Sample Paper-05 SUMMATIVE ASSESSMENT –II MATHEMATICS Class – X

Time allowed: 3 hours

Maximum Marks: 90

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 31 questions divided into four sections A, B, C and D.
- c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
- d) Use of calculator is not permitted.

Section A

- 1. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is not red?
- 2. Find the value of a, so that the point (3, a) lie on the line 2x 3y = 5.
- 3. Find the 25th term of the AP: -5, $\frac{-5}{2}$, 0, $\frac{5}{2}$
- 4. In given figure, the quadrilateral ABCD circumscribes a circle with centre 0. If $\angle AOB = 115^{\circ}$, then find $\angle COD$.



Section **B**

5. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre 0 (see figure). If $\angle AOB = 30^{\circ}$, find the area of the shaded region.



- 6. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys with the shape of a right circular cone mounted on a hemisphere of radius 3 cm. If the height of the toy is 12 cm, find the number of toys so formed.
- 7. Water flows through a circular pipe, whose internal diameter is 2 cm, at the rate of 0.7 m per second into a cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water in the cylindrical tank use in half an hour?
- 8. The graph of the polynomial $y = ax^2 + bx + c$ is shown in the figure. Write one value of $b^2 4ac$.



9. In an AP, the sum of first *n* terms is $\frac{3n^2}{2} + \frac{13}{2}n$. find its 2nd term.

10. If $d_1, d_2(d_2 > d_1)$ be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, then prove that $d_2^2 = c^2 + d_1^2$.

Section C

- 11. Prove that in a right-angled triangle, the mid-point of the hypotenuse is equidistant from the vertices.
- 12. The line joining the points (2,-1) and (5,-6) is bisected at P. If P lies on the line 2x + 4y + k = 0, find the value of k.
- 13. In the given figure, sectors of two concentric circles of radii 7 cm and 3.5 cm are shown. Find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



- 14. The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 6 a.m. and 6.05 a.m. $\left(\text{Use } \pi = \frac{22}{7}\right)$
- 15. Find the number of coins 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.
- 16. Solve the quadratic equation by using quadratic formula: $\sqrt{2}x^2 \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$
- 17. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
- 18. In the figure, if AB = AC, then prove that BE = EC.



- 19. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 m high building finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite side of the bird. Find the distance of the bird from the girl.
- 20. A die is thrown once. Find the probability of getting:
 - (i) a prime number
 - (ii) a number lying between 2 and 6
 - (iii) an odd number

Section D

21. Draw any quadrilateral ABCD. Construct another quadrilateral AB'C'D' similar to the quadrilateral ABCD with each side equal to $\frac{4}{5}$ th of the corresponding side of quadrilateral ABCD. Write the steps of exection also

ABCD. Write the steps of construction also.

22. A vertical tower stands on a horizontal plane and surmounted by vertical flagstaff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flagstaff are α

and β respectively, Prove that the height of the tower is $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$.

- 23. Cards marked with numbers 1,2,3,...25 are placed in a box and mixed thoroughly and one card is drawn at random from the box, what is the probability that the number on the card is
 - (i) a prime number?
 - (ii) a multiple of 3 or 5?
 - (iii) an odd number?
 - (iv) neither divisible by 5 nor by 10?
 - (v) perfect square?
 - (vi) a two-digit number?
- 24. If A (4, 2), B (7, 6) and C (1, 4) are the vertices of a $\triangle ABC$ and AD is in median, prove that the median AD divides AABC into two triangles of equal areas.
- 25. A cone of radius 10cm divided into two parts by drawing a plane through the mid-point of its axis, parallel to its base. Compare the volume of the two parts.
- 26. A farmer connects a pipe of internal diameter 20cm from a canal into a cylindrical tank in his field which is 10m in diameter and 2m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?
- 27. Solve for $x: \frac{1}{x+4} \frac{1}{x-7} = \frac{11}{30}; \quad x \neq -4, 7$
- 28. Two years ago, a man's age was three times the square of his son's age. Three years hence, his age will be four times his son's age. Find their present ages.
- 29. A person donates money to a trust working for education of children and women in some villages. If the person donates Rs.5,000 in the first year and his donation increases by Rs.250 every year, find the amount donated by him in the eighth year and the total amount donated in eight years.
 - (a) Which mathematical concept is being used here?
 - (b) Write any two values the person mentioned here possess.
 - (c) Why do you think education of women is necessary for the development of a society?
- 30. The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.
- 31. With the vertices of a triangle ABC as centre, three circles are described, each touching the other two externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm, then find the radii of the circle.

