CHAPTER 9 Time, Speed and Distance

TIME, SPEED AND DISTANCE

Speed

The rate at which any moving body covers a particular distance is called its speed.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}; \ \text{Time} = \frac{\text{Distance}}{\text{Speed}};$$

$$\text{Distance} = \text{Speed} \times \text{time}$$

Unit :

SI unit of speed is metre per second (mps). It is also measured in kilometers per hour (kph) or miles per hour (mph).

Basic Conversions :

(i)
- 1 hour = 60 minutes = 60 × 60 seconds.
- 1 km = 1000 m
- 1 km = 0.6214 mile
- 1 mile = 1.609 km i.e. 8 km = 5 miles
- 1 yard = 3 feet
- 1 foot = 12 inches
- $1 \text{ km/h} = \frac{5}{18} \text{ m/sec,}$
- $1 \text{ m/sec} = \frac{18}{5} \text{ km/h}$
- $1 \text{ miles/hr} = \frac{22}{15} \text{ ft/sec}$

Shortcut Approach

- $\text{Average speed} = \frac{\text{Total Distance}}{\text{Total time}}$
- While travelling a certain distance (d), if a man changes his speed in the ratio m : n, then the ratio of time taken becomes n : m.
- If a certain distance (d), say from A to B, is covered at 'a' km/hr and the same distance is covered again say from B to A in 'b' km/hr, then the average speed during the whole journey is given by :

  $$\text{Average speed} = \left(\frac{2ab}{a+b}\right) \text{km/hr}$$

  Also, if t_1 and t_2 is time taken to travel from A to B and B to A respectively, the distance 'd' from A to B is given by :

  $$d = (t_1 + t_2)\left(\frac{ab}{a+b}\right)$$

  $$d = (t_1 - t_2)\left(\frac{ab}{b-a}\right)$$

  $$d = (b-a)\left(\frac{t_1 t_2}{t_1 - t_2}\right)$$

- If first part of the distance is covered at the rate of v_1 in time t_1 and the second part of the distance is covered at the rate of v_2 in time t_2, then the average speed is $\left(\frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}\right)$

Relative Speed

When two bodies are moving in same direction with speeds S_1 and S_2 respectively, their relative speed is the difference of their speeds.

i.e., Relative Speed = $S_1 - S_2$, If $S_1 > S_2$

$= S_2 - S_1$, if $S_2 > S_1$

When two bodies are moving in opposite direction with speeds S_1 and S_2 respectively, then their relative speed is the sum of their speeds.

i.e., Relative Speed = $S_1 + S_2$

EXAMPLE 1. The driver of a maruti car driving at the speed of 68 km/h locates a bus 40 metres ahead of him travelling in the same direction. After 10 seconds, the bus is 60 metres behind. The speed of the bus is.

Sol. Let speed of Bus = S_B km/h.

Now, in 10 sec., car covers the relative distance

$= (60 + 40)$ m = 100 m

$\therefore$ Relative speed of car $= \frac{100}{10} = 10$ m/s

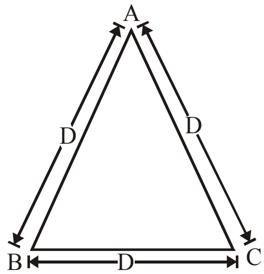
$= 10 \times \frac{18}{5} = 36 \text{ km/h}$

$\therefore \ 68 - S_B = 36$

$\Rightarrow S_B = 32 \text{ km/h}$

EXAMPLE 2. If a person goes around an equilateral triangle shaped field at speed of 10, 20 and 40 kmph on the first, second and third side respectively and reaches back to the starting point, then find his average speed during the journey.

Sol. Let the measure of each side of triangle is D km. The person travelled the distance from A to B with 10 kmph, B to C with 20 kmph and C to A with 40 kmph.



If T_{AB} = Time taken by the person to travel from A to B,
T_{BC} = Time taken by the person to travel from B to C and
T_{CA} = Time taken by the person to travel from C to A.
Then total time $= T_{AB} + T_{BC} + T_{CA}$

$$= \frac{D}{10} + \frac{D}{20} + \frac{D}{40} = D\left(\frac{8+4+2}{80}\right) = \frac{7D}{40}$$

Total distance travelled = D + D + D = 3D
Hence, average speed

$$= \frac{3D}{\frac{7D}{40}} = \frac{120}{7} = 17\frac{1}{7} \text{ kmph.}$$

EXAMPLE 3. Two guns were fired from the same place at an interval of 15 min, but a person in a bus approaching the place hears the second report 14 min 30 sec after the first. Find the speed of the bus, supposing that sound travels 330 m per sec.

Sol. Distance travelled by the bus in 14 min 30 sec could be travelled by sound in (15 min – 14 min 30 sec) = 30 second.

$\therefore$ Bus travels 330×30m in $14\frac{1}{2}$ min.

$\therefore$ Speed of the bus per hour

$$= \frac{330 \times 30 \times 2 \times 60}{29 \times 1000} = \frac{99 \times 12}{29} = \frac{1188}{29} = 40\frac{28}{29} \text{km/hr}$$

EXAMPLE 4. A hare sees a dog 100 m away from her and scuds off in the opposite direction at a speed of 12 km/h. A minute later the dog perceives her and gives chase at a speed of 16 km/h. How soon will the dog overtake the hare and at what distance from the spot where the hare took flight?

Sol. Suppose the hare at H sees the dog at D.

$\therefore$ DH = 100 m
Let K be the position of the hare where the dog sees her.
$\therefore$ HK = the distance gone by the hare in 1 min

$$= \frac{12 \times 1000}{60} \times 1\text{m} = 200 \text{ m}$$

$\therefore$ DK = 100 + 200 = 300 m
The hare thus has a start of 300 m.
Now the dog gains (16 – 12) or 4 km/h.

$\therefore$ The dog will gain 300 m in $\frac{60 \times 300}{4 \times 1000}$ min or $4\frac{1}{2}$ min.

Again, the distance gone by the hare in $4\frac{1}{2}$ min

$$= \frac{12 \times 1000}{60} \times 4\frac{1}{2} = 900\text{m}$$

$\therefore$ Distance of the place where the hare is caught from the spot H where the hare took flight = 200 + 900 = 1100 m

☞ Shortcut Approach

If two persons (or vehicles or trains) start at the same time in opposite directions from two points A and B, and after crossing each other they take x and y hours respectively to complete the journey, then

$$\frac{\text{Speed of first}}{\text{Speed of second}} = \sqrt{\frac{y}{x}}$$

EXAMPLE 5. A train starts from A to B and another from B to A at the same time. After crossing each other they complete their journey in $3\frac{1}{2}$ and $2\frac{4}{7}$ hours respectively. If the speed of the first is 60 km/h, then find the speed of the second train.

Sol.

$$\frac{\text{1st train's speed}}{\text{2nd train's speed}} = \sqrt{\frac{y}{x}} = \sqrt{\frac{2\frac{4}{7}}{3\frac{1}{2}}} = \sqrt{\frac{18}{7} \times \frac{2}{7}} = \frac{6}{7}$$

$$\therefore \quad \frac{60}{\text{2nd train's speed}} = \frac{6}{7}$$

$\Rightarrow$ 2nd train's speed = 70 km/h.

☞ Shortcut Approach

Usual speed : If a man changes his speed to $\frac{a}{b}$ of his usual speed, reachss his destination late/earlier by t minutes then,

$$\text{Usual time} = \frac{\text{Change in time}}{\left(\frac{b}{a} - 1\right)}$$

EXAMPLE 6. A boy walking at $\frac{3}{5}$ of his usual speed, reaches his school 14 min late. Find his usual time to reach the school.

Sol. Usual time $= \frac{14}{\frac{5}{3} - 1} = \frac{14 \times 3}{2} = 21$ min

EXAMPLE 7. A train after travelling 50 km, meets with an accident and then proceeds at $\frac{4}{5}$ of its former rate and arrives at the terminal 45 minutes late. Had the accident happened 20 km

further on, it would have arrived 12 minutes sooner. Find the speed of the train and the distance.

Sol. Let A be the starting place, B the terminal, C and D the places where the accidents to be placed.

By travelling at $\frac{4}{5}$ of its original rate the train would take $\frac{5}{4}$ of its usual time, i.e., $\frac{1}{4}$ of its original time more.

