Angles and Their Measurement

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Introduction

This is the one of the fundamental concept of "Trigonometry". Trigonometry is the branch of mathematics which deals with the measurement of sides and angles of a triangle. In modern days, its scope has been extended and it also includes the study of polygons and circles.



Concept of Angle in Geometry

We know that a line is a geometrical shape which extended infinitely in both directions.

P Suppose when a point P put anywhere in the above line, it will be converted into rays. These rays are responsible for the formation of an angle means "An angle is a geometrical figure made by two rays having common end point (called vertex)" these rays are called sides or arms of an angle.

An angle is represented by S a symbol "<". The way to represent it put vertex in the middle.



 \angle RPO, \angle SPQ, \angle RPS are the angles with vertex P.

The inclination of one ray to other represented by a number is called measurement of an angle. One of the most important aspect of geometry is that all the angles are between 0° and 360° and there is no meaning of negative angle.

Concept of Angles in Trigonometry

In case of trigonometry, angles may be positive or negative and of any magnitude.



Positive Angles

When we measure an angle in anti-clock wise direction it is always positive.



Negative Angles

If an angle measures in anti-clock wise direction then it is said to be negative angle.



Different Units of Measurement of an Angle

There are three system for measurement of an angle.

- 1. British System (Sexagesimal System)
- 2. French System (Centesimal System)
- 3. Circular Measure or Radian System

British System

It is also known as sexagesimal system. In this system a right angle is divided into 90 equal parts is called degrees.

 $1 \text{ right angle} = 90^{\circ}$ $1^{\circ} = 60'$ 1' = 60''

In the above, degree is divided into 60 equal parts known as minutes, and each minute is divided into 60 equal parts known as seconds.

Centesimal System

In this system a right angle is divided into 100 equal parts and each part is known as grade.

1 right angle	= 100g
1g	=100'
1'	=100"

Radian System

1 radian is defined as an angle which subtend at the centre of a circle by an arc whose length is equal to the length of radius.



PQ = R

We know that n is the ratio of circumference and diameter.

$$\frac{C}{D} = \pi$$

Relation between radian and degree, π radian = 180° It is also written as

$$\pi^{c} = 180^{o}$$

Relation between Angle, Length of an Arc and Radius of Circle

We know that the angle formed at the centre is proportional to the length of arc which subtend it. Means $\theta \, \alpha \, l$

(Here, θ represents angle and l represents the length of arc)



By the definition of radian.

Angle subtended by an arc of length r at the centre of circle = 1 radian.

- : Angle subtended by an arc of length 1 at the centre of circle = $\frac{1}{\pi}$ radian.
- : Angle subtended by an arc of length (. at the centre of circle = $\frac{\ell}{r}$



Relation among Different Units of Measurement of an Angle

1.
$$1^{\circ} = \frac{\pi}{180}$$
 radian
2. 1 radian $= \left(\frac{180}{180}\right)^{\circ}$

$$\left(\begin{array}{c} \pi \end{array}\right)^{s} = \left(\frac{\pi}{180^{\circ}}\right)^{c} = 1 \text{ degree}$$

It is convention that angles are always measured either in radian or in degree. Conversion of Some Common Angle in Degree into Radian

Degree	0^{o}	30°	45^{o}	60°	90°	180^{o}	360°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	2π



In the ancient times different geometrical structures were made. In these structures one of the most famous is the Great pyramid of Giza. It is the oldest and larges pyramid in the Giza. This was built as a tomb for fourth dynasty Egyptian king khafu and constructed over a 20 year period concluding around 2560 BC. The great pyramid was the tallest man-made structure in the world for over 3,800 years.

SUMMARY



- ↔ When we measure an angle in anti-clock wise direction it is always positive.
- If an angle measures in anti-clock wise direction then it is said to be negative angle.

$$\bullet \quad 1^{\circ} = \frac{\pi}{180} radian$$

•
$$1 radian = \left(\frac{\pi}{\pi}\right)^{\ell}$$

$$\theta = \frac{\ell}{r}$$

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Find the ratio of radii of two circles, if arc of equal length subtend angles 30° and 45° at their centre.