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(Solutions)

SECTION-A

 There are 3 + 5 = 8 balls in a bag. Out of these 8 balls, one can be chosen in 8 ways.
 ∴ Total number of elementary events = 8 Since the bag contains 5 black balls along with 3 red balls, therefore one black (not red) ball can be drawn in 5 ways.
 ∴ Favourable number of elementary events = 5

Hence P (getting a black ball) = $\frac{5}{8}$

2. Since (3, a) lies on the line 2x - 3y = 5 Then 2(3) - 3(a) = 5 Then 2(3) - 3(a) = 5 -3a = 5-6

$$-3a = -1$$
$$a = \frac{1}{3}$$

3. Here, $a = -5, d = -\frac{5}{2} - (-5) = \frac{5}{2}$

We know, $a_{25} = a + (25 - 1)d$

$$=(-5)+24\left(\frac{5}{2}\right)=-5+60=55$$

- 4. $\angle AOB = \angle COD$ (vertically opposite angle) Therefore, $\angle COD = 115^{\circ}$
- 5. Area of shaded region = Area of sector OAB Area of sector OCD

$$= \frac{30^{\circ}}{360^{\circ}} \times \frac{22}{7} \times 21 \times 21 - \frac{30^{\circ}}{360^{\circ}} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{1}{12} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{12} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{231}{2} - \frac{77}{6} = \frac{692 - 77}{6} = \frac{616}{6} = \frac{308}{3} \ cm^2$$

6. Volume of solid cylinder = $\pi r^2 h$

$$=\pi \left(\frac{12}{2}\right)^2 (15) = 540\pi \,\mathrm{cm}^3$$

Volume of one toy = Volume of conical portion + Volume of hemispherical portion

$$=\frac{1}{3}\pi r^{2}h+\frac{2}{3}\pi r_{1}^{3}$$

$$= \frac{1}{3}\pi(3)^{2} + (12-3) + \frac{2}{3}\pi(3)^{3}$$

$$27\pi + 18\pi = 45\pi$$

Number of toys = $\frac{\text{Volume of Cylinder}}{\text{Volume of one toy}} = \frac{540\pi}{45\pi} = 12$

7. Water flown out through the pipe in half an hour

$$= \pi \left(\frac{2}{2}\right)^2 (0.7 \times 100) (60 \times 30) \,\mathrm{cm}^3$$

Let the water level rise by x cm. Then,

$$\pi \left(\frac{2}{2}\right)^2 (0.7 \times 100)(60 \times 30) = \pi (40)^2 x$$

x = 78.75 cm

8. Since the graph intersects the x-axis at one point only, so the polynomial $ax^2 + bx + c$ has only one real zero. Consequently, the roots of the quadratic equation $ax^2 + bx + c = 0$ are real and equal.

$$\therefore b^{2} - 4ac = 0$$
9. $S_{n} = \frac{3n^{2}}{2} + \frac{13}{2}n$
Put $n = 1, 2, 3...$
 $S_{1} = \frac{16}{2} = 8$
 $S_{2} = 19$
 $a_{1} = s_{1} = 8$
 $a_{2} = S_{2} - S_{1} = 19 - 8 = 11$
10. $\left(\frac{d_{2}}{2}\right)^{2} = \left(\frac{d_{1}}{2}\right)^{2} + \left(\frac{c}{2}\right)^{2}$
 $\Rightarrow \quad d_{2}^{2} = c^{2} + d_{1}^{2}$

 \Rightarrow

11. Let A(2a,0), B(0,2b) and O(0,0) are the vertices of right-angled triangle Coordinate of

$$C\left(\frac{2a+0}{2},\frac{0+2b}{2}\right)$$

i.e. (a, b)



$$OC = \sqrt{a^2 + b^2}$$
$$AC = \sqrt{a^2 + b^2}$$
$$BC = \sqrt{a^2 + b^2}$$
Hence, C is Equidistant from the vertices

12.

