

ELECTRICAL CIRCUITS AND FIELDS TEST 3

Number of Questions: 25

Section Marks: 90

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The lines of force due to charged particles are
(A) Always curved (B) Always straight
(C) Sometimes curved (D) None of the above
2. If the sheet of Bakelite is inserted between the plates of an air capacitor, the capacitance will
(A) decrease (B) remains unchanged
(C) increase (D) become zero
3. The dissipation factor of a good dielectric is of the order of
(A) 0.0001 (B) 0.001
(C) 0.01 (D) 0.1
4. Which one of the following relation is correct?
(A) $\oint_l A \cdot dl = \int_s \nabla \cdot A ds$ (B) $\int_l \nabla \times A dl = \oint_s (A) ds$
(C) $\oint_s A \cdot ds = \int_v \nabla \times A dv$ (D) $\oint_s A \cdot ds = \int_v \nabla \cdot A dv$
5. "The total electric flux through any closed surface is equal to amount of charge enclosed".
The above statement is
(A) Ampere's law (B) Coloumb's law
(C) Gauss's law (D) Maxwell's first law
6. A field F is said to be conservative if
(A) $\nabla \times \vec{F} = 0$ (B) $\nabla \cdot F = 0$
(C) $\nabla \cdot \nabla F = 0$ (D) All the above
7. The electric field
 $E = 6 \cos(\omega t - \beta z) \hat{a}_x + 6 \cos(\omega t - \beta z + 60^\circ) \hat{a}_y$ has
(A) linear polarization
(B) left circular polarization
(C) elliptical polarization
(D) right circular polarization
8. A uniform plane wave incident on a sheet whose conductivity is $\sigma_m = 5.8 \times 10^7$ S/m at $f = 2$ GHz then reflection co-efficient is
(A) -1 (B) +1
(C) 0 (D) 0.5
9. The work done by a force $E = 3\hat{a}_x - 4\hat{a}_y + 2\hat{a}_z$ N/C in giving a 2 nC charge a displacement of $10\hat{a}_x + 2\hat{a}_y - 7\hat{a}_z$ meter is
(A) 4 J (B) 24 nJ
(C) 16 nJ (D) 8 J
10. EMW travelling in high loss medium at frequency f_1 has attained wavelength of λ_1 when frequency became four times what is the corresponding wavelength?
(A) $\frac{\lambda_1}{2}$ (B) $2\lambda_1$
(C) $\frac{\lambda_1}{4}$ (D) $4\lambda_1$
11. In a non-magnetic medium $E = 10 \cos(2\pi \times 10^8 t - 2x) \hat{a}_z$ V/m then total power crossing 40 cm² of plane $3x + y = 7$ is
(A) 2.53 W (B) 69.21 W
(C) 96.12 W (D) 52.3 W
12. Determine the energy density stored in free space by the field $2 \times 10^{-3} \hat{a}_x + 4 \times 10^{-3} \hat{a}_y$ T.
(A) $2\sqrt{5} \times 10^{-3}$ J/m² (B) 7.9 J/m³
(C) 4.8 J/m³ (D) 1.3 J/m²
13. Transform a vector $A = y\hat{a}_x - x\hat{a}_y + z\hat{a}_z$ into cylindrical co-ordinates
(A) $\hat{a}_\rho + \rho\hat{a}_\phi + z\hat{a}_z$ (B) $-\rho\hat{a}_\phi + z\hat{a}_z$
(C) $\rho\hat{a}_\rho + z\hat{a}_z$ (D) $\rho\hat{a}_\rho - \hat{a}_\phi + z\hat{a}_z$
14. A potential field is $V = 6x^2y - xyz$. The electric field at $P(-2, 1, 4)$ shall be
(A) $8\hat{a}_x + 32\hat{a}_y + 2\hat{a}_z$ (B) 0
(C) $32\hat{a}_y$ (D) $-28\hat{a}_x - 32\hat{a}_y - 2\hat{a}_z$
15. In a certain region current density is $J = (4z\hat{a}_x + 2x^2z\hat{a}_y + 4z^2\hat{a}_z) \sin(10^6 t)$ A/m. Then volume charge density is
(A) $(4z + 2x^2z + 4z^2) \sin(10^6 t)$ $\mu\text{C/m}^3$
(B) $-18x \cos(10^6 t)$ $\mu\text{C/m}^3$
(C) $4xz \sin(10^6 t)$ mC/m^3
(D) $-8z \cos(10^6 t)$ $\mu\text{C/m}^3$
16. The curl of vector field $A = \rho z \sin\phi \hat{a}_\rho + 3\rho z^2 \hat{a}_z$ at point $(3, 90^\circ, 2)$ is
(A) $9\hat{a}_\phi - 2\hat{a}_z$ (B) $2\hat{a}_\rho - 9\hat{a}_\phi + 2\hat{a}_z$
(C) $-9\hat{a}_\phi$ (D) $4\hat{a}_z$
17. Two free charges q and $9q$ are placed at a distance d apart a third charge Q is placed at a distance x from charge q such that the system is in equilibrium. Then charge Q is equal to
(A) $9q$ (B) $\frac{9q}{16}$
(C) $\frac{9q}{4}$ (D) $\frac{3q}{2}$