$\therefore$ $\frac{1}{4}$ of the usual time taken to travel the distance

CB = 45 min. ...(i)

and $\frac{1}{4}$ of the usual time taken to travel the distance

DB = (45 – 12) min ...(ii)

Subtracting (ii) from (i),

$\frac{1}{4}$ of the usual time taken to travel the distance

CD = 12 min.

$\therefore$ Usual time taken on travel 20 km = 48 min.

$\therefore$ Speed of the train per hour $= \frac{20}{48} \times 60$ or 25 km/h.

From (i), we have

Time taken to travel CB = 45 × 4 min = 3 hrs.

$\therefore$ The distance CB = 25 × 3 or 75 km.

Hence the distance AB = the distance (AC + CB)

= 50 + 75 or 125 km.

> **Shortcut Approach**
>
> A man covers a certain distance D. If he moves S_1 speed faster, he would have taken t time less and if he moves S_2 speed slower, he would have taken t time more. The original speed is given by
>
> $$\frac{2 \times (S_1 \times S_2)}{S_2 \sim S_1}$$

EXAMPLE 8. A man covers a certain distance on scooter. Had he moved 3 km/h faster, he would have taken 20 min less. If he had moved 2 km/h slower, he would have taken 20 min more. Find the original speed.

Sol. Speed $= \frac{2 \times (3 \times 2)}{3 - 2} = 12 \text{km/hr.}$

> **Shortcut Approach**
>
> If a person with two different speeds U & V cover the same distance, then required distance
>
> $$= \frac{U \times V}{U \sim V} \times \text{Difference between arrival time}$$
>
> Also, required distance $= \text{Total time taken} \times \frac{U \times V}{U + V}$

EXAMPLE 9. A boy walking at a speed of 10 km/h reaches his school 12 min late. Next time at a speed of 15 km/h reaches his school 7 min late. Find the distance of his school from his house?

Sol. Difference between the time = 12 – 7 = 5 min $= \frac{5}{60} = \frac{1}{12}$ hr

Required distance $= \frac{15 \times 10}{15 - 10} \times \frac{1}{12} = \frac{150}{5} \times \frac{1}{12} = 2.5$ km

> **Shortcut Approach**
>
> A policemen sees a thief at a distance of d. He starts chasing the thief who is running at a speed of 'a' and policeman is chasing with a speed of 'b' (b > a). In this case, the distance covered by the thief when he is caught by the policeman, is given by $d\left(\frac{a}{b-a}\right)$.

> **Shortcut Approach**
>
> A man leaves a point A at t_1 and reaches the point B at t_2. Another man leaves the point B at t_3 and reaches the point A at t_4, then they will meet at
>
> $$t_1 + \frac{(t_2 - t_1)(t_4 - t_1)}{(t_2 - t_1) + (t_4 - t_3)}$$

EXAMPLE 10. A bus leaves Ludhiana at 5 am and reaches Delhi at 12 noon. Another bus leaves Delhi at 8 am and reaches Ludhiana at 3 pm. At what time do the buses meet?

Sol. Converting all the times into 24 hour clock time, we get 5 am = 500, 12 noon = 1200, 8 am = 800 and 3 pm = 1500

Required time $= 500 + \frac{(1200 - 500)(1500 - 500)}{(1200 - 500) + (1500 - 800)}$

$= 500 + \frac{700 \times 1000}{700 + 700} = 1000 = 10$ am.

> **Shortcut Approach**
>
> Relation between time taken with two different modes of transport : $t_{2x} + t_{2y} = 2(t_x + t_y)$
>
> where,
>
> t_x = time when mode of transport x is used single way.
>
> t_y = time when mode of transport y is used single way.
>
> t_{2x} = time when mode of transport x is used both ways.
>
> t_{2y} = time when mode of transport y is used both ways.

EXAMPLE 11. A man takes 6 hours 30 min. in going by a cycle and coming back by scooter. He would have lost 2 hours 10

min by going on cycle both ways. How long would it take him to go by scooter both ways?

Sol. Clearly, time taken by him to go by scooter both way

$= 6h.30m - 2h.10m = 4h.20m = 4\frac{1}{3}h$

EXAMPLE 12. A man travels 120 km by ship, 450 km by rail and 60 km by horse taking altogether 13 hrs 30 min. The speed of the train is 3 times that of the horse and 1½ times that of the ship. Find the speed of the train.

Sol. If the speed of the horse is x km/hr; that of the train is 3x and that of the ship is $\frac{3x}{1\frac{1}{2}} = 2x$ km/hr

$\therefore \frac{120}{2x} + \frac{450}{3x} + \frac{60}{x} = \frac{27}{2}$

$\therefore \frac{60}{x} + \frac{150}{x} + \frac{60}{x} = \frac{27}{2} \quad \therefore \frac{270}{x} = \frac{27}{2}$

$\therefore x = 20 \quad \therefore$ Speed of the train = 60 km/hr.

EXAMPLE 13. Rajesh travelled from the city A to city B covering as much distance in the second part as he did in the first part of his journey. His speed during the second part was twice his speed during the first part of the journey. What is his average speed of journey during the entire travel ?

(1) His average speed is the harmonic mean of the individual speed for the two parts.

(2) His average speed is the arithmetic mean of the individual speed for the two parts.

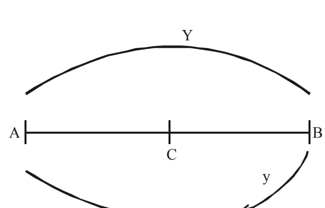
(3) His average speed is the geometric mean of the individual speeds for the two parts.

(4) Cannot be determined.

Sol. (1) The first part is ½ of the total distance & the second part is ½ of the total distance. Suppose, he travels at a km/hr speed during the first half & b km/hr speed during the second half. When distance travelled is the same in both parts of the journey, the average speed is gives by the formula $\frac{2ab}{a+b}$ i.e. the harmonic mean of the two speeds.

EXAMPLE 14. Two friends X and Y walk from A to B at a distance of 39 km, at 3 km an hour and $3\frac{1}{2}$ km an hour respectively. Y reaches B, returns immediately and meet x at C. Find the distance from A to C .

Sol. When Y meets X at C, Y has walked the distance AB + BC and X has walked the distance AC.



So, both X and Y have walked together a distance

$= 2 \times AB = 2 \times 39 = 78$ km .

The ratio of the speeds of X and Y is $3 : 3\frac{1}{2}$ i.e, $\frac{6}{7}$

Hence , the distance travelled by X = AC

$= \frac{6}{6+7} \times 78 = 36$ km

EXAMPLE 15. A man rides one-third of the distance from A to B at the rate of 'a' kmph and the remaining at the rate of '2b' kmph. If he had travelled at the uniform rate of 3c kmph, he could have rode from A to B and back again in the same time. Find a relationship between a, b and c.

Sol. Let the distance between A and B is X km and T_1 and T_2 be the time taken, then

$T_1 = \frac{X}{3a}, \; T_2 = \frac{2X}{6b} = \frac{X}{3b}, \quad T_1 + T_2 = \frac{X}{3}\left[\frac{a+b}{ab}\right]$

Let T_3 be the time taken in third case, then $T_3 = \frac{2X}{3c}$

$\Rightarrow \frac{2X}{3c} = \frac{X}{3ab}(a+b) \Rightarrow \frac{2}{c} = \frac{a+b}{ab} \Rightarrow c = \frac{2ab}{a+b}$

EXAMPLE 16. Two cyclists start from the same place to ride in the same direction. A starts at noon at 8 kmph and B at 1.30 pm at 10kmph. How far will A have ridden before he is overtaken by B ? Find also at what times A and B will be 5 km apart.

Sol. If A rides for X hours before he is overtaken, then B rides for (X – 15) hrs.

$\Rightarrow 8X = 10(X - 1.5) \Rightarrow X = 7.5$

$\Rightarrow$ A will have ridden 8×7.5 km or 60 km.

For the second part, if Y = the required number of hours after noon, then

$8X = 10(X - 1.5) \pm 5$

$\Rightarrow X = 10$ or 5 according as B is ahead or behind A.

$\Rightarrow$ The required times are 5 p.m. and 10 p.m.

EXAMPLE 17. Two men A and B start from a place P walking at 3 kmph and 3½ kmph respectively. How many km apart will they be at the end of 2½ hours ?

(i) If they walk in opposite directions ?

(ii) If they walk in the same direction ?

(iii) What time will they take to be 16 km apart if.

(a) they walk in opposite directions ?

