Engineering Mathematics Test I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. $\underset{x\to 0}{Lt}\dfrac{5^x-1}{\sqrt{x^2-x+1}-1}=$

(A) $\log_e 5$ (B) $2\log_e 5$

(C) $-2\log_e 5$ (D) 1

2. If $f(x) = x^2 + 5x - 13$, if $x < 1$,

$x - 8$ if $x \geq 1$, then $\underset{x\to 0}{Lt}\, f(x) =$

(A) –8 (B) –7

(C) 7 (D) does not exist

3. $f(x) = \dfrac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}$. for $x \neq 0$ is

$= 0$ for $x = 0$

(A) continuous at $x = 0$.

(B) not continuous at $x = 0$.

(C) continuous everywhere.

(D) none of these.

4. If $f(x) = a[x - 5] + b[x + 5]$ is continuous at $x = 5$, then the value of $a + b$ ($[x]$ is the greatest integer less than or equal to x) is

(A) 0 (B) 1

(C) 5 (D) 2

5. If $f(x) = \begin{cases}\dfrac{\sqrt{x}-3}{x-9}; \text{for } x\neq 9\\ 3k;\ \text{for } x=9\end{cases}$ is continuous everywhere,

then k is equal to ———.

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$

(C) $\dfrac{1}{9}$ (D) $\dfrac{1}{18}$

6. If $y = ax^2 + 3\log x + bx$ has its extreme value at $x = 1$ and $x = -1$, then the value of $2a - b$ is

(A) 3 (B) 0

(C) –1 (D) –3

7. If $xy = 6$, then find the minimum value of $2x + 3y$, $x, y \in R^+$.

(A) 12 (B) 15

(C) 9 (D) 6

8. Which of the following functions have neither maximum nor minimum?

(A) $\dfrac{1}{2x}$ (B) $4x + 7$

(C) e^{5x+1} (D) All the above

9. Find the maximum value of the function $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval $[-1, 2]$.

(A) $\dfrac{39}{16}$ (B) 2

(C) 21 (D) 18

10. Let $f(x)$ and $g(x)$ be two continuous functions in $[a, b]$ and differentiable in (a, b) and $g^1(x) \neq 0$ for any $x \in (a, b)$ then there exists at least one value $c \in (a, b)$, by Cauchy mean value theorem which of the following is true?

(A) $\dfrac{f^2(c)}{g^1(c)}=\dfrac{f(b)-g(a)}{g(b)+f(a)}$

(B) $\dfrac{g^1(c)}{f^1(c)}=\dfrac{f(b)+g(b)}{f(a)+f(b)}$

(C) $\dfrac{f^1(c)}{g^1(c)}=\dfrac{f^1(b)-f^1(a)}{g^1(b)-g^1(a)}$

(D) $\dfrac{f'(c)}{g'(c)}=\dfrac{f(b)-f(a)}{g(b)-g(a)}$

11. If the function $f(x) = (\sin 2x)\, e^{-2x}$ satisfies Rolle's theorem in the interval $\left(0,\dfrac{\pi}{2}\right)$, then the value of $c \in \left(0,\dfrac{\pi}{2}\right)$ such that $f'(c) = 0$ is

(A) $\dfrac{\pi}{8}$ (B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{6}$ (D) $\dfrac{\pi}{3}$

12. By using Lagrange's mean value theorem for $f(x) = x(x + 2)(x - 1)$, find c for $c \in (-1, 0)$.

(A) $\dfrac{-1}{3}$ (B) $\dfrac{-3}{4}$

(C) $\dfrac{-2}{3}$ (D) $\dfrac{-4}{5}$

13. If $u = \tan^{-1}\left(\dfrac{x^3-y^3}{x+y}\right)$, then the value of $x\dfrac{\vartheta u}{\vartheta x}+y\dfrac{\vartheta u}{\vartheta y}$ is

(A) 2 sin$u$ (B) 2 tan$u$

(C) sin2u (D) 2 cos2u

14. If $f(x, y)$ is a homogeneous function of degree n, then the value of

$y\dfrac{\vartheta^2 f}{\vartheta y^2}+x\dfrac{\vartheta^2 f}{\vartheta x\vartheta y}=$