18. An electric charge Q is placed in a dielectric medium. Which of the following Quantities are independent of the dielectric constant ϵ of the medium:
 (A) Electric potential and displacement
 (B) Electric field intensity and displacement density
 (C) Displacement density and electric displacement
 (D) Electric field intensity and displacement density
19. When a plane is incident normally from medium 1 (μ_o, ϵ_1) into medium 2 ($4\mu_o, \epsilon_2$), the electric field of the transmitted wave is -0.5 times the electric field of the reflected wave. Then the ratio of $\frac{\epsilon_1}{\epsilon_2}$ is
 (A) 10 (B) 0.1
 (C) 100 (D) 0.01
20. The xy -plane serves as the interface between two different media. Medium₁ ($z < 0$) is filled with a material whose $\mu_r = 3$ and medium₂ ($z > 0$) is filled with a material whose $\mu_r = 2$. If the interface carries current $2/\mu_o \hat{a}_y$ mA/m and $B_2 = 3\hat{a}_x + 7\hat{a}_z$ mwb/m². Then H_1 is
 (A) $\frac{14}{\mu_o} \hat{a}_x + \frac{6}{\mu_o} \hat{a}_z$ (B) $\frac{14}{\mu_o} \hat{a}_x$
 (C) $\frac{-14}{\mu_o} \hat{a}_x + \frac{6}{\mu_o} \hat{a}_z$ (D) $\frac{-0.5}{\mu_o} \hat{a}_x$
21. An EMW working in free space strikes a block of brass ($\sigma = 1.8 \times 10^7$ S/m) along its normal surface impedance of brass is $\eta_b = 0.08 \angle 45^\circ \Omega$. Calculate skin depth in brass?
 (A) $0.49 \mu\text{m}$ (B) $0.04 \mu\text{m}$
 (C) $0.98 \mu\text{m}$ (D) $0.69 \mu\text{m}$
22. In a lossless dielectric for $\eta = 30\pi\Omega$, $\mu_r = 1$ and $H = 0.5 \cos(\omega t - z) \hat{a}_x + 0.1 \sin(\omega t - z) \hat{a}_y$ A/m then $\vec{E}$ is
 (A) $15\pi \cos(75 \times 10^6 t - z) \hat{a}_x + 3\pi \sin(75 \times 10^6 t - z) \hat{a}_y$
 (B) $3\pi \sin(300 \times 10^6 t - z) \hat{a}_x + 15\pi \cos(300 \times 10^6 t - z) \hat{a}_y$
 (C) $3\pi \sin(75 \times 10^6 t - z) \hat{a}_x - 15\pi \cos(75 \times 10^6 t - z) \hat{a}_y$
 (D) $-15\pi \cos(300 \times 10^6 t - z) \hat{a}_x + 3\pi \sin(300 \times 10^6 t - z) \hat{a}_y$
23. A uniform plane wave propagating in a medium has $E = 4e^{-\alpha z} \cos(2 \times 10^8 t - \beta z) \hat{a}_x$ V/m. If the medium is characterized by $\epsilon_r = 1, \mu_r = 16$ and $\sigma = 5$ S/m. Then phase constant is
 (A) 2.67 rad/m (B) 76.2 rad/m
 (C) 20 rad/m (D) 100.2 rad/m
24. A point charge $Q_A = 2 \mu\text{C}$, is at $A(0, 0, 1)$ and $Q_B = -2 \mu\text{C}$ is at $B(0, 0, -1)$. Find electric field intensity along r co-ordinate $[E_r]$ at $p(1, 2, 3)$
 (A) 2070 (B) -206.81
 (C) -690 (D) -720
25. A uniform line charge $\rho_\ell = 30 \mu\text{C/m}^2$ on the z -axis. Then find $\vec{D}$ at $(3, 4, -5)$
 (A) $9 \hat{a}_x + 12 \hat{a}_y \mu\text{C/m}^2$
 (B) $1.5 \hat{a}_x - 2.2 \hat{a}_y \mu\text{C/m}^2$
 (C) $3.4 \hat{a}_x - 4.7 \hat{a}_y \mu\text{C/m}^2$
 (D) $0.56 \hat{a}_x + 0.76 \hat{a}_y \mu\text{C/m}^2$