A P (x,y) B (2,-1) (1:1) (5,-6) Coordinate of $P = \left(\frac{2+5}{2}, \frac{-1-6}{2}\right) = \left(\frac{7}{2}, \frac{-7}{2}\right)$

P lies on equation 2x + 4y + k = 0

$$\therefore \qquad 2\left(\frac{7}{2}\right) + 4\left(\frac{-7}{2}\right) + k = 0$$
$$\Rightarrow \qquad 7 - 14 + k = 0$$
$$\Rightarrow \qquad k = 7$$

13. Area of shaded region = $\pi(7)^2 \times \frac{30^\circ}{360^\circ} - \pi\left(\frac{7}{2}\right)^2 \times \frac{30^\circ}{360^\circ}$

$$=\frac{147}{48}\pi = \frac{147}{48} \times \frac{22}{7} = 9.625 \text{ cm}^2$$

14. Angle described by minute hand in 1 minute = $\frac{360^{\circ}}{60^{\circ}} = 6^{\circ}$

:. Angel described by the minute hand in 5 minutes = $6^{\circ} \times 5 = 30^{\circ}$

$$\therefore \quad \text{Required area} = \frac{22}{7} \times (\sqrt{21})^2 \times \frac{30^\circ}{360^\circ}$$
$$= \frac{22}{7} \times 21 \times \frac{1}{12}$$
$$= 5.5 \text{ cm}^2$$
15. Number of coins
$$= \frac{\text{Volume of cylinder}}{\text{Volume of one coin}}$$
$$= \frac{\pi \left(\frac{4.5}{2}\right)^2 (10)}{\pi \left(\frac{1.5}{2}\right)^2 (0.2)}$$
$$= \frac{5.0625 \times 10}{0.5625 \times 0.2} = 450$$
16. $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

16.
$$\sqrt{2x^2} - \frac{1}{\sqrt{2}x} + \frac{1}{\sqrt{2}x} = 0$$

 $\Rightarrow 2x^2 - 3x + 1 = 0$
Here, $a = 2, b = -3, c = 1$

$$\therefore \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore \qquad x = \frac{-(-3) \pm \sqrt{(-3)^3 - 4.(2).(1)}}{2(2)} = \frac{3 \pm \sqrt{9 - 8}}{4}$$

$$\Rightarrow \qquad x = \frac{3 \pm 1}{4} \qquad \Rightarrow \qquad x = \frac{3 + 1}{4}, \frac{3 - 1}{4}$$

$$\Rightarrow \qquad x = 1, \frac{1}{2}$$

17. Let the first term be a and common difference be d. Now, we have

 $a_{11} = 38 \implies a + (11-1)d = 38$ \Rightarrow a + 10d = 38..... (i) $a_{16} = 73$ $\Rightarrow a + (16 - 1)d = 73$ And \Rightarrow a + 15d = 73..... (ii) Now subtracting (ii) from (i), we have a + 10d = 38 $-a \pm 15d = -73$ Now, -5d = -35or 5d = 35 $\therefore d = \frac{35}{5} = 7$ Putting the value of d in equation (i), we have $a+10\times7=38 \Longrightarrow a+70=38$ ⇒a = 38-70 \Rightarrow a= -32 We have, a = -32 and d = 7Therefore, $a_{31} = a + (31 - 1)d$ $\Rightarrow a_{31} = a + 30d = (-32) + 30 \times 7 = -32 + 210$ $\Rightarrow a_{31} = 178$

18. Since Tangent segments from an external point to a circle are equal in length.

19. Positions of bird at A, boy at P and girl at B are as shown in figure. In $\triangle ABP$,

A (bird)
P (Boy)
AB

$$100 = \sin 30^{\circ} = \frac{1}{2}$$

 $\Rightarrow AD = \frac{100}{2} = 50m$
Also BC = DQ = 20m
 $\therefore AC = AB - BC = 50 - 20 = 30m$
In $\triangle ACQ, \frac{AC}{AQ} = \sin 45^{\circ}$
 $\Rightarrow \frac{30}{AQ} = \frac{1}{\sqrt{2}} \Rightarrow AQ = 30\sqrt{2}m$

Hence, the bird is $30\sqrt{2}$ m away from the girl.

