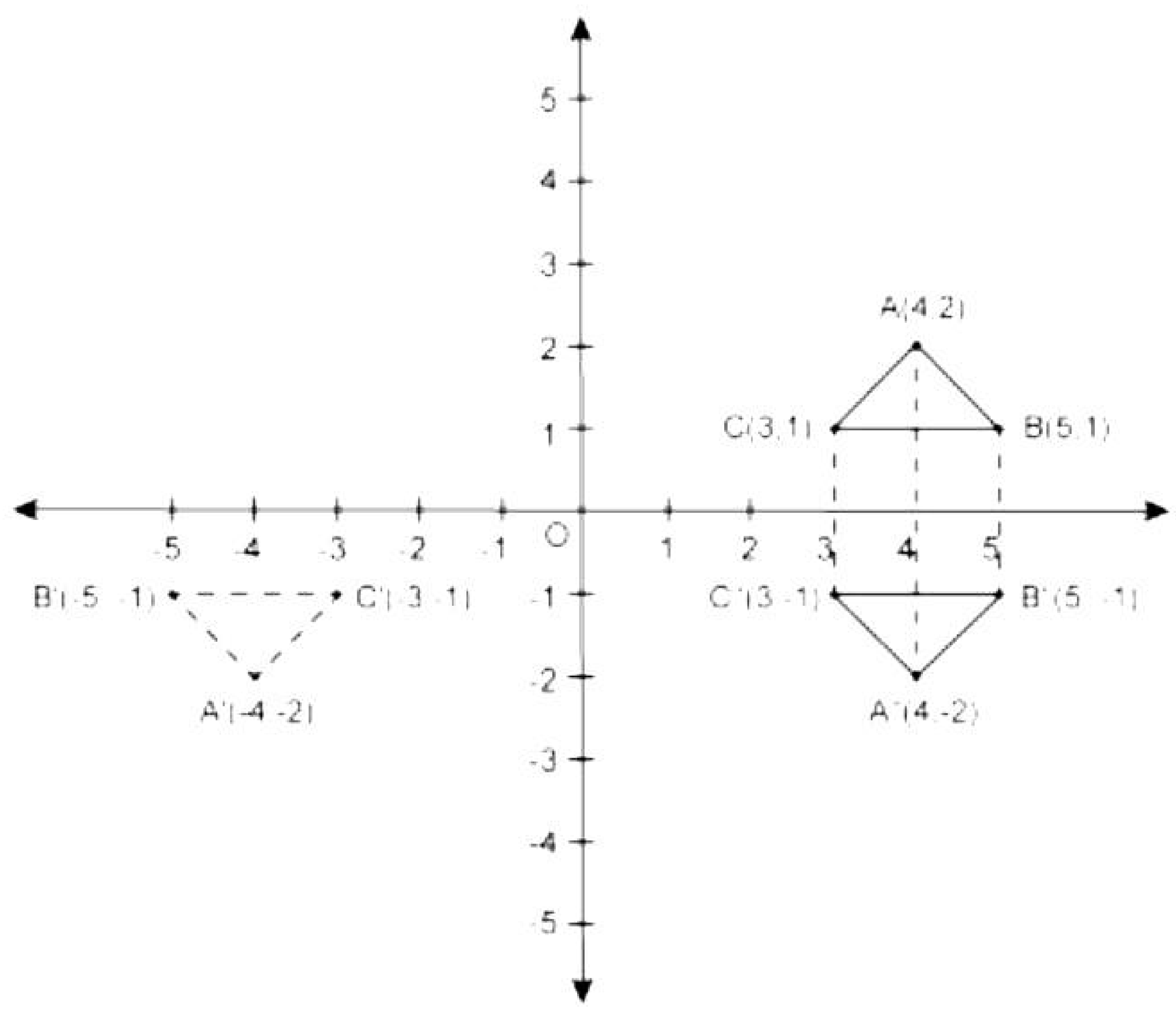
Then, possible outcomes, S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

$\Rightarrow n(S) = 20$

a) Let A be an event of getting a number divisible by both 2 and 5.
Now, since the LCM of 2 and 5 is 10.
So, 10 and 20 are the only numbers divisible by both 2 and 5.
$\Rightarrow$ Number of favourable outcomes is $n(A) = 2$
$\therefore$ Probability of getting a number which is divisible by 2 and 5 = P(A)
$= \frac{n(A)}{n(S)} = \frac{2}{20} = \frac{1}{10}$

b) Let B be an event of getting a number greater than 20.
The cards are numbered from 1 to 20.
$\Rightarrow$ Number of favourable outcomes = $n(B) = 0$
$\therefore$ Probability of getting a number greater than 20 = $P(B) = \frac{n(B)}{n(S)} = \frac{0}{20} = 0$

iii)



a) The points A'(−4, −2), B'(−5, −2) and C'(−3, −1) are reflections of the points A, B and C, respectively, in the origin. The triangle formed by the points A', B' and C' has been shown in the graph.

b) The points A''(4, −2), B''(5, −1) and C''(3, −1) are the reflections of the points A, B and C, respectively, in the x-axis. The triangle formed by the points A'', B'' and C'' has been shown in the graph.

c) The special name of the quadrilateral ABB''A'' is an isosceles trapezium and its area

$$= \frac{1}{2}(AA'' + BB'') \times \text{distance between } AA'' \text{ and } BB''$$

$$= \frac{1}{2}(4+2) \times 1$$

$$= 3 \text{ sq.units}$$