ANSWER KEYS

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|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1. A | 2. C | 3. A | 4. D | 5. C | 6. A | 7. B | 8. A | 9. C | 10. A |
| 11. C | 12. B | 13. B | 14. D | 15. D | 16. C | 17. B | 18. C | 19. D | 20. D |
| 21. C | 22. C | 23. D | 24. B | 25. D | | | | | |

HINTS AND EXPLANATIONS

1. Lines of force due to charged particles are always curved.



Choice (A)

2. $C \propto \epsilon_r$
 $\epsilon_r = 1$ for air
 $= 5$ for bakelite
 so capacitance will increase.

Choice (C)

3. Dissipation factor is $\frac{\sigma}{\omega\epsilon}$ for dielectric $\sigma \rightarrow 0$ Choice (A)

$$4. \oint_l A \cdot dl = \int \nabla \times A \cdot ds$$

$$\oint_s A \cdot ds = \int_v \nabla \cdot A \cdot dv \quad \text{Choice (D)}$$

5. $\Psi = \int D \cdot da = Q \rightarrow$ Gauss's law. Choice (C)

6. $\nabla \cdot \vec{F} = 0$ is said to be solenoidal

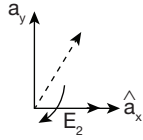
$\nabla \times \vec{F} = 0$ is said to be conservative (or) irrotational.

Choice (A)

$$7. E = 6\cos(\omega t - \beta z) \hat{a}_x + 6\cos(\omega t - \beta z + 60^\circ) \hat{a}_y$$

$$\text{If } \omega t - \beta z = 0 \Rightarrow E_1 = 6 \hat{a}_x + 3\hat{a}_y$$

$$\text{If } \omega t - \beta z = 30 \Rightarrow E_2 = \frac{6\sqrt{3}}{2} \hat{a}_x$$



E has left circular polarization.

Choice (B)

$$8. \frac{\sigma}{\omega\epsilon} \gg 1 \text{ so it is conductor, it acts as a short circuit}$$

$$Z_L = 0$$

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = -1$$

Choice (A)

$$9. W = \oint E \cdot dl = F \cdot dl$$

$$= [2 \times 10^{-9} [30 - 8 - 14]] = 16 \text{ nJ.}$$

Choice (C)

$$10. \text{ For high loss media } \frac{2\pi}{\lambda} = \beta a \sqrt{f} \Rightarrow \lambda_1 \sqrt{f_1} = \lambda_2 \sqrt{f_2}$$

$$\Rightarrow \lambda_1 \sqrt{f_1} = \lambda_2 \sqrt{4f_1}$$

$$\Rightarrow \lambda_2 = \lambda_1 \frac{1}{2}$$

Choice (A)