- 20. We have, the total number of possible outcomes associated with the random experiment of throwing a die is 6 (*i.e.*, 1, 2, 3, 4, 5, 6).
 - (i) Let *E* denotes the event of getting a prime number. So, favourable number of outcomes = 3 (*i.e.*, 2, 3, 5)

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

(ii) Let *E* be the event of getting a number lying between 2 and 6.

: Favourable number of elementary events (outcomes) = 3 (*i.e.*, 3,4, 5)

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

(iii) Let *E* be the event of getting an odd number.

:. Favourable number of elementary events = 3 (*i.e.*, 1, 3, 5)

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

21. Steps of construction:

- (a) Draw a quadrilateral ABCD.
- (b) Draw any ray AX making an acute angle with AB.
- (c) Locate 5 points X_1 , X_2 , X_3 , X_4 , X_5 on AX so that $AX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5$.
- (d) Join X₅B and draw a line B'X₄ parallel to X₅B.
- (e) Draw a line B'C' parallel to BC and C'D' parallel to CD.

Then AB'C'D' is the required quadrilateral.







In right-angled triangle OAB and OAC,

$$\frac{x}{H} = \cot \alpha$$

$$\Rightarrow x = H \cot \alpha$$

And $\frac{x}{H+h} = \cot \beta \Rightarrow x = (H+h) \cot \beta$
Equating value of x, we get
 $H \cot \alpha = (H+h) \cot \beta$

$$\Rightarrow H (\cot \alpha - \cot \beta) = h \cot \beta$$

$$\Rightarrow H = \frac{h \cot \beta}{\cot \alpha - \cot \beta}$$

- 23. Total no. of possible outcomes = 25
 (i) favourable cases are 2,3,5,7,11,13,17,19,23 which are 9 in number
 - :. Required probability = $\frac{9}{25}$
 - (ii) Multiple of 3 or 5

Favourable cases are 3,5,6,9,10,12,15,18,20,21,24,25, which are 12 in number

:. Required probability =
$$\frac{12}{25}$$

(iii) Favourable cases are 1,3,5,7,9,11,13,15,17,19,21,23,25, which are 13 in number

 \therefore Required probability = $\frac{13}{25}$

(iv) Favourable cases are 1,2,3,4,6,7,8,9,11,12,13,14,16,17,18,19,21,22,23,24, which are 20 in number

Required probability = $\frac{20}{25} = \frac{4}{5}$

(v) Perfect square numbers are1,4,9,16,25

Favourable cases are = 5

Required probability = $\frac{5}{25} = \frac{1}{5}$

- (vi) Two-digit numbers are10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25 =16 Required probability = $\frac{16}{25}$
- 24. Given: AD is the median on BC.

 \Rightarrow BD = DC

The coordinates of midpoint D are given by

$$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$$
 i.e., $\left(\frac{1+7}{2}, \frac{4+6}{2}\right)$

Coordinates of D are (4, 5).

Now, Area of triangle ABD = $\frac{1}{2} \Big[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) \Big]$ = $\frac{1}{2} \Big[4(6-5) + 7(5-2) + 4(2-6) \Big] = \frac{1}{2} [4+21-16]$ = $\frac{9}{2}$ sq. units Area of $\Delta ACD = \frac{1}{2} \Big[4(4-5) + 1(5-2) + 4(2-4) \Big]$

$$=\frac{1}{2}[-4+3-8] = -\frac{9}{2} = \frac{9}{2}$$
 sq. units

Hence, AD divides AABC into two equal areas.