(a) 2:3 (b) 4:3 (c) 3:2 (d) 3 :4 (e) None of these

Answer: (c)

Explanation

Suppose radii of circles be r_1 and r_2

then
$$\theta_1 = \frac{l}{r_1}$$
 and $\theta_2 = \frac{l}{r_2}$
where $\theta_1 = 30^\circ, \theta_2 = 45^\circ$
 $\frac{\theta_1}{\theta_2} = \frac{\frac{l}{r_1}}{\frac{l}{r_2}}$
 $\Rightarrow \frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} \Rightarrow \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} \Rightarrow \frac{r_1}{r_2} = \frac{45}{3\theta_2}^{3\circ}$
 $\Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$
 $r_1 : r_2 = 3:2$

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The measures of two angles of a triangle are $60^{\circ} 53' 51''$ and $51^{\circ} 22' 50''$ respectively. The measure of third angle in radian is_____.

- (a) 1.15 radian (b) 11 radian (c) 180 radian (d) 11.5 radian
- (e) None of these

Answer: (a)

Explanation

The sum of two angles = $[(62" 53' 51") + 51^{\circ} 22' 50"] = 114^{\circ} 16' 41"$

Third angle of triangle = $180^{\circ} - (140^{\circ} \ 16' \ 41'') = 65^{\circ} \ 43' \ 19''$

$$= \left(65 + \frac{43}{60} + \frac{19}{3600}\right)^0 = \frac{234000 + 2580 + 19}{3600} = \left(\frac{236599}{3600}\right)^0 = (65.722)^0$$

We know that

$$1^{\circ} = \frac{\pi}{180} \text{ radian}$$

$$(65.722)^{\circ} = \left(\frac{\pi}{180} \times 65.722\right) \text{ radian} = 1.15 \text{ radian (approx)}$$

If the ratiO of radii of two circles are in 5 : 4 then the angle subtend at the centre of circles by the same arc length be _____. (a) 75° 60° (b) 45° 60°

(a) 75°, 60°	(b) 45°, 60°
(b) 60°, 45°	(d) 60°, 75°
(e) None of these	

Answer: (d)

The distance travelled	y the tip of minute hand in 10 minutes when the length of it is 20 cm, is	
(a) 20.95m	(b) 20.95cm	
(c) 2.095cm	(d) 209.5cm	
(e) None of these		

Answer: (b)



Self Evaluation EST



Which one of the following statements is false for formula $\left| \theta = \frac{l}{r} \right|$? 1.

- (a) I is the length of arc
- (b) r is the radius of circle
- (c) θ is an angle at the centre whose measure is in degree
- (d) θ is an angle at the centre whose measure is in radian
- (e) None of these

2. If the measures of an angle in degree and radian be D and R respectively then which one of the following statements is true?

- (a) $\frac{R}{180} = \frac{D}{\pi}$
- (b) $\frac{R}{180} = \frac{R}{\pi}$ (d) $D \times \pi = 100 \times R$
- (c) $R \times 180 = D \times 2\pi$
- (e) None of these

3. Match the following:

А	В
(a) 180°	(i) 0.79 <i>radian</i>
(b) 1 ^{<i>c</i>}	(ii) 30°
(c) $\left(\frac{\pi}{6}\right)^c$	(iii) π radian
(d) 45° 20'10"	(iv) $\left(\frac{180}{\pi}\right)$ degree
(a) (a) - (iii)	(b) (b) - (iv)
(c) (c) - (ii)	(d) (d) - (i)
(e) None of these	

4. Find the diameter of a heavenly bodies, if it subtend an angle of 31' at the eye of person. The distance between earth and the heavenly body is 3, 8400 km.