$$11. P = E \times H = \frac{E_0^2}{\eta} \cos^2(2\pi \times 10^8 t - 2x) \hat{a}_x$$

$$P_{\text{avg}} = \frac{100}{\eta}$$

$$\Rightarrow \beta = \omega \sqrt{\mu\epsilon} \Rightarrow 2 = \frac{2\pi \times 10^7}{3 \times 10^8} \sqrt{\epsilon_r}$$

$$\eta = \frac{120\pi}{\sqrt{\epsilon_r}} = 4\pi \sqrt{\epsilon_r} = \frac{30}{\pi}$$

$$P_{\text{avg}} = \frac{25}{\pi r^2} \frac{W}{m^2} \hat{a}_x$$

On plane $3x + y = 7$ is $P = \int P_{\text{avg}} \cdot ds$

$$= \left(\frac{25}{\pi^2} \times 40 \right) \hat{a}_x \left(\frac{3\hat{a}_x + \hat{a}_y}{\sqrt{10}} \right)$$

$$= \frac{75 \times 40}{\pi^2 \sqrt{10}} = 96.12 \text{ W}$$

Choice (C)

$$12. \text{ Given that } B = 2 \times 10^{-3} \hat{a}_x + 4 \times 10^{-3} \hat{a}_y$$

$$\text{Energy density} = \frac{B^2}{2\mu_0}$$

$$= \frac{4 \times 10^{-6} + 16 \times 10^{-6}}{2 \times 4\pi \times 10^{-7}} = 7.9 \frac{\text{J}}{\text{m}^3}$$

Choice (B)

$$13. \hat{a}_x = \hat{a}_\rho \cos \phi - \hat{a}_\phi \sin \phi$$

$$\hat{a}_y = \sin \phi \hat{a}_\rho + \hat{a}_\phi \cos \phi$$

$$\Rightarrow y = \rho \sin \phi; \quad x = \rho \cos \phi$$

$$\Rightarrow \rho \sin \phi (\hat{a}_\rho \cos \phi - \hat{a}_\phi \sin \phi) - \rho \cos \phi (\sin \phi \hat{a}_\rho + \cos \phi \hat{a}_\phi) + z \hat{a}_z = A$$

$$\Rightarrow A = \hat{a}_\rho (\rho \sin \phi \cos \phi - \rho \sin \phi \cos \phi) - \hat{a}_\phi \rho (\sin^2 \phi + \cos^2 \phi) + z \hat{a}_z$$

$$\Rightarrow -\rho \hat{a}_\phi + z \hat{a}_z = A.$$

Choice (B)

$$14. E = -\nabla V$$

$$= - \left[\frac{\partial}{\partial x} (6x^2 y - xyz) + \frac{\partial}{\partial y} (6x^2 y - xyz) + \frac{\partial}{\partial z} (6x^2 y - xyz) \right]$$

$$= -[(12xy - yz) \hat{a}_x + (6x^2 - xz) \hat{a}_y + (-xy) \hat{a}_z] \text{ at } (-2, 1, 4)$$

$$= -[28 \hat{a}_x + 32 \hat{a}_y + 2 \hat{a}_z].$$

Choice (D)

$$15. \nabla \cdot J = - \frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow 8z \sin(10^6 t) dt = - \partial \rho_v$$

Integrating both sides

$$\Rightarrow \rho_v = -[8z \cos(10^6 t)] \mu\text{C/m}^3.$$

Choice (D)

$$16. \nabla \times A = \frac{1}{\rho} \begin{bmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \rho z \sin \phi & 0 & 3\rho z^2 \end{bmatrix}$$