(A) $n.\frac{\vartheta f}{\vartheta x}$ (B) $(n-1)\frac{\vartheta f}{\vartheta y}$

(C) $(n-1)\frac{\vartheta f}{\vartheta x}$ (D) $n.\frac{\vartheta f}{\vartheta y}$

15. If $u = \cos^{-1}\left(\frac{x^3+2y^3-4z^3}{x^7-y^7-z^7}\right)$, then the value of $\sum_{x,y,z} x\frac{\vartheta u}{\vartheta x}$ is

(A) $4\cot u$ (B) $-4\tan u$

(C) $-4\cos u$ (D) $-4\sin u$

16. If $z = \ln(x^2 + y^3)$, then

(A) $3x\frac{\vartheta z}{\vartheta x} = 2y\frac{\vartheta z}{\vartheta y}$

(B) $3x\frac{\vartheta z}{\vartheta x} + 2y\frac{\vartheta z}{\vartheta y} = 6$

(C) $x\frac{\vartheta z}{\vartheta x} - y\frac{\vartheta z}{\vartheta y} = 0$

(D) $2y\frac{\vartheta z}{\vartheta x} + 3x\frac{\vartheta z}{\vartheta y} = 6$

17. The value of the definite integral $\int_0^1 (x^2-2x+3)e^{-xdx}$ is ———.

(A) $3 + 4e$ (B) $3 - 4e$

(C) $3 + \frac{4}{e}$ (D) $3 - \frac{4}{e}$

18. $\int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\operatorname{cosec} x}{\sec x + \operatorname{cosec} x}\, dx =$

(A) $\frac{7\pi}{4}$ (B) $\frac{7}{2}$

(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{2}$

19. $\int_0^{\infty} \frac{x^2}{(1+x^2)^{\frac{5}{2}}} dx =$

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

20. Evaluate $\int_1^4 \frac{dx}{(x-2)(x-3)}$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) diverges

21. $\int_0^{\infty} \frac{dx}{9+x^2} =$

(A) $\frac{\pi}{2}$ (B) —

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

22. The area bounded by the cure $y^2 = x + 2$ and $y = x - 4$ (in sq units) is

(A) $20\frac{5}{6}$ (B) $15\frac{5}{6}$

(C) $12\frac{2}{3}$ (D) $18\frac{3}{4}$

23. The area bounded by the curves $y^2 = 9ax$ and $x^2 = 9ay$ is

(A) $54a^2$

(B) $48a^2$

(C) $36a^2$

(D) $27a^2$

24. The volume of the solid formed by the revolution of the area A about y – axis is

(A) $\iint_A 2\pi dxdy$ (B) $\iint_A 2\pi y dxdy$

(C) $\iint_A \pi x^2 dxdy$ (D) $\iint_A \pi y^2 dxdy$

25. The taylor's series expansion of $2e^x + 3\sin x$ about $x = 0$ is

(A) $2 + 5x + x^2 + \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots$

(B) $2 + 5x + x^2 - \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots\ldots$

(C) $2 + 5x + x^2 + \frac{x^3}{6} - \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots\ldots$

(D) $2 + 5x + x^2 + \frac{x^3}{+} + \frac{x^4}{12} + \frac{x^5}{24} + \ldots\ldots\ldots$

Answer Keys

1. C	**2.** B	**3.** B	**4.** A	**5.** D	**6.** D	**7.** A	**8.** D	**9.** C	**10.** D
11. A	**12.** C	**13.** C	**14.** B	**15.** A	**16.** B	**17.** D	**18.** A	**19.** B	**20.** D
21. C	**22.** A	**23.** D	**24.** A	**25.** B					

Hints and Explanations

1. $\underset{x\to 0}{Lt}\frac{5^x-1}{\sqrt{x^2-x+1}-1}\times\frac{\sqrt{x^2-x+1}+1}{\sqrt{x^2-x+1}+1}$