(e) None of these

Find the degree measure of the angle subtended at the centre of a circle of radius 50 cm by an arc of 5. length 11 cm. (a) 12°36' (b)12°6'36" (d)12°36" (c) 12°6'0 (e) None of these A bus is travelling on circular track of radius 750 m at the speed of 33 km/h. The angle does it turn in 10 6. second _____. (a) $\frac{11}{90}$ radian (b) $\frac{11}{135}$ radian (c) $\frac{11}{150}$ radian (d) $\frac{11}{45}$ radian (e) None of these 7. Find the radius of a circle in which a central angle of 60° intercepts an arc of 132 cm. (a) 2.2cm (b) 126cm (c) 6.908cm (d) 132cm (e) None of these 8. Find the length of minor arc of a circle whose diameter is 100 cm and length of chord is 50 cm. (a) 50n cm (b) $50 \times 3\pi$ cm (c) $\frac{50\pi}{3}$ cm (d) $(50+3)\pi$ cm (e) None of these If the angles of a triangle are in the ratio 5:4:3 then the smallest angle in radian is _____. 9. (b) $\frac{3\pi}{4}$ radian (a) 45 radian (c) $\frac{7\pi}{4}$ radian (d) $\frac{4\pi}{16}$ radian (e) None of these 10. Which one of the following relations is false? (a) $\frac{D}{9} = \frac{20R}{\pi}$ (b) $\frac{3\pi}{2} = 270^{\circ}$ (c) $\frac{4\pi}{16} = 90^{\circ}$ (d) $\frac{\pi}{3} = 60^{\circ}$ (e) None of these **Answers – Self Evaluation Test**

1.

С

2.

В

3.

А

4.

В

5.

А

6.

С

7.

В

8.

С

9.

D

10. C

Self Evaluation Test SOLUTIONS

The measure of angle 9 must be in radian. Therefore, option (C) is correct and res: of the options is incorrect. 1.

The correct relation is 2.

 $\frac{D}{180} = \frac{R}{\pi}$

Therefore, option (B) is correct and rest of the options is incorrect.

3. $180^\circ = \pi$ radian

$$1^{c} = \left(\frac{180}{\pi}\right) \text{degree}$$
$$\left(\frac{\pi}{6}\right)^{c} = \left[\frac{\pi}{6} \times \frac{180^{30}}{\pi}\right] \text{degree} = 30 \text{ degree}$$
$$45^{\circ}20'10'' = 0.79 \text{ radian}$$

4.
$$Q = \frac{I}{r}$$
Here $Q = 31' = \left(\frac{31}{60} \times \frac{\pi}{180}\right)$ radian
$$\Rightarrow \frac{30}{60} \times \frac{\pi}{180} = \frac{PQ}{38400}$$

$$\Rightarrow PQ = \frac{31\pi \times 38400}{60 \times 180} km$$

$$\Rightarrow PQ = 3464.13 km$$

5.
$$\theta = \frac{I}{r}$$

 $\Rightarrow \theta = \left(\frac{11}{50}\right)$ radian
 $\therefore 1$ radian $= \left(\frac{180}{\pi}\right)$ degree
 $\therefore \frac{11}{50}$ radian $= \left(\frac{180}{\pi} \times \frac{11}{50}\right)$ degree

6. Speed of bus =
$$33 km / h = \frac{33^{11} \times 5}{18Q_6} = \frac{55}{6} m / s$$

The distance travelled by bus in 10 second = $M^5 \times \frac{55}{6_3} = \frac{5 \times 55}{3} m$
 $\therefore \theta = \frac{I}{r}$
 $\Rightarrow \theta = \frac{35^{11} \times 5}{3 \times 75Q_{25Q_{5q}}} = \frac{11}{150}$ radian

9. Find the angles of triangle and convert the smallest angle into radian.

$$10. \qquad \frac{4\pi}{16} = 45^{\circ}$$