(b) in the same direction ?

Sol. (i) When they walk in opposite directions, they will be

$\left(3 + 3\frac{1}{2}\right) = 6\frac{1}{2}$ km apart in 1 hour.

$\therefore$ In $2\frac{1}{2}$ hours they will be $6\frac{1}{2} \times \frac{5}{2} = 16\frac{1}{4}$ km apart.

(ii) If they walk in the same direction, they will be

$3\frac{1}{2}-3=\frac{1}{2}$km apart in 1 hour.

$\Rightarrow$ In $2\frac{1}{2}$ hours they will be $\frac{1}{2}\times\frac{5}{2}=1\frac{1}{4}$km apart.

(iii) Time to be 16 km apart while walking in opposite directions $=\dfrac{16}{3+3\frac{1}{2}}=2\dfrac{6}{13}$ hours.

But if they walk in the same direction,

time $=\dfrac{16}{3\frac{1}{3}-3}=32$ hours

TRAINS

A train is said to have crossed an object (stationary or moving) only when the last coach of the train crosses the object completely. It implies that the total length of the train has crossed the total length of the object.

Shortcut Approach

- Time taken by a train to cross a pole/a standing man $=\dfrac{\text{Length of train}}{\text{Speed of train}}$.
- Time taken by a train to cross platform/bridge etc. (i.e. a stationary object with some length) $=\dfrac{\text{length of train + length of platform/bridge etc.}}{\text{speed of train}}$.
- When two trains with lengths L_1 and L_2 and with speeds S_1 and S_2 respectively, then
  (a) When they are moving in the same direction, time taken by the faster train to cross the slower train $=\dfrac{L_1+L_2}{\text{difference of their speeds}}$.
  (b) When they are moving in the opposite direction, time taken by the trains to cross each other $=\dfrac{L_1+L_2}{\text{sum of their speeds}}$.
- Suppose two trains or two bodies are moving in the same direction at u km/hr and v km/hr respectively such that u > v, then their relative speed = (u – v) km/hr.
  If their lengths be x km and y km respectively, then time taken by the faster train to cross the slower train (moving in the same direction) $=\left(\dfrac{x+y}{u-v}\right)$ hrs.
- Suppose two trains or two bodies are moving in opposite directions at u km/hr and v km/hr, then their relative speed = (u + v) km/hr.
  If their lengths be x km & y km, then :
  time taken to cross each other $=\left(\dfrac{x+y}{u+v}\right)$ hrs.
- If a man is running at a speed of u m/sec in the same direction in which a train of length L meters is running at a speed v m/sec, then (v – u) m/sec is called the speed of the train relative to man. Then the time taken by the train to cross the man $=\dfrac{1}{v-u}$ seconds
- If a man is running at a speed of u m/sec in a direction opposite to that in which a train of length L meters is running with a speed v m/sec, then (u + v) is called the speed of the train relative to man.
  Then the time taken by the train to cross the man $=\dfrac{1}{v+u}$ seconds.
- If two trains start at the same time from two points A and B towards each other and after crossing, they take (a) and (b) hours in reaching B and A respectively. Then,
  A's speed : B's speed $=\left(\sqrt{b}:\sqrt{a}\right)$.

EXAMPLE 18. How long does a train 90 m long running at the rate of 54 km/h take to cross –

(a) a Mahatma Gandhi's statue?
(b) a platform 120 m long?
(c) another train 150 m long, standing on another parallel track?
(d) another train 160 m long running at 36 km/h in same direction?
(e) another train 160 m long running at 36 km/h in opposite direction?
(f) a man running at 6 km/h in same direction?
(g) a man running at 6 km/h in opposite direction?

Sol. (a) The statue is a stationary object, so time taken by train is same as time taken by train to cover a distance equal to its own length.

Now, 54 km/h $=54\times\frac{5}{18}=15\text{m/s}$

$\therefore$ Required time $=\frac{90}{15}=6$ sec.

(b) The platform is stationary of length = 120 m.
Length to be covered
= Length of the train + Length of the platform
= 90 + 120 = 210 m

$\therefore$ Required time $=\frac{210}{15}=14$ sec.

(c) Length to be covered
= Length of the train + length of the other train
= 90 + 150 = 240 m.

$\therefore$ Required time $= \frac{240}{15} = 16$ sec.

(d) Another train is moving in same direction.
Length to be covered
= Length of the train + length of the other train
$= 90 + 160 = 250$ m
Relative speed = 54 – 36 = 18 kmph.

$\therefore$ Required time $= \frac{250}{18 \times \frac{5}{18}} = 50$ sec.

(e) Another train is moving in opposite direction.
Length to be covered
= Length of the train + length of the other train
$= 90 + 160 = 250$ m
Relative speed = 54 + 36 = 90 kmph

$\therefore$ Required speed $= \frac{250}{\frac{5}{18} \times 90} = 10$ sec .

(f) The man is moving in same direction,
so Length to be covered = Length of the train,
and relative speed = speed of train – speed of man

$\therefore$ Required time $= \frac{90}{(54-6) \times \frac{5}{18}}$

$= \frac{90}{40} \times 3 = \frac{27}{4} = 6\frac{3}{4}$ sec.

(g) The man is moving in opposite direction, so
Length to be covered = Length of the train, and
relative speed = speed of train + speed of man

$\therefore$ Required time $= \frac{90}{(54+6) \times \frac{5}{18}} = \frac{27}{5} = 5\frac{2}{5}$ sec.

EXAMPLE 19. Two trains of equal lengths are running on parallel tracks in the same direction at 46 km/h and 36 km/h, respectively. The faster train passes the slower train in 36 sec. The length of each train is :

(a) 50 m **(b) 80 m**
(c) 72 m **(d) 82 m**
(e) None of these

Sol. (a) Let the length of each train be x metres.
Then, the total distance covered = (x + x) = 2x m

Relative speed $= (46 - 36) = 10$ km/h $= \frac{10 \times 5}{18}$ m/s

Now, $36 = \frac{2x \times 18}{50}$ or $x = 50$ m

EXAMPLE 20. A train 110 m in length travels at 60 km/h. How much time does the train take in passing a man walking at 6 km/h against the train ?

(a) 6 s **(b) 12 s**
(c) 10 s **(d) 18 s**
(e) None of these

Sol. (a) Relative speeds of the train and the man

$= (60 + 6) = 66$ km/h $= \frac{66 \times 5}{18}$ m/s

Distance = 110 m
Therefore, time taken in passing the men

$= \frac{110 \times 18}{66 \times 5} = 6$s

EXAMPLE 21. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

(a) 10 sec **(b) 12 sec**
(c) 14 sec **(d) cannot be determined**
(e) None of these

Sol. (b) Relative speed of the trains
$= (42 + 48)$ kmph = 90 kmph

$= \left(90 \times \frac{5}{18}\right)$ m/sec = 25 m/sec.

Time taken by the trains to pass each other
= Time taken to cover (137 + 163) m at 25 m/sec

$= \left(\frac{300}{25}\right)$ sec = 12 seconds.

☞ Shortcut Approach

If a train of length L m passes a platform of x m in t_1s, then time taken t_2s by the same train to pass a platform of length y m is given as

$$t_2 = \left(\frac{L+y}{L+x}\right) t_1$$

EXAMPLE 22. A train of length 250m, passes a platform of 350 m length in 50s. What time will this train take to pass the platform of 230m length.

Sol. Here, L = 250 m, x = 350 m, $t_1 = 50$s,
y = 230 m and t_2 = ?

$\therefore t_2 = \left(\frac{L+y}{L+x}\right) t_1 = \left(\frac{250+230}{250+350}\right) \times 50$

$= \frac{480}{600} \times 50 = 40$s

☞ **Shortcut Approach**

From stations P and Q, two trains start moving towards each other with the speeds a and b, respectively. When they meet each other, it is found that one train covers distance d more than that of another train. In such cases, distance between stations P and Q is given as $\left(\frac{a+b}{a-b}\right)\times d$.

EXAMPLE 23. From stations A and B, two trains start moving towards each other with the speeds of 150 km/h and 130 km/h, respectively. When the two trains meet each other, it is found that one train covers 20 km more than that of another train. Find the distance between stations A and B.