$= \underset{x\to 0}{Lt}\frac{(5^x-1)\left(\sqrt{x^2-x+1}+1\right)}{x^2-x+1-1}$

$= \underset{x\to 0}{Lt}\frac{(5^x-1)\sqrt{x^2-x+1}+1}{x(x-1)}$

$= \underset{x\to 0}{Lt}\frac{5^x-1}{x}\underset{x\to 0}{Lt}\frac{\sqrt{x^2-x+1}+1}{x-1}.$

$= \log_e 5\times\frac{2}{-1}=2\log_e 5$ Choice (C)

2. we know that $\underset{x\to 1}{Lt} f(x) = \underset{x\to 1^-}{Lt} f(x) = \underset{x\to 1^+}{Lt} f(x)$

$\underset{x\to 1^-}{Lt} f(x) = \underset{x\to 1^-}{Lt} (x^2 + 5x - 13) = -7$

$\underset{x\to 1^+}{Lt} f(x) = \underset{x\to 1^+}{Lt} (x - 8) = -7$

$\therefore \underset{x\to 1^-}{Lt} f(x) = \underset{x\to 1^+}{Lt} f(x) = -7$

$\Rightarrow \underset{x\to 1}{Lt} f(x) = -7.$ Choice (B)

3. $\underset{x\to 0^-}{Lt}\frac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}=\frac{0}{1-0}=0$ ----------- (1)

$\underset{x\to 0^+}{Lt}\frac{e^{\frac{1}{x}}}{1-e^{\frac{1}{x}}}=\underset{x\to 0^+}{Lt}\frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}\left(e^{\frac{-1}{x}}-1\right)}$

$=\underset{x\to 0^+}{Lt}\frac{1}{e^{\frac{-1}{x}}-1}=\frac{1}{0-1}=-1$ ------------- (2)

From (1) and (2) $\underset{x\to 0^-}{Lt} f(x) \neq \underset{x\to 0^+}{Lt} f(x)$

$\therefore$ $f(x)$ is not continuous at $x = 0$. Choice (B)

4. $f(x) = a[x - 5] + b\,[x + 5]$

$\therefore$ $f(x) = a\,([x] - 5) + b([x] + 5)$

$\underset{x\to 5^-}{Lt} f(x) = \underset{x\to 5^{-1}}{Lt} \{a([x] - 5) + b\,([x] + 5)$

$= -a + 9b$ ------ (1)

$\underset{x\to 5^+}{Lt} f(x) = \underset{x\to 5^+}{Lt} \{a([x] - 5) + b\,([x] + 5)\}$

$= 10\,b$ ---- (2)

As $f(x)$ is continuous at $x = 5$

$\Rightarrow \underset{x\to 5^-}{Lt} f(x) = \underset{x\to 5^+}{Lt} f(x)$

$\Rightarrow -a + 9b = 10b$

$\Rightarrow a + b = 0.$ Choice (A)

5. Given $f(x) = \begin{cases}\frac{\sqrt{x}-3}{x-9}, & \text{for } x\neq 9\\ 3k, & \text{for } x=9\end{cases}$

As $f(x)$ is continuous everywhere

$f(x)$ is continuous at $x = 9$

$\therefore \underset{x\to 9}{Lt} f(x) = f(9)$

$\Rightarrow \underset{x\to 9}{Lt}\left(\frac{\sqrt{x}-3}{x-9}\right)=3k$

$\Rightarrow \underset{x\to 9}{Lt}\left(\frac{\frac{1}{2\sqrt{x}}}{1}\right)=3k$ (By L'Hospital's Rule)

$\Rightarrow \frac{1}{2\sqrt{9}}=3k$

$\Rightarrow \frac{1}{6}=3k$

$\Rightarrow k=\frac{1}{18}.$ Choice (D)

6. Let $f(x) = ax^2 + 3\log x + bx$

$f^1(x) = 2ax + \frac{3}{x} + b$

Given at $x = -1$ and 1 $f(x)$ has extreme values

$\Rightarrow f^1(-1) = 0$ and $f^1(1) = 0$

$f^1(-1) = 2a\,(-1) + \frac{3}{-1} + b = 0$

$\Rightarrow -2a + b = 3$

$\Rightarrow 2a - b = -3.$ Choice (D)