25. Let OAB be the cone and OQ be its axis and P be the mid-point of OQ $\,$



Let OQ = h cm

Then
$$OP = PQ = \frac{h}{2}cm$$

And QB = 10cm Also $\triangle OPD \Box \triangle OQB$

$$\therefore \frac{OP}{OQ} = \frac{PD}{QB} = \frac{h/2}{h} = \frac{PD}{10cm}$$

 $\Rightarrow PD = 5cm$

(i) A smaller cone of radius = 5cm and height = h/2cm

(ii) Frustum of a cone in which

 $R = 10cm, r = 5cm, \text{ height} = \left(\frac{h}{2}\right)cm$ Volume of smaller cone = $\frac{1}{3}\pi 5 \times 5 \times \frac{h}{2} = \frac{25\pi h}{6} cm^3$ Volume of frustum of the cone = $\frac{1}{3}\pi \frac{h}{2} \left[(10)^2 + (5)^2 + 10 \times 5 \right] cm^3$ $=\left(\frac{175\pi h}{6}\right)cm^3$ Ratio of required volume = $\frac{25\pi h}{6}$: $\frac{175\pi h}{6}$ = 25:175 = 1:726. Rate of water flowing = $\frac{3000}{60 \times 60} m / \sec = \frac{5}{6} m / \sec$ In 1 second the water flows = $\frac{5}{6}m$ Internal diameter = $20cm = \frac{1}{5}m$ Volume of the water that flows through the pipe in one second = $\pi r^2 h$ $=\frac{22}{7}\times\left(\frac{1}{10}\right)^{2}\times\frac{5}{6}=\frac{110}{4200}=\frac{11}{420}m^{3}$ Volume of water in the tank = $\pi r^2 h \left| r = \frac{10}{2} = 5m, h = 2m \right|$ $=\frac{22}{7}\times\left(\frac{10}{2}\right)^2\times 2$ $=\frac{1100}{7}m^{3}$ 100 \therefore Time taken to fill the tank = $\frac{7}{11}$ 420 $=\frac{1100\times420}{7\times11}=100\times60$ seconds = 100 minutes = 1hour 40 minutes 27. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ $\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$ $\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$ $\Rightarrow -(x+4)(x-7) = 30$ $\Rightarrow -(x^2-7x+4x-28)=30$

$$\Rightarrow -(x^2 - 3x - 28) = 30$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

Here, $a = 1, b = -3, c = 2$
So, $b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1$

$$\Rightarrow 1 > 0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{1}}{2(1)}$$

$$\Rightarrow x = \frac{3 \pm 1}{2}$$

$$\Rightarrow x = 2, 1$$

28. Let the present age of the son be *x* years.

Three years hence, age of the son = (x+3) years.

:. Three years hence, man's age = 4(x+3) years

 \therefore Present age of man = $\lceil 4(x+3)-3 \rceil$ years = (4x+9) years

Two years ago, man's age = (4x+9-2) years = (4x+7) years

And son's age = (x-2) years

According to the question,

 $4x+7=3(x-2)^{2} \qquad \Rightarrow \qquad 4x+7=3x^{2}-12x+12$ $\Rightarrow \qquad 3x^{2}-16x+5=0 \qquad \Rightarrow \qquad 3x^{2}-15x-x+5=0$ $\Rightarrow \qquad 3x(x-5)-1(x-5)=0 \qquad \Rightarrow \qquad (x-5)(3x-1)=0$ $\Rightarrow \qquad x=5,\frac{1}{3}$ $\therefore \qquad x=5 \text{ and } 4x+9=29$

Hence, the present ages of the man and his son are 29 years and 5 years respectively. 29. The amount donated by the person each year forms an AP.

Here, a = 5,000, d = 250 We have to find a_8 and S_8

 $a_8 = a + Id = 5,000 + 7 \times 250 = Rs.6,750$

$$S_8 = \frac{8}{2} [2a + 7d]$$

 $=4(2 \times 5,000 + 7 \times 250) = 4 \times 11,750 = \text{Rs.47,000}$

- (a) Arithmetic progression.
- (b) Socially aware and responsible citizen.
- (c) Educating a woman means educating the whole family and; an educated society always develops.
- 30. : Tangent segments from an external point to a circle are equal in length.

$$\int_{a}^{b} \int_{a}^{b} \int_{a$$

31. Let AB = 9 cm, BC = 7 cm and CA = 6 cm

Then,
$$x + y = 9$$
(i)
 $y + z = 7$ (ii)
 $x + z = 6$ (iii)
Subtracting eq. (ii) from eq. (i), we get,
 $x - z = 2$ (iv)
Solving eq. (iv) and eq. (iii), we get,
 $x = 4, z = 2$
Putting the value of x in eq. (i), we get,
 $y = 5$
 \therefore $x = 4, y = 5, z = 2$

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