Sol. Here, a = 150km/h, b = 130 km/h and d = 20 km

According to the formula,

Distance between stations A and B $=\left(\frac{a+b}{a-b}\right)\times d$

$$=\left(\frac{150+130}{150-130}\right)\times 20=\frac{280}{20}\times 20=280\,\text{km}$$

☞ **Shortcut Approach**

- The distance between P and Q is (d) km. A train with (a) km/h starts from station P towards Q and after a difference of (t) hr another train with (b) km/h starts from Q towards station P, then both the trains will meet at a certain point after time T. Then,

$$T=\left(\frac{d\pm tb}{a+b}\right)$$

- If second train starts after the first train, then t is taken as positive.

  If second train starts before the first train, then t is taken as negative.

EXAMPLE 24. The distance between two stations P and Q is 110km. A train with speed of 20km/h leaves station P at 7:00 am towards station Q. Another train with speed of 25 km/h leaves station Q at 8:00 am towards station P. Then, at what time both trains meet?

Sol. Here, d = 110 km, t = 8 : 00 – 7 : 00 = 1 h

a = 20 km/h and b = 25 km/h

Time taken by trains to meet, $T=\left(\frac{d+tb}{a+b}\right)$

$$\Rightarrow \quad T=\frac{110+(1)(25)}{20+25}=\frac{135}{45}$$

$\Rightarrow \quad t = 3$ h

$\therefore$ They will meet at = 7 : 00 am + 3 h = 10 : 00 am.

☞ **Shortcut Approach**

The distance between two stations P and Q is d km. A train starts from P towards Q and another train starts from Q towards P at the same time and they meet at a certain point after t h. If train starting from P travels with a speed of x km/h slower or faster than another train, then

(i) Speed of faster train $=\left(\frac{d+tx}{2t}\right)$ km/h

(ii) Speed of slower train $=\left(\frac{d-tx}{2t}\right)$ km/h

EXAMPLE 25. The distance between two stations A and B is 138 km. A train starts from A towards B and another from B to A at the same time and they meet after 6 h. The train travelling from A to B is slower by 7 km/h compared to other train from B to A, then find the speed of the slower train?

Sol. Here, d = 138 km, t = 6 h and x = 7 km/h

$$\therefore \text{ Speed of slower train}=\frac{d-tx}{2t}=\frac{138-(6)(7)}{2(6)}$$

$$=\frac{138-42}{12}=\frac{96}{12}=8\,\text{km/h}$$

☞ **Shortcut Approach**

A train covers distance d between two stations P and Q in t_1 h. If the speed of train is reduced by (a) km/h, then the same distance will be covered in t_2 h.

(i) Distance between P and Q is

$$d=a\left(\frac{t_1t_2}{t_2-t_1}\right)\text{ km}$$

(ii) Speed of the train $=\left(\frac{at_2}{t_2-t_1}\right)$ km/h

EXAMPLE 26. A train covers distance between two stations A and B in 2h. If the speed of train is reduced by 6 km/h, then it travels the same distance in 3 h. Calculate the distance between two stations and speed of the train.

Sol. Here, t_1 = 2h, t_2 = 3h, a = 6 km/h and d = ?

(i) Distance between A and B is

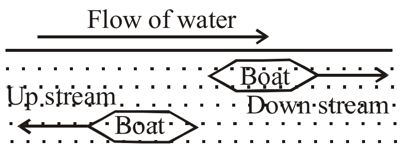
$$d=a\left(\frac{t_1\ t_2}{t_2-t_1}\right)\text{ km}$$

$$\Rightarrow \quad d=6\left(\frac{2\times 3}{3-2}\right)\Rightarrow d=36\,\text{km}$$

(ii) Speed of the train $=\frac{a\,t_2}{t_2-t_1}=\frac{6\times 3}{3-2}=18$ km/h

BOATS AND STREAMS

Stream : It implies that the water in the river is moving or flowing.
Upstream : Going against the flow of the river.
Downstream : Going with the flow of the river.
Still water : It implies that the speed of water is zero (generally, in a lake).



Let the speed of a boat (or man) in still water be X m/sec and the speed of the stream (or current) be Y m/sec. Then,

Shortcut Approach

- Speed of boat with the stream (or downstream or D/S) $= (X + Y)$ m/sec.
- Speed of boat against the stream (or upstream or U/S) $= (X - Y)$ m/sec.
- Speed of boat in still water is

$$X = \frac{(X+Y)+(X-Y)}{2} = \frac{\text{Upstream} + \text{Downstream}}{2}$$

- Speed of the stream or current is $Y = \frac{(X+Y)-(X-Y)}{2}$

$$= \frac{\text{Downstream} - \text{Upstream}}{2}$$

EXAMPLE 27. A boat is rowed down a river 28 km in 4 hours and up a river 12 km in 6 hours. Find the speed of the boat and the river.

Sol. Downstream speed is $\frac{28}{4} = 7$ kmph

Upstream speed is $\frac{12}{6} = 2$ kmph

Speed of Boat $= \frac{1}{2}$ (Downstream + Upstream Speed)

$$= \frac{1}{2}[7+2] = 4.5 \text{ kmph}$$

Speed of current $= \frac{1}{2}$ (Downstream–Upstream speed)

$$= \frac{1}{2}(7-2) = 2.5 \text{ kmph}$$

EXAMPLE 28. P, Q, and R are the three towns on a river which flows uniformly. Q is equidistant from P and R. I row from P to Q and back in 10 hours and I can row from P to R in 4 hours. Compare the speed of my boat in still water with that of the river.

(a) 4 : 3 **(b) 5 : 3**
(c) 6 : 5 **(d) 7 : 3**
(e) None of these

Sol. (c) Let the speed of the boat be v_1 and the speed of the current be v_2 and d be the distance between the cities.

Now, $\frac{d}{v_1+v_2} = 4$ and $\frac{d}{v_1-v_2} = 6$

$$\Rightarrow \frac{v_1+v_2}{v_1-v_2} = \frac{6}{4}$$

or $\frac{2v_1}{2v_2} = \frac{10}{2}$ or $\frac{v_1}{v_2} = 5:1$

Required ratio $= (5+1) : 5 = 6 : 5$

Shortcut Approach

A man can row X km/h in still water. If in a stream which is flowing of Y km/h, it takes him Z hours to row to a place and back, the distance between the two places is

$$\frac{Z(X^2 - Y^2)}{2X}$$

EXAMPLE 29. A man can row 6 km/h in still water. When the river is running at 1.2 km/h, it takes him 1 hour to row to a place and back. How far is the place?

Sol. Man's rate downstream $= (6 + 1.2) = 7.2$ km/h.
Man's rate upstream $= (6 - 1.2)$ km/h $= 4.8$ km/h.
Let the required distance be x km.

Then $\frac{x}{7.2} + \frac{x}{4.8} = 1$ or $4.8x + 7.2x = 7.2 \times 4.8$

$$\Rightarrow x = \frac{7.2 \times 4.8}{12} = 2.88\text{km}$$

SHORTCUT METHOD

$$\text{Required distance} = \frac{1 \times \left(6^2 - (1.2)^2\right)}{2 \times 6}$$

$$= \frac{36 - 1.44}{12} = \frac{34.56}{12} = 2.88 \text{ km}$$

Shortcut Approach

- A man rows a certain distance downstream in X hours and returns the same distance in Y hours. If the stream flows at the rate of Z km/h, then the speed of the man in still water is given by

$$\frac{Z(X+Y)}{Y-X} \text{km/hr}$$

- And if speed of man in still water is Z km/h then the speed of stream is given by

$$\frac{Z(Y-X)}{X+Y} \text{km/hr}$$

Shortcut Approach

If speed of stream is a and a boat (swimmer) takes n times as long to row up as to row down the river, then

Speed of boat (swimmer) in still water $= \frac{a(n+1)}{(n-1)}$

Note: *This formula is applicable for equal distances.*

EXAMPLE 30. Rajnish can row 12 km/h in still water. It takes him twice as long to row up as to row down the river. Find the rate of stream.

Sol. Here, speed of Rajnish in still water = 12 km/h

n = 2; Speed of stream (a) = ?

According to the formula,

$$\text{Speed in still water} = \frac{a(n+1)}{(n-1)}$$

$$\Rightarrow 12 = \frac{a(2+1)}{(2-1)}$$

$$\Rightarrow 3a = 12$$

$$\therefore a = \frac{12}{3} = 4\,\text{km/h}$$

EXAMPLE 31. Vikas can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of 3 km/h, find the speed of Vikas in still water.