7. Given $xy = 6 \Rightarrow y = \frac{6}{x}$

$2x + 3y = 2x + \frac{3\times 6}{x}$

Let $f(x) = 2x + \frac{18}{x}$

$f^1(x) = 2 - \frac{18}{x^2}$

$\Rightarrow f^{11}(x) = \frac{36}{x^3}$ which is always positive as x, y and R^+

For maximum or minimum $f^1(x) = 0$

$= 2 - \frac{18}{x^2} = 0$

$\Rightarrow x = 3$

$\therefore y = \frac{6}{x}$

$\Rightarrow y = 2$

$\therefore$ At $x = 3$ and $y = 2$, $2x + 3y$ has minimum value

The minimum value is $2(3) + 3(2) = 12.$ Choice (A)

8. Choice A: Let $f(x) = \frac{1}{2x} \Rightarrow f^1(x) \neq 0$ for any real value of x

Choice B: $f_1(x) = 4x + 7 \Rightarrow f_1^1(x) \neq 0$ for any real value of x

Choice C: $f_2(x) = e^{5x+1} \Rightarrow f_2^1(x) \neq 0$ for any real value of x

$\therefore$ None of the options have either maximum or minimum values. Choice (D)

9. Let $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$

$f^1(x) = 12x^3 - 6x^2 - 12x + 6$

$f^1(x) = 0 \Rightarrow 2x^3 - x^2 - 2x + 1 = 0$

$(x - 1)(x + 1)(2x - 1) = 0$

$\Rightarrow x = \pm 1, \frac{1}{2}$

Max of $f(x)$ in $[-1, 2]$ = Max $\{f(-1), f\left(\frac{1}{2}\right), f(1), f(2)\}$

$= \text{Max}\left\{-6, \frac{39}{16} 2,\ 21\right\} = 21.$ Choice (C)

10. Standard result of Cauchy mean value theorem. Choice (D)

11. Given $f(x) = e^{-2x} \sin 2x$

Clearly $f(x)$ is continuous on $\left[0, \frac{\pi}{2}\right]$ and differentiable in $\left(0, \frac{\pi}{2}\right)$ and $f(0) = f\left(\frac{\pi}{2}\right)$ then there exists $c \in \left(0, \frac{\pi}{2}\right)$ such that $f^1(c) = 0$

$f^1(x) = \sin 2x\,(-2e^{-2x}) + 2\cos 2x e^{-2x}$

$f^1(c) = 2e^{-2c}(\cos 2c - \sin 2c) = 0$

$\Rightarrow \sin 2c = \cos 2c$ or $\tan 2c = 1$

$\Rightarrow c = \frac{\pi}{8} \in \left(0, \frac{\pi}{2}\right)$

The required value of $c = \frac{\pi}{8}$. Choice (A)

12. Given $f(x) = x(x + 2)(x - 1)$

Clearly $f(x)$ is continuous on $[-1, 0]$ and differentiable on $(-1, 0)$

$a = -1, b = 0;$

$f(b) = 0, f(a) = (-1)(1)(-2) = 2$

$f^1(x) = 3x^2 + 2x - 2$ and $f^1(c) = 3c^2 + 2c - 2$

By Lagrange's mean value theorem

$$f^1(c) = \frac{f(b) - f(b)}{b - a}$$

$\Rightarrow 3c^2 + 2c - 2 = \frac{0 - 2}{1}$

$\Rightarrow 3c^2 + 2c = 0$

$c(3c + 2) = 0 \Rightarrow c = 0$ or $c = \frac{-2}{3}$ As $c \neq 0$,

$c = \frac{-2}{3} \in (-1, 0).$ Choice (C)

13. $U = \tan^{-1}\left(\frac{x^3 - y^3}{x + y}\right) \Rightarrow \tan u = \frac{x^3 - y^3}{x + y}$

Let $x = kx$ and $y = ky$ then

$\tan u = \frac{k^3(x^3 - y^3)}{k(x + y)} = k^2\left(\frac{x^3 - y^3}{x + y}\right) = f(x)$ say

