

(Olympiad Champs Notes)

Factions and Operation on Factions

Real – Life Example

- ❖ Time is a very good example of fractions. Time is divided into various denominations of hours, minutes and seconds. For example, 1 hour has 60 minutes or it can also be said that 1 hour can be divided into 60 equal fractions each being equal to 1 minute.

LEARNING OBJECTIVES

This lesson will help you to:

- ❖ Learn and study about finding fractional part of a collection.
- ❖ Learn to compare one and more fractions.
- ❖ Learn to study about identifying equivalent fractions.
- ❖ Study and learn to estimate the degree of closeness of a fraction to known fractions ($\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$).
- ❖ Understand and learn to use decimal fractions in the context of units of length and money.
- ❖ Understand and study about expressing a given fraction into decimal notation and vice-versa.
- ❖ Understand the fractional part of a collection.
- ❖ Comparison of fractions.
- ❖ Identifying equivalent fractions.

QUICK CONCEPT REVIEW

- ❖ Fractional Part: The fractional part of a number is the part of the number that appears after the decimal point.
- ❖ A fraction is a way of representing division of a 'whole' into parts.
- ❖ A fraction is a way of representing division of a 'whole' into parts. It has the form

$$\frac{\text{Numerator}}{\text{Denominator}}$$

Where the

Numerator = Number of parts chosen

And the

Denominator = Total number of the parts

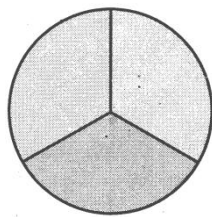
Amazing Fact

- ❖ The number Pi (the ratio of the circumference to the diameter of a circle) can't be expressed as a fraction. When written as a decimal it never repeats and never ends.

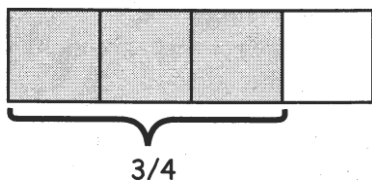
Here is Pi written to 50 decimal

Places: 3.1415926535897932384624338327950288419716939937510

Example: Fraction $\frac{1}{3}$ is shown by the pie chart below. The pie is divided into 3 equal parts. The green part is equal to one third of the pie, thus $\frac{1}{3}$.



Part of a whole



- ✓ the top number (the numerator) says how many parts the whole is divided into.
 - ✓ the bottom number (the denominator) says how many you have.
- Comparing fractions: Fractions are compared to see if one fraction is equal to ($=$), greater than ($>$) or smaller than ($<$) the other fraction.
- While comparing the fractions, if the fraction are like fractions, the fraction with bigger numerator is greater.
 - If the fractions are not like fractions, convert fraction into like fraction using the LCM of the denominator and then compare.

Steps to compare two fractions :

Step 1: Find a common denominator by taking L.C.M of all different denominators.

Step 2: Make equivalent fractions with the new denominator.

Step 3: Compare the numerators.

Example: Compare the fractions $\frac{3}{5}$ and $\frac{7}{9}$.

Step 1: L.C.M of 5 and 9 =45

Step 2: $3/5 = \frac{3}{5} \times \frac{9}{9} = 27/45$; $7/9 = \frac{7 \times 5}{9 \times 5} = 35/45$

Step 3: Since $27 < 35$ so $3/5 < 7/9$.

Note: If the numerator is same, the fraction with greater denominator is smaller.

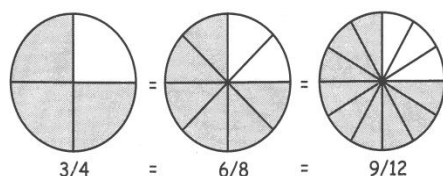
Shortcut Method: To compare the two fractions, simply cross multiply them.

Historical preview

- ❖ In ancient Rome. Fractions were written using words instead of numbers.
- ❖ Fractions were firstly used in the Indus Valley civilization, followed by the Egyptians and the Greeks.
- ❖ By about 500 AD, the Indians had developed a system from a way of writing called brahmi, which had nine symbols and a zero. In India fractions were written very much like we do now, with one number (the numerator) above another (the denominator), but without a line. It was the Arabs who added the line (sometimes drawn horizontally, sometimes on a slant) which we now use to separate the numerator and denominator.

Equivalent Fraction: To find equivalent fractions, multiply the numerator and denominator by the same number (except zero).

Equivalent fractions may look different, but they have the same value.



The fractions three-fourths, six-eighths, and nine-twelfths are equivalent.

Estimating the degree of closeness of a fraction

- ❖ A fraction is closest to $1/2$ when the denominator is about twice the numerator.
Example: $2/5$, $5/26$, $8/15$ etc.
- ❖ A fraction is closest to $1/4$ when the denominator is about four times the numerator.
Example: $3/14$, $4/17$, $7/26$ etc.