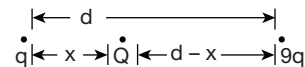
$$= \frac{1}{\rho} [-\rho \hat{a}_\phi [3z^2 - \rho \sin \phi] + \hat{a}_z (\rho z \cos \phi)]$$

$$= [\hat{a}_\phi [\rho \sin \phi - 3z^2] + \hat{a}_z z \cos \phi]_{(3, 90^\circ, 2)}$$

$$= \hat{a}_\phi [3 - 12] + \hat{a}_z (0) = -9 \hat{a}_\phi$$

Choice (C)

$$17.$$



Given that system in equilibrium

Force on Q is

$$\Rightarrow \frac{qQ}{4\pi\epsilon_0 x^2} = \frac{9qQ}{4\pi\epsilon_0 (d-x)^2}$$

$$\Rightarrow d = 4x = x = \frac{d}{4}$$

$$\text{Force on } q \text{ is } \frac{9q^2}{4\pi\epsilon_0 d^2} = \frac{qQ}{4\pi\epsilon_0 X^2}$$

$$\Rightarrow Q = \frac{9}{16} q.$$

Choice (B)

$$18. \text{ Displacement density } D = \frac{Q}{4\pi r^2}$$

$$\text{Displacement } \Psi = \int D \cdot da = Q.$$

Choice (C)

$$19. E_t = -0.5 \epsilon_r$$

$$\Rightarrow \frac{2\eta_2}{\eta_2 + \eta_1} = +\frac{1}{2} \left(\frac{\eta_1 - \eta_2}{\eta_2 + \eta_1} \right)$$

$$\Rightarrow 5\eta_2 = \eta_1$$

$$\Rightarrow 5\sqrt{\frac{4\mu_0}{\epsilon_2}} = \sqrt{\frac{\mu_0}{\epsilon_1}}$$

$$\Rightarrow 10\sqrt{\frac{\epsilon_1}{\epsilon_2}} = 1 \quad \frac{\epsilon_1}{\epsilon_2} = 10^{-2} = 0.01.$$

Choice (D)

$$20. (H_1 - H_2) \times a_{n12} = K$$

$$H_2 = \frac{B_2}{\mu_2} = \frac{1}{2\mu_0} (3\hat{a}_x + 7\hat{a}_z)$$

$$H_1 = \frac{B_1}{\mu_1} = \frac{1}{3\mu_0} (B_1\hat{a}_x + B_2\hat{a}_y + B_3\hat{a}_z)$$

$$\Rightarrow \frac{1}{6\mu_0} \{ (2B_1 - 9)\hat{a}_x + 2B_2\hat{a}_y + (2B_3 - 21)\hat{a}_z \} \times \hat{a}_z$$

$$= \frac{2\hat{a}_y}{\mu_0}$$

$$[\hat{a}_x \times \hat{a}_z = -\hat{a}_y]$$

$$\Rightarrow \frac{1}{6\mu_0} (9 - 2B_1) = \frac{2}{\mu_0}$$

$$B_1 = -1.5, B_2 = 0$$

$$\Rightarrow H_1 = \frac{-1.5}{\mu_1} = \frac{-1.5}{3\mu_0} = \frac{-0.5}{\mu_0} \hat{a}_x$$

Choice (D)

$$21. \delta = \sqrt{\frac{2}{w\mu\epsilon}}$$

$$\eta_s = R_s + jX_s = \sqrt{\frac{w\mu}{2\sigma}} (1 + j)$$

$$= \frac{0.08}{\sqrt{2}} (1 + j)$$

$$\Rightarrow R_s \cdot \delta = \frac{1}{\sigma}$$

$$= \frac{\sqrt{2}}{1.8 \times 10^7 \times 0.08} = 0.98 \mu\text{m}.$$

Choice (C)

$$22. \eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

$$\Rightarrow 30\pi = \frac{120\pi}{\sqrt{\epsilon_r}} \epsilon_r = 16$$

$$\Rightarrow w\sqrt{\mu\epsilon} = \beta \Rightarrow w = \frac{1 \times 3 \times 10^8}{4} = 75 \text{ MHz}$$

$$\Rightarrow \frac{E_x}{H_y} = \eta = \frac{-E_y}{H_x} = \frac{120\pi}{4} = 30\pi$$

$$\Rightarrow E_x = 30\pi \times 0.1 \sin(75 \times 10^6 t - z) \hat{a}_x$$

$$= 3\pi \sin(75 \times 10^6 t - z) \hat{a}_x$$

$$\Rightarrow E_y = -\eta H_x = -0.5 \times 30\pi \cos(75 \times 10^6 t - z) \hat{a}_y$$

$$= -15\pi \cos(75 \times 10^6 t - z) \hat{a}_y$$

$$E = E_x + E_y = 3\pi \sin(75 \times 10^6 t - z) \hat{a}_x - 15\pi \cos(75 \times 10^6 t - z) \hat{a}_y$$