$\therefore$ f is a homogeneous function of degree 2

$\therefore$ By Euler's theorem $x \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = = n(f)$ (n is degree of f)

$= x.\frac{\partial(\tan u)}{\partial x} + y\frac{\partial(\tan u)}{\partial y} = 2 \tan u$

$= x \sec^2 u \,.\, \frac{\partial u}{\partial x} + y\ \sec^2 u \frac{\partial u}{\partial y}\ \frac{\partial u}{\partial y} = 2 \tan u$

$\therefore\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$ Choice (C)

14. Given $f(x, y)$ is a homogeneous function of degree n then by Euler's theorem we know that x $\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ $= nf$ ------- (1)

Differentiating (1) w. r. to y partially we have

$$x.\ \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} + y.\frac{\partial^2 f}{\partial y^2} = n.\frac{\partial f}{\partial y}$$

$\Rightarrow y\frac{\partial^2 f}{\partial y^2} + x\frac{\partial^2 f}{\partial x \partial y} = (n-1)\frac{\partial f}{\partial y}$ Choice (B)

15. $U = \cos^{-1}\left(\frac{x^3 + 2y^3 - 4z^3}{x^7 - y^7 - z^7}\right)$

$\Rightarrow \cos u = \frac{x^3 + 2y^3 - 4z^3}{x^7 - y^7 - z^7} = f(\text{say})$

Clearly f is a homogeneous function of degree – 4

$\therefore$ By Euler's theorem $x.\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = n.f$

$\therefore\ x\frac{\partial(\cos u)}{\partial x} + y\frac{\partial(\cos u)}{\partial y} + z\frac{\partial(\cos u)}{\partial z} s = -4.\cos u$

$= -\left(x.\frac{\partial u}{\partial x}.(\sin u) + y.\frac{\partial u}{\partial y}(\sin u) + z\frac{\partial u}{\partial z}(\sin u)\right)$

$= -4 \cos u$

$\therefore\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 4 \cot u.$ Choice (A)

16. Given $z = \text{in}\,(x^2 + y^3)$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{2x}{x^2+y^3} \text{ and } \frac{\partial z}{\partial y} = \frac{3y^2}{x^2+y^3}$$

Consider $3x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y} = \frac{6x^2}{x^2+y^3} + \frac{6y^3}{x^2+y^3}$

$$= \frac{6(x^2+y^3)}{x^2+y^3} = 6.$$

Choice (B)

17. We have $\int_0^1 (x^2 - 2x + 3)e^{-x}dx$

$= (x^2 - 2x + 3)(-e^{-x}) - (2x - 2)(e^{-x}) + 2(-e^{-x})_0^1$

$= (-2e^{-1} - 0 - 2e^{-1}) - (-3 + 2 - 2)$

$= 3 - 4e^{-1} = 3 - \frac{4}{e}$

Choice (D)

18. We know that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Let $I = \int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\text{cosec}\, x}{\sec x + \text{c osec}\, x}$

$$= \int_0^{\frac{\pi}{2}} \frac{3\sec\left(\frac{\pi}{2} - x\right) + 4\,\text{c osec}\left(\frac{\pi}{2} - x\right)}{\sec\left(\frac{\pi}{2} - x\right) + \text{cosec}\left(\frac{\pi}{2} - x\right)}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{3\text{cosec}\, x + 4\sec x}{\text{cosec}\, x + \sec x}dx$$

$I + I$

$$= \int_0^{\frac{\pi}{2}} \frac{3\sec x + 4\,\text{cosec}\, x}{\text{cosec}\, x + \text{cosec}\, x}dx + \int_0^{\frac{\pi}{2}} \frac{3\text{cosec}\, x + 4\sec x}{\sec x + \text{cosec}\, x}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{3(\text{cosec}x + \sec x) + 4(\sec x + \text{cosec}x)}{\sec x + \text{cosec}x}dx$$

$$= \int_0^{\frac{\pi}{2}} 7dx = 7(x)_0^{\frac{\pi}{2}} = 7\frac{\pi}{2}$$

$\therefore \quad 2I = 7\frac{\pi}{2}$

$\Rightarrow \quad I = 7\frac{\pi}{4}$

Choice (A)