Sol. By the formula,

$$\text{Vikas's speed in still water} = \frac{3(9+6)}{9-6} = 15\,\text{km/h}$$

☞ Shortcut Approach

If a man capable of rowing at the speed (u) m/sec in still water, rows the same distance up and down a stream flowing at a rate of (v) m/sec, then his average speed through the journey is

$$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}} = \frac{(u-v)(u+v)}{u}$$

EXAMPLE 32. Two ferries start at the same time from opposite sides of a river, travelling across the water on routes at right angles to the shores. Each boat travels at a constant speed though their speeds are different. They pass each other at a point 720m from the nearer shore. Both boats remain at their sides for 10 minutes before starting back. On the return trip they meet at 400m from the other shore. Find the width of the river.

(a) 1760m **(b) 1840m**
(c) 2000m **(d) Cannot be found**
(e) None of these

Sol. (a)

Let the width of the river be x.

Let a, b be the speeds of the ferries.

$$\frac{720}{a} = \frac{(x-720)}{b} \qquad \text{......... (i)}$$

$$\frac{(x-720)}{a} + 10 + \frac{400}{a} = \frac{720}{b} + 10 + \frac{(x-400)}{b} \qquad \text{......... (ii)}$$

(Time for ferry 1 to reach other shore + 10 minute wait + time to cover 400m)

= Time for freely 2 to cover 720m to other shore + 10 minute wait + Time to cover (x – 400m))

Using (i), we get $\frac{a}{b} = \frac{720}{(x-720)}$

Using (ii), $\frac{(x-320)}{a} = \frac{(x+320)}{b} \Rightarrow \frac{a}{b} = \frac{(x-320)}{(x+320)}$

On, solving we get, x = 1760m

EXAMPLE 33. A man rows 27km with the stream and 15km against the stream taking 4 hours each time. Find this rate per hour in still water and the rate at which the stream flows.

Sol. Speed with the stream $= \frac{27}{4} = 6\frac{3}{4}$ kmph

∴ Speed against the stream $= \frac{15}{4} = 3\frac{3}{4}$ kmph.

∴ Speed of the man in still water

$$= \frac{1}{2}\left(6\frac{3}{4} + 3\frac{3}{4}\right) = 5\frac{1}{4}\,\text{kmph}$$

$$\therefore \text{Speed of the stream} = \frac{1}{2}\left(6\frac{3}{4} - 3\frac{3}{4}\right) = 1.5\,\text{kmph}$$

EXAMPLE 34. On a river, B is between A and C and is also equidistant from A and C. A boat goes from A to B and back in 5 hours 15 minutes and from A to C and back in 7 hours. How long will it take to go from C to A if the river flows from A to C?

Sol. If the speed in still water is x kmph and speed of the river is y kmph, speed down the river = x + y and speed up the river = x – y.

$$\therefore \frac{d}{x+y} + \frac{d}{x-y} = 5\frac{1}{4} \qquad \text{............. (1)}$$

$$\frac{2d}{x+y} = 7 \qquad \text{............. (2)}$$

Multiplying (1) by 2, we get $\frac{2d}{x+y} + \frac{2d}{x-y} = 10\frac{1}{2}$

$$\Rightarrow 7 + \frac{2d}{x-y} = \frac{21}{2} \qquad \left[\because \frac{2d}{x-y} = 7\right]$$

$$\Rightarrow \frac{2d}{x-y} = 3\frac{1}{2}$$ hours = Time taken to travel from C to A.

☞ Shortcut Approach

If boat's (swimmer's) speed in still water is a km/h and river is flowing with a speed of b km/h, then average speed in going to a certain place and coming back to starting point is given by $\frac{(a+b)(a-b)}{a}$ km/h.

EXAMPLE 35. Ramesh rows in still water with speed of 4.5 km/h to go to a certain place and to come back. Find his average speed for the whole journey, if the river is flowing with a speed of 1.5 km/h.

Sol. Here, a = 4.5 km/h, b = 1.5 km/h

$$\text{Average speed} = \frac{(a+b)(a-b)}{a}$$

$$= \frac{(4.5+1.5)(4.5-1.5)}{4.5} = \frac{6\times 3}{4.5} = \frac{18}{4.5} = 4\,\text{km/h}$$

EXERCISE

1. A car finishes a journey in ten hours at the speed of 80 km/hr. If the same distance is to be covered in eight hours how much more speed does the car have to gain?
(a) 8 krn/hr (b) 10 km/hr
(c) 12 km/hr (d) 16 krn/hr
(e) None of these

2. Two cars *A* and *B* are running towards each other from different places 88 km apart. If the ratio of the speeds of the cars *A* and *B* is 5 : 6 and the speed of the car *B* is 90 km per hour then after how long will the two meet each other?
(a) $26\frac{2}{3}$ minutes (b) 24 minutes
(c) 32 minutes (d) 36 minutes
(e) None of these

3. Train '*A*' leaves Mumbai Central for Lucknow at 11 am, running at the speed of 60 kmph. Train '*B*' leaves Mumbai Central for Lucknow by the same route at 2 pm on the same day, running at the speed of 72 kmph. At what time will the two trains meet each other?
(a) 2 am on the next day
(b) 5 am on the next day
(c) 5 pm on the next day
(d) 2 pm on the next day
(e) None of these

4. A motor starts with the speed of 70 kmph with its speed increasing every two hours by 10 kmph. In how many hours will it cover 345 kms?
(a) $2\frac{1}{4}$ hours (b) $4\frac{1}{2}$ hours
(c) 4 hours 5 minutes (d) Cannot be determined
(e) None of these

5. A boat takes 3 hours to travel from place *M* to *N* downstream and back from *N* to *M* upstream. If the speed of the boat in still water is 4 km/hr, what is the distance between the two places?
(a) 8 km (b) 12 km
(c) 6 km (d) Data inadequate
(e) None of these

6. A boat has to travel upstream 20 km distance from point *X* of a river to point *Y*. The total time taken by boat in travelling from point *X* to *Y* and *Y* to *X* is 41 minutes 40 seconds. What is the speed of the boat?
(a) 66 km/hr (b) 72 km/hr
(c) 48 km/hr (d) Data inadequate
(e) None of these

7. The speed of a car increases by 2 km after every hour. If the distance travelled in the first hour was 35 km, what was the total distance travelled in 12 hours?
(a) 522 km (b) 456 km
(c) 556 km (d) 482 km
(e) None of these

8. A boat covers a distance of 30 km downstream in 2 hours while it take 6 hours to cover the same distance upstream. If the speed of the current is half of the speed of the boat then what is the speed of the boat in km per hour?
(a) 15 kmph (b) 5 kmph
(c) 10 kmph (d) Data inadequate
(e) None of these

9. A man starts walking. He walked 2 km in the first hour. Then he walked two-thirds of the distance of the previous hour in each next hour. If he walked continuously then how long could he walk maximum?
(a) 60 km (b) 6 km
(c) 12 km (d) 8 km
(e) None of these

10. Starting with the initial speed of 30 km/hr, the speed is increased by 4 km/hour every two hours. How many hours will it take to cover a distance of 288 km?
(a) 4 (b) 6
(c) 12 (d) 8
(e) None of these

11. With a uniform speed a car covers a distance in 8 hours. Were the speed increased by 4 km/hr the same distance could be covered in $7\frac{1}{2}$ hours. What is the distance covered?
(a) 640 km (b) 480 km
(c) 420 km (d) Cannot be determined
(e) None of these

12. A 300-metre-long train crosses a platform in 39 seconds while it crosses a signal pole in 18 seconds. What is the length of the platform?
(a) 320 metres (b) 650 metres
(c) 350 metres (d) Data inadequate
(e) None of these

13. A 260-metre-long train crosses a 120-metre-long wall in 19 seconds. What is the speed of the train?
(a) 27 km/hr (b) 49 km/hr
(c) 72 km/hr (d) 70 km/hr
(e) None of these

14. A 270-metre-long train running at the speed of 120 kmph crosses another train running in opposite direction at the speed of 80 kmph in 9 secs. What is the length of the other train?
(a) 240 metres (b) 320 metres
(c) 260 metres (d) 230 metres
(e) None of these

15. A monkey ascends a greased pole 12 metres high. He ascends 2 metres in first minute and slips down 1 metre in the alternate minute. In which minute, he reaches the top ?
(a) 21st (b) 22nd
(c) 23rd (d) 24th
(e) None of these