Choice (C)

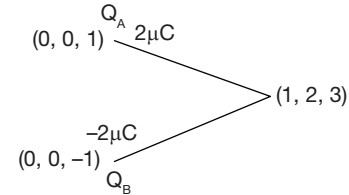
$$23. \frac{\sigma}{\omega\epsilon} = \frac{5 \times 36\pi}{2 \times 10^8 \times 10^{-9}} = 900\pi \gg 1$$

$$\text{it represents a good conductor } \infty = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{2 \times 10^8 \times 4\pi \times 10^{-7} \times 16 \times 5}{2}} = 100.2 \text{ rad/m}.$$

Choice (D)

24.



$$\vec{E} = \frac{2 \times 10^{-6}}{4\pi\epsilon_0} \left[\frac{(1-0)\hat{a}_x + (2-0)\hat{a}_y + (3-1)\hat{a}_z}{\sqrt{1^2 + 2^2 + 4^2}} \right]$$

$$- \frac{2 \times 10^{-6}}{4\pi\epsilon_0} \left[\frac{(1-0)\hat{a}_x + (2-0)\hat{a}_y + (3+1)\hat{a}_z}{\sqrt{1^2 + 2^2 + 4^2}} \right]$$

$$= \frac{2 \times 10^{-6}}{4\pi\epsilon_0} \left[\left(\frac{1}{3} - \frac{1}{\sqrt{21}} \right) \hat{a}_x + \left(\frac{2}{3} - \frac{2}{\sqrt{21}} \right) \hat{a}_y + \left(\frac{2}{3} - \frac{4}{\sqrt{21}} \right) \hat{a}_z \right]$$

$$\vec{E} = \frac{2 \times 10^{-6}}{4\pi\epsilon_0} [0.115\hat{a}_x + 0.25\hat{a}_y - 0.206\hat{a}_z]$$

$$r = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$\phi = \tan^{-1} 2 = 63.43^\circ$$

$$\theta = \cos^{-1} \left(\frac{3}{\sqrt{14}} \right) = 36.7^\circ$$

$$\overline{E_r} = \overline{E} \cdot \hat{a}_r = 2070 \hat{a}_x \cdot \hat{a}_r + 4140 \hat{a}_y + 4140 \hat{a}_y \cdot \hat{a}_r - 3708 \hat{a}_z \cdot \hat{a}_r$$

$$= [2070 \sin \theta \cos \phi + 4140 \sin \theta \sin \phi - 3708 \cos \theta] \hat{a}_r$$

$$= [553.33 + 2212.86 - 2973] \hat{a}_r$$

$$= -206.81 \hat{a}_r$$

Choice (B)

$$25. \quad \overline{D} = \epsilon \overline{E} = \frac{\rho_L}{2\pi d} \hat{a}_{12}$$

z axis are $x = 0$ and $y = 0$

P_2 is $(3, 4, -5)$ and $P_1(0, 0, -5)$

$$\hat{a}_{12} = (3-0)\hat{a}_x + (4-0)\hat{a}_y = 3\hat{a}_x + 4\hat{a}_y \quad d = |\hat{a}_{12}| = 5$$

$$\overline{D} = \frac{30 \times 10^{-6}}{2\pi \times 5} \left[\frac{3\hat{a}_x + 4\hat{a}_y}{5} \right]$$

$$= 0.56 \hat{a}_x + 0.76 \hat{a}_y \frac{\mu\text{C}}{\text{m}^2}.$$

Choice (D)