19. $\int_0^\infty \frac{x^2}{(1+x^2)^{\frac{5}{2}}}dx$ Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$ $L.L$

When $x = 0$ $\theta = 0$ and $U.L$ when $x = \infty$, $\theta = \frac{\pi}{2}$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\tan^2\theta.\sec^2\theta}{(1+\tan^2\theta)^{\frac{5}{2}}}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^2\theta.\sec^2\theta}{\cos^2 0.\sec^5\theta}d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2\theta.\cos\theta d\theta = \left.\frac{\sin^3\theta}{3}\right]_0^{\frac{\pi}{2}} = \frac{1}{3}$$

Choice (B)

20. $\int_1^4 \frac{dx}{(x-2)(x-3)}$

$$= \int_1^2 \frac{dx}{(x-2)(x-3)} + \int_2^3 \frac{dx}{(x-2)(x-3)} + \int_3^4 \frac{dx}{(x-2)(x-3)}$$

$I = I_1 + I_2 + I_3$ (say)

Clearly as I has discontinuity at $x = 2$, I_1 diverges

As I_1 diverges I also diverges

$\therefore \int_1^4 \frac{dx}{(x-2)(x-3)}$ diverges.

Choice (D)

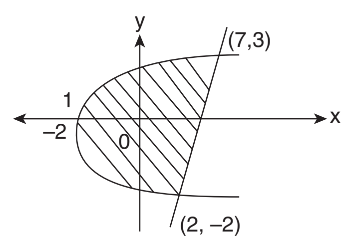
21. $\int_0^\infty \frac{dx}{9+x^2} = \underset{a\to\infty}{Lt} \int_0^a \frac{dx}{3^2+x^2} = \underset{a\to\infty}{Lt} \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^a$

$\underset{a\to\infty}{Lt} \frac{1}{3}\tan^{-1}\left(\frac{a}{3}\right) = \frac{\pi}{6}$

The given improper integral converges to $\frac{\pi}{6}$

Choice (C)

22.



$y^2 = x + 2$ and $y = x - 4$

The above two intersect at (2, –2) and (7, 3).

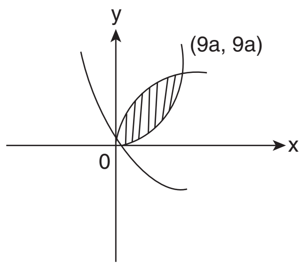
The area bounded $= \int_{-2}^{+3} \left[(y+4) - (y^2 - 2)\right]dy$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{3} + 2y\right]_{-2}^3$$

$= \frac{125}{6} = 20\frac{5}{6}$ sq units.

Choice (A)

23.



Given $y^2 = 9ax$ and $x^2 = 9ay$

The above two curves intersect at (0, 0) and $(9a, 9a)$.

The required area $= \int_0^{9a}\left(\sqrt{9ax} - \frac{x^2}{9a}\right)dx$

$$= 3\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{9a} - \frac{1}{9a}\left(\frac{x^3}{3}\right)_0^{9a}$$

$$= 2\sqrt{a}\left((9a)^{\frac{3}{2}}\right) - \frac{1}{27a}(9a)^3$$

$= 54a^2 - 27a^2 = 27a^2$. Choice (D)

24. Standard Result. Choice (A)

25. We have $2e^x + 3\sin x$

$$= 2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}.....\right) + 3\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}.....\right)$$

$$= \left(2 + 2x + 2\frac{x^2}{2!} + 2.\frac{x^3}{3!} + 2\frac{x^4}{4!} + 2\frac{x^5}{5!} + ...\right)$$

$$+ \left(3x - 3\frac{x^3}{3!} + 3.\frac{x^5}{5!} - 3\frac{x^7}{7!} + ...\right)$$

$= 2 + 5x + x^2 - \frac{x^3}{3!} + \frac{x^4}{12} + \frac{x^5}{24} + ...$ Choice (B)