16. A man walks a certain distance and rides back in $6\frac{1}{4}$h. He can walk both ways in $7\frac{3}{4}$h. How long it would take to ride both ways ?
(a) 5 hours (b) $4\frac{1}{2}$hours
(c) $4\frac{3}{4}$hours (d) 6 hours
(e) None of these

17. There are 20 poles with a constant distance between each pole. A car takes 24 seconds to reach the 12th pole . How much time will it take to reach the last pole?
(a) 25.25 s (b) 17.45 s
(c) 35.75 s (d) 41.45 s
(e) None of these

18. A man is walking at a speed of 10 km per hour. After every kilometre, he takes rest for 5 minutes. How much time will be take to cover a distance of 5 kilometres?
(a) 48 min. (b) 50 min.
(c) 45 min. (d) 55 min.
(e) None of these

19. On a journey across Bombay, a tourist bus averages 10 km/h for 20% of the distance, 30 km/h for 60% of it and 20 km/h for the remainder. The average speed for the whole journey was
(a) 10 km/h (b) 30 km/h
(c) 5 km/h (d) 20 km/h
(e) None of these

20. In a 800 m race around a stadium having the circumference of 200 m, the top runner meets the last runner on the 5th minute of the race. If the top runner runs at twice the speed of the last runner, what is the time taken by the top runner to finish the race ?
(a) 20 min (b) 15 min
(c) 10 min (d) 5 min
(e) None of these

21. A man walks half of the journey at 4 km/h by cycle does one third of journey at 12 km/h and rides the remainder journey in a horse cart at 9 km/h, thus completing the whole journey in 6 hours and 12 minutes. The length of the journey is
(a) 36 km (b) $\frac{1332}{67}$ km
(c) 40 km (d) 28 km
(e) None of these

22. R and S start walking each other at 10 AM at the speeds of 3 km/h and 4 km/h respectively. They were initially 17.5 km apart. At what time do they meet?
(a) 2 : 30 PM (b) 11 : 30 AM
(c) 1 : 30 PM (d) 12 : 30 PM
(e) None of these

23. A train does a journey without stoppage in 8 hours, if it had travelled 5 km/h faster, it would have done the journey in 6 hours 40 minutes. Find its original speed.
(a) 25 km/h (b) 40 km/h
(c) 45 km/h (d) 36.5 km/h
(e) None of these

24. A train leaves station X at 5 a.m. and reaches station Y at 9 a.m. Another train leaves station Y at 7 a.m. and reaches station X at 10: 30 a.m. At what time do the two trains cross each other ?
(a) 7 : 36 am (b) 7 : 56 am
(c) 8 : 36 am (d) 8 : 56 am
(e) None of these

25. Cars C_1 and C_2 travel to a place at a speed of 30 and 45 km/h respectively. If car C_2 takes $2\frac{1}{2}$ hours less time than C_1 for the journey, the distance of the place is
(a) 300 km (b) 400 km
(c) 350 km (d) 225 km
(e) None of these

26. If I walk at 4 km/h, I miss the bus by 10 minutes. If I walk at 5 km/h, I reach 5 minutes before the arrival of the bus. How far I walk to reach the bus stand ?
(a) 5 km (b) 4.5 km
(c) $5\frac{1}{4}$ km / h (d) Cannot be determined
(e) None of these

27. A goods train leaves a station at a certain time and at a fixed speed. After 6 hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.
(a) 36 kmph (b) 40 kmph
(c) 30 kmph (d) 42 kmph
(e) None of these

28. Without stoppages, a train travels certain distance with an average speed of 80 km/h, and with stoppages, it covers the same distance with an average speed of 60 km/h. How many minutes per hour the train stops ?
(a) 15 (b) 18
(c) 10 (d) 16
(e) None of these

29. If a man walks to his office at 3/4 of his usual rate, he reaches office 1/3 of an hour later than usual. What is his usual time to reach office.
(a) $\frac{1}{2}$ hr (b) 1 hr
(c) $\frac{3}{4}$ hr (d) 2 hrs
(e) None of these

30. If a man walks to his office at 5/4 of his usual rate, he reaches office 30 minutes early than usual. What is his usual time to reach office.
(a) 2 hrs (b) $2\frac{1}{2}$ hr s
(c) 1 hr 50 min (d) 2 hrs 15 min
(e) None of these

31. A train running between two stations A and B arrives at its destination 10 minutes late when its speed is 50 km/h and 50 minutes late when its speed is 30km/h. What is the distance between the stations A and B ?
(a) 40 km (b) 50 km
(c) 60 km (d) 70 km
(e) None of these

32. A car travels 25 km an hour faster than a bus for a journey of 500 km. If the bus takes 10 hours more than the car, then the speeds of the bus and the car are
(a) 25 km/h and 40 km/h respectively
(b) 25 km/h and 60 km/h respectively
(c) 25 km/h and 50 km/h respectively
(d) 25 km/h and 70 km/h respectively
(e) None of these

33. A train consists of 12 boggies, each boggy 15 metres long. The train crosses a telegraph post in 18 seconds. Due to some problem, two boggies were detached. The train now crosses a telegraph post in
(a) 18 sec (b) 12 sec
(c) 15 sec (d) 20 sec
(e) None of these

34. A man started running at a distance of 225 metre from the train. If the speed of the man is 6 km/h, then how much time should the train wait so that the man will be just able to catch it ?
(a) $2\frac{1}{4}$ min (b) 3 min
(c) $4\frac{1}{4}$ min (d) $4\frac{1}{2}$ min
(e) None of these

35. A man sitting in a train which is travelling at 50 kmph observes that a goods train, travelling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.
(a) 62 kmph (b) 58 kmph
(c) 52 kmph (d) 50 kmph
(e) None of these

36. Two trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is:
(a) 2 : 3 (b) 4 : 3
(c) 6 : 7 (d) 9 : 16
(e) None of these

37. A train 75 metres long overtook a man who was walking at the rate of 6 km/h and passed him in 18 seconds. Again, the train overtook a second person in 15 seconds. At what rate was the second person travelling ?
(a) 3 km/h (b) 2.5 km/h
(c) 4 km/h (d) 1.5 km/h
(e) None of these

38. A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of a 120 metre long train running at 45 kmph in the same direction. In how much time will the train pass the jogger?
(a) 3.6 sec (b) 18 sec
(c) 36 sec (d) 72 sec
(e) None of these

39. Two trains are running at 40 km/h and 20 km/h respectively in the same direction. Fast train completely passes a man sitting in the slower train in 5 seconds. What is the length of the fast train?
(a) 23 m (b) $23\frac{2}{9}$ m
(c) 27 m (d) $27\frac{7}{9}$ m
(e) None of these

40. Two trains, 130 and 110 metres long, while going in the same direction, the faster train takes one minute to pass the other completely. If they are moving in opposite direction, they pass each other completely in 3 seconds. Find the speed of trains.
(a) 30 m/s, 40 m/s (b) 32 m/s, 48 m/s
(c) 40 m/s, 44 m/s (d) 38 m/s, 42 m/s
(e) None of these

41. A train overtakes two person who are walking in the same direction in which the train is going , at the rate of 2 kmph and 4 kmph and passes them completely in 9 and 10 seconds respectively. The length of the train is:
(a) 45 m (b) 50 m
(c) 54 m (d) 72 m
(e) None of these

42. Local trains leave from a station at an interval of 15 minutes at a speed of 16 km/h. A man moving from opposite side meets the trains at an interval of 12 minutes. Find the speed of the man.
(a) 4 km/h (b) 3.5 km/h
(c) 4.5 km/h (d) 3 km/h
(e) None of these

43. Local trains leave from a station at an interval of 14 minutes at a speed of 36 km/h. A man moving in the same direction along the road meets the trains at an interval of 18 minutes. Find the speed of the man.
(a) 8 km/h (b) 7 km/h
(c) 6 km/h (d) 5.8 km/h
(e) None of these

44. A train overtakes two persons walking along a railway track. The first one walks at 4.5 km/h. The other one walks at 5.4 km/h. The train needs 8.4 and 8.5 seconds respectively to overtake them. What is the speed of the train if both the persons are walking in the same direction as the train?
(a) 66 km/h (b) 72 km/h
(c) 78 km/h (d) 81 km/h
(e) None of these

45. Two trains running in opposite directions cross a man standing on the platform in 27 seconds and 17 seconds respectively and they cross each other in 23 seconds. The ratio of their speeds is:
(a) 1 : 3 (b) 3 : 2
(c) 3 : 4 (d) 2 : 1
(e) None of these

46. Two trains each of 120 m in length, run in opposite directions with a velocity of 40 m/s and 20 m/s respectively. How long will it take for the tail ends of the two trains to meet each other during the course of their journey :
(a) 20 s (b) 3 s
(c) 4 s (d) 5 s
(e) None of these

Directions (Qs. 47-48): Answer the following questions on the basis of the information given below:

(i) Trains A and B are travelling on the same route heading towards the same destination. Train B has already covered a distance of 220 km before train A started.