Decimal Fractions

Decimal fraction is a fraction with denominator 10, 100, 1000 etc. For example fractions $1/10$, $1/100$, $1/1000$ can be written as 0.1, 0.01, 0.001 respectively. 0.1, 0.01, 0.001 are decimal fractions.

Each decimal fraction has two parts- a whole number and a decimal. The point or dot that separates both the parts is called a decimal point. The number of digits in the decimal part gives the number of decimal places.

Example: 234.5623

In this number 234 is a whole number and 5623 is a decimal number.

This number has 4 decimal places.

❖ Decimal fractions are very useful in the context of units of length and money.

Example 1: A log of wood of length 5 m is given below. It is to be cut into 10 equal parts.



The length of each part represents $\frac{1}{10}$ of the length of the whole wood.

So, the length of each part of a 5m long log of wood is $5\text{m} \times \frac{1}{10} = \frac{5}{10}\text{m}$. $\frac{5}{10}\text{m}$ can also be written as 0.5.

Misconcept/ concept

Misconcept : The common misconception is that decimals and fractions are different types of numbers. Hence there is no equivalent fraction for any decimal.

Concept : Taking an example of a decimal 4.422 which is to be converted to its equivalent fraction. Now this decimal can be expressed as 4 and the fraction $\frac{422}{1000}$ or $\frac{4422}{1000}$. These are fraction equivalent. These can be simplified by dividing both numerator and denominator by 2 to give $4.422 = \frac{2211}{500}$ or $4(\frac{211}{500})$.

Misconnect: The value of $A \div \frac{1}{B}$ is equivalent to $A \div B$ and hence has value A/B . Taking an example, the value of $3 \div \frac{1}{4}$ is equivalent to $3 \div 4$ and hence has $\frac{3}{4}$ or 0.75.

Example 2: Rs. 5 is to be distributed equally among 10 students.

So, Rs. 5 is to be divided into 10 equal parts i.e. each student gets Rs. $\frac{5}{10}$ or Rs. 0.50.

Expressing a decimal as a fraction and vice-versa

Decimal into Fraction

Write the decimal as the numerator without the decimal point and then write the denominator as 1 followed by as many zeroes as there are decimal places in the decimal.

Example: $0.8 = \frac{8}{10}$, $0.25 = \frac{25}{100}$, $0.2147 = \frac{2147}{10000}$ etc.

Fraction into Decimal

The fractions having denominators 10, 100 or 1000 can easily be converted into decimals. Place the decimal point from right to left in the numerator after as many digits as there are zeroes in the denominator.

Example: $\frac{5}{100} = 0.05$, $\frac{4}{1000} = 0.004$, $\frac{3}{10} = 0.3$ etc.

The fractions with denominators other than 10, 100 or 1000 can also be converted to decimals by first converting their denominators to 10, 100 or 1000 and then following the same steps as above.

Example: $\frac{1}{5} = \frac{1}{5} \times \frac{2}{2} = \frac{2}{10} = 0.2$.

More about Fractions:

Equivalent Fractions: Fractions having the same value, even though they may look different.

Example: $\frac{1}{2}$ and $\frac{2}{4}$ both have the same value because they both are "half". Like Fractions: Fractions having the same denominator.

Example: $\frac{1}{2}$ and $\frac{3}{2}$ are like fractions having the same denominator as 2.

While comparing the fractions, if the fractions are like fractions, the fraction with bigger numerator is greater.

If the fractions are not like fractions, convert fractions into like fractions using the LCM of the denominator and then compare.

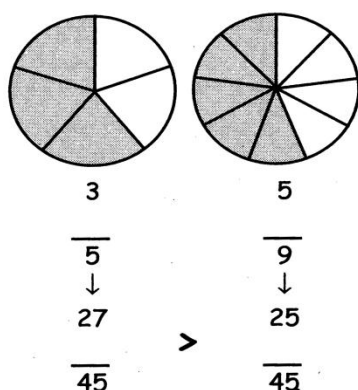
Important points

These conversions must be kept in mind when dealing with decimal fractions.

1 meter = 100 centimeter

1 kilometer = 1000 meter

1 rupee = 100 paise



Example: $\frac{3}{5}$ and $\frac{5}{9}$ are unlike fractions LCM for 5 and 9 is 45

Like fractions are $\frac{27}{45}$ and $\frac{25}{45}$

$\frac{27}{45} > \frac{25}{45}$

I.e. $\frac{3}{5} > \frac{5}{9}$

Facts: For a fraction, a denominator cannot be 0.