(ii) The two trains meet each other 11 hours after the start of train A.

(iii) Had the trains been travelling towards each other (from a distance of 220 km), they would have met after one hour.

47. What is the speed of train B in kmph?
(a) 100 (b) 180
(c) 116 (d) Data inadequate
(e) None of these

48. What is the speed of train A in kmph?
(a) 102 (b) 80.5
(c) 118 (d) Data inadequate
(e) None of these

ANSWER KEY

1	(e)	6	(d)	11	(b)	16	(c)	21	(a)	26	(a)	31	(b)	36	(b)	41	(b)	46	(c)
2	(c)	7	(e)	12	(c)	17	(d)	22	(d)	27	(a)	32	(c)	37	(a)	42	(a)	47	(a)
3	(b)	8	(c)	13	(c)	18	(b)	23	(a)	28	(a)	33	(c)	38	(c)	43	(a)	48	(e)
4	(b)	9	(b)	14	(d)	19	(d)	24	(b)	29	(b)	34	(a)	39	(d)	44	(d)		
5	(d)	10	(d)	15	(b)	20	(c)	25	(d)	30	(b)	35	(a)	40	(d)	45	(b)		

Hints & Explanations

1. (e) Distance covered by the car = 80 × 10 = 800 km

$\therefore$ Speed $= \frac{800}{8} = 100 \text{ km/hr}$

$\therefore$ Speed gain = 100 – 80 = 20 km/hr

2. (c) Speed of the car $A = \frac{5}{6} \times 90 = 75$ km/hr

$\therefore$ Reqd time $= \frac{88}{90+75} \times 60 = 32$ minutes

3. (b) Distance covered by train A before the train B leaves Mumbai Central = 60 × 3 = 180 km

$\therefore$ Time taken to cross each other $= \frac{180}{12} = 15$ hours

$\therefore$ Reqd time = 2 pm + 15 = 5 am on the next day

4. (b) Distance covered in first two hours = 70 × 2 = 140 km
Distance covered in next two hours = 80 × 2 = 160 km
Distance covered in first four hours
140 + 160 = 300 km

Remaining distance = 345 – 300 = 45 km. Now, this distance will be covered at the speed of 90 km/hr.

$\therefore$ Time taken $= \frac{45}{90} = \frac{1}{2}$ hour

Total time = 4 + 1/2 = $4\frac{1}{2}$ hours.

5. (d) Let the distance between M and N and the speed of current in still water be d km and x km/hr respectively.

According to the question, $\frac{d}{4+x} + \frac{d}{4-x} = 3$

In the above equation we have only one equation but two variables, hence, can't be determined.

6. (d) Let x be the speed of the boat.
and y the speed of the current.

$$\therefore \frac{20}{x-y} + \frac{20}{x+y} = \frac{25}{36}$$

In this equation there are two variables, but only one equation, so, the value of 'x' cannot be determined.

7. (e) This is the problem of arithmetic progression (AP) with the first term (a) = 35, common difference (d) = 2 and total no. of terms (n) = 12. The sum of this series will be total distance travelled.

$$\text{Sum } (S_n) = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{12}{2}\{70 + 11 \times 2\}$$

$$= \frac{12 \times 92}{2} = 552 \text{ km}$$

8. (c) Here downstream speed = 15 km/hr and upstream speed = 5 km/hr

$\therefore$ Speed of the boat $= \frac{15+5}{2} = 10$ km/h

9. (b) Required distance

$$= 2\left[1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots\right]$$

$$= 2 \times \frac{1}{1 - \frac{2}{3}} = 2 \times 3 = 6 \text{ km.}$$

10. (d) Hour Speed (km/h) Distance travelled (in km)

Hour	Speed (km/h)	Distance travelled (in km)
2	30	60
2	34	68
2	38	76
2	42	84
8		288

Hence, the required time = 8 hours

11. (b) Here $\frac{D}{7.5} - \frac{D}{8} = 4$

(where D is the distance in km)

$\Rightarrow$ $0.5\,D = 4 \times 8 \times 7.5$

$\Rightarrow$ $D = 2 \times 4 \times 8 \times 7.5 = 480$ km

12. (c) When a train crosses a platform, it crosses a distance equal to the sum of the length of the platform and that of the train. But when a train crosses a signal pole, it crosses the distance equal to its length only.
Here, time taken by the train to cross a signal pole = 18 seconds

Hence, speed of the train $= \frac{300}{18}$ m/sec

The train takes 21(= 39 – 18) seconds extra in order to cross the platform.

Hence, length of platform $= \frac{21 \times 300}{18} = 350$m

13. (c) Speed of train $= \frac{260 + 120}{19} \times \frac{18}{5} = 72$ km/hr.

14. (d) Relative speed = 120 + 80 kmph $= 200 \times \frac{5}{18}$ m/sec

$$t = \frac{\text{Distance}}{\text{Speed}} = \frac{(270 + x) \times 9}{500}$$

or $270 + x = \frac{9 \times 500}{9}$ $x = 500 - 270 = 230$ m

15. (a) In 2 minutes, he ascends = 1 metre
$\therefore$ 10 metres, he ascends in 20 minutes.
$\therefore$ He reaches the top in 21st minute.

16. (c) We know that, the relation in time taken with two different modes of transport is

$t_{\text{walk both}} + t_{\text{ride both}} = 2\,(t_{\text{walk}} + t_{\text{ride}})$

$$\frac{31}{4} + t_{\text{ride both}} = 2 \times \frac{25}{4}$$

$$\Rightarrow t_{\text{ride both}} = \frac{25}{2} - \frac{31}{4} = \frac{19}{4} = 4\frac{3}{4} \text{ hrs}$$

17. (d) Let the distance between each pole be x m.
Then, the distance up to 12th pole = 11 x m

$$\text{Speed} = \frac{11x}{24} \text{ m/s}$$

Time taken to covers the total distance of 19x

$$= \frac{19x \times 24}{11x} = 41.45 \text{ s}$$

18. (b) Rest time = Number of rest × Time for each rest
= 4 × 5 = 20 minutes
Total time to cover 5 km

$$= \left(\frac{5}{10} \times 60\right) \text{ minutes} + 20 \text{ minutes} = 50 \text{ minutes.}$$

19. (d) Let the average speed be x km/h.
and Total distance = y km. Then,

$$\frac{0.2}{10}y + \frac{0.6}{30}y + \frac{0.2}{20}y = \frac{y}{x}$$

$$\Rightarrow x = \frac{1}{0.05} = 20 \text{ km/h}$$

20. (c) After 5 minutes (before meeting), the top runner covers 2 rounds i.e., 400 m and the last runner covers 1 round i.e., 200 m.
$\therefore$ Top runner covers 800 m race in 10 minutes.

21. (a) Let the length of the journey = x km.

$$\therefore \text{ Journey rides by horse cart } = x\left(1 - \frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{1}{6}x \text{ km.}$$

Then, total time taken to complete journey $= \frac{31}{5}$ hr

$$\Rightarrow t_1 + t_2 + t_3 = \frac{31}{5}$$

$$\Rightarrow \frac{x}{2}\times\frac{1}{4}+\frac{x}{3}\times\frac{1}{12}+\frac{x}{6\times 9}=\frac{31}{5}$$

$$\Rightarrow x=\frac{31}{5}\times\frac{216}{37}=36.2\text{km}\approx 36\text{km}$$

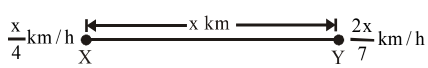
22. (d) Let after t hours they meet then,
$3t+4t=17.5 \Rightarrow t=25$
$\therefore$ Time $=10$ am $+2.5$ h $=12:30$pm

23. (a) Let original speed = S km/h
Here, distance to be covered is constant

$$\therefore S\times 8=(S+5)\left(\frac{20}{3}\right)$$

$$\Rightarrow 8S-\frac{20}{3}S=\frac{100}{3} \quad \Rightarrow S=\frac{100}{4}=25 \text{ km/h}$$

24. (b) Let the distance between X and Y be x km. Then, the speed of A is $\frac{x}{4}$ km/h and that of B is $\frac{2x}{7}$ km/h.



Relative speeds of the trains

$$=\left(\frac{x}{4}+\frac{2x}{7}\right)=\frac{15x}{28} \text{ km/h}$$

Therefore the distance between the trains at 7 a.m.

$$=x-\frac{x}{2}=\frac{x}{2}\text{ km}$$

Hence, time taken to cross each other

$$=\frac{\frac{x}{2}}{\frac{15x}{28}}=\frac{x}{2}\times\frac{28}{15x}=\frac{14}{15}\times 60=56\text{min}$$

Thus, both of them meet at 7 : 56 a.m.

25. (d) Let C_1 takes t hrs. Then,
$\because$ Distance is same.

$$\therefore 30t=45\left(t-\frac{5}{2}\right)$$

$$\Rightarrow t=\frac{15}{2}\text{hrs}$$

$$\therefore \text{Distance}=30\times\frac{15}{2}=225\text{km}$$

26. (a) $d=\text{product of speed}\left[\frac{\text{difference of time}}{\text{difference of speed}}\right]$

$$d=\frac{4\times 5}{60}\left[\frac{10-(-5)}{5-4}\right]$$ [Here, –ve sign indicates before the schedule time]

$\Rightarrow d=5$ km

27. (a) Let the speed of the goods train be x kmph.
Distance covered by goods train in 10 hours
= Distance covered by express train in 4 hours.
$\therefore 10x=4\times 90$ or $x=36$.
So, speed of goods train = 36 kmph.

28. (a) Due to stoppages, it covers 20 km less .

Time taken to cover $20\text{km}=\frac{20}{80}\text{h}=\frac{1}{4}\text{h}$

$$=\frac{1}{4}\times 60\text{ min}=15\text{ min}$$

29. (b) If new speed is $\frac{a}{b}$ of original speed, then

$$\text{usual time}\times\left(\frac{b}{a}-1\right)=\text{change in time}$$

$$\therefore \text{ usual time}\times\left(\frac{4}{3}-1\right)=\frac{1}{3}$$

$$\Rightarrow \text{usual time}=\frac{1}{3}\times 3=1\text{ hr}$$

30. (b) $\text{usual time}\times\left(\frac{4}{5}-1\right)=\frac{-30}{60}$

$$\Rightarrow \text{usual time}=\frac{1}{2}\times 5=2\frac{1}{2}\text{hrs}$$

31. (b) Let the distance between the two stations be x km.

Then, $\frac{x}{50}-\frac{10}{60}=\frac{x}{30}-\frac{50}{60}$

$$\Rightarrow \frac{x}{50}-\frac{1}{6}=\frac{x}{30}-\frac{5}{6}$$

or $\frac{x}{30}-\frac{x}{50}=\frac{2}{3}$ or $x=50\text{km}$

Thus distance between the station A and B = 50 km

32. (c) Let the speed of the bus be x km / h.
then speed of the car = (x + 25) km / h

$$\therefore \frac{500}{x}=\frac{500}{x+25}+10$$

$\Rightarrow x^2+25x-1250=0 \Rightarrow x=25$
Thus speed of the bus = 25 km/h
Speed of the car = 50 km/h
Alternative:
Difference in speeds 25 km / hr is in only option (c).

33. (c) Length of train = 12 × 15 = 180 m.

Then, speed of train $=\frac{180}{18}=10\text{ m/s}$

Now, length of train $=10\times 15=150\text{m}$

$$\therefore \text{Required time}=\frac{150}{10}=15\text{sec}.$$

34. (a) $\text{Time}=\frac{225}{6\times\frac{5}{18}}=135\text{sec}=2\frac{1}{4}\text{ min}.$

35. (a) Relative speed $= \left(\frac{280}{9}\right) m/sec = \left(\frac{280}{9} \times \frac{18}{5}\right)$ kmph

$= 112$ kmph.

$\therefore$ Speed of goods train = (112 – 50) kmph = 62 kmph.

36. (b) Let us name the trains as A and B. Then,

(A's speed) : (B's speed) = $\sqrt{b} : \sqrt{a} = \sqrt{16} : \sqrt{9} = 4 : 3$.

37. (a) Let actual speed of train $= S_T$ km/h.

Then, $S_T - 6 = \frac{75}{18} \times \frac{18}{5} = 15$

$\Rightarrow \quad S_T = 21$ km/h

Now, let speed of second man $= S_m$

$21 - S_m = \frac{75}{15} \times \frac{18}{5} = 18$

$\Rightarrow S_m = 3$ km/h

38. (c) Speed of train relative to jogger

$= (45 - 9)$ km/h $= 36$ km/h

$= \left(36 \times \frac{5}{18}\right) m/sec = 10$ m/sec.

Distance to be covered = (240 + 120) m = 360 m.

$\therefore$ Time taken $= \left(\frac{360}{10}\right)$ sec = 36 sec.

39. (d) Relative speed = (40 – 20) km/h

$= \left(20 \times \frac{5}{18}\right) m/sec = \left(\frac{50}{9}\right) m/sec$.

Length of faster train

$= \left(\frac{50}{9} \times 5\right) m = \frac{250}{9} m = 27\frac{7}{9}$ m.

40. (d) Let speed of trains are S_1 m/s and S_2 m/s.

Then, $S_1 - S_2 = \frac{130 + 110}{60} = 4$ ...(i)

and $S_1 + S_2 = \frac{130 + 110}{3} = 80$ ...(ii)

on solving (i) and (ii), we get

$S_1 = 42$ m/s, $S_2 = 38$ m/s

41. (b) Let actual speed of train = S m/sec

and length of train = L m.

Then, $S - \frac{2 \times 5}{18} = \frac{L}{9}$

$\Rightarrow 9S = L + 5$ ...(i)

and $S - 4 \times \frac{5}{18} = \frac{L}{10}$

$\Rightarrow 90S = 9L + 100$ ...(ii)

By (i) & (ii), we get L = 50 m.

42. (a) Let speed of man = S km/h. Then,

Distance covered in 15 min = Distance covered in 12 min

$16 \times \frac{15}{60} = \frac{12}{60}[16 + S]$

$\Rightarrow 16 + S = 20 \Rightarrow S = 4$ km/h

43. (a) Let speed of man = S km/h. Then,

$36 \times \frac{14}{60} = \frac{18}{60}[36 - S] \Rightarrow 36 - S = 28 \Rightarrow S = 8$ km/h.

44. (d) 4.5 km/h $= \left(4.5 \times \frac{5}{18}\right)$ m/sec = 1.25 m/sec,

& 5.4 km/h $= \left(5.4 \times \frac{5}{18}\right)$ m/sec = 1.5 m/sec.

Let the speed of the train be S m/sec.

Then, $(S - 1.25) \times 8.4 = (S - 1.5) \times 8.5$

$\Rightarrow 8.4S - 10.5 = 8.5S - 12.75 \Rightarrow 0.1S = 2.25 \Rightarrow S = 22.5$.

$\therefore$ Speed of the train $= \left(22.5 \times \frac{18}{5}\right)$ km/h = 81 km/h.

45. (b) Let the speeds of the two trains be S_1 m/sec and S_2 m/sec respectively. Then, length of the first train $= 27S_1$ metres, and length of the second train $= 17S_2$ metres.

$\therefore \frac{27S_1 + 17S_2}{S_1 + S_2} = 23 \Rightarrow 27S_1 + 17S_2 = 23S_1 + 23S_2$

$\Rightarrow 4S_1 = 6S_2 \Rightarrow \frac{S_1}{S_2} = \frac{3}{2}$.

46. (c) Relative speed of the trains = (40 + 20) = 60 m/s

Distance = (120 + 120) = 240 m

Time taken by trains to cross each other completely

$= \frac{240}{60} = 4s$

(47-48)

Let the speeds of train A and B be V_A and V_B respectively.

$V_A - V_B = \frac{220}{11} \Rightarrow \frac{220}{11} \Rightarrow V_A - V_B = 20$ .... (i)

$V_A + V_B = \frac{220}{1} \Rightarrow V_A + V_B = 220$ .... (ii)

Solving the equations (i) and (ii), we get V_A = 120 km/hr and V_B = 100 km/hr

47. (a) 48. (e)